Monitoring a building using deconvolution interferometry. I: Earthquake-data analysis

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ABSTRACT
For health monitoring of a building, we need to separate the response of the building to an earthquake from the imprint of soil-structure coupling and from wave propagation below the base of the building. Seismic interferometry based on deconvolution, where we deconvolve the wavefields recorded at different floors, is a technique to extract this building response and hence estimate velocity of the wave which propagates inside the building. Deconvolution interferometry also allows us to estimate the damping factor of the building. Compared with other interferometry techniques, such as crosscorrelation and crossehance interferometry, deconvolution interferometry is the most suitable technique to monitor a building using earthquake records. For deconvolution interferometry, we deconvolve the wavefields recorded at all levels with the waves recorded at a target receiver inside the building. This receiver behaves as a virtual source, and we retrieve the response of a cut-off building, a short building which is cut off at the virtual source. Because the cut-off building is independent from the structure below the virtual source, the technique might be useful for estimating local structure and local damage. We apply deconvolution interferometry to 17 earthquakes recorded during two weeks at a building in Fukushima, Japan and estimate time-lapse changes in velocity and normal-mode frequency. As shown in a previous study, the change in velocity correlates with the change in normal-mode frequency. We compute the velocities from both traveling waves and the fundamental mode using coda-wave interferometry. These velocities have a negative correlation with the maximum acceleration of the observed earthquake records.

Key words: time-lapse monitoring, seismic interferometry, deconvolution, earthquake, building

1 INTRODUCTION

The response of a building to an earthquake has been studied since the early 1900s [e.g., Biot, 1933; Sezawa and Kanai, 1935; Carde, 1936]. We can estimate the frequencies of the fundamental and higher modes of buildings using ambient and forced vibration experiments (Trifunac, 1972; Ivanović et al., 2000; Kohler et al., 2005; Clinton et al., 2006; Michel et al., 2008). Due to the shaking caused by major earthquakes, the frequencies of normal modes decrease (Trifunac et al., 2001a; Kohler et al., 2005); the reduction is mostly temporary (a few minutes) and healing occurs with time, but some reduction is permanent (Clinton et al., 2006), who found more than 20% temporal reduction and 4% permanent reduction in the fundamental frequency of the motion of the Millikan Library located at the California Institute of Technology after the 1987 M6.1 Whittier Narrows earthquake. The reduction in the frequency logarithmically correlates with the maximum acceleration of observed records (Clinton et al., 2006). Precipitation, strong wind, temperature, reinforcement, and
heavy weight loaded in a building also change the frequencies of normal modes (Kohler et al., 2005; Clinton et al., 2006). Because these frequencies are related to both the building itself and the soil-structure coupling, we have to consider soil-structure interactions (Şafak, 1995) and nonlinearities in the response of the foundation soil (Trifunac et al., 2001a,b). Normal-mode frequencies estimated from observed records are, therefore, not suitable for health monitoring of a building in isolation of its environment (Todorovska and Trifunac, 2008b).

Snieder and Şafak (2006) show that one can estimate an impulse response independent from the soil-structure coupling and the complicated wave propagation (e.g., attenuation and scattering) below the bottom receiver by using seismic interferometry based on deconvolution. Seismic interferometry is a technique to extract the Green’s function which accounts for the wave propagation between receivers (Lobkis and Weaver, 2001; Derode et al., 2003; Snieder, 2004; Paul et al., 2005; Snieder et al., 2006b; Wapenaar and Fokkema, 2006). Seismic interferometry can be based on cross-correlation, deconvolution, and crosscoherence (Snieder et al., 2009; Wapenaar et al., 2010). Deconvolution interferometry is a useful technique for monitoring structures especially in one dimension (Nakata and Snieder, 2011, 2012a,b). Because deconvolution interferometry changes the boundary condition at the base of the building, we are able to extract the pure response of the building regardless of its coupling to the subsurface (Snieder and Şafak, 2006; Snieder et al., 2006a).

Deconvolution interferometry has been applied to earthquake records observed in a building to retrieve the velocity of traveling waves and attenuation of the building (Oyunchimeg and Kawakami, 2003; Snieder and Şafak, 2006; Kohler et al., 2007; Todorovska and Trifunac, 2008a,b) (some studies call the method impulse response function or normalized input-output minimization). Todorovska and Trifunac (2008b) use 11 earthquakes occurring over a period of 24 years to monitor the fundamental frequency of a building after applying deconvolution interferometry. The fundamental frequencies they estimated from the interferometry are always higher than the frequencies obtained from the observed records because the frequencies computed from the observed records are affected by both the building itself and soil-building coupling, while the frequencies estimated using the interferometry are only related to the building itself. Oyunchimeg and Kawakami (2003) apply short-time moving-window interferometry to an earthquake record to estimate the velocity reduction of a building during an earthquake. Prieto et al. (2010) apply deconvolution interferometry to ambient vibrations after normalizing amplitudes per frequency using the multitaper method (Thomson, 1982) to estimate the traveling-wave velocity and damping factor.

In this study, we apply deconvolution interferometry to 17 earthquakes observed at a building in Japan over a period of two weeks and monitor the changes in velocity of the building. This study is based on the work of Snieder and Şafak (2006); furthermore, we extend the deconvolution interferometry as proposed by Snieder and Şafak (2006) to deconvolution with the waveforms recorded at an arbitrary receiver, compare this with crosscorrelation and crosscoherence interferometry, and use interferometry for monitoring a building in Japan. First, we introduce our data: geometry of receivers, locations of the building and epicenters of earthquakes used, observed waveforms, and shapes of the normal modes extracted from observed records. We also introduce the equations of interferometry based on deconvolution, crosscorrelation, and crosscoherence. We further indicate the deconvolved waveforms obtained from one earthquake and estimate a velocity as well as a quality factor \( Q \). Next, we apply deconvolution interferometry to all observed earthquakes and monitor the change in velocity of the building using coda-wave interferometry (Snieder et al., 2002). In a companion paper, we apply the interferometry to ambient vibrations.

2 BUILDING AND EARTHQUAKES

The building (rectangle in Figure 1) in which we recorded vibrations is located in the Fukushima prefecture, Japan. Continuous seismic vibrations were recorded by Suncoh Consultants Co., Ltd. for two weeks using 10 microelectromechanical-systems (MEMS) accelerometers, which were developed by Akebono Brake Industry Co., Ltd., and 17 earthquakes were observed during the two weeks (Table 1 and Figure 1). In this

![Figure 1](image-url)
Table 1. Origin times, magnitudes, and hypocenter locations of recorded earthquakes estimated by the Japan Meteorological Agency (JMA). The earthquakes are numbered sequentially according to their origin times. Maximum acceleration is the observed maximum amplitude of the MEMS accelerometers at the first floor in the 120 s following the origin time of each earthquake.

<table>
<thead>
<tr>
<th>No.</th>
<th>Origin time</th>
<th>M&lt;sub&gt;JMA&lt;/sub&gt;</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Depth (km)</th>
<th>Maximum acceleration (m/s&lt;sup&gt;2&lt;/sup&gt;)</th>
</tr>
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<tr>
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<td>5/31/11 16:12:20.2</td>
<td>3.9</td>
<td>37.1983</td>
<td>141.0900</td>
<td>32</td>
<td>0.145</td>
</tr>
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<td>37.6583</td>
<td>141.7617</td>
<td>44</td>
<td>0.088</td>
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<td>37.1167</td>
<td>140.8400</td>
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</tr>
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<tr>
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<td>5.5</td>
<td>36.9900</td>
<td>141.2100</td>
<td>30</td>
<td>1.923</td>
</tr>
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The (left) EW and (right) NS vertical cross sections of the building and the positions of receivers (triangles). Elevations denote the height of each floor from ground level. We put receivers on stairs 0.19 m below each floor except for the basement (on the floor) and the first floor (0.38 m below). Receiver M2 is located between the first and second floors. Horizontal-receiver components are aligned with the EW and NS directions.

Study, we focus on processing of the earthquake records, and we analyze ambient vibrations in the companion paper. The building includes eight stories, a basement, and a penthouse (Figure 2). We installed receivers on the stairs, located 20 m from the east side and at the center between the north and south sides. The sampling interval of the records is 1 ms, and the receivers have three components. Here, we use two horizontal components which are aligned with the east-west (EW) and north-south (NS) directions.

Figures 3a–3d illustrate the observed waveforms and their power spectra of earthquake No. 5, which gives the greatest acceleration to the building. Figures 3e and 3f show the spectrogram of the motion at the fourth floor computed with the continuous-wavelet transform (Torrence and Compo, 1998). Higher frequencies quickly attenuate and the fundamental mode is dominant for later times in Figures 3e and 3f. The frequency of the fundamental mode in the EW component (1.17 Hz) is higher than the frequency in the NS component (0.97 Hz) because the EW side of the building is longer than the NS side. Both components have large amplitudes at around 0.5 Hz between 14 and 21 s. Since the 0.5-Hz component is localized in time (Figure 3e and 3f), it corresponds to a surface wave that moves the entire building. However, because the frequency of the surface wave is less than that of the fundamental mode of the building, it does not excite waves that propagate within the building.

Figure 4 illustrates the shapes of the normal-mode displacement computed from the real part of the Fourier spectra at different floors. We calculate displacement from acceleration using numerical integration (Schiff and Bogdanoff, 1967). Just as for the fundamental mode, the frequencies of overtones in the EW component are also higher than those in the NS component. Although the displacements of both components in mode 1 (the fundamental mode) are almost the same, the NS-component displacement is larger than that of the EW component in modes 2 and 3. The amplitude of mode 1 is much larger than the amplitudes of other modes.
Figure 3. Unfiltered waveforms of earthquake No. 5 recorded at the building in (a) the EW component and (b) the NS component, and (c, d) their power spectra. (e, f) Spectrogram computed with continuous-wavelet transformed waveforms recorded at the fourth floor. Time 0 s represents the origin time of the earthquake. We preserve relative amplitudes of the EW and NS components.

3 DECONVOLUTION WITH AN ARBITRARY RECEIVER

By deconvolving observed earthquake records, we obtain the impulse response of a building (Oyunchimeg and Kawakami, 2003; Snieder and Şafak, 2006; Kohler et al., 2007; Todorovska and Trifunac, 2008a). When the height of the building is \( H \), the recorded signal of an earthquake in the frequency domain at an arbitrary receiver at height \( z \) is given by Snieder and Şafak (2006):

\[
 u(z) = \sum_{m=0}^{\infty} S(\omega) R_m(\omega) \left\{ e^{ik(2mH+z)} e^{-\gamma |k|(2mH+z)} + e^{ik(2(m+1)H-z)} e^{-\gamma |k|(2(m+1)H-z)} \right\} \\
 = S(\omega) \left\{ e^{ikz} e^{-\gamma |k|z} + e^{ik(2H-z)} e^{-\gamma |k|(2H-z)} \right\} \\
 \frac{1}{1 - R(\omega) e^{2ikH} e^{-2\gamma |k|H}}. \tag{1}
\]
where \( S(\omega) \) is the incoming waveform to the base of the building, \( R(\omega) \) the reflection coefficient related to the coupling of the ground and the base of the building, \( k \) the wavenumber, \( \gamma \) the attenuation coefficient, and \( i \) the imaginary unit. We use the absolute value of wavenumbers in the damping terms because the waves attenuate regardless of whether the wavenumber is positive or negative. In expression 1, we assume the wave vertically propagates in the building (one-dimensional propagation) with constant amplitude and wavenumber, and without internal reflections. The constant wavenumber implies that we assume constant velocity \( c \) because \( k = \omega/c \). The incoming waveform \( S(\omega) \) includes the source signature of the earthquake and the effect of propagation such as attenuation and scattering along the path from the hypocenter of the earthquake to the base of the building. The attenuation coefficient \( \gamma \) is defined as

\[
\gamma = \frac{1}{2Q},
\]

with \( Q \) the quality factor (Aki and Richards, 2002).

For \( m = 0 \) in the first line in expression 1, the first term \( S(\omega)e^{ikz}e^{-\gamma kz} \) indicates the incoming upgoing wave and the second term \( S(\omega)e^{ik(2H-z)}e^{-\gamma k(2H-z)} \) the downgoing wave, which is reflected off the top of the building. The index \( m \) represents the number of reverberations between the base and top of the building.

As we deconvolve a waveform recorded by a receiver at \( z \) with a waveform observed by another receiver at \( z_0 \), from expression 1 we obtain

\[
D(z, z_0, \omega) = \frac{u(z)}{u(z_0)} = \frac{S(\omega)}{S(\omega)} \left\{ e^{ikz}e^{-\gamma k(z)} + e^{ik(2H-z)}e^{-\gamma k(2H-z)} \right\}
\]

\[
S(\omega) \left\{ e^{ikz}e^{-\gamma k(z)} + e^{ik(2H-z)}e^{-\gamma k(2H-z)} \right\}
\]

\[
= \sum_{n=0}^{\infty} (-1)^n \left\{ e^{ik(2n(H-z_0)+z-z_0)}e^{-\gamma k[(2n(H-z_0)+z-z_0)]} + e^{ik(2n(H-z_0)+2H-z-z_0)}e^{-\gamma k[(2n(H-z_0)+2H-z-z_0)]} \right\},
\]

where we use a Taylor expansion in the last equality. In equation 3, the receiver at \( z_0 \) behaves as a virtual source. Equation 3 may be unstable because of the spectral division. In practice we use a regularization parameter \( \epsilon \) (Yilmaz, 2001, Section 2.3):

\[
D(z, z_0, \omega) = \frac{u(z)}{u(z_0)} \approx \frac{u(z)u^*(z_0)}{|u(z_0)|^2 + \epsilon(|u(z_0)|^2)},
\]

where * is a complex conjugate and \( |u(z_0)|^2 \) the average power spectrum of \( u(z_0) \). In this study we use \( \epsilon = 1\% \).

Note that these deconvolved waves are independent of the incoming waveform \( S(\omega) \) and the ground coupling \( R(\omega) \). When we consider substitutions \( S(\omega) \rightarrow 1, R(\omega) \rightarrow -1, H \rightarrow H - z_0 \), and \( z \rightarrow z-z_0 \), equation 1 reduces to equation 3. These conditions indicate the physical properties of the deconvolved waveforms: impulse response \( S(\omega) \rightarrow 1 \), perfect reflection at the virtual source \( (R(\omega) \rightarrow -1) \), and a small building \( (H \rightarrow H - z_0) \) and \( z \rightarrow z-z_0 \) as we discuss later.

When \( z > z_0 \), equation 3 describes a wave that is excited at \( z_0 \) and reverberates between \( z_0 \) and the top of the building. Using a normal-mode analysis (equation A4 in Appendix A), the fundamental mode of equation 3 in the time domain is given by

\[
D(z, z_0, t) = \frac{4\pi c}{H - z_0}e^{-\gamma \omega_0 t}\sin(\omega_0 t)\cos\left(\frac{\omega \theta}{c} - \frac{H - z}{e}\right),
\]

where \( \omega_0 = \pi c / \{2(H - z_0)\} \). The period of the fundamental mode is, thus,

\[
T_0 = \frac{4(H - z_0)}{c},
\]

which corresponds to the period of the fundamental mode of the building that is cut off at \( z_0 \) (cut-off building: Figure 5a). According to equation 3 the polarity change resulting from reflection at \( z_0 \) is given by \((-1)^n\), the reflection coefficient at the virtual source is \(-1\). Therefore, the cut-off building is only sensitive to the properties of the building above \( z_0 \), and the reconstructed wave motion in the cut-off building has the potential to estimate local structure and local damage instead of structure and damage for the entire building.

When \( z < z_0 \) and \( n = 0 \) in equation 3, we...
obtain two waves: an acausal upgoing wave from \( z \) to \( z_a \) \((e^{ik(z-z_a)}e^{-\gamma|k|(z-z_a)})\) and a causal downgoing wave from \( z \) to \( z \) \((e^{i(2H-z-z_a)}e^{-\gamma|k|(2H-z-z_a)})\). Waves for \( n \geq 1 \) in equation 3 account for the reverberations between \( z_a \) and the top of the building. Because \( D(z_a, z_a, \omega) = 1 \), the deconvolved waveforms at \( z_a \) is a delta function in the time domain \((D(z_a, z_a, t) = \delta(t))\); hence \( D(z_a, z_a, t) = 0 \) for \( t \neq 0 \) (clamped boundary condition (Snieder et al., 2006a, 2009)). The upgoing and downgoing waves interfere destructively at \( z_a \).

Although we assume, for simplicity, a constant velocity in equation 1, we can apply deconvolution interferometry to wavefields observed at a building with smoothly varying velocities. When the velocity \( c \) varies with height, the local wavenumber does so as well, and using the WKBJ approximation for the phase, expression 1 generalizes to

\[
u(z) = \frac{U}{1 - R(\omega) \exp \left( 2i \int_0^z k(z)dz \right) \exp \left( -2\gamma \int_0^z |k(z)|dz \right)}
\]

with

\[
U = S(\omega) \left[ \exp \left( i \int_0^z k(z)dz \right) \exp \left( -\gamma \int_0^z |k(z)|dz \right) + \exp \left\{ i \left( \int_0^H k(z)dz + \int_z^H k(z)dz \right) \right\} \times \exp \left\{ -\gamma \left( \int_0^H |k(z)|dz + \int_z^H |k(z)|dz \right) \right\} \right],
\]

and equation 3 generalizes in this case to

\[
D(z, z_a, \omega) = \sum_{n=0}^{\infty} (-1)^n \times \left[ \exp \left\{ i \left( 2n \int_{z_a}^z k(z)dz + \int_{z_a}^z z(z)dz \right) \right\} \right. \\
\left. \times \exp \left\{ -\gamma \left( 2n \int_{z_a}^H |k(z)|dz + \int_{z_a}^H |k(z)|dz \right) \right\} + \exp \left\{ i \left( (2n+1) \int_{z_a}^H k(z)dz + \int_{z_a}^H k(z)dz \right) \right\} \\
\times \exp \left\{ -\gamma \left( (2n+1) \int_{z_a}^H |k(z)|dz + \int_{z_a}^H |k(z)|dz \right) \right\} \right] \times 2
\]

As for equation 3, equation 8 represents the waves in a cut-off building at \( z_a \), and the period of the fundamental mode of the cut-off building depends on the slowness averaged between \( z_a \) and \( H \). Note that when \( z > z_a \), \( D(z, z_a, \omega) \) in equation 8 is only related to \( k(z) \) above \( z_a \).

When \( z_a \) is at the first floor \( (z_a = 0) \) or at the top of the building \( (z_a = H) \), equation 3 corresponds to equations 26 or 21 in Snieder and Şafak (2006), respectively, which we rewrite here as

Figure 5. Schematic shapes of the fundamental mode retrieved by using seismic interferometry. (a) Fundamental mode retrieved by deconvolving wavefields with a motion recorded at \( z_a \) (equation 3). (b) Fundamental mode retrieved by deconvolving wavefields with a motion recorded at the first floor (equation 9).

\[
D(z, 0, \omega) = \sum_{n=0}^{\infty} (-1)^n \times \left\{ e^{ik(2(n+1)H-z)} e^{-\gamma|k|(2(n+1)H-z)} + e^{ik(2nH-z)} e^{-\gamma|k|(2nH-z)} \right\},
\]

\[
D(z, H, \omega) = \frac{1}{2} \left\{ e^{ik(H-z)} e^{-\gamma|k|(H-z)} + e^{-ik(H-z)} e^{\gamma|k|(H-z)} \right\}.
\]
4 CROSSCORRELATION AND CROSSCOHERENCE INTERFEROMETRY

In the previous section, we focused on seismic interferometry based on deconvolution. Let us consider seismic interferometry based on crosscorrelation [e.g., Schuster et al., 2004] and crosscoherence [e.g., Nakata et al., 2011]; these two methods are the widest-applied technique and the earliest application (Aki, 1957), respectively.

4.1 Crosscorrelation

From equation 1, the crosscorrelation of \( u(z) \) and \( u(z_a) \) is

\[
C(z, z_a, \omega) = u(z)u^*(z_a) = |S(\omega)|^2 \frac{e^{ikz}e^{-\gamma|k|z} + e^{ik(2H-z)}e^{-\gamma|k|(2H-z)}}{1 - R(\omega)e^{2ikH}e^{-2\gamma|k|H} - R^*(\omega)e^{-2ikH}e^{-2\gamma|k|H} + |R(\omega)|^2e^{-4\gamma|k|H}}.
\]

(11)

In contrast to the deconvolution (equation 3), equation 11 depends on the incoming wave \( S(\omega) \) and the ground coupling \( R(\omega) \), and does not create a clamped boundary condition \((z_a, z, \omega) \neq 1\). Because of the presence of the reflection coefficient \( R(\omega) \) and the power spectrum \( |S(\omega)|^2 \), it is much more complicated to estimate the properties (e.g., traveling-wave velocity and attenuation) of the building from crosscorrelation than from deconvolution.

When \( z_a = 0 \) and \( z = H \), equation 11 reduces to

\[
C(z, 0, \omega) = |S(\omega)|^2 \frac{e^{ikz}e^{-\gamma|k|z} + e^{ik(2H-z)}e^{-\gamma|k|(2H-z)} + e^{-ik(2H-z)}e^{-\gamma|k|(2H+z)}}{1 - R(\omega)e^{2ikH}e^{-2\gamma|k|H} - R^*(\omega)e^{-2ikH}e^{-2\gamma|k|H} + |R(\omega)|^2e^{-4\gamma|k|H}} \frac{e^{-2\gamma|k|H}}{1 + R(\omega)e^{2ikH}e^{-2\gamma|k|H} - R^*(\omega)e^{-2ikH}e^{-2\gamma|k|H} + |R(\omega)|^2e^{-4\gamma|k|H}},
\]

(12)

(13)

respectively. If \( R(\omega) = 0 \) (no reflection at the base), equation 13 is, apart from the prefactor \( 2|S(\omega)|^2e^{-2\gamma|k|H} \), the same as equation 10.

4.2 Crosscoherence

Crosscoherence is defined as frequency-normalized crosscorrelation:

\[
CH(z, z_a, \omega) = \frac{u(z)u^*(z_a)}{|u(z)||u(z_a)|} \approx \frac{u(z)u^*(z_a)}{|u(z)||u(z_a)| + e^*(|u(z)||u(z_a)|)}
\]

(14)

Similar to equation 4, we use a regularization parameter \( \epsilon^* \) in the last equality in practice. In this study, we use \( \epsilon^* = 0.1\% \). For mathematical interpretation, using Taylor expansions of \( \sqrt{1 + x} \) and \( 1/\sqrt{1 + x} \) for \( x < 1 \), the crosscorrelation between \( u(z) \) and \( u(z_a) \) is given by

\[
CH(z, z_a, \omega) = \frac{u(z)u^*(z_a)}{|u(z)||u(z_a)|} = \frac{u(z)u^*(z_a)}{\sqrt{u(z)u^*(z)}} \sqrt{u(z)u^*(z_a)} = \frac{\sqrt{u(z)}\sqrt{u^*(z_a)}}{\sqrt{u(z)}\sqrt{u(z_a)}} = \frac{\sqrt{u(z)}\sqrt{u^*(z_a)}}{\sqrt{u(z)}\sqrt{u(z_a)}}
\]

\[
\approx \left[ 1 + \frac{1}{2} \sum_{n=1}^{\infty} \frac{e^{i2nk(H-z)}}{n!} \frac{e^{-2n\gamma|k|(H-z)}}{A_{n-1}} \right] \left[ 1 + \frac{1}{2} \sum_{n=1}^{\infty} \frac{e^{-i2nk(H-z)}}{n!} \frac{e^{-2n\gamma|k|(H-z)}}{A_{n-1}} \right] \left[ 1 + \sum_{n=1}^{\infty} \frac{e^{i2nk(H-z)}}{n!} \frac{e^{-2n\gamma|k|(H-z)}}{A_{n}} \right] \left[ 1 + \sum_{n=1}^{\infty} \frac{e^{-i2nk(H-z)}}{n!} \frac{e^{-2n\gamma|k|(H-z)}}{A_{n}} \right],
\]

(15)

(16)

where \( A_0 = 1 \) and \( A_n = \frac{(2n-1)!}{(-2)^n} \). As for deconvolution interferometry, equation 15 does not depend on \( S(\omega) \) and \( R(\omega) \). For a complex number \( z = re^{i\phi} \), the square root is defined as \( \sqrt{z} = \sqrt{r}e^{i\phi/2} \). Furthermore, the waveforms of crosscorrelation interferometry satisfies a clamped boundary condition at \( z = z_a \) \((CH(z_a, z_a, \omega) = 1) \), hence \( CH(z_a, z_a, t) = \delta(t) \), and \( CH(z_a, z_a, t) = 0 \) for \( t \neq 0 \).

For \( z_a = 0 \) and \( z_a = H \), equation 15 simplifies to
For crosscorrelation interferometry, we calculate the amplitudes of the traveling waves in Appendix B using Taylor expansions and consider deconvolution, crosscorrelation, and crosscoherence are the same. The spectral amplitude is different, though, and this leads to different waveforms in the time domain. We numerically compute synthetic waveforms excited at 0 m by an impulsive source \((S(\omega) = 1)\) based on equation 1 shown in Figure 6a. In the computation, we use the following parameters: \(H = 100\) m, \(R(\omega) = 0.5\), \(Q = 3000\), and \(c = 200\) m/s. In applying seismic interferometry, we compute deconvolution \(\langle u(z)u^*(0) \rangle / \langle |u(0)|^2 \rangle\): Figure 6b), crosscorrelation \((u(z)u^*(0))\): Figure 6c), and crosscoherence \(\langle u(z)v^*(0) \rangle / \langle |u(z)||v(0)| + \epsilon\langle |u(z)||u(0)| \rangle)\): Figure 6d), where \(\epsilon = 1\% \) and \(\epsilon' = 0.1\%\), using the synthetic waveforms shown in Figure 6a. In Figures 6b–6d, the virtual source is at \(z = 0\) m. Deconvolved waveforms (Figure 6b) arrive at the same time as the waves in the synthetic records, but the polarization is reversed when the wave is reflected at \(z = 0\) m due to the clamped boundary condition. In crosscorrelation interferometry (Figure 6c), the causal waves arrive at the same time as the waves in the synthetic records (Figure 6a), and the acausal waves are kinematically identical to the time-reversed causal waves. Although for simplicity we use \(S(\omega) = 1\) in Figure 6, the in-
Figure 6. (a) Synthetic waveforms based on equation 1. We numerically calculate waveforms with an impulse response \(S(\omega) = 1\) at \(t = 0\) s at 0 m, \(R(\omega) = 0.5\), \(Q = 3000\), \(H = 100\) m, and \(c = 200\) m/s. Interferometric waveforms by computing deconvolution (panel b: equation 9), crosscorrelation (panel c: equation 12), and crosscoherence (panel d: equation 16) using waveforms shown in panel (a). The virtual source for interferometry is at the 0-m receiver. We apply a bandpass filter 0.5-1-30-40 Hz after computing each waveform. The circles in each panel highlight four waves which are discussed in the main text. The numbers near each arrow indicate the ratio of the amplitude difference between two waves highlighted by the circles apart from the attenuation expected from the traveling distance at the correct velocity. To estimate the ratio of amplitude in panel (d), we ignore wavefields which attenuate faster than \(e^{-6\gamma|k(H-z)|}\), and \(CH_1 = 2/(4-e^{-4\gamma|k(H-z)|}-e^{-3\gamma|k(H+z)|})\) where \(z = 40\) m. Amplitudes in each panel are normalized by the amplitude of the first highlighted wave (at \(t=0.2\) s). The gray line in panel (d) shows the wave which propagates at 66.67 m/s.

coming wave complicates the crosscorrelated waveforms when we use real earthquake data, and picking the arrival times of the traveling waves may be difficult in that case. Crosscoherence interferometry creates traveling waves which propagate at slower velocities than true velocity \(c = 200\) m/s. In Figure 6d, the gray line highlights the wave \(-e^{-k(2H-3z)}e^{-2\gamma|k(H-z)|}/2\), which travels with one third of the true wave speed (66.7 m/s). To estimate the velocity of the traveling waves, therefore, deconvolution interferometry is useful.

We highlight the amplitudes of the waves in Figure 6 with the circles. A comparison of Figures 6a and 6b shows that the ratios of the amplitudes of the synthetic records and deconvolved waves within the first two circles are the same, but the ratios in the second and third circles are different. The reflection coefficient at 0 m of the synthetic records is \(R(\omega)\) while the reflection coefficient of waves obtained by deconvolution interferometry is \(-1\); see the numbers next to the arrows in Figures 6a and 6b. The difference between the reflection coefficients implies that deconvolved waveforms are independent from the ground coupling, and the decay of amplitudes of the waves are only related to the attenuation of the building. The ratios of the amplitudes of the waves highlighted by the circles in crosscorrelation interferometry are the same as those in the synthetic records; see the numbers next to the arrows in Figures 6a and 6c. Hence, both the building and the soil-structure cou-
crosscorrelation interferometry in Appendix B. For crosscoherence interferometry we ignore wavefields which attenuate faster than $e^{-2\gamma |k|(H-z)}$.

The space variable is denoted by $z$ and the velocity and attenuation of the building. Crosscorrelation interferometry do satisfy the wave equation. Equations 11–18, Table 2, and Figure 6, conclude that deconvolution interferometry is suitable for application to earthquake records to estimate the velocity and attenuation of the building. Crosscorrelated waveforms depend on the incoming wave $S(\omega)$ and the ground coupling $R(\omega)$. Crosscoherence interferometry creates pseudo events, and the decay of amplitudes of waveforms reconstructed by crosscoherence is not exponentially depending on the traveled distances. Therefore, these types of interferometry are not appropriate to estimate velocity and attenuation.

Snieder et al. (2006a) show that the wavefields obtained from deconvolution interferometry satisfy the same wave equation as the wavefield of the real building for an external source. Using this idea, we explain why crosscoherence interferometry creates unphysical events. Following Snieder et al. (2006a), we denote the linear differential operator that defines the wave propagation by $L(z)$ (e.g., for the one-dimensional wave equation $L(z) = d^2/dz^2 + \omega^2/c^2(z)$). The operator acts on the space variable $z$. For an internal source at $z_0$, the wavefield $u(z)$ (equation 1) satisfies $L(z)u(z) = F(z_0)$ where $F$ is the excitation at $z_0$. For an external source, the other hand, $u(z)$ satisfies $L(z)u(z) = 0$; this homogeneous equation applies to earthquake data. Applying the operator $L(z)$ to equations 3, 11, and 14, respectively, gives

\[
L(z)D(z, z_0, \omega) = L(z) \frac{u(z)}{u(z_0)} = \frac{1}{u(z_0)}L(z)u(z) = 0, \tag{19}
\]

\[
L(z)C(z, z_0, \omega) = L(z)u(z)u^*(z_0) = \{L(z)u(z)\} u^*(z_0) = 0, \tag{20}
\]

\[
L(z)CH(z, z_0, \omega) = L(z)\left( \frac{u(z)u^*(z_0)}{|u(z)||u(z_0)|} \right) = \left( \frac{u^*(z_0)}{|u(z_0)|} \right) L(z) \left( \frac{u(z)}{|u(z)|} \right) \neq 0, \tag{21}
\]

Table 2. Amplitudes of traveling wave fields obtained from observed records and computed by seismic interferometry based on deconvolution, crosscorrelation, and crosscoherence for $z_0 = 0$ (equations 1, 9, 12, and 16). We compute the amplitudes of crosscorrelation interferometry in Appendix B. For crosscoherence interferometry we ignore wavefields which attenuate faster than $e^{-2\gamma |k|(H-z)}$.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Observed record</th>
<th>Deconvolution</th>
<th>Crosscorrelation</th>
<th>Crosscoherence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^{ikz}$</td>
<td>$S(\omega)e^{-</td>
<td>k</td>
<td>z}$</td>
<td>$e^{-</td>
</tr>
<tr>
<td>$e^{ik(2H-z)}$</td>
<td>$S(\omega)e^{-</td>
<td>k</td>
<td>(2H-z)}$</td>
<td>$e^{-</td>
</tr>
<tr>
<td>$e^{ik(2H+z)}$</td>
<td>$S(\omega)R(\omega)e^{-</td>
<td>k</td>
<td>(2H+z)}$</td>
<td>$-e^{-</td>
</tr>
<tr>
<td>$e^{ik(4H-z)}$</td>
<td>$S(\omega)R(\omega)e^{-</td>
<td>k</td>
<td>(4H-z)}$</td>
<td>$-e^{-</td>
</tr>
</tbody>
</table>

\[
C_1 = |S(\omega)|^2 e^{-|k|z} \left\{ 1 + R(\omega)e^{-2\gamma |k|H} \right\} \left\{ \sum_{n=0}^{\infty} |R(\omega)|^2 e^{-4n\gamma |k|H} \right\}
\]

\[
C_2 = |S(\omega)|^2 e^{-|k|(2H-z)} \left\{ 1 + R(\omega)e^{-2\gamma |k|H} \right\} \left\{ \sum_{n=0}^{\infty} |R(\omega)|^2 e^{-4n\gamma |k|H} \right\}
\]

\[
C_3 = |S(\omega)|^2 R(\omega)e^{-|k|(2H+z)} \left\{ 1 + R(\omega)e^{-2\gamma |k|H} \right\} \left\{ \sum_{n=0}^{\infty} |R(\omega)|^2 e^{-4n\gamma |k|H} \right\}
\]

\[
C_4 = |S(\omega)|^2 R(\omega)e^{-|k|(4H-z)} \left\{ 1 + R(\omega)e^{-2\gamma |k|H} \right\} \left\{ \sum_{n=0}^{\infty} |R(\omega)|^2 e^{-4n\gamma |k|H} \right\}
\]

5 DECONVOLVED WAVEFORMS GENERATED FROM AN EARTHQUAKE

As an illustration of the data analysis, we first show the application of deconvolution interferometry to the records of earthquake No. 5. We first estimate whether the reflection point of the traveling wave is at R1 or R2 because the building has a penthouse (Figure 2). Figure 7 shows waveforms deconvolved by the motion recorded at the first floor (equation 9) for the EW and NS components. We apply a 0.4–0.5–45–50 Hz sine-squared bandpass filter to the deconvolved waveforms. Because the physical property at the basement is different from the
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Figure 7. Deconvolved waveforms, in which the virtual source is at the first floor, of earthquake No. 5 after applying a bandpass filter 0.4-0.5-45-50 Hz in the (a) EW and (b) NS components. Gray lines indicate the arrival time of traveling waves with the velocity that is estimated from the least-squares fitting of the first upgoing and downgoing waves. We repeat the gray lines after the second traveling waves based on equation 9. Solid gray lines highlight the waves in the positive polarization and dashed gray lines the waves in the negative polarization.

To estimate the velocity of the traveling wave and the location of the reflection point, we compute the travel-time curve using a least-squares fitting of the picked travel times on the first upgoing and downgoing waves at each floor (the first two solid gray lines in Figure 7). For picking the travel times, we seek the maximum amplitude time in each traveling wave. We repeatedly draw the reverberating travel-wave paths based on equation 9 using the velocity estimated from the first upgoing and downgoing waves (in Figure 7). To avoid large uncertainties, we use the picked travel times between floors one through five in the EW component and between floors one through six in the NS component because at these floors the positive amplitudes of the upgoing and downgoing waves do not overlap. Both travel-time curves in the EW and NS components indicate that the waves reflect off the top of the penthouse (R2), and the velocity is $214 \pm 9$ m/s in the EW direction and $158 \pm 7$ m/s in the NS direction, respectively, where the uncertainties are one standard deviation of measurements. Because the NS side is shorter, the velocity in the NS component is slower. The deconvolved waveforms in the NS component show large deviation from expected arrival times shown in the gray lines in Figure 7b, which indicates that the frequency dispersion in the NS component is larger than in the EW component. In the following, we focus on the EW-component analysis.

In Appendix C, we apply crosscorrelation and crosscoherence interferometry to records in the EW component. Because of the power spectrum of the incoming wave, we cannot obtain traveling waves using crosscorrelation interferometry (Figure C1c). We can estimate the velocity of traveling waves from the waveforms created by crosscoherence interferometry, but cannot estimate attenuation because the fundamental mode is not reconstructed (Figure C1d).

Next, we deconvolve the wavefields with the motion recorded by the receiver at the fourth floor (Figure 8), where the fourth-floor receiver behaves as a virtual source and satisfies a clamped boundary condition; then we apply the same bandpass filter as used in Figure 7. We can interpret waveforms in Figure 8 in two ways, which are explained using Figures 8a and 8b. We obtain upgoing and downgoing waves, which interfere at the fourth floor. At the circle in Figure 8a, the upgoing wave from the bottom and downgoing wave from the top cancel, and the deconvolved waveform at the fourth floor vanishes for nonzero time, which is due to the fact that the waveform at the virtual source is a band-limited delta function.

The fourth floor also behaves as the reflection point with reflection coefficient $-1$ (equation 3), which means we can separate the building to two parts: above and
Figure 8. (a) Deconvolved waveforms, in which the virtual source is at the fourth floor, of earthquake No. 5 after applying a bandpass filter 0.4-0.5-45-50 Hz in the EW component. The gray lines indicate the travel paths expected from the velocity 195 m/s and equation 3. Solid gray lines highlight the waves in the positive polarization and dashed gray lines the waves in the negative polarization. The circle indicates the point where the positive and negative polarization waves cancel. (b) The same waveforms shown in panel (a) but omitting deconvolved waveforms lower than the fourth floor. When we focus on the cut-off building above the fourth floor, the reflection coefficient at the circle is $-1$.

Figure 9. Deconvolved waveforms, in which the virtual source is at the eighth floor, of earthquake No. 5 after applying a bandpass filter 0.4-0.5-12-16 Hz in the EW component. The gray lines show the travel time of the waves propagating at 210 m/s. The positions of the lines are estimated from equation 3. The thick lines have positive polarization and the dashed line negative polarization.

below the virtual source. Figure 8b shows the building above the virtual source. At the circle in Figure 8b, the downgoing wave with positive polarization from the top is perfectly reflected as the negative-polarization upgoing wave. Since we obtain an upgoing wave from the virtual source and reverberations between the fourth floor and the top of the building, this example of interferometry creates the response of a cut-off building that is independent from the structure below the fourth floor (see equation 3 and Figure 8b). Similar to Figure 7, the fundamental mode for the cut-off building (equation 3 and Figure 5a) is dominant for later times in Figure 8b. Note the similarity between Figures 7a and 8b; both figures show traveling waves and fundamental mode. The period of the normal mode in Figure 8b is shorter than in Figure 7a as is expected from equation 6. Interestingly, because the cut-off building is independent from the structure below the fourth floor, this fictitious building is useful for detecting local structure and local damage of the building.

Applying a least-squares fit of the travel times of the first upgoing wave at the first to fourth floors ($n = 0$ and $0 \leq z \leq z_0$ in equation 3), we obtain the velocity of traveling waves to be $195 \pm 25$ m/s. To avoid large uncertainties, we use the travel times at the first to fourth floors to estimate the velocity. At these floors, the upgoing waves are well separated from the downgoing waves. For the cut-off building, by estimating the velocity from the deconvolved waveforms at the floors only below or above the virtual source, we can obtain the velocity which is only related to the structure below or above the virtual source because the virtual source satisfies the clamped boundary conditions. The structure between the first and fourth floors (below the virtual source) contributes to the estimation of this velocity. This is the main reason why the mean velocities estimated from Figures 7 and 8 differ, but this discrepancy is not statistically significant.

We apply deconvolution interferometry to the mo-
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Figure 10. Natural logarithm of the envelopes (thin line) and linear fitting using the least-squares method (thick line). We show envelopes at only the middle-second to eighth floors because the first floor is a virtual source and the basement floor has a different physical condition.

6 MONITORING A BUILDING USING 17 EARTHQUAKES

Using the 17 earthquakes recorded in the two weeks (Table 1 and Figure 2), we monitor the change in the shear-wave velocity of the building. Figure 11 illustrates the waveforms which are deconvolved by the wave recorded at the first floor of each earthquake (left panel) and the power spectra of the deconvolved waveforms (right panel). The virtual source is at the first floor (similar to Figure 7a). The frequency component around 1.5 Hz shows the fundamental mode and around 5 Hz the first overtone. From the bottom to top traces for each earthquake, the traces are aligned from the first to eighth floors, and the waves propagating between the bottom and the top are visible. Comparing the fundamental-mode waves for later times among earthquakes, we can roughly estimate changes in velocity from a visual inspection, e.g., the velocities in earthquakes 5, 8, 10, and 12 are slower. The earthquakes, which show slower velocity, indicate lower normal-mode frequencies as shown by Todorovska and Trifunac (2008b). The ratio of the reductions in velocity and frequency are almost the same.

The amplitude of each resonant wave provides an estimate of attenuation. For example, the attenuation is strong for earthquake No. 5 because the amplitude of the fundamental mode fades away at around 2.5 s. For some earthquakes, although the fundamental mode is dominant at later times, deconvolved waveforms still show upgoing and downgoing waves (e.g., at 2.5 s of earthquake No. 15), which implies either that the attenuation at higher frequency is relatively weak at the time these earthquakes occurred, or that the overtones are strongly excited. We estimate the velocity of traveling waves using the method that is the same as for Figure 7a (the black symbols in Figure 12). The black marks in Figure 12b illustrate a negative correlation between the velocities and the maximum acceleration of observed records.

To estimate velocities, we also apply coda-wave interferometry as developed by Snieder et al. (2002) to deconvolved waveforms. Coda-wave interferometry allows us to estimate a relative velocity change from two waveforms by computing crosscorrelation. Coda-wave interferometry has been applied to multiplets [e.g., Poupinet et al., 1984; Snieder and Vrijlandt, 2005] and to waveforms which are obtained by seismic interferometry [e.g., Sens-Schonfelder and Wegler, 2006]. By using coda-wave interferometry, we estimate velocities from the deconvolved waves between 1-3 s in Figure 11. The waves in the time interval are mostly the fundamental mode. We choose earthquake No. 5 as a reference and estimate the relative velocity for each earthquake from the reference earthquake. In coda-wave interferometry, we stretch and interpolate one waveform and compute a correlation coefficient (CC) with a reference waveform ($u_{ref}$) in the time domain (Figure 13) (Lobkis...
and Weaver, 2003; Hadziioannou et al., 2009; Weaver et al., 2011):}

\[
CC(\alpha) = \frac{\int_{t_1}^{t_2} u(t(1-\alpha))u_{ref}(t)dt}{\sqrt{\int_{t_1}^{t_2} u^2(t(1-\alpha))dt \int_{t_1}^{t_2} u_{ref}^2(t)dt}},
\]

(22)

where \(t_1\) and \(t_2\) denote the time window, and in this study we use 1-3 s. At the maximum of \(CC(\alpha)\),

\[
\alpha = \frac{v - v_{ref}}{v_{ref}},
\]

(23)

where \(v\) and \(v_{ref}\) are the velocities at each earthquake and the reference earthquake, respectively. For comput-
Figure 12. (a) Velocities estimated from traveling waves (black) and by coda-wave interferometry using the stretching method (gray) of each earthquake. The error bars of the velocities estimated from traveling waves (black) are one standard deviation of individual arrival times, and the bars in the stretching method are calculated by $\sqrt{\sigma_v^2 + \sigma_r^2}$, where $\sigma_v$ is the standard deviation of the velocity measurements estimated from traveling waves at different floors in earthquake No. 5, and $\sigma_r$ is the standard deviation of the relative velocity measurements between each earthquake and earthquake No. 5 estimated by the stretching method at different floors. The gray symbols in Figure 12b indicate that the velocities obtained by the stretching method also have a negative correlation with the acceleration, but the slope is steeper than that for the traveling waves. Since the waves in 1-3 s are mostly fundamental mode and the main difference between the traveling waves and the fundamental mode is the frequency (the traveling waves contain higher frequencies than the fundamental mode), the difference in slopes indicates dispersion. The steeper slope of the gray symbols in Figure 12b indicates that the imprint of acceleration is stronger for lower frequencies than for higher frequencies.

7 CONCLUSION

We obtain impulse responses of the building and their changes in velocity by applying deconvolution interferometry to 17 earthquake records. We estimate the reflection point of the traveling wave, which is at the top of the penthouse, from the deconvolved waveforms. Since the shape of the ground plan of the building is rectangular, the velocities of the traveling wave in two orthogonal horizontal components are different. According to the properties of deconvolution, the responses are independent from the soil-structure coupling and the effect of wave propagation below the bottom receiver. Because the cut-off building is independent of the structure below the virtual source, one might be able to use the cut-off building to investigate local structure and local damage. Crosscorrelation interferometry cannot separate the building response from the soil-building coupling and the wave propagation below the virtual source. Crosscoherence interferometry produces unphysical wavefields propagating at slower velocity than the true wave speed of the real building, and the attenuation of the waveforms obtained from crosscoherence do not correspond to the travel distance of the waves. Hence, in contrast to deconvolution interferometry, these types of interferometry are not appropriate.
Figure 13. (a) Correlation coefficient (CC) as a function of $\alpha$ (equation 22) between deconvolved waveforms computed from earthquakes No. 5 and No. 9 at the eighth floor. Dashed arrows point to the maximum CC value and its value of $\alpha$. For computing CC, we use only the waveforms from 1.0 s to 3.0 s. (b) Deconvolved waveforms at the eighth floor of earthquakes No. 5, No. 9, and No. 9 with stretching for $\alpha = 0.21$ (see panel (a)).

for applying to earthquake records for estimating velocities and attenuation of buildings. We estimate velocities from both traveling waves and the fundamental mode of deconvolved waveforms. The velocities estimated from each earthquake and maximum acceleration have a negative correlation.

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We thank sponsor companies of the Consortium Project on Seismic Inverse Methods for Complex Structures. We thank JMA for providing the earthquake catalog. We are grateful to Diane Witters for her help in preparing this manuscript.

REFERENCES

Trifunac, M. D., 1972, Comparisons between ambient and forced vibration experiments: Earthquake Eng. and Structural Dynamics, 1, 133–150.
APPENDIX A: NORMAL-MODE ANALYSIS OF DECONVOLUTION INTERFEROMETRY

In equation 3, we analyze the deconvolution interferometry based on superposition of traveling waves using a Taylor expansion. Here, we analyze equation 3 based on summation of normal-mode waves while using contour integration as following the procedure proposed by Snieder and Şafak (2006). Applying the inverse Fourier transform to \( D(z, z_a, \omega) \) and using the relationship \( k = \omega/c \), the deconvolved response in the time domain is given by

\[
D(z, z_a, t) = \int_{-\infty}^{\infty} e^{-i\omega(1-t/2)} e^{-\gamma|\omega|/2} + e^{-i\omega(t-\frac{2H-z}{c})} e^{-\gamma|\omega|/2} d\omega. \tag{A1}
\]

For \( t > (2H - z)/c \), the locations of poles \( (\omega_l) \) of the integrand in equation A1 are

\[
\omega_l = \omega_l(\pm 1 - i\gamma), \quad (l = 0, 1, 2, \ldots)
\]

where the normal-mode frequencies are given by

\[
\omega_l = \frac{(l + \frac{1}{2})\pi c}{H - z_a} \tag{A3}
\]

Using the residue theorem, equation A1 can be written as the summation of normal-mode wavefields:

\[
D(z, z_a, t) = \frac{4\pi c}{H - z_a} \sum_{l=0}^{\infty} (-1)^l e^{-\gamma|\omega_l|^2} \sin(\omega_l t) \cos \left( \frac{\omega_l H - z}{c} \right). \tag{A4}
\]

APPENDIX B: AMPLITUDE OF CROSSCORRELATION INTERFEROMETRY

In this appendix, we compute the amplitude of the crosscorrelated waveforms in equation 12. Using Taylor expansions, we rewrite the equation 12 as

\[
C(z, 0, \omega) = |S(\omega)|^2 \left\{ e^{ikz} e^{-\gamma|k|z} + e^{ikz} e^{-\gamma|k|(4H - z)} + e^{ik(2H - z)} e^{-\gamma|k|(2H - z)} + e^{-ik(2H - z)} e^{-\gamma|k|(2H + z)} \right\}
\times \left\{ \sum_{n=0}^{\infty} (R(\omega))^n e^{2inkH} e^{-2n\gamma|k||H|} \right\} \left\{ \sum_{m=0}^{\infty} (R^*(\omega))^m e^{-2imkH} e^{-2m\gamma|k||H|} \right\}. \tag{B1}
\]

Here, we only focus on the waves with phases \( e^{ikz} \), \( e^{ik(2H - z)} \), \( e^{ik(2H + z)} \), and \( e^{-ik(4H - z)} \), and define their amplitudes as \( C_1 \), \( C_2 \), \( C_3 \), and \( C_4 \), respectively. From equation B1, \( C_1 \) is obtained by

\[
C_1 e^{ikz} = |S(\omega)|^2 \left\{ e^{ikz} e^{-\gamma|k|z} \sum_{n=0}^{\infty} (R(\omega))^n e^{2inkH} e^{-2n\gamma|k||H|} \right\} + e^{-ik(2H - z)} e^{-\gamma|k|(2H + z)} \sum_{n=0}^{\infty} (R(\omega))^{n+1} e^{2(n+1)kH} e^{-2(n+1)\gamma|k||H|} \right\}.
\]

\[
C_1 = |S(\omega)|^2 e^{-\gamma|k|z} \left\{ 1 + R(\omega) e^{-4\gamma|k||H|} \right\} \left\{ \sum_{n=0}^{\infty} |R(\omega)|^{2n} e^{-4n\gamma|k||H|} \right\}. \tag{B2}
\]

Similar to expression B2, we obtain \( C_2 \), \( C_3 \), and \( C_4 \) as
Appendix C: Applying crosscorrelation and crosscoherence interferometry to real data

In Figure C1, we apply crosscorrelation and crosscoherence interferometry to real data to draw similar figures as Figure 7a. When we use the same bandpass filter as for Figure 7, crosscorrelation interferometry enhances the fundamental-mode frequency, and only the fundamental mode is visible (Figure C1a). This is caused by the power spectrum of the incoming wave (equation 12). When we cut the fundamental-mode frequency, the first-
higher mode is dominant in the crosscorrelated waveforms (Figure C1c). Therefore, we cannot estimate the velocity of traveling waves.

Figure C1b illustrates upgoing and downgoing waves from $t = 0$ s, and the waves propagate at the same velocity as the traveling waves reconstructed by deconvolution interferometry (Figure 7a). However, these propagating waves may be affected by the wave which propagates at slower velocities. The fundamental mode is not clear in Figure C1b. This is due to the spectral ratio used in crosscoherence interferometry, as a result the amplitude in later times is much smaller than the amplitude of deconvolution interferometry (Table 2). Because of the frequency we used, the wave propagating with slower velocity is not clear (the dashed gray line in Figure C1d). The negative amplitudes around the dashed line might be related to the wave with slower velocity.