Monitoring a building using deconvolution interferometry. II: Ambient-vibration analysis

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ABSTRACT
Application of deconvolution interferometry to earthquake data recorded inside a building is a powerful technique for monitoring parameters of the building, such as velocities of traveling waves, frequencies of normal modes, and intrinsic attenuation. In this study, we apply interferometry to ambient-vibration data, instead of using earthquake data, to monitor a building. The time continuity of ambient vibrations is useful for temporal monitoring. We show that because multiple sources simultaneously excite vibrations inside the building, the deconvolved waveforms obtained from ambient vibrations are non-zero for both positive and negative times, unlike the purely causal waveforms obtained from earthquake data. We develop a string model to qualitatively interpret the deconvolved waveforms. Using the synthetic waveforms, we find that the traveling waves obtained from ambient vibrations propagate with the correct velocity of the building, and the amplitude decay of the deconvolved waveforms depends on both intrinsic attenuation and ground coupling. The velocities estimated from ambient vibrations are more stable than those computed from earthquake data. Since the acceleration of the observed earthquake records varies depending on the strength of the earthquakes and the distance from the hypocenter, the velocities estimated from earthquake data vary because of the nonlinear response of the building. From ambient vibrations, we extract the wave velocity due to the linear response of the building.

Key words: time-lapse monitoring, deconvolution interferometry, ambient noise, building, normal mode

1 INTRODUCTION
Spectral analysis using forced vibrations and/or earthquakes is a common technique to estimate frequencies of normal modes, mode shapes, and viscous damping parameters of a building (Kanai and Yoshizawa, 1961; Trifunac, 1972; Trifunac et al., 2001a,b; Clinton et al., 2006). These parameters are useful for risk assessment and for estimating the response of a building to earthquakes (Michel et al., 2008). The sources listed above are sometimes inappropriate to use for temporal monitoring a building because of the lack of data continuity. Ambient vibrations, caused by sources within the building, are more suitable for monitoring a building because of the quasi-continuous nature of these vibrations (Trifunac, 1972; Ivanović et al., 2000). In this study we use seismic interferometry to analyze ambient vibrations recorded inside a building in the Fukushima prefecture in Japan.

Using seismic interferometry, we can reconstruct waves that propagate from one receiver to another. Seismic interferometry was invented by Aki (1957) and Claerbout (1968), and has been well-developed over the last decade [e.g., Lobkis and Weaver, 2001; Derode et al., 2003; Snieder, 2004b; Wapenaar, 2004; Schuster, 2009; Snieder et al., 2009; Tsai, 2011]. One can apply seismic interferometry to active sources [e.g., Bakulin and Calvert, 2006; Wegler et al., 2006; Mehta et al., 2008; van der Neut et al., 2011] or to earthquake data [e.g., Sawazaki et al., 2009; Yamada et al., 2010; Nakata and Snieder, 2011, 2012a,b]. These latter studies used interferometry for monitoring purposes. One can also apply interferometry to noise caused by production [e.g., Miyazawa et al., 2008], drilling [e.g., Vasconcelos and Snieder, 2008a,b], and traffic [e.g., Nakata et al., 2011], and to non-specific vibrations (so-called ambient vibration or ambient noise) [e.g., Sens-Schönfelder and We-
In a companion paper (henceforth called Part I: Nakata et al. (2013)), we analyze earthquake data, recorded over the same time period in the same building as in this study, using seismic interferometry. Although several studies apply interferometric approaches to earthquake data recorded in a building [e.g., Snieder and Safak, 2006; Snieder et al., 2006; Kohler et al., 2007; Todorovska and Trifunac 2008a,b], few studies apply this technique to ambient vibrations (Prieto et al., 2010). As we explain below, by applying seismic interferometry to ambient vibrations recorded in a building, we not only achieve continuous monitoring in time but also obtain information of the ground coupling and linear response of the building, which we cannot estimate from earthquake data.

We first introduce ambient-vibration data and deconvolved waveforms computed from the observed data. Next, we analytically and qualitatively interpret the deconvolved waveforms using traveling-wave and normal-mode analyses. Then we monitor the building using ambient vibrations based on the interpretation.

2 DECONVOLUTION ANALYSIS USING AMBIENT VIBRATION

We present data acquisition, pre-processing for deconvolution interferometry, and the interferometry using ambient-vibration data in this section. Data are observed in the same building over the same time period as for the earthquake data in Part I (Figure 1). Pre-processing has an important role for obtaining reliable correlograms (Bensen et al., 2007), and here we focus on the pre-processing to exclude large amplitudes caused by earthquakes and human activities.

2.1 Observed records

The building we used is in the Fukushima prefecture, Japan (the rectangle in Figure 1). Continuous ambient seismic vibrations were recorded by Suncoh Consultants Co., Ltd. for two weeks (May 31–June 14, 2011) using 10 MEMS accelerometers developed by Akebono Brake Industry Co., Ltd. The building has eight stories, a basement, and a penthouse (Figure 2). Based on the analysis in Part I, the waves, which propagate vertically inside the building, reflect off the top of the penthouse (R2 in Figure 2). The sampling interval of the accelerometers is 1 ms, and the receivers have vertical, east-west (EW) horizontal, and north-south (NS) horizontal components. In this study we focus on the EW horizontal component to extract horizontal modes.

Figure 3 illustrates the root-mean-square (RMS) amplitude computed over a moving window with a duration of 30 s of unfiltered seismic records observed for the two weeks. The hours of operation of the building are from 8AM to 6PM on weekdays, when the RMS amplitude is elevated. On the weekends we observe lower RMS amplitudes (June 4, 5, 11, and 12 are weekends). The vibrations are probably induced by human activities, elevators, air conditioners, computers, traffic near the building, and other sources. Amplitudes at the upper floors are stronger due to the shape of the fundamental mode of the building (see Figure 4 in Part I for the shape of the fundamental mode). Stronger amplitudes at the first floor compared with nearby floors may be caused by vibrations from traffic outside the building and/or many visitors to that floor. Because the amplitudes at the basement are much smaller than the other
floors, we do not interpret the records at the basement in this study.

2.2 Pre-processing

Before applying deconvolution interferometry, we exclude large-amplitude intervals from the continuous records because we focus on ambient vibrations. Large amplitudes are excited by earthquakes and human interference, such as people touching the accelerometers. Since receivers are often located at places where people can touch them (e.g., on stairs), a technique proposed here to exclude the human interference is useful. To exclude large-amplitude waves, we apply a data-weighting procedure based on the standard deviation of data recorded for one hour in which the data do not include significant earthquakes or human interference (Wegler and Sens-Schönfelder, 2007). When one receiver records a larger amplitude than the threshold, the samples of all receivers at that time are set to zero since we need the waveforms at the same time at all sensors for the deconvolution analysis. After someone touches a receiver, the DC component on the seismograms may change. We subtract the DC component from the data of every 30 s and discard data when the DC component changes during that time interval. Similar to large amplitudes, we exclude time intervals when one receiver indicates a change in the DC component.

2.3 Deconvolution analysis using two-week ambient vibration

We apply deconvolution interferometry to ambient-vibration records observed inside the building. Here, we stack deconvolved waveforms over the two weeks in which data were collected. In the later section Monitoring the building using ambient vibration, we stack over four-day intervals for monitoring purposes. We deconvolve each 30-s ambient-vibration record with the first-floor record and then stack the waveforms over the two-week interval:

$$D(z, t) = \sum_{n=1}^{N} \left[ \mathcal{F}^{-1} \left\{ \frac{u_n(z, \omega)}{u_n(0, \omega)} \right\} \right]$$

$$\approx \sum_{n=1}^{N} \left[ \mathcal{F}^{-1} \left\{ \frac{u_n(z, \omega)u_n^*(0, \omega)}{|u_n(0, \omega)|^2 + \alpha |u_n(0, \omega)|^2} \right\} \right],$$

(1)

where $N$ is the number of 30-s intervals (40,080 in this study), $u_n(z, \omega)$ the $n$th wavefield in the frequency domain recorded at $z$ ($z = 0$ is the first floor), $\omega$ the angular frequency, $t$ time, $\mathcal{F}^{-1}$ the inverse Fourier transform, $*$ the complex conjugate, $\langle |u_n|^2 \rangle$ the average power spectrum of $u_n$, and $\alpha = 0.5\%$ a regularization parameter stabilizing the deconvolution (Clayton and Wiggins, 1976). Our Fourier convention is $f(t) = \int_{-\infty}^{\infty} F(\omega)e^{-i\omega t}d\omega$. We apply a bandpass filter, 1.3-1.5-15-20 Hz, to the deconvolved waveforms (Figure 4).

In Figure 4, we obtain traveling waves and the fun-
Figure 5. (a) Synthetic waveforms obtained from one source inside a building (expressions 2 and 3) and (b) waveforms of panel (a) after deconvolution with the waves observed at \( z = 0 \) m. The source is located at \( z_s = 13 \) m and excites waves at \( t = 0.2 \) s. The gray lines in panel (b) show the arrival times of the traveling waves based on expressions 4 and 5. The solid and dashed gray lines respectively illustrate the terms \( e^{ik(z-2z_s)}e^{-\gamma|b|(z-2z_s)} \) and \( \text{Re} e^{ikz}e^{-\gamma|b|z} \), and their reverberations. The amplitudes of panels (a) and (b) are normalized after applying the same bandpass filter as used in Figure 4.

3 DISCUSSION OF THE DECONVOLVED WAVEFORMS

In this section, we interpret the deconvolved waveforms in Figure 4 using a mathematical description and synthetic waveforms based on traveling waves and normal modes. The goal of this section is to understand why we obtain both causal and acausal waves after applying interferometry to ambient vibrations, to reconstruct the waveforms using synthetic computation, and to determine to what degree we can estimate the velocity of traveling waves and the quality factor from ambient vibrations. The main differences of deconvolved waveforms obtained from ambient vibrations and earthquakes are that for ambient vibrations, sources are inside the building and more than one source simultaneously excites inside and outside the building. We consider deconvolved waveforms computed from one source inside the building based on traveling waves and from multiple sources based on normal modes.
3.1 One source inside the building

To analyze deconvolved waveforms obtained from one source inside the building, we employ the same assumptions as equation 1 in Part I: vertically propagating waves in the building, constant amplitude and wavenumber, no torsional waves, and no internal reflections. Based on Snieder and Şafak (2006) and Part I, when a source is at height $z_s$, the observed record at an arbitrary receiver at height $z$ is

$$u(z > z_s, \omega) = \frac{X}{1 - Re^{2ikH}e^{-2\gamma |k|H}}$$

for $z > z_s$, and

$$u(z < z_s, \omega) = \frac{X'}{1 - Re^{2ikH}e^{-2\gamma |k|H}}$$

for $z < z_s$. Here, $S(\omega)$ is the source function, $R$ the reflection coefficient at the base of the building, $k$ the wavenumber, $\gamma$ the attenuation coefficient, $H$ the height of the building, and $i$ the imaginary unit. The attenuation coefficient is defined by $\gamma = 1/(2Q)$ (Aki and Richards, 2002). The numerators $X$ and $X'$ are given by

$$X = e^{ik(z-z_s)}e^{-\gamma |k|(z-z_s)} + e^{-ik((2H-z-z_s)}e^{-\gamma |k|(2H-z-z_s)}$$

$$+ R \left( e^{ik(z-z_s)}e^{-\gamma |k|(z-z_s)} + e^{-ik((2H-z-z_s)}e^{-\gamma |k|(2H-z-z_s)} \right),$$

$$X' = e^{ik(z-z_s)}e^{-\gamma |k|(z-z_s)} + e^{-ik((2H-z-z_s)}e^{-\gamma |k|(2H-z-z_s)}$$

$$+ R \left( e^{ik(z-z_s)}e^{-\gamma |k|(z-z_s)} + e^{-ik((2H-z-z_s)}e^{-\gamma |k|(2H-z-z_s)} \right),$$

respectively.

The waveforms recorded at height $z$ deconvolved with the waveform recorded at the first floor ($z = 0$) are

$$D(z > z_s, \omega) = \frac{u(z > z_s)}{u(z = 0)} = \frac{e^{ik(z-z_s)}e^{-\gamma |k|(z-z_s)} + Re^{ikz}e^{-\gamma |k|z}}{1 + R \left( e^{ik((H-z)}e^{-2\gamma |k|(H-z)} \right)} \sum_{n=0}^{\infty} (-1)^n \left( e^{2n\gamma |k|H} \right),$$

(4)

$$D(z < z_s, \omega) = \frac{u(z < z_s)}{u(z = 0)} = \frac{e^{-ikz}e^{-\gamma |k|z} + Re^{-ikz}e^{-\gamma |k|z}}{1 + R},$$

(5)

From expressions 4 and 5, the deconvolved waveforms obtained from one source inside the building are dependent on the ground coupling; this is in contrast to the case when sources are outside the building (i.e., earthquakes). Interestingly, although the deconvolved waveforms retrieved from external sources are only related to the structure of the building (Part I), the waveforms from internal sources are governed by both the structure of the building and the ground coupling (through the reflection coefficient $R$).

We numerically compute synthetic observed records based on expressions 2 and 3 (Figure 5a) and deconvolve these records with the waveform recorded at $z = 0$ m (Figure 5b). The model parameters to compute the waveforms in Figure 5a are $H = 39$ m, $R = -0.6$, $Q = 30$, and $c = 270$ m/s, where $c$ is the velocity of the traveling wave in the building. The waves are excited at $z_s = 13$ m at $t = 0.2$ s. The gray lines in Figure 5b indicate the arrival times of traveling waves estimated from expression 4 for above the source ($z > z_s$) and expression 5 for below the source ($z < z_s$). After deconvolution we obtain acausal waves in Figure 5b. These waves correspond to the term $e^{ik(z-z_s)}e^{-\gamma |k|(z-z_s)}$ in expression 4 and $e^{-ikz}e^{-\gamma |k|z}$ in expression 5. Note that these acausal waves exist only for a time interval $-z_s/c < t < 0$ ($-0.05 s < t < 0 s$ in Figure 5b) and that the waves are not symmetric in time. Therefore, one source in the building does not explain the symmetry between the acausal and causal waves in Figure 4.
3.2 Multiple sources

Using the normal-mode theory (Snieder, 2004b, Ch. 20), we compute the deconvolved waveforms obtained from multiple sources to qualitatively interpret the waveforms in Figure 4. We can express waves using either the summation of traveling waves or normal modes [Dahlen and Tromp, 1998, Ch. 4; Snieder and Şafak, 2006]. Equations 2–5 are based on traveling waves, and these equations depend on the location of sources. We have to modify all terms in the numerators of equations 2 and 3, and choose equation 2 or 3 depending on the locations of receiver and source. On the other hand, the normal-mode analysis is suitable for multiple sources inside the building because source terms are separated from other terms (see for example equation 20.69 in Snieder (2004b)).

The model for our normal-mode analysis is a one-dimensional string model that includes radiation damping (Snieder, 2004b, Ch. 20.10). This model consists of an open-ended light string with mass density \( \rho \) connected to a heavy string with density \( \rho_b \gg \rho \) at \( z = 0 \) (Figure 6). The wave propagation in the light and heavy strings represents the propagation in the building and the subsurface, respectively. Although the string model is primitive, the model qualitatively accounts for the wave propagation in the building because of three reasons: 1) we are only interested in the building, 2) the effect of the ground for the building is limited to the coupling at \( z = 0 \), and 3) we assume no waves return after the waves propagate to the ground. The ratio of the densities of the light and heavy strings is related to the reflection coefficient at the connection of the strings (at the base of the building) (Coulson and Jeffrey, 1977, Ch. 2):

\[
R = \sqrt{\frac{\rho - \rho_b}{\rho + \rho_b}} = \frac{1 + \epsilon}{1 + \epsilon},
\]

where

\[
\epsilon = \sqrt{\rho/\rho_b}.
\]

We carry out a perturbation analysis for this small dimensionless parameter.

The eigenfunctions and eigenfrequencies of this string model to first order in \( \epsilon \) for the mode \( m \) (\( m = 0, 1, \cdots \)) are given by

\[
u_m(z) = \sin \left\{ \pm \left( m + \frac{1}{2} \right) \frac{\pi z}{H} \right\}
- i \epsilon \frac{H - z}{H} \cos \left\{ \pm \left( m + \frac{1}{2} \right) \frac{\pi z}{H} \right\},
\]

\[
\omega_m^{(r)} = \left\{ \pm \left( m + \frac{1}{2} \right) \pi - i \epsilon \right\} \frac{c}{H},
\]

respectively (see Appendix A). Because this string model does not include the intrinsic attenuation of the building, the eigenfrequency in expression 9 does not incorporate the attenuation. The superscript in expression 9 indicates that the complex eigenfrequency accounts only for the radiation loss. Snieder and Şafak (2006) derive the eigenfrequency \( \omega_m^{(a)} \) with the intrinsic attenuation, but without radiation damping:

\[
\omega_m^{(a)} = \left( m + \frac{1}{2} \right) \frac{\pi c}{H} (\pm 1 - i \gamma).
\]

Comparing expressions 9 and 10, we account for the intrinsic attenuation and the radiation damping using the following eigenfrequency:

\[
\omega_m^{(ar)} = \left\{ \pi \left( m + \frac{1}{2} \right) (\pm 1 - i \gamma) - i \epsilon \right\} \frac{c}{H},
\]

where we assume the intrinsic attenuation to be weak and ignore a cross term between the intrinsic attenuation and radiation damping. In expression 11, the first term \( (\pi (m+1/2)c/H) \) is the frequency in case there is no intrinsic attenuation \( (\gamma = 0) \) and the building has a rigid boundary at the bottom \( (R = -1) \). The second term \( (\pm i \epsilon (m+1/2)c/H) \) accounts for the intrinsic attenuation, and the third term \( (-i \epsilon c/H) \) accounts for the radiation loss at the base of the building. The waveforms in this string model with the intrinsic attenuation are given by the summation of normal modes (Snieder, 2004b):

\[
u(z, \omega) = \sum_{m=0}^{\infty} \frac{u_m(z) \int u_{m}^{*}(z') F(z') dz'}{\left( \omega_m^{(ar)} \right)^2 - \omega^2},
\]
Figure 7. Synthetic deconvolved waveforms using three-hour random vibrations as sources after applying the same bandpass filter as in Figure 4. Panels (a)–(i) are computed by adopting different quality factors $Q$ and reflection coefficients $R$ (see lower-left of each panel). Gray lines indicate the arrival time of the traveling wave with the velocity used for the modeling ($c = 270$ m/s). The scale of the amplitudes at each panel is the same.

where $F$ indicates the forces that excite the vibrations.

We numerically compute the synthetic records using expression 12 for various values of the quality factor $Q$ and the reflection coefficient $R$ with fixed parameters: $H = 39$ m and $c = 270$ m/s. We use random sources (random amplitude, phase, and location) and compute three-hour random-source synthetic observed records. Then we deconvolve the waveforms with the records at the floor at $z = 0$ (Figure 7). All panels in Figure 7 show waves for both positive and negative times, which is consistent with the deconvolved waveforms in Figure 4. Especially for $|t| > 1$ s, the waveforms in Figure 7 are similar in character to those in Figure 4. For $|t| < 0.3$ s, we obtain the traveling waves, propagating with the same velocity as used for the modeling ($c = 270$ m/s; compare the waveforms and the gray lines in Figure 7).

The waveforms are increasingly asymmetric in time as the reflection coefficient differs from $R = -1$, or as the anelastic damping increases (see for example Figures 7def or 7beh). From Figure 7, we learn that the amplitude decay of the waveforms is related to the intrinsic attenuation and the boundary condition. Based on the similarity of the waveforms in Figures 4 and 7, the reflection coefficient and the quality factor of the real building are likely close to those in Figures 7a–7e. Because we can estimate $Q^{(a)}$ independent from the ground coupling using the earthquake data (Part I), the deconvolution using ambient vibrations is potentially useful for estimating $R$. However, to estimate $R$, we need a more quantitative analysis, which is a topic of future work. Also, for waveform matching this string model may be too simple. We conclude that the estimated velocity from the waveforms in Figure 4 indicates the true velocity of the traveling wave in the building, and the quality factor estimated from the amplitude decay of the
4 MONITORING THE BUILDING USING AMBIENT VIBRATION

For monitoring the velocity of the traveling waves, we need to know the minimum time length to obtain stable waveforms. To determine this time interval, we compute the convergence of the deconvolved waveforms as a function of the stacking duration \( h \) using a RMS misfit as used by Prieto et al. (2010):

\[
\text{Misfit}(z, h) = \sqrt{\frac{\int_{t_a}^{t_b} \{D_h(z, t) - D_{all}(z, t)\}^2 dt}{\int_{t_a}^{t_b} \{D_{all}(z, t)\}^2 dt}}, \quad (13)
\]

where \( t_a \) and \( t_b \) define the time interval to compute the misfit (-1.5 s and 1.5 s in this study), \( h \) the stacking duration, \( D_h \) the deconvolved waves stacked over time period \( h \), and \( D_{all} \) the deconvolved waveforms obtained from the entire data set recorded during the two weeks. If the RMS misfit is small, the deconvolved waveform \( D_h \) is similar to the deconvolved waveforms obtained from the entire data set.

Figure 8 indicates the convergence of deconvolved waveforms with respect to the stacking duration. In Figure 8a, we use both daytime (8AM–6PM) and nighttime (6PM–8AM) data. Since the RMS misfit is lower than 5% when we use the ambient-vibration data longer than 96 hours, we decide that stacking over 96 hours is sufficient to obtain stable deconvolved waveforms. The RMS misfit in Figure 8a increases during some night-times. However, since for example the RMS misfits at \( h = 66 \) are smaller than the misfits at \( h = 52 \) at all floors, the vibrations in nighttime also contribute to the convergence. We also compute the waveforms using daytime data only and estimate the RMS misfit (Figure 8b). Interestingly, although Figure 8b shows rapid convergence to 10%, we need about 40 hours (equivalent to four days) to obtain the RMS misfit lower than 5%. In Figure 8, we show the RMS misfits for 122 hours (panel a) and for 52 hours (panel b), which are equivalent because 122 hours include 52 hours of daytime and 70 hours of nighttime.

Figure 9 shows the deconvolved waveforms using the data recorded during both daytime and nighttime (same data as used in Figure 8a), and Figure 10 using the data recorded daytime only (same data as used in Figure 8b). In Figure 9, we stack the data over four-day intervals (96 hours) and overlap these intervals over two days. From the waveforms in Figure 9, we estimate the velocity of the traveling waves using the same method as Figure 4. The estimated velocities are stable during the two weeks, and the uncertainty is about 6 m/s, which is smaller than the uncertainty in the velocity estimated from earthquake data (Figure 12 in Part I). For earthquakes, the estimated velocities vary more than for ambient vibrations, and the acceleration of the observed records also varies (Figure 12b in Part I). These variations indicate that the velocities estimated from earthquakes include nonlinear effects. The velocities estimated from ambient vibrations are not affected by nonlinearity because the acceleration of the observed records is small and does not vary much. Therefore, ambient vibration is appropriate for monitoring the velocity of traveling waves in the linear regime. Deconvolved waveforms in Figure 10 are similar to those in Figure 9, and the difference in estimated velocities are not statistically significant.
5 CONCLUSION

We retrieve traveling waves inside the building by applying seismic interferometry to ambient-vibration data. In contrast to the case when sources are only outside the building, deconvolved waves obtained from ambient vibrations are nonzero for both positive and negative times, which is explained by that multiple sources simultaneously excite inside the building. Based on the normal-mode analysis, we synthetically reconstruct waveforms that are qualitatively similar to the real data using the simple string model. The velocity estimated from the synthetic waveforms with this model is the same as the true velocity although the attenuation estimated from the decay of the amplitude with time is not equal to the intrinsic attenuation of the building. Since the amplitude decay is also influenced by radiation losses at the base of the building, we are, in principle, able to estimate both quality factors and reflection coefficients separately from the amplitude of the waveforms, which requires a more accurate model than the string model used here. For monitoring the building, we find the time interval to obtain stable waveforms using the convergence test, and we need deconvolved ambient vibrations averaged over four days to obtain stable waveforms for this building. The velocity estimated from ambient-vibration data is more stable than that from earthquake data because the ambient vibrations are due to the linear response of the building.

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Figure 10. (Left) Time-lapse deconvolved waveforms averaged over 40 hours with a 20-hour overlap using ambient vibration recorded in daytime only. We have applied the same bandpass filter as used in Figure 4. (Right) Shear-wave velocities estimated from the traveling waves in the left panel. The width of each box indicates one standard deviation of estimated velocities at each floor.

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APPENDIX A: EIGENFUNCTIONS AND EIGENFREQUENCIES OF THE STRING MODEL

In this appendix, we derive the eigenfunctions (expression 8) and eigenfrequencies (expression 9) of the string model (Figure 6) using a perturbation analysis in the small parameter $\epsilon$ (expression 7). The normal modes of the unperturbed string ($\epsilon = 0$, $\rho_g = \infty$, and $R = -1$) are given by

$$u(z > 0) = \sin k^{(u)} z, \quad (A1)$$
$$u(z < 0) = 0, \quad (A2)$$

where $k^{(u)}$ is the unperturbed wavenumber. The parameter $\epsilon$ accounts for the coupling of the light string to the heavy string (expression 6). For the unperturbed model ($\epsilon = 0$), the string has infinite mass for $z < 0$, and hence it does not move. When $\epsilon \neq 0$, the waveforms, which include perturbed waves, are expressed by

$$u(z > 0) = \sin k z + A \cos k z, \quad (A3)$$
$$u(z < 0) = B e^{-i k g z}, \quad (A4)$$

respectively. The coefficients $A$ and $B$ depend on $\epsilon$. According to expression A4, waves are radiated downward in the lower (heavy) part of the string (the thick line in Figure 6). The ratio of the wavenumbers in the light and heavy strings ($k/k_g$) is given by Ch. 2 in Coulson and Jeffrey (1977):

$$k/k_g = \epsilon. \quad (A5)$$

The boundary conditions of the model are $\partial u/\partial z = 0$ at $z = H$, and $u$ and $\partial u/\partial z$ are continuous at $z = 0$.

From expressions A3 and A4 and the boundary conditions, we obtain

$$\frac{k}{k_g} \sin k H + i \cos k H = \frac{\epsilon}{2 i} (e^{i k H} - e^{-i k H}) + \frac{i}{2} (e^{i k H} + e^{-i k H}) = 0, \quad (A6)$$

where we use expression A5 for $k/k_g$. From expression A6, we obtain

$$e^{i k H} = -\frac{1 - \epsilon}{1 + \epsilon}. \quad (A7)$$

Applying a first-order Taylor expansion in $\epsilon$ to the wavenumber in expression A7, we obtain the wavenumber for mode $m$:

$$k_m = \left\{ \pm \left( m + \frac{1}{2} \right) \pi - i \epsilon \right\} \frac{1}{H}, \quad (A8)$$

where the real and imaginary of $k_m$ are the unperturbed and perturbed parts of the wavenumber of mode $m$, respectively. The perturbation of the wavenumber caused by the radiation damping ($-i \epsilon/H$) is constant for all modes. The eigenfrequency $\omega_m$ that corresponds to this wavenumber is given by expression 9.

From expressions A3 and A8, the waveform (eigenfunction) for the mode $m$ within the light string is given by

$$u_m(z) = \sin (k_m z) + A \cos (k_m z)$$
$$= \sin (k_m z) + \frac{\cos (k_m H)}{\sin (k_m H)} \cos (k_m z)$$
$$= \frac{\cos (k_m (H - z))}{\sin (k_m H)}, \quad (A9)$$

where we use the boundary condition $\partial u/\partial z = 0$ at $z = H$ at the second equality. Using Taylor expansions to first order in $\epsilon$ in the sine and cosine functions, we derive the eigenfunction shown in expression 8.