Optimization of time reversal focusing through deconvolution

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ABSTRACT

In this study a technique is demonstrated to optimize the ability of time reversal to both spatially and temporally focus, or compress, elastic wave energy. This method allows for optimization of time reversal focusing for any number of channels by calculating the required time reversed signal to be rebroadcast from the time reversal mirror element using only the original recorded scattered signal from that element. The speed, simplicity and robust nature of this process for any number of time reversal mirror elements, including only a single channel, provide the ability to enhance applications currently using time reversal for tasks such as communications, nondestructive evaluation, lithotripsy and imaging, among others. It is also demonstrated theoretically and numerically that good temporal focusing implies that the radius in the spherically symmetric part of the spatial focus is small.

Key words: time reversal, signal processing, communications

1 INTRODUCTION

Time Reversal (TR) in acoustics has been the focus of much research (Parvulescu, 1961; Fink, 1997; Larmat et al., 2010). This has led to many applications in a wide variety of fields, such as medicine, communications, nondestructive evaluation (NDE), seismology, etc. In many of these applications, the ability to use TR depends upon its ability to precisely compress the measured scattered waveforms to a point in both time and space, ideally a delta function, $\delta(t)$, which is commonly referred to as TR focusing. The desire to enhance the TR process to focus wave energy has lead researchers to develop a variety of ways to accomplish this. Some techniques use arrays of input transducers, measure the wave field with an array near the desired focal spot, and then optimize the spatial and temporal focusing (Tanter et al., 2000, 2001; Montaldo et al., 2004; Vignon et al., 2006; Gallot et al., 2011; Bertaix et al., 2004; Roux and Fink, 2000; Aubry et al., 2001; Jonsson et al., 2004). Other methods use an array of input transducers and optimize the temporal focusing at an output transducer (Daniels and Heath, 2005; Qiu et al., 2006; Blomgren et al., 2008; Zhou et al., 2006; Zhou and Qiu, 2006). Some of these techniques require, or at least benefit greatly from, large arrays, while others may enhance temporal focusing at the expense of spatial localization.

We explore a simple approach and use deconvolution, a primitive, though robust, version of the inverse filter (Tanter et al., 2000; Gallot et al., 2011), to give an input signal that gives a delta function $\delta(t)$ as an output at the focal point, and show laboratory experiments with ultrasound that the temporal focusing thus obtained is superior to that obtained by using time reversal. By scanning the sample near the focal point, we find that this improved temporal focusing is accompanied by a better spatial focusing. We then show theoretically that improved temporal focusing implies enhanced spatial focusing for the spherical average of the focus. The method described in this paper enhances both spatial and temporal focusing for any number of array elements, including a single channel. In contrast to previous applications of the inverse filter (Tanter et al., 2000, 2001; Gallot et al., 2011), we also explore the effectiveness of using deconvolution for small time reversal mirrors (TRM), i.e., using 1-8 channels, and in the case of limited available bandwidth.

2 DECONVOLUTION AS AN INVERSE FILTER

The time reversal process relies on the assumption that from a recorded impulse response, or Green function
$G_{AB}$ between two points $A$ and $B$, a simple reversal in time of that signal and rebroadcast of it back into the same propagating medium, from $A$ to $B$ or vice versa, will focus an impulsive function $\delta(t)$ at the appropriate point in space ($A$ or $B$); or

$$\int_{-\infty}^{\infty} G_{AB}(\tau)G_{AB}(\tau - t) d\tau \approx \delta(t). \quad (1)$$

where reciprocity has been used to replace the Green function $G_{BA}$ with $G_{AB}$. Expression (1) states that the autocorrelation of $G_{AB}(t)$ is ideally equal to a delta function. In practice, actual TR experiments cannot recreate a true delta function as one or more conditions necessary to satisfy eq. (1) are typically violated. For example, due to either time, physical memory, or attenuation constraints, it is not possible to record for an infinitely long period of time. Also, the Green functions here are assumed to contain an infinite bandwidth, which is routinely not the case in experiments. For this reason we, as others before us, asked the question: what would be required to most closely approach an impulse-like TR focus given typical experimental limitations?

Going back to eq. (1), we can rewrite this (using a convolution notation, rather than the integral form) as

$$f(t) = g(t) \ast r(t) \approx \delta(t), \quad (2)$$

where $f(t)$ is the acoustic wave signal measured at the source, $r(t)$ is the signal send from the source, and $g(t)$ is the signal that is back propagated to the source. Here we restrict ourselves to looking only at signals between the two points $A$ and $B$ and thus have removed them from the notation. We also change from a Green function notation so as not to imply that we necessarily have an infinite bandwidth available. From eq. (2), we aim to approximate the focused signal $f(t)$ to a delta function by assuming that the recorded impulse response $r(t)$ contains information enough to produce an optimized signal $g(t)$ to rebroadcast in our TR focusing procedure.

Deconvolution equates to inverse filtering by transforming to the frequency domain, thus eq. (2) becomes

$$f(\omega) = g(\omega) \times r(\omega) \approx 1, \quad (3)$$

which can easily be manipulated to provide $g(\omega)$ simply from measuring $r(\omega)$ (or rather from the FFT of $r(t)$). As such, we can now calculate the optimized signal which should produce the most impulse-like TR focus from

$$g(\omega) = \frac{1}{r(\omega)} = \frac{r^*(\omega)}{|r(\omega)|^2}. \quad (4)$$

This expression gives a mathematical expression for $g(\omega)$, but this may not be practical for experimental use in the event that there is a limited bandwidth, or more precisely if $r = 0$ at any frequency. To avoid this we simply add a constant to the denominator to ensure that we never divide by 0, hence we replace eq. 4 by

$$g(\omega) = \frac{r^*(\omega)}{|r(\omega)|^2 + \epsilon}, \quad (5)$$

where $\epsilon$ is a constant related to the original received signal as

$$\epsilon = \gamma \times \text{mean}(|r(\omega)|^2). \quad (6)$$

We used an arbitrary value of $\gamma = 1$ for the experiments described in the next section.

As the above derivation has been performed in the frequency domain, it is necessary to transform back to the time domain to recover the optimized signal, $g(t)$ to be used in the time reversal experiments. It is worth noting that this procedure produces a $g(t)$ that can be used directly for rebroadcasting, i.e., there is no need to perform a TR operation $g(t) \rightarrow g(-t)$ to obtain a focused signal. While this completes the deconvolution optimization procedure used in this paper, it is only a portion of the inverse filter procedure as defined by Tanter et al. (2000), and utilized most recently by Gallot et al. (2011). Performing the full inverse filter procedure requires the singular value decomposition (SVD) of the matrix of filtered waveforms. For reasons that will be addressed in the following section this step is neither necessary nor appropriate for this study.

3 EXPERIMENTAL VALIDATION

The purpose of this study is to optimize TR in typical experimental situations, specifically those where only a few channels are used with reverberant signals in a closed cavity. With this in mind, it is prudent to test the methodology experimentally, as presented above. To do this, we used a sample made of aluminum, approximately 1000cm³ volume though irregularly shaped. A
source transducer (2 cm PZT disk) was attached to the sample using epoxy. This defines, and fixes, our source position \( A \). The receiver, a non-contact laser vibrometer (PSV 301, OFV 5000 controller, Polytec Inc.) was positioned at a point \( B \) on the surface. As we cannot perform a classical TR experiment with a laser vibrometer, since it cannot act as both receiver and source, we performed reciprocal TR experiments, i.e., where the original source and back propagation signals are emitted from the same location/transducer, thus focusing the wave energy to the original receiver location. This use of TR in a reciprocal sense is now a widely used form of TR, and indeed is the method originally used by Parvelescu (Parvulescu, 1961) in the first known demonstration of TR in acoustics (also known as phase conjugation). It is due to the use of reciprocal TR that SVD is unnecessary. The SVD step of the inverse filter procedure is applicable to the classical TR procedure as its purpose is to remove correlated noise that may exist simultaneously in all of the received signals. Reciprocal TR requires that each received signal \( r(t) \) is obtained individually, i.e., not simultaneously. For this reason there is no expected correlation of noise from one measurement of \( r(t) \) to the next.

The original impulse, a 5\( \mu \)s shaped toneburst of 200kHz, was broadcast from the source transducer using a 12-bit arbitrary waveform generator with a conversion rate of 10MHz. The response to this impulse was recorded by the laser vibrometer (5mm/s/V, 250kHz bandwidth) using a 14-bit digitizer also sampling at 10MHz. The generator and digitizer were synced such that the source impulse was centered in a 3.2768ms window, thus the first portion of the received signal \( r(t) \) contains no causal signal. This also means that the TR focus will occur at 1.6384ms during the rebroadcast.

In fig. 1, both the reversed original response \( r(-t) \) and the result of the \( \text{optimization} \) procedure \( g(t) \) can be seen. Each of these signals were used independently to achieve a TR focus. These focused signals, corresponding to the back propagation of these signals, can be found in fig. 2 (a). There are a few key points to be noted in these two figures. First, note the acausal signal, i.e., signal appearing after \( t = 1.64 \text{ms} \), in the \( \text{optimized} \) signal. This portion of the signal is necessary for the reconstruction of a symmetric focal signal. Arbitrarily setting \( g(t > 1.64 \text{ms}) = 0 \) does not affect the impulse-like focus at 1.6384ms in fig. 2 (a). However, it does destroy the symmetry that is seen in fig. 2 (a) for the \( \text{optimized} \) focus.

In fig. 2, the two important features of note are: i) the temporal compression, and ii) maximum achievable amplitudes. The normalizing of the focal signals (and the original source signal) shown in fig. 2 (a) provide a clear demonstration of the enhanced temporal compression. This can be quantified by calculating the total energy near the focal time (50 samples centered at 1.6384ms, corresponding to the duration of the original source) and normalizing by the total energy in the signal. Performing this metric for these signals, we find that the original source has 100% of the energy in this 50 sample window, while the standard and \( \text{optimized} \) focused signals contain ~ 15% and 70% of the total energy respectively. Fig. 2 (b) shows further evidence in the lack of energy in the \( \text{optimized} \) signal (red) at all times away from the focal window, whereas the standard TR focal signal exhibits temporal side lobes symmetrically around the focal time. The second point to be noted from these figures can be seen in fig. 2 (b). The maximum achieved focal amplitude for the \( \text{optimized} \) process is approximately one half of that achieved by the standard TR process, a fact that we observe in all tests of this technique for all numbers of channels 1-8.

While figs. 1 and 2 effectively illustrate the \( \text{optimization} \) of temporal compression, albeit at the expense of the amplitude, they do not in any way address the spatial extent of the focus. To explore this, the same laser vibrometer was used in a scanning mode to measure the wavefield in a 5 \( \times \) 5cm\(^2\) region around the focus point (i.e., location \( B \)) with a resolution of 2mm. The images in fig. 3 show the wavefield measured at the time of focus (\( t = 1.6384 \text{ms} \)) for both the standard and \( \text{optimized} \) TR process. The standard TR process in fig. 3 (a) shows the typical focal spot of approximately \( \lambda/2 \) with a fringe, or ring, appearing at approximately 2\( \lambda \) from the focal point. Using the \( \text{optimized} \) signal \( g(t) \) from fig. 1, on the other hand, appears to have a smaller spatial extent, see fig. 3 (b), but most notably lacks any structure away from the central focus. Normalizing the two images and performing a simple subtraction (standard - \( \text{optimized} \)) verifies the more peaked nature of the \( \text{optimized} \) process.
Figure 3. Images of the spatial extent of the focus. (a) the standard TR focus, corresponding to the blue signals in figs. 1 and 2; (b) the optimized focus, corresponding to the red signals in figs. 1 and 2; and (c) difference of normalized images (a)-(b).

both the enhanced temporal and spatial compression can be quantified by examining the amount of energy in the focal region, defined as 10 µs (the full width of the source pulse) centered around 1.6384 ms, and the diffraction limit (λ/2) as a spatial definition. In both cases, spatial and temporal, the deconvolution method enhances the energy compression over the standard TR procedure.

Both the enhanced temporal and spatial compression can be quantified by examining the amount of energy in the focal region, defined as 10 µs (the full width of the source pulse) centered around 1.6384 ms, and the diffraction limit (λ/2) as a spatial definition. In both cases, spatial and temporal, the deconvolution method enhances the energy compression over the standard TR procedure. When performing these same tests for variable number of channels (1-8) the results are consistent showing the maximal compression is achieved at approximately 4-5 TRM elements (for either deconvolution or standard TR), after which time total amplitude increases linearly with number of channels (not shown) but further enhancement to focusing is minimal (fig. 4).

To accomplish this study as a function of TRM elements, the same experimental procedure as previously described was used for up to 8 individual transducers affixed at 8 unique locations on the aluminum sample. For 1 channel, the values in Fig. 4 result from an average of the temporal and spatial metrics across the results from the 8 individual channels. The results for N channels come from summing the data of N channels to produce the two metrics and then averaging these metric values across various combinations of N of the 8 channels. The results for 8 channels is a summation of all 8 individual data to produce one value each for the temporal and spatial metrics.

In conclusion, the main objective for the use of de-
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4 IMPROVED TEMPORAL FOCUSING LEADS TO IMPROVED SPATIAL FOCUSING

In this section, we will show that better temporal focusing implies better spatial focusing for the spherical average of the focus. We begin by first considering the wave field near its focal spot at \( r = 0 \) and consider the medium to be locally homogeneous in that region. The solution of the Helmholtz equation in a homogeneous medium can be written as

\[
p(r, \theta, \varphi, \omega) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm,j_l}(kr)Y_{lm}(\theta, \varphi), \tag{7}
\]

see table 8.2 of ref. Arfken and Weber (2001). In this expression \( j_l \) denotes the spherical Bessel function, \( Y_{lm} \) the spherical harmonics, and \( k = \omega/c \). According to expressions (11.147) and (11.148) of ref. Arfken and Weber (2001), \( j_0(0) = 0 \) for \( l \geq 1 \) and \( j_0(0) = 1 \). This means that at the focal point \( r = 0 \) only the terms \( l = m = 0 \) contribute. Using that \( Y_{00} = 1/\sqrt{4\pi} \) (table 12.3 of ref. Arfken and Weber (2001)), this means that at the focal point

\[
p(r = 0, \theta, \varphi, \omega) = \frac{a_{00}}{\sqrt{4\pi}}. \tag{8}
\]

The properties of the wave field at the focal point thus only depend on the coefficient \( a_{00} \). Since the \( l = m = 0 \) component of the spherical harmonics expansion gives the spherically symmetric component of the wave field, the properties of the wave field at the focal point can only bear a relation to the spherically symmetric component of the wave field. The properties of temporal focusing can thus only be related to the spherically symmetric component of the spatial focusing.

Because of this, we restrict ourselves to the spherically symmetric component \( (l = m = 0) \) of the wave field at the focal point. Using that \( j_0(kr) = \sin(kr)/kr \), the spherically symmetric component of expression (7) is given by

\[
p(r, \omega) = p_0 e^{-ikr} - e^{ikr}, \tag{9}
\]

with \( p_0 = -a_{00}/(2ik\sqrt{4\pi}) \). The coefficient \( p_0 \) depends on frequency. Using the Fourier convention \( f(t) = \int p_0(\omega)e^{-i\omega t}d\omega \), and using that \( k = \omega/c \), expression (9) corresponds, in the time domain, to

\[
p(r, t) = \frac{f(t + r/c) - f(r - r/c)}{r}. \tag{10}
\]

In this expression, \( f(t + r/c) \) denotes the wave that is incident on the focus, and \( f(t - r/c) \) the outgoing wave once it has passed through the focus. The field at the focus follows by Taylor expanding \( f(t \pm r/c) \) in \( r/c \) and taking the limit \( r \to 0 \), this gives

\[
p(r = 0, t) = \frac{2}{c} f'(t). \tag{11}
\]

In this expression and the following, the prime denotes the time derivative. Expression (11) states that the wave field at the focus is the time derivative of the incoming wave field.

Equation (11) gives the temporal properties of the focus. In order to get the spatial properties, we consider the wave field near the focal point at time \( t = 0 \). Setting \( t = 0 \) in expression (10), and making a third order Taylor expansion of \( f(\pm r/c) \) gives

\[
p(r, t = 0) = \frac{2}{c} f'(0) + \frac{r^2}{3c^3} f''''(0). \tag{12}
\]

Using equation (11) to eliminate \( f \), gives

\[
p(r, t = 0) = \frac{1}{6c^3} f''''(r = 0, t = 0)r^2. \tag{13}
\]

This is a parabolic approximation for the wave field near the focus, with coefficients related to the wave field and the focus and its second time derivative. The radius \( R \) of the focal spot can be estimated by setting the left hand side of equation (13) equal to zero, this gives

\[
R = \sqrt[3]{\frac{-p(r = 0, t = 0)}{p''''(r = 0, t = 0)}}. \tag{14}
\]

A good temporal focusing means that the temporal curvature of the wave field at the focal point is strong, this means that \( -p''''/p \) is large, and that the radius \( R \) is small. Good temporal focusing thus implies that the radius in the spherically symmetric part of the spatial focus is small.

5 CONCLUSION

Here we have introduced a simple method for determining the optimal signal to be used in a TR experiment in order to maximally compress the focus in both time and space. This method has been experimentally verified, albeit without fully varying all possible parameters. A more detailed study is currently underway to quantify the effects due to variations in available bandwidth,
and material properties (e.g., homogeneous vs. heterogeneous media, range of attenuations, wave speeds, etc.)

While temporal and spatial focusing have been enhanced with this method, there is a cost in the maximum achieved amplitude. This fact, coupled with the effect of the acausal portion of $g(t)$ may lead one to think simplistically about $g(t)$ as containing a signal to focus the energy at the appropriate time and place, and another signal that, simultaneously, actively cancels noise at all other times and locations. Thus the optimized focus is cleaner and more impulse-like, but not as strong, as some of the energy being transmitted is being exerted in suppressing sidelobes, spatially and temporally.

We have also shown, theoretically and numerically, that if one has good temporal focusing, the radius in the spherically symmetric part of the spatial focus is small. This concept can turn out to be useful in future research because it allows one to work on improving the temporal focusing in order to improve the spatial focusing.

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**REFERENCES**


