Spectral element modeling of fault-plane reflections and their sensitivity to stacking and migration errors

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ABSTRACT

Simulating the 2D elastic wave equation via the spectral element method (SEM) has advantages over other modeling techniques for studying seismic reflectivity associated with faults. For instance, irregular geometries can be easily accommodated and slip boundary conditions are naturally implemented. We run through a complete modeling exercise incorporating both SEM forward modeling of shot gathers over a realistically-sized numerical model containing a normal fault and processing of the simulated data to reconstruct post-stack time-migrated images of the kind that are routinely interpreted. To gain insight into the pitfalls involved in interpreting, for instance, amplitudes on post-stack time-migrated data, we develop a simple theory for gauging the amplitude and phase errors resulting from stacking and migration velocity errors. We utilize our modeling capabilities to test the theoretical results pertaining to stacking velocity errors. We find that the character of fault-plane reflections are relatively more robust to stacking and migration errors than the reflections from flat-lying layers and conclude that, in order to gauge stacking and migration errors, information on the acquisition geometry is critical. Though the robust nature of the fault-plane reflections we model may be a special case for post-stack migration, the claim that fault-plane reflections are less sensitive to stacking errors should hold more generally for pre-stack migration.

Key words: weak formulation, stack response, fault reflectivity

INTRODUCTION

Seismic data acquisition and processing have evolved to the point that fault-plane reflections are often imaged under favorable conditions, such as above-salt in the Gulf of Mexico (Liner, 1999). Reflections originating from fault-zones may hold important information about fluid movement along faults or the capacity of a fault to act as a seal (Haney et al., 2004a). Faults have long stumped interpreters by virtue of their split-personality as effective hydrocarbon traps and pathways for hydrocarbons to move from deep kitchens into shallower, economically producible reservoirs. Any light that seismic data shed on the situation would be useful.

To gain a stronger grasp on the factors at play in causing fault-plane reflectivity and errors that can deteriorate their imaging, we have pursued a complete numerical study of wave interaction with fault models. By complete, we mean that we do not simply model the entire measured (elastic) wavefield with high fidelity, but process the data back into its time-migrated image, which is at what point many geoscientists in the oil industry gain access to and begin examining seismic data. We model the wavefield with an implementation of the spectral element method (SEM) written by Dimitri Komatitsch and Jean-Pierre Vilotte at the Institut de Physique du Globe in Paris, France. Further improvements have been made to original code by the third author of this paper in the course of his graduate work (Ampuero, 2002). Processing of the wavefield output by the SEM code has been accomplished in Seismic Unix (Stockwell, 1997).
This paper is divided into two main parts. In the first section, we briefly describe the theory behind SEM. We devote the rest of the paper to modeling the response of post-stack time-migrated waveforms to stacking and migration errors using SEM to generate synthetic seismograms over a simple normal fault model. An array-based theory, similar to what has recently been presented by Gausiland (2004), is presented to quantify the observed phase-shifts and degraded amplitudes in the mis-stacked data. We first apply our theoretical results to reflections from flat-lying layers and then examine the changes in fault-plane reflectivity due to incorrect stacking velocities. We observe that the shape of the fault-plane reflections are relatively more robust in the presence of stacking velocity errors than the associated layer reflections.

THE SPECTRAL ELEMENT METHOD

Numerical modeling of wave propagation in the earth can be based on the weak or strong forms of the elastodynamic equations of motion. The spectral element method (SEM) combines favorable aspects of both approaches. For instance, since SEM is rooted in the weak form, it naturally handles general geometries and exotic boundary conditions. In the finite-difference method, it is notoriously difficult to implement, for instance, a linear-slip boundary condition (Coates & Schoenberg, 1995). On the other hand, SEM does not require the inversion of a large matrix, a property long identified with finite-difference (strong form) methods. Formally, this last property of SEM means that its mass matrix is diagonal and its computational cost is relatively small. Note that this is not the same as mass-lumping, which has been used to diagonalize finite-element schemes. Haney and Snieder (2003) concluded their paper on finite-element modeling by asking whether a technique, based on the weak form, could avoid matrix inversion to increase computational efficiency; SEM is just that technique. SEM has the additional property of spectral convergence, meaning that, as the polynomial order of the basis-functions is increased, the numerical error goes down exponentially.

The term "spectral element" indicates that SEM is a mixture of finite-element and spectral methods (Komatitsch & Vilotte, 1998). As a result, there are two parameters relevant to the mesh in SEM: the size of the elements and polynomial degree \( n - 1 \), where \( n \) is the number of zero crossings) of the basis functions used within each element. Komatitsch and Tromp (2003) refer to these parameters when they speak of the global mesh and the local mesh. Concerning the local mesh, there is a known trade-off between accuracy and numerical cost (Seriani & Priolo, 1994) which suggests that polynomial degrees between 5 and 10 be used within the elements. For the numerical examples in this paper, we use a polynomial degree of eight. We made this choice based on the optimal criterion for polynomial degree and the fact that SEM typically works with Lagrange polynomials. The zero-crossings of Lagrange polynomials are irregularly spaced over the interval \([-1,1]\) (which is mapped onto the sides of the elemental mesh). The exception to the irregular spacing occurs for Lagrange polynomials of even degree when a zero-crossing always occurs at 0 on the interval \([-1,1]\). Since, in our current implementation of SEM, sources and receivers can only exist at the zero crossings (they are the control points in a Gauss-Lobatto-LeGendre integration in SEM), an extra equally-spaced point exists for an even polynomial degree.

MODELING OF A FAULT

As an example of the ability of SEM to model seismic scale structures, we present a complete modeling/processing sequence for a simple fault model. The
SEM forward modeling has been run in serial (one node for each shot) on a 32 processor pentium IV xeon (3.0 GHz) cluster. All of the processing has been performed on a workstation using the Seismic Unix package (Stockwell, 1997). Fig. 1 depicts the geometry of the model and Table 1 shows the material properties of the various layers. The normal fault we model has a vertical throw of 20 m, a typical value for a small growth fault in the Gulf of Mexico (Rowan et al., 1998). The model we present here, called Model A, has been introduced by Townsend et al. (1998) to study changes in seismic attributes due to faults disrupting the lateral continuity of events.

We mesh the interior of the computational grid shown in Fig. 1 using a freely available mesh program developed by INRIA called EMC2. The program can be downloaded at:

http://www-rocoq.inria.fr/gamma/cdrom/ww/emc2/eng.htm

For this example and others in this paper, we used a semi-structured mesh since the geometries are not overly complex. Though the mesh has some structure, it honors the sharp edges at the fault. After initial construction of the mesh, the quadrangle elements are regularized so that their shapes mimic squares as closely as possible.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness (m)</th>
<th>( V_P ) (m/s)</th>
<th>( \rho ) (kg/m(^3))</th>
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<tr>
<td>7</td>
<td>850</td>
<td>2800</td>
<td>2250</td>
</tr>
</tbody>
</table>

Table 1. Model A for our SEM examples. \( \sqrt{3} \)

We show a typical shot record in the left panel of Fig. 2. Since the SEM code is elastic, both primary and converted waves show up on the vertical component of the displacement seismograms. It should be noted that, if we had introduced a physically realistic vertical velocity gradient in the uppermost layer, ray bending would have served to separate the \( P \)- and \( S \)-waves more effectively into the vertical and horizontal components. We relied on model-specific muting of the converted waves in order to proceed with conventional \( P \)-wave time-processing. We first subtracted the direct waves by running a homogeneous subsurface simulation with the elastic properties of layer 1 (see Fig. 1 and Table 1). After this step, we performed the geometrical spreading correction, NMO, constant-velocity DMO, and stacked to simulate zero-offset data. The stacked section is shown in the middle panel of Fig. 2. Note the significant diffracted energy coming from the sharp cor-

Figure 2. Three stages of data processing. On the top is a shot record from the SEM code after subtracting off the direct waves, leaving the purely up-going wavefield. The middle panel depicts the data in the midpoint domain after geometric-spreading, NMO, DMO, and stack. Note the diffractions from the bed terminations at the fault plane. Finally, the migrated image is shown on the bottom, with the fault plane clearly illuminated. The migration, though constant velocity, was depth converted.
ners at the fault. With the simulated zero-offset section, we performed constant velocity migration using the velocity of the overburden (layer 1). Hence, a source of error in this simulation came from the slight undermigration of the deepest reflectors. We chose to migrate with constant velocity to truly represent a time-migrated section that has nonetheless been depth converted in the right panel of Fig. 2. With our full suite of forward modeling and processing capabilities, we decided to apply SEM to a study of stacking and velocity errors; however, before presenting the results of the modeling with stacking velocity errors, we develop an array-theory based approach for analyzing the impact of erroneous stacking velocities on the migrated waveforms. This analysis is necessary to be able to say anything about the errors inherent in interpreting a post-stack time-migrated image as is done in Haney et al. (2004a).

THE STACK RESPONSE

An understanding of the filtering action of stacking is necessary to gauge the reliability of amplitudes and phases in a post-stack time-migrated image. Recently, the amplitudes of post-stack time-migrated fault-plane reflections have been shown to qualitatively correspond to a section of a growth fault in the Gulf of Mexico that seals (Haney et al., 2004a). Furthermore, Haney et al. (2004b) demonstrate that a localized strongly reflecting section of a growth fault in the same minibasin (South Eugene Island Block 330) coincides with a proposed fluid pulse caught in the act of ascending the fault (Losh et al., 1999; Revil & Cathles, 2002). The most significant source of error in the amplitudes and phases of these post-stack time-migrated images, besides the neglect of ray-bending and anisotropy, probably arises due to miss-stacking. Errors in the migration velocities should mainly result in mis-positioning of the reflections, a problem ameliorated by the use of a dip-filter that locally searches for the maximum coherence direction and manual picking of the fault-plane reflections, whether these lie at the correct spatial position (bed terminations) or not. In this section, we investigate the errors due to incorrect stacking velocities. The same array-theory based methods we use to study the amplitude degradation and phase shifts due to mis-stacking are applied in a later section of this paper to errors in migration velocities.

As discussed by Gausland (2004), the result of stacking is not limited to the often quoted factor of $1/\sqrt{n}$ reduction in noise. Stacking is a powerful frequency and wavenumber filter. To illustrate the stack response, suppose that in a CMP gather there is a single event with a zero-offset waveform $f(t)$ at zero-offset-two-way-time $T_0$ which has hyperbolic moveout with true NMO-velocity $v_T$ and a wavelet that does not change with offset. The event is stacked with a hyperbola at $T_0$ with a stacking velocity $v_{st}$ not necessarily equal to $v_T$. When $v_{st}$ does not equal $v_T$, a time shift $\Delta t_k$ occurs at the $k$-th offset trace before stacking. Hence, the normalized stacking response, neglecting NMO-stretch, is

$$g(t) = \frac{1}{2n+1} \left[ f(t - \Delta t_{-n}) + \cdots + f(t - \Delta t_{-1}) + f(t - \Delta t_1) + \cdots + f(t - \Delta t_n) \right],$$

$$\Delta t_k = \sqrt{T_0^2 + \frac{k^2 h^2}{v_T^2}} - \sqrt{T_0^2 + \frac{k^2 h^2}{v_{st}^2}},$$

where $h$ is the offset spacing (assumed to be regular) and the subscript $k$ represents the $k$-th offset trace. The case of linear moveout ($T_0 = 0$) is described by Lu (1993) in similar terms. Denoting the Fourier transforms of $f(t)$ and $g(t)$ as $F(\omega)$ and $G(\omega)$, the transform of equation (1) may be written

$$G(\omega) = F(\omega)K(\omega),$$

where the transfer function, $K$, is

$$K(\omega) = \frac{1}{2n+1} \sum_{k=-n}^{n} e^{i\omega \Delta t_k}.$$

At this point, since we are in the frequency domain, NMO-stretch could easily be included as an amplitude factor multiplying the exponentials in the series (Dunkin & Levin, 1973; Yilmaz, 1987); however, in the interest of simplicity, we do not account for it. For the particular case of linear moveout, the series in equation (4) is a geometric series and can be evaluated exactly. The series is geometric because the respective time delays are regularly spaced ($\Delta t_k = k\Delta t_1$). When the moveout is nonlinear, for instance hyperbolic, the time delays are not regularly spaced and the series in equation (4) cannot be evaluated exactly.

We proceed by transforming equation (4) into an integral and evaluating it by the method of stationary phase (Born & Wolf, 1980; Bleistein et al., 2001). The approach for moving from the finite series of equation (4) to a Fresnel integral is identical to that in Snieder (2004). First, note that the fold of the CMP-gather, $2n+1$, is related to the spreadlength, $L_s$, and the offset-spacing, $h$

$$2n+1 = \frac{L_s + h}{h}.$$

Using the definition from equation (5), the finite series of equation (4) may be rewritten as

$$K(\omega) = \frac{1}{L_s + h} \sum_{k=-n}^{n} e^{i\omega \Delta t_k} h.$$

The finite series in equation (6) looks like a discretized

\[ \text{equation} \]

\[ \text{equation} \]
integral over offset. Formally taking the limit of continuous sources/receivers, \( n \to \infty \) and \( h \to 0 \), and allowing the discrete variable \( k \) to become the continuous variable \( x \) results in

\[
K(\omega) = \frac{1}{L_s} \times \int_{-L_s/2}^{L_s/2} \exp \left\{ i \omega \left( \sqrt{T_0^2 + \frac{x^2}{v_p^2}} - \sqrt{T_0^2 + \frac{x^2}{v_t^2}} \right) \right\} dx. \tag{7}
\]

We simplify the evaluation of the integral in equation (7) by letting the slowness go to infinity, \( L_s \to \infty \). This simplification avoids accounting for Cornu’s spiral in the Fresnel integral (Born & Wolf, 1980). Denoting \( I(\omega) = K(\omega)L_s \) as a scaled version of the transfer function gives

\[
I(\omega) = \int_{-\infty}^{\infty} \exp \left\{ i \omega \left( \sqrt{T_0^2 + \frac{x^2}{v_p^2}} - \sqrt{T_0^2 + \frac{x^2}{v_t^2}} \right) \right\} dx. \tag{8}
\]

This type of integral can be approximately evaluated by the method of stationary phase (Born & Wolf, 1980). In this limit, \( I(\omega) \) is given by

\[
I(\omega) = \sqrt{2\pi}e^{-\pi/4} \left[ \frac{\partial^2 \phi}{\partial x^2} \right]^{-1/2}_{x_{st}} e^{i\phi(x_{st})}, \tag{9}
\]

where \( x_{st} \) is the stationary point of the phase function \( \phi(x) \) of equation (8) that is given by

\[
\phi(x) = \omega \left( \sqrt{\frac{T_0^2}{v_t^2} + \frac{x^2}{v_p^2}} - \sqrt{\frac{T_0^2}{v_t^2} + \frac{x^2}{v_t^2}} \right). \tag{10}
\]

In equation (9), the subscript \( x_{st} \) indicates the quantity is to be evaluated at the stationary point. To find the stationary point, we set the \( x \)-derivative of the phase function to zero

\[
\frac{\partial \phi}{\partial x} = \omega \left[ \frac{1}{v_t^2 \sqrt{\frac{T_0^2}{v_t^2} + \frac{x^2}{v_p^2}}} - \frac{1}{v_t^2 \sqrt{T_0^2 + \frac{x^2}{v_t^2}}} \right] = 0. \tag{11}
\]

From equation (11), we identify the stationary point \( x_{st} = 0 \).

After calculating the second \( x \)-derivative of the phase function and evaluating it at the stationary point, the scaled transfer function \( I(\omega) \) can be expressed, in the stationary phase approximation, as

\[
I(\omega) = \sqrt{\frac{2\pi T_0}{|\omega|}} \frac{\exp(i \text{sgn}(v_{st} - v_t) \pi/4)}{|(v_{st}^2 - v_t^2)^{-1/2}|}. \tag{12}
\]

Equation (12) states that, when an event is stacked with a velocity that is not the true velocity, a phase shift of \( \pm 45^\circ \) results. The phase shift is positive or negative depending on whether the stacking velocity is higher or lower than the true velocity. The amplitude of \( I(\omega) \) scales with \( \sqrt{T_0} \) and as a quasi-hyperbola as a function of stacking velocity, \(|(v_{st}^2 - v_t^2)^{-1/2}|^2 \). The amplitude response is also inversely proportional to \( \sqrt{\omega} \). Hence, stacking errors cause the stacked waveform to be lower in frequency than it should be - an effect identical to stacking NMO-stretched waveforms. Note that the factor of \( 1/\sqrt{\omega} \) does not apply at \( v_{st} = v_t \). From our construction of the series in equation (4), the phase delays \( \Delta \) are all zero for \( v_{st} = v_t \) and stacking does not change the frequency content of the waveform. We venture that, for finite frequencies, the frequency filtering aspect of mis-stacking vanishes gradually as \( v_{st} \to v_t \).

As a result of the \( \pm 45^\circ \) phase shifts introduced by stacking errors, incorrectly stacked events acquire a time advance or delay (Gausland, 2004). Such a property suggests a complimentary method of performing velocity analysis. Traditional semblance-based velocity analysis exploits the fact that the amplitude of the stack maximizes at the true velocity. This is evident from equation (12) since the amplitude of \( I(\omega) \rightarrow \infty \) when \( v_{st} = v_t \). Alternately, the time advance/delay caused by the phase shift vanishes for the correct stacking velocity. Cross-correlation of waveforms with different stacking velocities could provide the time-lag as a function of stacking velocity. Instead of picking “bullseyes” on the semblance contour plot, zero-crossings would be picked on a time-advance/delay plot.

<table>
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<th>survey parameter</th>
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<td>erroneous stacking velocity, ( v_{st} )</td>
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</tr>
<tr>
<td>dominant frequency</td>
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</tr>
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Table II. Parameters for studying stacking errors

**NUMERICAL TEST OF MIS-STACKING**

We illustrate the impact of stacking velocity errors on a post-stack migrated image with a numerical example, using the SEM code. Our example does not satisfy the approximations we made above to obtain analytic results for the stack response - there is finite spread length, discrete receivers, and finite frequency. To evaluate the stack response, we numerically calculate the exact transfer function \( K(\omega) \) - the finite series appearing in equation (4). Table II summarizes the parameters from our numerical survey that are necessary to evaluate the exact transfer function. Note that our offset spacing is two times the shot spacing (see Fig. 1) and that the velocity of layer 1 serves as the true NMO velocity. We generate two tests for the stacking velocity - one with the true NMO velocity (2000 m/s) and another with a -10% error (1800 m/s). The material properties for the model
investigated here are slightly different than those used earlier in this paper and are shown in Table III. The geometry of this model, Model B, is the same in Fig. 1.

<table>
<thead>
<tr>
<th>Layer</th>
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<th>$V_p$ (m/s)</th>
<th>$\rho$ (kg/m$^3$)</th>
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<tr>
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<td>2000</td>
<td>2000</td>
</tr>
</tbody>
</table>

Table III. Model B for our SEG examples. $V_S = V_P/\sqrt{3}$

We first simulated the stack response exactly from equation (4). Both the amplitude and phase of the stacked waveform are displayed in Fig. 3 as a function of stacking velocity. When $v_{st} = v_{tr}$, the stacked amplitude is maximized and the phase shift is zero. Away from the true velocity, though, significant amplitude degradation and phase error occur. We highlight in Fig. 3 the amplitude and phase we expect at an erroneous value of $v_{st} = 1800$ m/s with circles. For such a -10% error in stacking velocity, the stacked amplitude of a flat reflector should be reduced by a factor of $\sim 0.6$
relative to the amplitude at the correct stacking velocity and have acquired a phase shift of $\sim 60^\circ$. For our survey parameters, it is evident that we are far from the geometrical optics limit, when the phase shifts away from the true stacking velocity should be $\pm 45^\circ$. This demonstrates that, away from the geometrical optics limit, the error due to stacking velocity errors are mostly determined by the survey and acquisition geometry.

To test the array-based $f$-$x$ domain theory for stacking errors, we ran through the complete sequence of SEG and processing with Model B and formed migrated images both with and without stacking velocity errors. Note that a stacking velocity error implies that both the NMO and constant-velocity DMO steps had erroneous (-10%) velocities. The migration velocity for the following examples is kept as the correct migration velocity. Two migrated images are displayed in Fig. 4. In comparing the migration without stacking errors to the migration with, the degradation of the image with errors is noticeable.

To quantify the error in the images, we first took a slice from the migrated image shown by the white dashed lines in Fig. 4. This slice of the migrated image is shown as a solid line in Fig. 5. Also plotted in Fig. 5 is the convolutional model obtained from our far-field wavelet and the known reflectivity series. Good agreement exists between the zero-offset migration and the convolutional model, as there should be.

In the top panel of Fig. 6, we plot the same convolutional model as in Fig. 5, but now show the slice of the migration with incorrect stacking velocity in the solid line. As discussed before, amplitude degradation and phase shift is evident between the convolutional model and the migration with erroneous stacking velocity. As a further check if the array-based theory for stacking errors accurately describes the migrated waveform in the top panel of Fig. 6, we multiplied the far-field wavelet by 0.6, phase-shifted it by $60^\circ$, and recomputed the convolutional model. These factors come from the circles at -10% error in Fig. 3 for the flat reflector. The damped and shifted version of the convolutional model is plotted in the bottom panel of Fig. 6 together with the migration with the erroneous stacking velocities. Indeed, by virtue of the excellent match of the two curves in the bottom plot of Fig. 6, incorrect stacking velocities have caused the amplitude and phase response of the migrated waveform to be as that predicted by the exact array-based theory (not in the geometrical optics limit).

We now turn our attention to the impact of the stacking velocity errors on the fault-plane reflections instead of the flat layers. Shown in Fig. 7 are dip-filtered versions of the migrations both with and without the errors in stacking velocity. We dip-filter the images in order to isolate the reflectivity from the fault plane, as in Haney et al. (2004a). The main difference here is that the dip-filter is in the $f$-$k$ domain and not the $t$-$x$ do-

\[ \text{Figure 5. An overlap of the correct stacking velocity migration (solid) and the convolutional model (dashed) along the white, dashed line of Fig. 4.} \]

\[ \text{Figure 6. On the top: an overlap of the incorrect stacking velocity migration (solid) and the correct convolutional model (dashed) along the white, dashed line of Fig. 4. Note the disparity in the amplitudes and phases of the two plots. On the bottom: an overlap of the incorrect stacking velocity migration (solid) and the incorrect convolutional model (dashed) calculated by dampening and phase shifting the source wavelet by the factors predicted in Fig. 3.} \]
main. Note that, due to the rotation of dipping reflectors after migration, the correct direction to slice the fault-plane reflections is perpendicular to the fault, as indicated by the white arrow.

In Fig. 8 we show the slices taken from the dip-filtered fault-plane reflections for both cases of stacking velocity error and no error and compare them to the same far-field wavelet. Compared to the flat layer reflections examined previously, the fault-plane reflection is relatively insensitive to a stacking velocity error of -10%, as predicted by considering their stack response in Fig. 3. This phenomenon can be understood since, when not in the geometrical optics limit, the fault-plane reflection has a faster moveout and is therefore less sensitive to small errors in the stacking velocity than a flat reflector with slower moveout. In geometrical optics, both flat reflectors and fault-plane reflections would have identical errors since the erroneous moveout curves would only be tangent to the true moveout curve at a point. With finite-frequencies, the faster moveout of the fault-plane reflections means that an erroneous moveout curve still intersects a larger swath of the curve, near its stationary point, than for a flat reflector. Fault-plane reflections may be more difficult to observe in non-ideal
acquisition geometries (strike-line instead of dip-line), but once their reflection has been captured, their reconstructed finite-frequency waveform is relatively robust in shape and amplitude. Particularly if velocity analysis is performed on the reflections from flat layers (since there are more of these than fault-plane reflections), the associated error in the velocity picks is reduced when considering the reflections from the fault-plane.

THE MIGRATION RESPONSE

We have already touched on the filtering action of migration in our discussion on the stack response, specifically with equation (12). The connection with the migration response exists for $t_{tr} \to \infty$. When an event has a true stacking velocity that is infinite and it is stacked in the offset domain with a hyperbolic trajectory described by a finite velocity, an analogy exists with diffraction summation of a flat reflector in the midpoint domain. To pursue this further, in the case of $v_{tr} \to \infty$, equation (12) becomes

$$I(\omega) = v_{st} \sqrt{\frac{2\pi T_0}{\omega}} \exp(-i\pi/4).$$

(13)

Two of the three corrections made to simple diffraction summation for Kirchhoff migration are shown in equation (13). The factor $v_{st} \sqrt{T_0}$ in the transfer function requires multiplying the result of diffraction summation by $1/v_{st} \sqrt{T_0}$ to recover the true waveform. Yilmaz (1987) calls this the 2D geometrical spreading factor. Furthermore, to recover the true waveform after diffraction summation requires multiplying by $\exp(i\pi/4)\sqrt{\omega}$ (a half-derivative). This correction is called the wavelet shaping factor (Yilmaz, 1987). The obliquity factor does not appear in equation (13) - this is a result of the reflector being flat for infinite velocity.

In the rest of this section, we briefly generalize the above connection with migration for the case of a dipping reflector and recover the obliquity factor, which was not present for a flat reflector. Moreover, we are able to confirm the well-known fact that the amplitudes and phases of post-stack time-migrated waveforms from planar features are virtually insensitive to migration velocity errors. The same cannot be said for diffractions, which can contain errors due to incorrect migration velocities just as stacked waveforms contained errors due to mis-stacking.

We want to study the phase and amplitude distortion of a waveform at $x_{in}, t_{in}$ (in the zero-offset section) after it is migrated to an output point $x_{out}, t_{out}$. Since we are primarily interested in fault-plane reflections, we assume a single dipping event in the zero-offset section with a time dip

$$p = 2 \sin \theta / v_{tr},$$

(14)

where $\theta$ is the dip angle and $v_{tr}$ is the true migration velocity. When the zero-offset section is migrated with a velocity $v_m$ not necessarily equal to $v_{tr}$, the hyperbola that is tangent to the reflector at $x_{in}, t_{in}$ is described by

$$t_{hyp}(x) = \sqrt{t_{in}^2 (1 - p^2 v_m^2/4) + \frac{4(x - x_{in} + pv_m^2 t_{in}/4)}{v_m^2}},$$

(15)

where the output points are

$$t_{out} = t_{in} \sqrt{1 - p^2 v_m^2 /4},$$

$$x_{out} = x_{in} - pv_m^2 t_{in}/4.$$

The equation describing the dipping reflector in the zero-offset section is

$$t_{ref}(x) = px + t_{in} - p x_{in}.$$

(18)

From equations (15) through (18), the diffraction summation transfer function can be written

$$K(\omega) = \frac{1}{2n+1} \sum_{h=-n}^{n} e^{i\Delta t_h},$$

(19)

where now $2n+1$ is the number of midpoints in the migration aperture and $\Delta t_h$ are the time shifts given by

$$\Delta t_h = t_{ref}(kh) - t_{hyp}(kh),$$

(20)

with $h$ representing the spacing between midpoints.

Moving from the finite series to an integral expression as we did earlier for stacking, the diffraction summation response is given, for the case of infinite aperture and continuous midpoints, by

$$I(\omega) = \int_{-\infty}^{\infty} \exp[i\omega(t_{ref}(x) - t_{hyp}(x))] dx,$$

(21)

where $t_{ref}(x)$ and $t_{hyp}(x)$ are defined above. We note that, just as we ignored NMO-stretch for the stack response, we neglect the analogous stretch due to migration. The stretch caused by migration is manifested through rotation and steepening of a reflector in the migrated image. Its origins are the same as NMO-stretch.

Applying the method of stationary phase to the integral in equation (21) yields a stationary point at $x_{in}$. The resulting expression for the scaled transfer function $I(\omega)$ in the infinite frequency limit is

$$I(\omega) = \frac{\sqrt{2\pi} \exp(-i\pi/4)}{2} \frac{t_{in}}{t_{out}} \frac{v_m \sqrt{t_{in}}}{\sqrt{\omega}}.$$
the migrated waveform to departures of the migration velocity \( v_m \) from the true velocity \( v_t \).

Comparing the stacking response, equation (12), and the diffraction stack response, equation (22), it is evident that the diffraction stack for a reflector is essentially one side of the stacking response. Since, as witnessed in the numerical modeling of fault-plane reflections earlier, both diffractions and dipping reflections occur at a fault-plane, caution should be taken in interpreting post-stack time-migrated amplitudes originating from a fault plane if it is unclear whether they are diffractions or reflections. Access of the interpreter to an unmigrated section, showing the fault-related diffractions, could help to diagnose areas of less certainty.

**DISCUSSION**

From our numerical example, we have found that the reflected waveform from the fault-plane is seemingly less sensitive to stacking velocity errors than the reflections from the layers. This certainly holds true for our model and, from the array-based theory for stacking errors, it seems that it should hold more generally. However, we have ignored the impact of muting on the stack response. Since muting selects a larger offset range with increasing time and fault-plane reflections have a larger two-way reflection-time, fault-plane reflections should have a larger pre-stack offset range than a layer reflection at the same depth. This did not affect our model since the muting happened to be chosen in such a way that no new offsets arose in the CMP-gathers from the time of the layer reflection until the fault-plane reflection. Introducing more offsets into fault-plane reflections should make them more sensitive to stacking errors. Hence the competing effects of higher moveout velocity and larger offset range should cancel themselves out. The same may not be true in general for pre-stack migration, where the relevant time is the migration time (same for flat or dipping reflectors) instead of the two-way reflection time.

**CONCLUSION**

We have performed a complete numerical modeling experiment by utilizing an SEM implementation of the 2D elastic wave equation and processing the resulting waveforms into their time-migrated image. Since non-zero offset waveforms are available in the simulations, we tested a simple array-based theory of the stack response. Future simulations will study the differences in fault-plane reflections due to across-fault juxtaposition versus fault-plane reflections due to the fault acting as a lateral seal (Haney et al., 2004a). Random noise will also be introduced into the pre-stack data to gauge its effect on the imaging. We will also look to extend the array-based theory for post-stack migration, described in this paper, to study the sensitivities of DMO and pre-stack migration to velocity errors.

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APPENDIX A: MODELING OF A LINEAR-SLIP INTERFACE

We chose SEM to simulate fault reflectivity for its ability to allow a free-form mesh and in order to include the possibility of slip at interfaces in our numerical models. As evidence of SEM’s ability to handle challenging boundary conditions, it has recently been applied to wave propagation near a fluid-solid interface (Komatitsch et al., 2000). However, interfacial slip had not yet, to our knowledge, been implemented in SEM. In fact, Komatitsch and Tromp (2003) claimed in the description of their SEM code that “at every internal boundary, both the displacement and the traction need to be continuous”, in clear contradiction to slip.

For a normally incident P-wave, the linear-slip boundary condition can be expressed as (Schoenberg, 1980)

\[ u_i^+ - u_i^- = \eta_N \sigma_{zz} \]  
\[ \sigma_{zz}^+ = \sigma_{zz}^- \]

(A1)  \hspace{1cm} (A2)

where the superscript (-) refers to the side of the interface on which the wave is incident, (+) the other side of the interface, \( u_i \) is the displacement in the direction of propagation, and \( \sigma_{zz} \) is the normal stress. The parameter \( \eta_N \) is called the normal compliance and quantifies the degree of slip along the interface. For \( \eta_N = 0 \), the interface is welded, and for \( \eta_N = \infty \), it is a free surface. The boundary condition described by equations (A1) and (A2) can be obtained in the limit of a thin, weak layer in welded contact with its surrounding rock. Linear-slip has been suggested as a good model for scattering from faults and fractures (Coates & Schoenberg, 1995). With this in mind, it is important not to confuse the slip model in equations (A1) and (A2) with slip that occurs along a fault during an earthquake. The linear-slip model entails some slipping at the interface that is the order of particle displacements during the passage of a seismic wave (\( \sim 10^{-6} \) m). Active, earthquake-generating faults typically slip on a length scale 3 to 4 orders of magnitude larger (\( \sim 10^{-3} \) - \( 10^{-2} \) m). Earthquake slip is also hysteretic, whereas interfaces undergoing linear-slip return to their equilibrium state after the seismic wave has moved on.

To implement the linear-slip model in SEM, the weak form of the equation of motion is needed

\[ \int \int \rho \ddot{u} \ddot{\phi} + \int \int \nabla \phi : c : \nabla u - \tau \phi = 0, \]

(A3)

where \( u \) is the displacement, \( \rho \) is the density, \( \tau \) is the traction on the boundary of the computational domain, \( c \) is the elastic stiffness tensor, and \( \phi \) is the test function. The semi-colons in equation (A3) represent tensor multiplication. After discretizing the displacement, equation (A3) can be written as a matrix equation

\[ M \ddot{u} = -Ku + B \tau, \]

(A4)

with \( M \) and \( K \) the mass and stiffness matrices, respectively. The last term is non-zero only on the part of the boundary where slip occurs; this is described by the matrix \( B \). The essence of this implementation is that two separate meshes on either side of the slip discontinuity (let’s call them mesh 1 and mesh 2) are put into communication via the last term in equation (A4). To get linear-slip between the two meshes along a direction normal to their contact, we substitute equation (A1) for the traction into equation (A4) for meshes 1 and 2 to get two matrix equations

\[ M_1 \ddot{u}_1 = -K_1 u_1 + \eta_N^1 B_1 (u_1^+ - u_1^-) \]
\[ M_2 \ddot{u}_2 = -K_2 u_2 - \eta_N^1 B_2 (u_1^+ - u_2^-), \]

(A5)

where the asymmetry of the ±-signs between the two last terms is in accordance with Newton’s third law and the superscript \( \pm \) means the normal component of the displacement. In the formulation we have outlined here, the slip law, equation (A1), enters into the equation of motion by a substitution of the slip for the traction at the fault. A weak formulation of the first-order velocity-stress equations, instead of the second-order wave equation, results in the opposite substitution: the slip emerges from the equations by applying the weak form, and substitution for the traction is necessary to impose the slip (Haney & Snieder, 2003).

The SEM implementation of equation (A5) utilizes an explicit Newmark scheme whose algorithm consists of a predictor, a solver, and a corrector:
for \( n=1:N \)

**PREDICTOR**
\[
\begin{align*}
\dot{u}_1(n+1) &= u_1(n) + \Delta t \cdot v_1(n) + \frac{1}{2} \Delta t^2 \ddot{u}_1(n) \\
\dot{u}_2(n+1) &= u_2(n) + \Delta t \cdot v_2(n) + \frac{1}{2} \Delta t^2 \ddot{u}_2(n)
\end{align*}
\]

**SOLVER**
solve both equations in (A5) for \( \ddot{u} \) using updates at 
\( n+1 \) not difficult since \( M \) is diagonal

**CORRECTOR**
\[
\begin{align*}
\dot{v}_1(n+1) &= \dot{u}_1(n+1) + \frac{1}{2} \Delta t \ddot{u}_1(n+1) \\
\dot{v}_2(n+1) &= \dot{u}_2(n+1) + \frac{1}{2} \Delta t \ddot{u}_2(n+1)
\end{align*}
\]

end

where, in the above algorithm, \( n \) stands for the previous 
time step, \( \ddot{u} \) is the acceleration, \( \Delta t \) is the time step 
increment, and \( v \) is the velocity.

Waveforms computed with this implementation are 
displayed in Fig. A1. Our model is an elongated 2D 
block, whose side boundaries, shown as dashed in 
Fig. A1, are periodic and whose upper and lower 
boundaries are absorbing. Consistent with the periodic 
boundary condition, we excite a unidirectional plane \( P \)-
wave from the base of the block. In front of the plane wave, 
we measure the wavefield with ten receivers. A slip interface 
cuts through the center of the block, between receivers 5 
and 6, which is characterized by a normal compliance of 
\( 2.2 \times 10^{-6} \) m/Pa. The slip interface has also a shear 
compliance, but the \( P \)-wave is incident normally and excites 
no shear. The media on either side of the slip interface 
are identical with density, \( P \)-velocity, and \( S \)-velocity of 
\( 2300 \) kg/m\(^3\), \( 2000 \) m/s, and \( 1000 \) m/s, respectively. The 
plane wave source waveform is a Ricker wavelet with a 
dominant frequency of 10 Hz.

We plot the resulting wavefield in the left portion 
of Figure A1. For the value of slip used in this example, 
the slip interface virtually acts as a stress-free 
surface, almost totally reflecting the incident wave. As 
a result, only a small fraction of the energy transmits 
through the interface. From the analytic expressions for 
the frequency-dependent reflection and transmission 
coefficients at such a slip interface (Chaisri & Krebes, 
2000), we find \( R_{P,P} = 0.95 \) and \( T_{P,P} = 0.30 \) for an 
incident frequency of 10 Hz. These values are qualitatively 
in agreement with the displayed waveforms in 
Figure A1. A more rigorous benchmarking will quantitatively 
verify our technique for inserting slip interfaces into SEM.

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**Figure A1.** Numerical simulation of a normally incident \( P \)-
wave scattering from a linear-slip interface. On the top is the 
model, showing ten numbered receiver locations and a plane 
wave incident from the upper end of the model. The linear-
slip interface is between the receivers 5 and 6. On the bottom 
are seismograms taken at each of the receivers. At \( t = 6 \) s, 
the incident wave reflects from the linear-slip interface. The 
media above and below the linear-slip interface are identical 
and, therefore, no reflection would occur had there been no slip.