Anisotropic velocity analysis of P-wave reflection and borehole data

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by

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Efficient application of the modern imaging technology requires development of velocity-analysis methods that take anisotropy into account. In the thesis, I present time- and depth-domain algorithms for anisotropic parameter estimation using P-wave reflection and VSP (vertical seismic profiling) data.

First, I introduce a nonhyperbolic moveout inversion technique based on the velocity-independent layer-stripping (VILS) method of Dewangan and Tsvankin (2006). The layer stripping of moveout parameters in the conventional method is replaced by the more stable stripping of reflection traveltimes. Then, to estimate the interval parameters of TTI (transversely isotropic with a tilted symmetry axis) models composed of homogeneous layers separated by plane dipping interfaces, I develop 2D and 3D inversion algorithms based on combining reflection moveout with borehole information. These algorithms help build an accurate initial TTI model for migration velocity analysis.

To perform parameter estimation for more complicated heterogeneous TTI media, I develop a 2D P-wave ray-based tomographic algorithm. The symmetry-direction velocity $V_{P0}$ and the anisotropy parameters $\epsilon$ and $\delta$ are iteratively updated on a rectangular grid, while the symmetry-axis tilt $\nu$ is obtained by setting the symmetry axis orthogonal to the reflectors. To ensure stable reconstruction of parameter fields, reflection data are combined with walkaway VSP traveltimes. To improve the convergence of the inversion algorithm, I propose a three-stage model-updating procedure that gradually relaxes the constraints on the spatial variations of $\epsilon$ and $\delta$. Geologic constraints are incorporated into tomography by designing appropriate regularization terms.

Synthetic tests for models with a “quasi-factorized” TTI syncline (i.e., $\epsilon$ and $\delta$ are constant inside the TTI layer) and a TTI thrust sheet are used to identify conditions for stable parameter estimation. Then the performance of the regularized joint tomography of reflec-
tion and VSP data is examined for two sections of the more complicated TTI model produced by BP that contain an anticline and a salt dome. Finally, the algorithm is applied to a 2D line from 3D OBS (ocean bottom seismic) data acquired at Volve field in the North Sea.
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company easy and orderly.
Velocity analysis is a key step in seismic processing and imaging of subsurface structures. With the emergence of reverse time migration (RTM) and other robust prestack depth migration (PSDM) algorithms, it has become critical to build accurate velocity models because the output of PSDM is highly sensitive to velocity errors. Increasing demand for better resolution and illumination in complex geologic structures has made migration velocity analysis (MVA) one of the most important research areas in seismic exploration over the past two decades.

Because most subsurface formations are anisotropic, ignoring anisotropy in P-wave processing causes imaging and interpretation errors (e.g., Alkhalifah & Larner, 1994; Alkhalifah et al., 1996; Vestrum et al., 1999). In order to avoid such anisotropy-induced distortions as poor focusing of dipping events and mispositioning of horizontal and dipping reflectors, it is necessary to construct anisotropic, heterogeneous velocity models. For example, in VTI (transversely isotropic with a vertical symmetry axis) media, kinematics of P-wave propagation is governed by three Thomsen (1986) parameters: the vertical velocity $V_{P0}$ and the anisotropy coefficients $\epsilon$ and $\delta$. Estimation of all three parameters is challenging because of the tradeoffs between $V_{P0}$, $\epsilon$, and $\delta$ and insufficient sensitivity of reflection traveltimes to the interval anisotropy parameters.

The anisotropy parameters in the unmigrated time domain are often obtained by layer-stripping techniques or traveltime tomography. P-wave time-domain signatures in VTI media are fully controlled by the normal-moveout (NMO) velocity from a horizontal reflector ($V_{nmo}$) and the anellipticity coefficient $\eta$ (Alkhalifah & Tsvankin, 1995; Tsvankin, 2005). A common way of estimating the interval values of $V_{nmo}$ and $\eta$ in layered VTI models is from nonhyperbolic moveout inversion of long-spread reflection data (the maximum offset should reach two
reflector depths) followed by Dix-type differentiation (Grechka & Tsvankin, 1998b; Tsvankin, 2005). Despite the relative simplicity of this Dix-type method, the interval $\eta$-estimation suffers from instability caused by the tradeoff between the effective parameters $V_{\text{nmo}}$ and $\eta$ and by the subsequent error amplification in the layer-stripping process (Grechka & Tsvankin, 1998b). A more detailed review of published time-domain parameter-estimation algorithms can be found in chapter 2.

While VTI models adequately describe P-wave kinematics in most horizontally stratified, unfractured sediments, the symmetry axis typically is tilted in dipping shale layers near salt domes and in fold-and-thrust belts such as the Canadian Foothills (Charles et al., 2008; Isaac & Lawton, 1999; Vestrum et al., 1999). Transverse isotropy with a tilted symmetry axis (TTI) is also caused by obliquely dipping penny-shaped fractures embedded in an isotropic background medium (Dewangan & Tsvankin, 2006a). P-wave velocities and traveltimes in TTI media can be described using the symmetry-direction velocity $V_{p0}$ and the parameters $\epsilon$ and $\delta$ defined with respect to the symmetry axis, whose orientation is specified by the tilt angle $\nu$ with the vertical and the azimuth $\beta$ (for VTI, $\nu = 0^\circ$). Deviation of the symmetry axis from the vertical causes serious complications in velocity analysis, especially if the axis orientation is unknown.

Although P-wave reflection moveout provides enough information for time-domain processing (such as NMO correction and time migration) in VTI media, it is typically insufficient to constrain the velocity $V_{p0}$ and the depth scale of TI models. Therefore, to resolve the parameters required for depth imaging, P-wave moveout typically has to be combined with borehole data or shear modes (SS- or PS-waves). A review of existing results on the joint inversion of PP- and PS-data for TI media can be found in chapter 3. Despite the valuable information provided by PS-waves for parameter estimation, acquisition and processing of mode-converted data is more expensive and difficult than that of pure PP reflections.

Layer-based algorithms cannot always properly account for spatial parameter variations. Therefore, more general time-domain techniques, such as traveltime tomography, have been
devised for reconstructing vertically and laterally heterogeneous velocity fields. The emergence of ray-tracing algorithms for arbitrarily anisotropic media (e.g., Gajewski & Pšenčík, 1987; Vavryčuk, 2001) has made it possible to invert for 2D and 3D heterogeneous velocity models (Billette & Lambaré, 1998; Zhou et al., 2008).

Besides the general problem of ambiguity (nonuniqueness), the performance of time-domain methods (e.g., traveltime tomography) is hampered by data quality. Prestack data are contaminated by correlated noise and distorted by wave-propagation phenomena, which causes difficulties in traveltime picking and moveout analysis. In contrast, migrated data have a much higher signal-to-noise ratio along with sufficient sensitivity to the velocity field. Therefore, migration velocity analysis (MVA), which operates in the migrated domain, has become the most common tool for velocity model-building (Deregowski, 1990; Etgen, 1990; Fowler, 1988; Liu, 1997; van Trier, 1990).

A review of existing MVA algorithms for TTI media can be found in chapter 5. Most current techniques either rely on relatively simple model representation (e.g., layer- or block-based models, Behera & Tsvankin, 2009) or simplify the inversion by keeping the anisotropy parameters $\epsilon$ and $\delta$ fixed and updating only the symmetry-direction velocity $V_{P0}$ defined on a grid (Charles et al., 2008; Huang et al., 2008). To ensure stable parameter estimation, the orientation of the symmetry axis typically has to be known.

In the thesis, I develop time- and depth-domain algorithms designed to enhance the accuracy and stability of anisotropic parameter estimation using P-wave reflection and VSP (vertical seismic profiling) data. Chapters 2, 3, and 4 present time-domain layer-based techniques for interval parameter estimation, and Chapters 5, 6, and 7 focus on the methodology and applications of 2D post-migration gridded tomography for heterogeneous TTI media.

In contrast to Dix-type techniques that operate with effective moveout parameters, in Chapter 2 (based on the paper by Wang & Tsvankin, 2009) I employ velocity-independent layer stripping (VILS) developed by Dewangan & Tsvankin (2006c) to produce the interval traveltime function without knowledge of the velocity field. After extending VILS to 3D,
I apply it to interval parameter estimation in azimuthally anisotropic media using wide-azimuth, long-spread data. The robustness of the VILS-based algorithm is demonstrated on synthetic tests with noise contaminated data from layered orthorhombic media. Then the 3D version of the method is applied to wide-azimuth P-wave data from Rulison field in Colorado.

The most common way of constraining the inversion of P-wave reflection data is by including borehole information. For example, velocities measured by check shots or well logs in a vertical borehole help resolve the VTI parameters $\epsilon$ and $\delta$ (Sexton & Williamson, 1998). In Chapters 3 and 4 (from the papers by Wang & Tsvankin, 2010, 2011), I develop 2D and 3D inversion algorithms based on combining conventional-spread P-wave moveout with well data for models composed of homogeneous TTI layers separated by plane dipping interfaces. These methods make it possible to estimate the interval parameters $V_{P0}$ and $\delta$ near the borehole, whereas the parameter $\epsilon$ can be obtained from nonhyperbolic moveout analysis of reflection data (if the dips are gentle). Then an anisotropic velocity model can be constructed by interpolation between wells or extrapolation away from a well.

2D P-wave tomographic algorithm for more complicated heterogeneous TTI media is presented in Chapter 5 (based on the papers by Wang & Tsvankin, 2012a,b). To estimate the parameters $V_{P0}$, $\epsilon$, and $\delta$ defined on rectangular grids, reflection data are inverted jointly with walkaway VSP traveltimes. Here, I adopt the common assumption that the symmetry axis is perpendicular to the reflectors, so that the tilt field (in 2D) can be computed from a depth image. Each iteration of the tomographic algorithm aims to simultaneously remove the residual moveout in common-image gathers produced by prestack Kirchhoff depth migration and minimize the VSP traveltime misfit. The objective function includes structure-guided regularization that imposes geologic constraints on the model to ensure more stable inversion. In contrast to most existing algorithms, the proposed method properly accounts for both vertical and lateral variations of $\epsilon$ and $\delta$. 
Synthetic tests of the tomographic algorithm introduced in chapter 5 are performed in Chapter 6. First, a model containing a “quasi-factorized” TTI syncline (i.e., $\epsilon$ and $\delta$ are constant inside the TTI layer) is used to investigate the conditions necessary for stable inversion. Then the method is tested on the synthetic data of Zhu et al. (2007) generated for a model that includes a TTI thrust sheet. Finally, the joint tomography is applied to two sections of the TTI model produced by BP, one with an anticline and the other with a salt dome. The BP model is representative of typical offshore structures encountered in the Gulf of Mexico and other important exploration areas.

Chapter 7 presents application of the TTI tomography to Volvo OBS (ocean-bottom seismic) data from the North Sea provided by Statoil. The P-wave reflection data are combined with check-shot traveltimes from two boreholes in the joint inversion to construct a heterogeneous TTI model. Most material from chapters 6 and 7 is included in the paper by Wang & Tsvankin (2012a).
Moveout analysis of long-spread P-wave data is widely used to estimate the key time-processing parameter $\eta$ in layered VTI (transversely isotropic with a vertical symmetry axis) media. Inversion for interval $\eta$ values, however, suffers from instability caused by the tradeoff between the effective moveout parameters and by subsequent error amplification during Dix-type layer stripping.

Here, I propose an alternative approach to nonhyperbolic moveout inversion based on the velocity-independent layer-stripping (VILS) method of Dewangan and Tsvankin. I also develop the 3D version of VILS and apply it to interval parameter estimation in orthorhombic media using wide-azimuth, long-spread data. If the overburden is laterally homogeneous and has a horizontal symmetry plane, VILS produces the exact interval traveltime-offset function in the target layer without knowledge of the velocity field. Hence, Dix-type differentiation of moveout parameters employed in existing techniques is replaced by the much more stable layer stripping of reflection traveltimes. The interval traveltimes are then inverted for the moveout parameters using the single-layer nonhyperbolic moveout equation.

The superior accuracy and stability of our algorithm is illustrated on ray-traced synthetic data for typical VTI and orthorhombic models. Even small correlated noise in reflection traveltimes causes substantial distortions in the interval $\eta$ values computed by the conventional Dix-type differentiation. In contrast, the output of VILS proves to be insensitive to mild correlated travelt ime errors. The algorithm is also tested on wide-azimuth P-wave reflection data recorded above a fractured reservoir at Rulison field in Colorado. The interval moveout parameters estimated by VILS in the shale layer above the reservoir are more plausible and less influenced by noise than those obtained by the Dix-type method.
2.1 Introduction

Traveltime analysis of surface reflection data yields effective moveout parameters for the whole section above the reflector. However, for purposes of migration velocity analysis, AVO (amplitude-variation-with-offset) inversion, and seismic fracture characterization, it is necessary to obtain the interval properties of a target layer. Most existing approaches to interval parameter estimation, both in isotropic and anisotropic media, are based either on layer stripping (e.g., Dix, 1955; Grechka & Tsvankin, 1998b; Grechka et al., 1999) or tomographic inversion (e.g., Grechka et al., 2002a; Stork, 1992).

The conventional Dix (1955) equation, derived for horizontally layered isotropic media, helps to compute the interval normal-moveout (NMO) velocity using the NMO velocities for the reflections from the top and bottom of a layer. The Dix equation remains valid for horizontally layered VTI media; also, it was generalized by Alkhalifah and Tsvankin (1995) for dipping reflectors overlaid by a laterally homogeneous VTI overburden. For 3D wide-azimuth data from layered azimuthally anisotropic media, the effective NMO velocity can be obtained by Dix-type averaging of the interval NMO ellipses (Grechka et al., 1999).

NMO velocity, however, is often insufficient to build the velocity field for anisotropic media, even in the time domain. This explains the importance of using nonhyperbolic (long-spread) reflection moveout in anisotropic parameter estimation. P-wave long-spread reflection moveout in a horizontal VTI layer can be described by the following nonhyperbolic equation (Alkhalifah & Tsvankin, 1995; Tsvankin, 2005):

\[
 t^2 = t_0^2 + \frac{x^2}{V_{nmo}^2} - \frac{2\eta x^4}{V_{nmo}^2[t_0^2V_{nmo}^2 + (1 + 2\eta)x^2]},
 \]

where \(x\) is the offset, \(t_0\) is the two-way zero-offset reflection traveltime, \(V_{nmo}\) is the normal-moveout velocity which controls the conventional-spread reflection moveout of horizontal P-wave events, and \(\eta\) is the “anellipticity” coefficient responsible for the deviation from hyperbolic moveout at long offsets. For stratified VTI media, the moveout parameters become effective quantities for the stack of layers above the reflector. Many implementations of non-
hyperbolic moveout inversion for VTI media (e.g., Alkhalifah, 1997; Grechka & Tsvankin, 1998b; Toldi et al., 1999) are based on equation 2.1 which represents a simplified version of the more general Tsvankin–Thomsen (1994) equation. Accurate estimation of $V_{\text{nmo}}$ and $\eta$ makes it possible to carry out all P-wave time-domain processing steps, which include NMO and DMO (dip-moveout) corrections and time migration.

An alternative algorithm for $\eta$ estimation operates with the dip dependence of P-wave NMO velocity (Alkhalifah & Tsvankin, 1995). Although the DMO inversion is relatively stable, its application is more complicated and requires the presence of dipping reflectors under the formation of interest (Tsvankin, 2005).

Nonhyperbolic moveout inversion for the parameters $V_{\text{nmo}}$ and $\eta$ usually involves a 2D semblance scan on long-spread data (the maximum offset should reach two reflector depths) from a horizontal reflector. Despite its relative simplicity, this method suffers from instability caused by the tradeoff between $V_{\text{nmo}}$ and $\eta$ (Alkhalifah, 1997; Grechka & Tsvankin, 1998b). Grechka & Tsvankin (1998b) found that even small traveltime errors, which could be considered as insignificant in data processing, may cause large errors in the effective parameter $\eta$. For layered media, this error is amplified in the layer-stripping process, which may cause unacceptable distortions in the interval $\eta$ values. The effective $\eta$ function is often smoothed prior to application of the Dix-type equations, but smoothing does not remove the source of instability in the interval $\eta$ estimation.

The Alkhalifah–Tsvankin (1995) equation was extended to wide-azimuth data by taking into account the azimuthal variation of the NMO velocity and parameter $\eta$ (Vasconcelos and Tsvankin, 2006; Xu and Tsvankin, 2006). Here, we consider P-wave data from azimuthally anisotropic media with orthorhombic symmetry typical for fractured reservoirs (Bakulin et al., 2000; Grechka & Kachanov, 2006; Schoenberg & Helbig, 1997). Nonhyperbolic moveout of P-waves in an orthorhombic layer with a horizontal symmetry plane is governed by the azimuths of the vertical symmetry planes, the symmetry-plane NMO velocities ($V_{\text{nmo}}^{(1)}$ and $V_{\text{nmo}}^{(2)}$) responsible for the NMO ellipse, and three anellipticity coefficients $\eta^{(1,2,3)}$ (Grechka &
Tsvankin, 1999a). Since the symmetry-plane NMO velocities and parameters $\eta^{(1,2,3)}$ depend on the fracture compliances and orientation (Bakulin et al., 2000), nonhyperbolic moveout inversion can help in building physical models for reservoir characterization. Also, the parameters $V_{\text{nmo}}^{(1,2)}$ and $\eta^{(1,2,3)}$ are sufficient to perform all P-wave time-processing steps in orthorhombic models (Grechka & Tsvankin, 1999a).

For layered orthorhombic media, the parameters of the Alkhalifah–Tsvankin equation become effective quantities, and the interval values of $\eta^{(1,2,3)}$ can be estimated by a generalized Dix-type differentiation scheme based on the results of Vasconcelos & Tsvankin (2006) and Xu & Tsvankin (2006). However, this procedure is hampered by the same instability problems as the ones discussed above for the $\eta$-inversion in layered VTI media.

Here, I propose to overcome the shortcomings of Dix-type techniques by employing the velocity-independent layer-stripping (VILS) method of Dewangan & Tsvankin (2006c). This layer-stripping algorithm, which operates with reflection traveltimes, produces accurate interval long-spread reflection moveout, which can then be inverted for the layer parameters. I review the 2D version of VILS designed for VTI media and introduce a 3D implementation for wide-azimuth data from orthorhombic media. Numerical tests demonstrate that, in contrast to Dix-type inversion, our method remains robust in the presence of typical correlated noise in reflection traveltimes. Finally, I apply the algorithm to nonhyperbolic moveout analysis of wide-azimuth P-wave data acquired over a fractured reservoir at Rulison field in Colorado.

### 2.2 Velocity-independent layer stripping

The velocity-independent layer-stripping algorithm of Dewangan & Tsvankin (2006c) is based on the so-called “PP + PS = SS” method (Grechka & Tsvankin, 2002b). VILS is entirely data-driven and, if the model assumptions are satisfied, does not require knowledge of the velocity field anywhere in the medium.
2.2.1 Layer stripping in 2D

Figure 2.1 shows 2D ray trajectories of pure-mode (non-converted) reflections from the top and bottom of the target zone overlaid by a laterally homogeneous overburden. The incidence plane is supposed to represent a symmetry plane for the model as a whole, so that wave propagation is two-dimensional; this assumption becomes unnecessary in the 3D extension of the method discussed below. While the target zone can be heterogeneous with interval curved interfaces, each layer in the overburden has to be laterally homogeneous with a horizontal symmetry plane. Then the raypath of any reflection from the top of the target zone is symmetric with respect to the reflection point (e.g., points T or R in Figure 2.1).

![Figure 2.1: 2D diagram of the layer-stripping algorithm for pure-mode reflections (after Dewangan & Tsvankin, 2006c). Points T and R are located at the bottom of the laterally homogeneous overburden. The leg $x^{(1)}T$ is shared by the target reflection $x^{(1)}TQRx^{(2)}$ and the overburden event $x^{(1)}T(x^{(3)}); the leg $Rx^{(2)}$ is shared by the reflections $x^{(1)}TQRx^{(2)}$ and $x^{(2)}Rx^{(4)}$.

As discussed by Dewangan & Tsvankin (2006c), equalizing time slopes on common-receiver gathers at the source location $x^{(1)}$ can be used to identify the overburden reflection $x^{(1)}T(x^{(3)}$ that has the same horizontal slowness as the reflection $x^{(1)}TQRx^{(2)}$ from the bottom of the target layer. This means that the reflections $x^{(1)}T(x^{(3)} and $x^{(1)}TQRx^{(2)} share the
downgoing leg \( x^{(1)}T \). Likewise, we can find the overburden reflection \( x^{(2)}R \times x^{(4)} \) that has the same upgoing leg \( R \times x^{(2)} \) as the target event \( x^{(1)}TQR \times x^{(2)} \). Since any reflection path in the overburden is symmetric with respect to the reflection point, the interval reflection traveltime \( t_{\text{int}} \) in the target zone can be computed as

\[
t_{\text{int}}(T, R) = t_{\text{eff}}(x^{(1)}, x^{(2)}) - \frac{1}{2} \left[ t_{\text{ovr}}(x^{(1)}, x^{(3)}) + t_{\text{ovr}}(x^{(2)}, x^{(4)}) \right],
\]

where the superscripts \( \text{eff} \) and \( \text{ovr} \) refer to the target event \( x^{(1)}TQR \times x^{(2)} \) and the reflections from the bottom of the overburden, respectively. The corresponding source-receiver pair \((T, R)\) has the following horizontal coordinates:

\[
x_T = \frac{x^{(1)} + x^{(3)}}{2}, \quad x_R = \frac{x^{(2)} + x^{(4)}}{2}.
\]

Equations 2.2 and 2.3 yield the interval reflection moveout function in the target zone without any information about the velocity model.

2D interval moveout-inversion methods based on ideas similar to those behind VILS were developed by Van der Baan & Kendall (2002, 2003) and Fowler et al. (2008). In contrast to VILS, however, these methods assume the invariance of the horizontal slowness (ray parameter) along each ray, which implies that the target zone (not just the overburden) has to be laterally homogeneous. Van der Baan & Kendall (2002, 2003) and Fowler et al. (2008) implemented their algorithms for P-wave data from horizontally layered VTI media.

### 2.2.2 3D layer stripping for wide-azimuth data

The 3D version of the layer-stripping algorithm does not impose any restrictions on the properties (anisotropy, heterogeneity) of the target zone, but each layer in the overburden still has to be laterally homogeneous with a horizontal symmetry plane. For wide-azimuth data (Figure 2.2), identifying the target and overburden reflections with the same ray segments requires estimating two orthogonal horizontal slowness components from reflection time slopes. In Figure 2.2, the horizontal slownesses of the target (eff) and overburden (ovr)
Figure 2.2: 3D diagram of the layer-stripping algorithm. Points T and R are located at the bottom of the laterally homogeneous overburden. The sources and receivers ($x^{(1)}$, $x^{(2)}$, $x^{(3)}$ and $x^{(4)}$) are placed at the surface but not necessarily along a straight line. The reflection point Q is located at the bottom of the target layer, which can be arbitrarily anisotropic and heterogeneous. The leg $x^{(1)}T$ is shared by the target event $x^{(1)}TQRx^{(2)}$ and the overburden reflection $x^{(1)}Tx^{(3)}$; the leg $Rx^{(2)}$ is shared by the reflections $x^{(1)}TQRx^{(2)}$ and $x^{(2)}Rx^{(4)}$. 


reflections at location \( \mathbf{x}^{(1)} = [x_1^{(1)}, x_2^{(1)}] \) can be obtained from

\[
p_i^{\text{eff}}(\mathbf{x}^{(1)}, \mathbf{x}^{(2)}) = \frac{\partial t^{\text{eff}}(\mathbf{x}, \mathbf{x}^{(2)})}{\partial x_i} \bigg|_{\mathbf{x} = \mathbf{x}^{(1)}}, \quad (i = 1, 2)
\]

and

\[
p_i^{\text{ovr}}(\mathbf{x}^{(1)}, \mathbf{x}^{(3)}) = \frac{\partial t^{\text{ovr}}(\mathbf{x}, \mathbf{x}^{(3)})}{\partial x_i} \bigg|_{\mathbf{x} = \mathbf{x}^{(1)}}, \quad (i = 1, 2).
\]

Using equations 2.4 and 2.5, we find the location \( \mathbf{x}^{(3)} \), for which the time slopes (horizontal slownesses) of the two events are identical,

\[
p_i^{\text{eff}}(\mathbf{x}^{(1)}, \mathbf{x}^{(2)}) = p_i^{\text{ovr}}(\mathbf{x}^{(1)}, \mathbf{x}^{(3)}), \quad (i = 1, 2).
\]

Therefore, the reflections \( \mathbf{x}^{(1)}\text{TQRx}^{(2)} \) and \( \mathbf{x}^{(1)}\text{Tx}^{(3)} \) have the common leg \( \mathbf{x}^{(1)}\text{T} \). The same operation applied at point \( \mathbf{x}^{(2)} \) helps to find the overburden reflection \( \mathbf{x}^{(2)}\text{Rx}^{(4)} \) that shares the upgoing leg \( \text{Rx}^{(2)} \) with the target event \( \mathbf{x}^{(1)}\text{TQRx}^{(2)} \). The interval reflection traveltime can then be obtained from equation 2.2. Since \( \text{T} \) and \( \text{R} \) represent the midpoints of the corresponding source-receiver pairs, their horizontal coordinates can be easily found from \( \mathbf{x}^{(1)} \), \( \mathbf{x}^{(2)} \), \( \mathbf{x}^{(3)} \), and \( \mathbf{x}^{(4)} \).

Thus, the velocity-independent layer-stripping algorithm makes it possible to construct interval moveout functions in both 2D and 3D. Because reflection traveltimes can be estimated with relatively high accuracy, VILS helps to avoid the stability problems in Dix-type inversion caused by the tradeoffs between the effective moveout parameters.

Similar to the original version of the PP+PS=SS method, our layer-stripping algorithm operates with reflection traveltimes. Grechka & Dewangan (2003) developed an efficient implementation of the PP+PS=SS method by replacing traveltime analysis with a convolution of recorded PP and PS traces. Their technique can be adapted to compute interval P-wave reflection data using the reflections from the top and bottom of the target layer. Although the convolution of recorded traces cannot produce the correct amplitudes, the constructed arrivals should have the kinematics of P-wave primary reflections and, therefore, are suitable for interval moveout analysis.
2.3 Tests on synthetic data

Next, I test the layer-stripping algorithm on 2D and 3D long-spread P-wave data generated by kinematic ray tracing (Gajewski & Pšenčík, 1987) for VTI and orthorhombic models. Reflected arrivals on the synthetic seismograms are obtained by placing the Ricker wavelet at the corresponding reflection traveltime. The interval moveout parameters in the target layer are estimated from both our method and the Dix-type equations. To compare the stability of the two techniques in the presence of correlated noise, I add several noise functions to the input traveltimes.

2.3.1 2D inversion for VTI media

For stratified VTI media composed of \( N \) horizontal layers, long-spread P-wave traveltime is described by equation 2.1 with effective moveout parameters (Grechka & Tsvankin, 1998b; Tsvankin, 2005):

\[
t^2(N) = t_0^2(N) + \frac{x^2}{V_{nmo}^2(N)} - \frac{2\eta(N) x^4}{V_{nmo}^2(N) \{ t_0^2(N) V_{nmo}^2(N) + [1 + 2\eta(N)] x^2 \} }. \tag{2.7}
\]

The effective NMO velocity is found from the Dix equation,

\[
V_{nmo}^2(N) = \frac{1}{t_0(N)} \sum_{i=1}^{N} (V_{nmo}^{(i)})^2 t_0^{(i)}, \tag{2.8}
\]

where \( t_0^{(i)} \) and \( V_{nmo}^{(i)} \) are the interval values in layer \( i \). The effective parameter \( \eta \) is approximately given by

\[
\eta(N) = \frac{1}{8} \left\{ \frac{1}{V_{nmo}^4(N) t_0(N)} \left[ \sum_{i=1}^{N} (V_{nmo}^{(i)})^4 (1 + 8\eta^{(i)} t_0^{(i)}) - 1 \right] \right\}. \tag{2.9}
\]

The best-fit effective parameters \( V_{nmo} \) and \( \eta \) for the top and bottom of a layer of interest are usually obtained by applying semblance-based nonhyperbolic moveout inversion to long-spread P-wave data. Then the interval \( V_{nmo} \) can be computed from equation 2.8,

\[
(V_{nmo}^{(i)})^2 = \frac{V_{nmo}^2(i) t_0(i) - V_{nmo}^2(i - 1) t_0(i - 1)}{t_0(i) - t_0(i - 1)}, \tag{2.10}
\]
while equation 2.9 yields the interval $\eta$:

$$\eta^{(i)} = \frac{1}{8(V_{nmo}^{(i)})^4} \left[ g(i)t_0(i) - g(i-1)t_0(i-1) \frac{t_0(i) - t_0(i-1)}{t_0(i) - t_0(i-1)} - (V_{nmo}^{(i)})^4 \right] ; \quad (2.11)$$

$$g(N) \equiv V_{nmo}^4(N)[1 + 8\eta(N)].$$

Although equations 2.1 and 2.7 provide a good approximation for nonhyperbolic moveout in VTI media, the estimated $\eta$ is sensitive to small errors in $V_{nmo}$ even if the maximum offset-to-depth ratio ($x_{max}/D$) is between two and three. The tradeoff between the effective $V_{nmo}$ and $\eta$ (along with the slight bias of the nonhyperbolic moveout equation) causes substantial instability in the $\eta$ estimation, which is amplified in the Dix-type layer stripping based on equation 2.11 (Grechka & Tsvankin, 1998b).

### 2.3.1.1 Model 1

The first numerical test was performed for the three-layer VTI model with the parameters listed in Table 2.1 (Figure 2.3a; $x_{max}/D = 2$ for the bottom of the model). Both VILS and the Dix-type method were used to estimate the interval parameters $V_{nmo}$ and $\eta$ in the third layer. Although the $\eta$ values in this model are moderate, the traveltime curves for the top and bottom of the target layer noticeably deviate from the hyperbolic moveout approximation at large offsets (Figure 2.3b).

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness (km)</th>
<th>$t_0$ (s)</th>
<th>$V_{nmo}$ (km/s)</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>0.7</td>
<td>0.70</td>
<td>2.10</td>
<td>0.00</td>
</tr>
<tr>
<td>Layer 2</td>
<td>0.3</td>
<td>0.25</td>
<td>2.52</td>
<td>0.10</td>
</tr>
<tr>
<td>Layer 3 (target)</td>
<td>0.5</td>
<td>0.39</td>
<td>2.78</td>
<td>0.20</td>
</tr>
</tbody>
</table>

To reconstruct the reflection traveltimes from the top and bottom of the target (third) layer, I employed 2D semblance search for the effective parameters $V_{nmo}$ and $\eta$ based on equation 2.7. (Note that equations 2.1 and 2.7 are equivalent in terms of semblance analysis.) Then VILS was applied to compute the interval traveltime, which was inverted for the
interval parameters using least-squares fitting of moveout equation 2.1 (see the flow chart in Figure 2.4). The interval value of $\eta$ estimated by VILS is quite accurate, with the error (just 0.02) mostly caused by the slight bias of equation 2.1 discussed by Grechka and Tsvankin (1998).

The semblance analysis for the top and bottom of the target layer also provided input data for the Dix-type differentiation described above. However, in contrast to VILS, the Dix-type algorithm operates with the effective moveout parameters, not traveltimes. As a result, small distortions in the effective $\eta$ estimates get amplified in the layer-stripping procedure, which leads to an error of 0.06 in the interval $\eta$ value.

### 2.3.1.2 Error analysis

To study the influence of realistic noise on the interval parameter estimation, I added random, linear and sinusoidal time errors to the reflection moveout from the bottom of the
target layer. The traveltimes from the top of the target were left unchanged. As before, the input data for both VILS and the Dix-type method were obtained from 2D semblance search based on equation 2.7.

Since both methods employ semblance analysis, they remain reasonably stable in the presence of random noise. For random errors with the magnitude approaching 10 ms (Figure 2.5), the interval $\eta$ estimated by VILS is distorted by less than 0.02, while the Dix-type method produces $\eta$ errors in the range of 0.05–0.08.

The second type of noise used in our tests is linear, which can simulate long-period static errors. For a relatively large error that changes from 6 ms at zero offset to -6 ms at the maximum offset, VILS estimates the interval $V_{\text{nmo}}$ and $\eta$ with errors of 4% and 0.07, respectively. The distortions in $V_{\text{nmo}}$ and $\eta$ after the Dix-type layer stripping are much larger (15% and 0.34, respectively), which makes the inversion practically useless. These results are consistent with the analysis in Grechka and Tsvankin (1998) who demonstrate that linear time noise of a somewhat smaller magnitude may cause errors in the effective $\eta$ close to
Figure 2.5: Random traveltime error with the maximum magnitude close to 10 ms.

0.1. The Dix-type procedure increases such errors by a factor that depends on the relative thickness of the target layer (i.e., on the ratio of its thickness and depth).

Next, I contaminated the data with a sinusoidal time function designed to emulate short-period static errors: \( t = A \sin(n\pi x / x_{\text{max}}) \). The interval parameter-estimation results for different values of \( A \) and \( n \) are listed in Table 2.2. The error in the interval \( \eta \) produced by VILS reaches only 0.08 even for \( A = 8 \text{ ms} \), while the Dix-type method breaks down for \( A \geq 3 \text{ ms} \).

Table 2.2: Influence of correlated noise on the interval parameter estimation for the third layer in model 1. A sinusoidal error function \( (t = A \sin(n\pi x / x_{\text{max}})) \) was added to the traveltimes from the bottom of the layer. The percentage error in the interval velocity \( V_{\text{nmo}} \) and the absolute error in the interval \( \eta \) are estimated by VILS and the Dix-type method for different combinations of \( A \) and \( n \).

<table>
<thead>
<tr>
<th>Parameters of error function</th>
<th>( A = 3 \text{ ms}, n = 3 )</th>
<th>( A = 3 \text{ ms}, n = 2 )</th>
<th>( A = 8 \text{ ms}, n = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inversion error</td>
<td>( V_{\text{nmo}} ) (%)</td>
<td>( \eta )</td>
<td>( V_{\text{nmo}} ) (%)</td>
</tr>
<tr>
<td>VILS</td>
<td>0.6</td>
<td>0.01</td>
<td>0.0</td>
</tr>
<tr>
<td>Dix</td>
<td>11</td>
<td>0.19</td>
<td>8.2</td>
</tr>
</tbody>
</table>

These tests clearly demonstrate the superior stability of VILS in the presence of typical correlated noise in reflection traveltimes. Even relatively small time errors can cause sub-
stantial distortions in the effective moveout parameters, which propagate with amplification into the interval $\eta$-values. In contrast, percentage errors in the traveltimes themselves are insignificant, which ensures the high accuracy of the interval moveout produced by VILS.

### 2.3.1.3 Influence of lateral heterogeneity

The layer-stripping procedure in VILS is based on the assumption that each layer in the overburden is laterally homogeneous and has a horizontal symmetry plane. Lateral velocity gradients or dipping interfaces make the raypaths of overburden events asymmetric with respect to the reflection point, which may cause errors in equations 2.2 and 2.3. Note that lateral heterogeneity above the reflector also violates the assumptions behind the Dix-type method (Alkhalifah and Tsvankin, 1995; Grechka and Tsvankin, 1998).

To evaluate the influence of mild dips in the overburden on interval parameter estimation, I tilted the most shallow reflector in model 1 by $10^\circ$ (Figure 2.6). Then I generated noise-free synthetic data and applied both VILS and the Dix-type method without taking the dip into account. The interval parameters $V_{nmo}$ and $\eta$ in the third layer estimated by the VILS are distorted by only 2% and 0.05, respectively. In contrast, the Dix-type method produces more significant errors in the interval values, reaching 6% in $V_{nmo}$ and 0.15 in $\eta$. This and other tests indicate that VILS is much less sensitive to mild lateral heterogeneity than the Dix differentiation.

### 2.3.2 3D inversion for orthorhombic media

The azimuthally-dependent P-wave reflection moveout in a horizontal orthorhombic layer can be well-approximated by equation 2.1 with azimuthally varying parameters $V_{nmo}$ and $\eta$ (Vasconcelos & Tsvankin, 2006; Xu & Tsvankin, 2006):

$$t^2(x, \alpha) = t_0^2 + \frac{x^2}{V_{nmo}^2(\alpha)} - \frac{2\eta(\alpha)x^4}{V_{nmo}^2(\alpha)[t_0^2V_{nmo}^2(\alpha) + (1 + 2\eta(\alpha))x^2]}, \quad (2.12)$$
Figure 2.6: Three-layer VTI model used to evaluate the influence of mild dip (10°) in the overburden on the inversion results. Except for the dip, all medium parameters are the same as those in model 1 (Table 2.1). The lateral extent of the model is 4 km; the CMP (common midpoint) used for parameter estimation is located in the middle.

where $\alpha$ is the source-to-receiver azimuth. The azimuthally-dependent NMO velocity is obtained from the equation of the NMO ellipse:

$$V_{nmo}^{-2}(\alpha) = \frac{\sin^2(\alpha - \varphi)}{[V_{nmo}^{(1)}]^2} + \frac{\cos^2(\alpha - \varphi)}{[V_{nmo}^{(2)}]^2};$$  \hspace{1cm} (2.13)

$\varphi$ is the azimuth of the $[x_1, x_3]$ symmetry plane, and $V_{nmo}^{(1)}$ and $V_{nmo}^{(2)}$ are the NMO velocities in the vertical symmetry planes $[x_2, x_3]$ and $[x_1, x_3]$, respectively. The parameter $\eta$ is approximately given by (Pech & Tsvankin, 2004):

$$\eta(\alpha) = \eta^{(1)} \sin^2(\alpha - \varphi) + \eta^{(2)} \cos^2(\alpha - \varphi) - \eta^{(3)} \sin^2(\alpha - \varphi) \cos^2(\alpha - \varphi),$$  \hspace{1cm} (2.14)

where $\eta^{(1)}$, $\eta^{(2)}$, and $\eta^{(3)}$ are the anellipticity coefficients defined in the $[x_2, x_3]$, $[x_1, x_3]$, and $[x_1, x_2]$ symmetry planes, respectively.

For layered orthorhombic media, all moveout parameters become effective quantities. The semi-axes and orientation of the effective NMO ellipse (equation 2.13) can be obtained from the generalized Dix equation by averaging the interval NMO ellipses (Grechka et al., 1999). If the vertical symmetry planes in different layers are misaligned, the principal directions for
the effective parameter $\eta$ are described by a separate azimuth, $\varphi_1$ (Xu & Tsvankin, 2006):

$$
\eta(\alpha) = \eta^{(1)} \sin^2(\alpha - \varphi_1) + \eta^{(2)} \cos^2(\alpha - \varphi_1) - \eta^{(3)} \sin^2(\alpha - \varphi_1) \cos^2(\alpha - \varphi_1).
$$

(2.15)

The effective parameter $\eta$ for each azimuth $\alpha$ can be computed from the VTI equation 2.9, since kinematic signatures in each vertical plane of layered orthorhombic media can be approximately described by the corresponding VTI equations (Tsvankin, 1997, 2005). Then the parameters $\eta^{(1)}$, $\eta^{(2)}$, $\eta^{(3)}$ and $\varphi_1$ are found by fitting the effective $\eta$ values for a wide range of azimuths to equation 2.15.

To implement both VILS and the Dix-type inversion for layered orthorhombic media, I use the 3D nonhyperbolic semblance algorithm of Vasconcelos & Tsvankin (2006) based on equations 2.12, 2.13, and 2.15. The best-fit effective moveout parameters $V_{\text{nmo}}^{(1,2)}$, $\eta^{(1,2,3)}$, $\varphi$ and $\varphi_1$ for the top and bottom of the target layer are found by multidimensional semblance search using the full range of available offsets and azimuths. For purposes of the Dix-type layer stripping, the interval NMO ellipse is obtained from the generalized Dix equation (Grechka et al., 1999), and the interval $\eta$ value for each azimuth is computed from the VTI equation 2.11. Finally, the interval parameters $\eta^{(1,2,3)}$ are estimated by fitting equation 2.14 to the azimuthally varying $\eta$ values.

The long-spread, wide-azimuth reflection traveltimes produced by the nonhyperbolic semblance analysis serve as the input data for VILS (see the flow chart in Figure 2.4). To apply VILS to 3D wide-azimuth data (Figure 2.2), it is also necessary to estimate the horizontal slowness components at the source and receiver locations. In principle, the horizontal projection of the slowness vector (equations 2.4 and 2.5) can be computed from reflection traveltimes on common-shot or common-receiver gathers. A more stable and efficient option, however, is to express the horizontal slownesses as functions of offset and azimuth through the best-fit moveout parameters using equation 2.12. Despite the parameter tradeoffs, equation 2.12 provides sufficient accuracy for long-spread P-wave moveout and, therefore, for the traveltime derivatives. After the interval traveltimes are computed by VILS, the interval
parameters $\varphi$, $V_{nmo}^{(1,2)}$, and $\eta^{(1,2,3)}$ are obtained from the single-layer moveout inversion.

### 2.3.2.1 Model 2

The 3D parameter-estimation algorithm was first tested on an orthorhombic layer overlaid by VTI and isotropic layers (Table 2.3). VILS and the Dix-type method were applied to long-spread ($x_{\text{max}}/D = 2$ for the bottom of the model), wide-azimuth data from the top and bottom of the orthorhombic layer. Since the model is laterally homogeneous, synthetic data were generated for a single source location and a full ($180^\circ$) range of source-receiver azimuths. To ensure the stability of 3D nonhyperbolic moveout inversion, the receivers were placed on 19 lines with an azimuthal interval of $10^\circ$. As illustrated by Figure 2.7, the azimuthal anisotropy in the target layer makes the traveltimes from its bottom vary with azimuth. Without traveltime noise, both methods give similar accuracy in the interval moveout parameters.

Table 2.3: Interval parameters of a three-layer model used to test the 3D layer-stripping algorithm (model 2).

<table>
<thead>
<tr>
<th></th>
<th>Layer 1</th>
<th>Layer 2</th>
<th>Layer 3 (target)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetry type</td>
<td>ISO</td>
<td>VTI</td>
<td>Orthorhombic</td>
</tr>
<tr>
<td>Thickness (km)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$t_0$ (s)</td>
<td>0.50</td>
<td>0.41</td>
<td>0.39</td>
</tr>
<tr>
<td>$V_{nmo}^{(1)}$ (km/s)</td>
<td>2</td>
<td>2.49</td>
<td>3.18</td>
</tr>
<tr>
<td>$V_{nmo}^{(2)}$ (km/s)</td>
<td>2</td>
<td>2.49</td>
<td>2.64</td>
</tr>
<tr>
<td>$\eta^{(1)}$</td>
<td>0</td>
<td>0.05</td>
<td>0.2</td>
</tr>
<tr>
<td>$\eta^{(2)}$</td>
<td>0</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>$\eta^{(3)}$</td>
<td>0</td>
<td>0</td>
<td>0.13</td>
</tr>
<tr>
<td>$\varphi$ ($^\circ$)</td>
<td></td>
<td></td>
<td>30</td>
</tr>
</tbody>
</table>

### 2.3.2.2 Error analysis

As before, I added linear and sinusoidal time errors to the reflection moveout from the bottom of the target layer in model 2. For the azimuthally invariant linear error that changes from 6 ms at zero offset to -6 ms at the maximum offset, the interval parameters $V_{nmo}^{(1,2)}$ and
Figure 2.7: Synthetic long-spread P-wave reflections from the top and bottom of layer 3 (target) in model 2 (Table 2.3). The seismograms are computed in the two orthogonal vertical symmetry planes of the target orthorhombic layer.

$\eta^{(1,2,3)}$ estimated by VILS are distorted by no more than 3% and 0.06, respectively. The Dix-type method produces much larger errors reaching 9% in $V_{nmo}^{(1,2)}$ and 0.16 in $\eta^{(1,2,3)}$. The errors in the symmetry-plane azimuth $\phi$ for both methods are negligible.

I also contaminated the traveltimes with the noise function of the form $t(x, \alpha) = A \sin(n \pi x / x_{max}) \sin m \alpha$ (Table 2.4). The coefficients $n$ and $m$ control the period of the error in the radial and azimuthal directions, respectively. In general, inversion errors tend to be higher when $m$ is an even number. If the error function does not vary with azimuth ($m = 0$), both methods give more accurate results for even values of $n$, which agrees with the conclusions of Xu & Tsvankin (2006). As illustrated by several examples in Table 2.4, even for noisy data with $A = 10$ ms VILS produces errors in the interval $\eta^{(1,2,3)}$ not exceeding 0.09, while the Dix-type method distorts the $\eta$-parameters by up to 0.22 and the NMO velocities by 10%.
Table 2.4: Influence of correlated noise on the interval parameter estimation for the third layer in model 2. A sinusoidal error function \( t = A \sin(n \pi x/x_{\text{max}}) \sin m\alpha \) was added to the traveltimes from the bottom of the layer. The maximum percentage error in the interval velocities \( V_{nmo}^{(1,2)} \) and the maximum absolute error in the interval \( \eta^{(1,2,3)} \) are estimated by VILS and the Dix-type method. The errors in the azimuth \( \varphi \) do not exceed 0.5° for both methods.

<table>
<thead>
<tr>
<th>Inversion error</th>
<th>( V_{nmo} ) ( % )</th>
<th>( \eta )</th>
<th>( V_{nmo} ) ( % )</th>
<th>( \eta )</th>
<th>( V_{nmo} ) ( % )</th>
<th>( \eta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>VILS</td>
<td>1.2 0.04</td>
<td></td>
<td>1.0 0.02</td>
<td></td>
<td>2.4 0.09</td>
<td></td>
</tr>
<tr>
<td>Dix</td>
<td>4.4 0.09</td>
<td></td>
<td>2.3 0.08</td>
<td></td>
<td>10 0.22</td>
<td></td>
</tr>
</tbody>
</table>

It is interesting that despite the complexity of orthorhombic symmetry, the Dix-type method gives much better results for model 2 than for the VTI model 1 (compare Table 2.2 and Table 2.4). Most likely, this improvement is explained by wide azimuthal coverage in 3D inversion, which creates redundancy and makes the estimation of the effective moveout parameters more stable.

Next, I studied the influence of the thickness of the target layer in model 2 on the inversion results. Any layer-stripping method inevitably becomes less accurate as the layer of interest gets thinner. I added the sinusoidal error with \( A = 3 \text{ ms}, n = 3 \) and \( m = 0 \) to the traveltimes from the bottom of the target and reduced its thickness until it reached 0.15 km (Table 2.5). VILS gives acceptable results for both the interval \( V_{nmo} \) and \( \eta \) when the thickness exceeds 0.25 km (i.e., when the thickness-to-depth ratio exceeds 0.2), while the error in \( V_{nmo} \) estimated by the Dix-type method approaches 10%. However, VILS breaks down for the target layer that is only 0.15 km thick.

Table 2.5: Errors of the interval \( V_{nmo}^{(1,2)} \) and \( \eta^{(1,2,3)} \) for two different thicknesses of the target layer in model 2 (Table 3). The traveltimes from the bottom of the target were contaminated by the sinusoidal noise function with \( A = 3 \text{ ms}, n = 3 \) and \( m = 0 \).

<table>
<thead>
<tr>
<th>Thickness (km)</th>
<th>0.35</th>
<th>0.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inversion error</td>
<td>( V_{nmo} ) ( % )</td>
<td>( \eta )</td>
</tr>
<tr>
<td>VILS</td>
<td>2.2 0.05</td>
<td></td>
</tr>
<tr>
<td>Dix</td>
<td>7.5 0.13</td>
<td></td>
</tr>
</tbody>
</table>
2.3.2.3 Model 3

The third model includes the target orthorhombic layer beneath the overburden composed of isotropic and orthorhombic layers (Table 2.6). Note that the vertical symmetry planes in the two orthorhombic layers are misaligned, so the azimuthally varying parameter $\eta$ from the bottom of the target is described by equation 2.15. The vertical variation of the symmetry-plane azimuths does not cause any complications in the application of VILS, as long as the overburden is laterally homogeneous and has a horizontal symmetry plane.

Table 2.6: Interval parameters of a three-layer model that includes two orthorhombic layers with misaligned vertical symmetry planes (model 3).

<table>
<thead>
<tr>
<th>Layer 1</th>
<th>Layer 2</th>
<th>Layer 3 (target)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetry type</td>
<td>ISO</td>
<td>ORTH</td>
</tr>
<tr>
<td>Thickness (km)</td>
<td>0.3</td>
<td>0.7</td>
</tr>
<tr>
<td>$t_0$ (s)</td>
<td>0.30</td>
<td>0.58</td>
</tr>
<tr>
<td>$V_{nmo}^{(1)}$ (km/s)</td>
<td>2</td>
<td>2.56</td>
</tr>
<tr>
<td>$V_{nmo}^{(2)}$ (km/s)</td>
<td>2</td>
<td>3.06</td>
</tr>
<tr>
<td>$\eta^{(1)}$</td>
<td>0</td>
<td>0.05</td>
</tr>
<tr>
<td>$\eta^{(2)}$</td>
<td>0</td>
<td>0.07</td>
</tr>
<tr>
<td>$\eta^{(3)}$</td>
<td>0</td>
<td>0.02</td>
</tr>
<tr>
<td>$\phi$ (°)</td>
<td>30</td>
<td>70</td>
</tr>
</tbody>
</table>

The accuracy of VILS and the Dix-type method for noise-free data is similar, which was also the case for model 2. The sinusoidal error $t = A \sin(n\pi x/x_{max})$ applied to the traveltimes from the bottom of the target layer produces much more significant distortions in the output of the Dix-type differentiation compared to VILS. For instance, when $A = 6$ ms and $n = 3$, the maximum errors in the interval $V_{nmo}$ and $\eta^{(1,2,3)}$ estimated by VILS are 4% and 0.09 (respectively), while the corresponding errors of the Dix-type method reach 13% and 0.25.

2.4 Field-data example

The 3D VILS algorithm was applied to wide-azimuth P-wave data acquired by the Reservoir Characterization Project (a research consortium at Colorado School of Mines) at Rulison field, a basin-centered gas accumulation in South Piceance Basin, Colorado. The reservoir
(Williams Fork formation) is capped by the UMV (Upper Mesaverde) shale, which served as the target layer in our study (Figure 2.8).

![Stratigraphic column of Rulison field (after Xu & Tsvankin, 2007). The gas-producing reservoir is bounded by the UMV shale (the target layer in this study) and the Cameo coal.](image)

**Figure 2.8:** Stratigraphic column of Rulison field (after Xu & Tsvankin, 2007). The gas-producing reservoir is bounded by the UMV shale (the target layer in this study) and the Cameo coal.

Xu & Tsvankin (2007) applied a comprehensive anisotropic processing sequence to the data and analyzed the effective and interval NMO ellipses, as well as the azimuthal AVO response. I used the same data set, which was acquired in 2003 and preprocessed for purposes of azimuthal moveout and AVO analysis. Since the subsurface structure is close to horizontally layered (Figure 2.9), the moveout equations discussed above should give an accurate description of reflection traveltimes. As suggested by Xu & Tsvankin (2007), I combined CMP gathers into 5x5 superbins to increase the azimuthal and offset coverage. The moveout inversion was carried out in the center of the RCP survey area (Figure 2.10), where the coverage is sufficient for minimizing the influence of the acquisition footprint.

Because the maximum offset-to-depth ratio at the bottom of the reservoir is close to unity (on average for the study area), nonhyperbolic moveout inversion cannot be applied to the reservoir formation. Therefore, I performed parameter estimation for the UMV shale,
Figure 2.9: Seismic section across the middle of the survey area at Rulison field (after Xu & Tsvankin, 2007).

Figure 2.10: P-wave fold for the 16.8x16.8 m bin size at Rulison field (after Xu & Tsvankin, 2007). The rectangle in the center marks the study area of our paper.
the layer between Mesaverde Top and the top of the reservoir (Figure 2.8). In the center of the study area, the offset-to-depth ratio at the bottom of the shale is between 1.9 and 2.2. To estimate the interval moveout parameters, I used the VILS and Dix-type algorithms for layered orthorhombic media discussed above.

Our tests show that the NMO ellipticity is small for both the top and bottom of the target shale layer over most of the area. Therefore, the principal directions of the effective and interval NMO ellipses are poorly constrained by the data. However, as long as the offset-to-depth ratio is close to two, the parameters \( \eta^{(1,2,3)} \) can be estimated in a reliable fashion. The interval values of \( \eta^{(1,2,3)} \) for two superbin gathers near the center of the area are listed in Table 2.7.

Table 2.7: Interval parameters \( \eta^{(1,2,3)} \) estimated for two superbin gathers in the center of the study area at Rulison field.

<table>
<thead>
<tr>
<th>Superbin</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \eta^{(1)} )</td>
<td>( \eta^{(2)} )</td>
</tr>
<tr>
<td>VILS</td>
<td>0.38</td>
<td>0.47</td>
</tr>
<tr>
<td>Dix</td>
<td>0.74</td>
<td>1.24</td>
</tr>
</tbody>
</table>

Although there is no independent information about the actual anellipticity parameters in the field, the values produced by VILS are much more plausible than those computed from the Dix-type equations. First, the Dix-derived interval parameters \( \eta^{(1,2,3)} \) are too large for shale formations and lie outside the range suggested by laboratory and field studies (Tsvankin, 2005; Wang, 2002). The interval value of \( \eta^{(2)} \) for the first superbin even exceeds unity. Also, horizontal shale layers typically exhibit weak (if any) azimuthal anisotropy, unless they are fractured. Available geologic information for Rulison field and the small eccentricity of the NMO ellipses suggest that the symmetry of the UMV shale over most of the study area is close to VTI. This implies that the difference between the parameters \( \eta^{(1)} \) and \( \eta^{(2)} \), as well as the magnitude of \( \eta^{(3)} \), should be relatively small, which is in agreement with the output of VILS. The values of \( \eta^{(2)} \) for both superbins produced by the Dix-type method, however, are much larger than those of \( \eta^{(1)} \).
To test the stability of both methods, I also added a linear time error (from 4 ms at zero offset to -4 ms at the maximum offset for each azimuth) to the reflection moveout from the bottom of the shale layer in the second superbin. The interval parameters $\eta^{(1,2,3)}$ estimated by VILS change only by -0.06, -0.07, and 0.01, respectively, while the corresponding variations produced by the Dix-type method are much larger (-0.12, -0.21, and 0.13). Hence, VILS is more stable than the Dix-type method in the presence of correlated time errors, as was established above for the synthetic data.

The NMO ellipticity in the UMV shale is pronounced only near the east boundary of the study area (Xu & Tsvankin, 2007). Due to the small offset-to-depth ratio (between 1 and 1.3) for the bottom of the shale layer near the area boundary, the inverted parameters $\eta^{(1,2,3)}$ are unstable and are likely to contain large errors. The interval velocities $V^{(1,2)}_{nmo}$ and the azimuth $\varphi$ estimated by both methods for two adjacent superbin gathers in the area of substantial NMO ellipticity are listed in Table 2.8. As before, I added a linear time error (from 2 ms at zero offset to -2 ms at the maximum offset for each azimuth) to the traveltimes from the bottom of the shale in the second superbin, which causes a deviation of about 3% in the effective velocities $V^{(1,2)}_{nmo}$. As a result, the interval $V^{(1,2)}_{nmo}$ and $\varphi$ estimated by the Dix-type method change by 13%, 16% and 4°, respectively. The sensitivity of VILS to the time error is much lower, with the NMO velocities changing by less than 8% and $\varphi$ by 1°. Despite the superior performance by VILS, the errors in $V^{(1,2)}_{nmo}$ are relatively large because of the small thickness of the target layer (the thickness-to-depth ratio is about 0.2) and significant uncertainty in semblance analysis (the average semblance is about 0.4). Still, this result shows that it may be beneficial to apply VILS to interval NMO-velocity estimation from conventional-spread data.

### 2.5 Conclusions

I combined the velocity-independent layer-stripping method (VILS) with nonhyperbolic moveout inversion to estimate the interval parameters of VTI and orthorhombic media. While Dix-type differentiation algorithms operate with effective moveout parameters, VILS is
Table 2.8: Interval NMO ellipses for two superbin gathers near the east boundary of the study area.

<table>
<thead>
<tr>
<th>Superbin</th>
<th>( V_{nmo}^{(1)} ) (km/s)</th>
<th>( V_{nmo}^{(2)} ) (km/s)</th>
<th>( \phi ) (°)</th>
<th>( V_{nmo}^{(1)} ) (km/s)</th>
<th>( V_{nmo}^{(2)} ) (km/s)</th>
<th>( \phi ) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VILS</td>
<td>4.22</td>
<td>3.99</td>
<td>115</td>
<td>4.33</td>
<td>3.84</td>
<td>122</td>
</tr>
<tr>
<td>Dix</td>
<td>4.26</td>
<td>3.82</td>
<td>104</td>
<td>4.41</td>
<td>3.91</td>
<td>139</td>
</tr>
</tbody>
</table>

Based on layer stripping of reflection traveltimes. If the overburden is laterally homogeneous and has a horizontal symmetry plane, VILS produces exact interval traveltimes without any information about the velocity field. Then the interval traveltime function is inverted for the relevant parameters of the target layer using moveout equations for a homogeneous medium.

Because effective traveltimes are much better constrained by reflection data than effective moveout parameters, VILS gives more stable interval parameter estimates than Dix-type techniques. In particular, our synthetic tests on noise-contaminated data confirm that VILS can substantially increase the accuracy of nonhyperbolic moveout inversion for the interval time-processing parameter \( \eta \) in VTI media. The addition of small linear or sinusoidal time errors causes pronounced distortions in the effective \( \eta \) values, which get further enhanced by Dix-type layer stripping. In contrast, the interval moveout function produced by VILS is weakly sensitive to moderate levels of noise in the input traveltimes, which ensures a higher stability of the interval \( \eta \) estimates. Our tests show that VILS remains sufficiently accurate for VTI media in the presence of mild lateral heterogeneity (e.g., dips of up to 10°) in the overburden.

I also discussed an extension of VILS to 3D wide-azimuth P-wave data from azimuthally anisotropic models composed of orthorhombic and TI layers. To identify the target and overburden reflections that share the same ray segments, I obtain the horizontal slowness components from the best-fit effective moveout parameters, which helps to avoid direct differentiation of traveltimes and reduces the computational cost. Then the interval moveout produced by VILS is inverted for the azimuths of the vertical symmetry planes, symmetry-direction NMO velocities, and the anellipticity parameters \( \eta^{(1,2,3)} \). Wide azimuthal coverage
helps to increase the stability of \( \eta \) estimation using 3D Dix-type layer stripping. Still, numerical testing clearly demonstrates the higher accuracy of VILS for typical orthorhombic models, including those with the depth-varying azimuths of the vertical symmetry planes.

The 3D version of the method was also successfully tested on wide-azimuth P-wave reflections from an anisotropic shale layer at Rulison field in Colorado. For long-spread superbin gathers in the center of the study area, VILS yields more plausible and stable values of the interval parameters \( \eta^{(1,2,3)} \) than the Dix-type method. Near the east boundary of the study area, where the offset-to-depth ratio is smaller and the \( \eta \)-parameters are poorly constrained, application of VILS helps to obtain a better estimate of the interval NMO ellipse.

It should be mentioned that the superior accuracy of VILS is achieved at the expense of its somewhat higher (compared to the Dix-type algorithms) computational cost. In addition to matching reflection time slopes at the surface, it is necessary to carry out nonhyperbolic moveout inversion not only for recorded reflection events, but also for the interval moveout function. However, our implementation of VILS for horizontally layered media is much more efficient than the general version of the method because time slopes are calculated directly from the moveout parameters.
CHAPTER 3
STACKING-VELOCITY INVERSION WITH BOREHOLE CONSTRAINTS FOR
TILTED TI MEDIA

Transversely isotropic models with a tilted symmetry axis (TTI) play an increasingly
important role in seismic imaging, especially near salt bodies and in active tectonic areas.
Here, I present a 2D parameter-estimation methodology for TTI media based on combining
P-wave normal-moveout (NMO) velocities, zero-offset traveltimes, and reflection time slopes
with borehole data that include the check-shot traveltimes as well as the reflector depths
and dips.

For a dipping TTI layer with the symmetry axis confined to the dip plane of the reflector,
simultaneous estimation of the symmetry-direction velocity $V_{P0}$, the anisotropy parameters
$\epsilon$ and $\delta$, and the tilt $\nu$ of the symmetry axis proves to be ambiguous despite the borehole
constraints. If the symmetry axis is orthogonal to the reflector, $V_{P0}$ and $\delta$ can be recovered
with high accuracy, even when the symmetry axis deviates by $\pm 5^\circ$ from the reflector normal.
The parameter $\epsilon$, however, cannot be constrained for dips smaller than $60^\circ$ without using
nonhyperbolic moveout.

To invert for the interval parameters of layered TTI media, I apply 2D stacking-velocity
inversion supplemented with the same borehole constraints. The dip planes in all layers
are assumed to be aligned, and the symmetry axis is set orthogonal to the reflector in
each layer. Information about reflector dips can be replaced with near-offset walkaway VSP
(vertical seismic profiling) traveltimes. Tests on noise-contaminated data demonstrate that
the algorithm produces stable estimates of the interval parameters $V_{P0}$ and $\delta$, if the range of
dips does not exceed $30^\circ$. Our method can be used to build an accurate initial TTI model for
post-migration reflection tomography and other techniques that employ migration velocity
analysis.
3.1 Introduction

Ignoring anisotropy in P-wave processing causes imaging and interpretation errors, such as mispositioning of horizontal and dipping reflectors (e.g., Alkhalifah & Larner, 1994; Alkhalifah et al., 1996; Vestrum et al., 1999). While many widely used migration algorithms have been extended to transversely isotropic (TI) media, constructing an accurate anisotropic velocity model remains a challenging problem. For TI models with a vertical symmetry axis (VTI), the depth-domain P-wave velocity field is controlled by the vertical velocity \( V_{P0} \) and the Thomsen (1986) parameters \( \epsilon \) and \( \delta \). To resolve all three parameters individually, P-wave moveout typically has to be combined with borehole data or shear modes (SS- or PS-waves) (Sexton & Williamson, 1998; Tsvankin & Grechka, 2000).

Vertical transverse isotropy has proved to be adequate for most horizontally stratified, unfractured sediments. However, in progradational clastic or carbonate sequences, as well as in the presence of obliquely dipping fractures, the symmetry axis is tilted (Figure 3.1). Also, transverse isotropy with a tilted symmetry axis (TTI) is an appropriate model for dipping shale layers near salt domes and in fold-and-thrust belts such as the Canadian Foothills (Behera & Tsvankin, 2009; Charles et al., 2008; Huang et al., 2008; Isaac & Lawton, 1999; Vestrum et al., 1999). The parameters \( V_{P0}, \epsilon, \) and \( \delta \) for TTI media are defined in the rotated coordinate system with respect to the symmetry axis, whose orientation is described by the tilt angle \( \nu \) with the vertical and the azimuth \( \beta \).

In principle, the symmetry-axis orientation and the interval parameters \( V_{P0}, \epsilon, \) and \( \delta \) of a TTI layer can be estimated from wide-azimuth P-wave data (Grechka & Tsvankin, 2000), if the medium is not close to elliptical (i.e., \( \epsilon \neq \delta \)). Stable inversion, however, requires at least two NMO ellipses from interfaces with different orientations (e.g., a horizontal and a dipping reflector). Also the tilt \( \nu \) of the symmetry axis has to exceed 30° and the reflector dip \( \phi \) should be between 30° and 80° (Grechka & Tsvankin, 2000). If shear data are available, the addition of the SV-wave NMO ellipse from a horizontal reflector helps increase the inversion accuracy and makes parameter estimation possible for elliptically anisotropic media. Still, combining
horizontal SV-wave events with P-wave data does not remove the above constraints on $\nu$ and $\phi$ (Grechka & Tsvankin, 2000).

Grechka et al. (2002a) develop a multicomponent inversion algorithm for interval parameter estimation in layered TI media using wide-azimuth PP and PS (or SS) reflection data. For relatively large tilt angles $\nu$ and reflector dips, multicomponent, multiazimuth reflection data can be used to build anisotropic models for depth processing. However, parameter estimation is still ambiguous for a wide range of small and moderate angles $\nu$ and $\phi$ (Figure 3.2), mainly because of the multimodal nature of their misfit function.

To carry out parameter estimation for a horizontal TTI layer, Dewangan & Tsvankin (2006a) apply the PP+PS=SS method (Grechka & Dewangan, 2003; Grechka & Tsvankin, 2002b) to reflection traveltimes of PP- and PS-waves. They implement nonlinear inversion of the NMO velocities and zero-offset traveltimes of the recorded PP-waves and computed SS-waves combined with the moveout-asymmetry attributes of the PS(PSV)-wave.$^{1}$ The method of Dewangan & Tsvankin (2006a) remains accurate for a wide range of tilts, except

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$^{1}$The moveout of PS-waves is asymmetric if the traveltime does not stay the same when the source and receiver are interchanged. In a horizontal TTI layer, this asymmetry is caused by the tilt of the symmetry axis.
Figure 3.2: Illustration of the uniqueness of depth-domain parameter estimation for TI media using wide-azimuth, multicomponent data (after Grechka et al., 2002a). The tilt and azimuth of the symmetry axis are assumed to be unknown, even for VTI and HTI models.

for “quasi-VTI” models with $\nu < 10^\circ$.

In a sequel paper, Dewangan & Tsvankin (2006b) extend this algorithm to a dipping TTI layer with the symmetry axis orthogonal to the layer’s bottom. In that model, the moveout asymmetry of PS-waves is caused not just by the tilted symmetry axis, but also by the reflector dip. Despite the fixed axis orientation, parameter estimation is stable only for significant tilts ($\nu > 30^\circ - 40^\circ$), with the anisotropy parameter $\delta$ constrained more tightly than $\epsilon$.

Although PS-waves provide valuable information for velocity model building in the depth domain, they are not routinely acquired in exploration. Also, processing of modeconverted data is much more difficult than that of pure PP reflections due to such inherent features of PS-waves as the raypath and moveout asymmetry, polarity reversals, and low amplitudes at small offsets. Also, it is often challenging to identify PP and PS events from the same interface because of their different reflectivities and of the depth-varying $V_P/V_S$ ratio.
Here, I present a 2D inversion methodology for a stack of homogeneous TTI layers based on combining conventional-spread P-wave moveout with borehole information. P-wave NMO velocities, reflection slopes, and zero-offset traveltimes are supplemented with check-shot traveltimes and reflector depths and dips. First, I introduce a semi-analytic inversion procedure for a single TTI layer above a dipping interface and show that the medium parameters cannot be resolved without constraining the tilt of the symmetry axis. Then I develop joint inversion of moveout and borehole data for a stack of TTI layers with the symmetry axis orthogonal to the lower boundary of each layer. Whereas the reflector depths have to be known, dip information can be replaced with VSP traveltimes for nonzero offsets. Synthetic tests with a realistic level of Gaussian noise illustrate the stability of estimating the interval parameters $V_P^0$, $\epsilon$, and $\delta$.

### 3.2 Inversion for a single TTI layer

I start by considering the simple model of a homogeneous TTI layer above a plane dipping reflector. To make the problem 2D, the symmetry axis is assumed to be confined to the dip plane (Figure 3.3). The tilt angle $\nu$ is taken positive, if the symmetry axis is rotated counterclockwise from the vertical. P-wave surface data provide the zero-offset reflection time $t_0$, the reflection slope (horizontal slowness) $p$ on the zero-offset time section, and the NMO velocity $V_{\text{nmo}}$. Because the layer is homogeneous, $t_0$, $p$, and $V_{\text{nmo}}$ can be estimated for a single common midpoint (CMP). It is assumed that the depth and dip of the reflector are measured at a borehole location along with the P-wave group velocity obtained from check shots.

#### 3.2.1 Arbitrary axis orientation

##### 3.2.1.1 Inversion methodology

The exact P-wave phase-velocity function in TI media expressed through the Thomsen parameters is given by (Tsvankin, 1996, 2005)
Figure 3.3: Dipping TTI layer with the CMP at the head of a vertical borehole. The arrow marks the symmetry axis; the reflector dip is $\phi_b$. a) AB is the zero-offset raypath; the phase and group angles of the zero-offset ray with the vertical are $\phi_b$ and $\tilde{\psi}_0$, respectively. b) The phase-velocity vector of the vertical (check-shot) ray makes the angle $\tilde{\theta}_{cs}$ with the vertical.

\[
\frac{V^2}{V_{P_0}^2} = 1 + \epsilon \sin^2 \theta - \frac{f}{2} + \frac{f}{2} \sqrt{1 + \frac{4 \sin^2 \theta (2 \delta \cos^2 \theta - \epsilon \cos 2\theta) + 4 \epsilon^2 \sin^4 \theta}{f^2}},
\]

(3.1)

where $\theta$ is the phase angle with the symmetry axis (assumed to be positive for counterclockwise rotation), $V_{P_0}$ is the symmetry-direction velocity, and

\[
f \equiv 1 - \frac{V_{S_0}^2}{V_{P_0}^2};
\]

(3.2)

$V_{S_0}$ is the symmetry-direction velocity of S-waves. Because the influence of $V_{S_0}$ on P-wave kinematics is negligible, the value of $f$ can be set to a constant using a typical $V_{P_0}/V_{S_0}$ ratio (e.g., $V_{P_0}/V_{S_0} = 2$). Therefore, the phase velocity $V$ represents a function of the four medium parameters ($V_{P_0}$, $\epsilon$, $\delta$, and $\nu$) and the phase angle $\tilde{\theta}$ with the vertical:

\[
V = f_1(V_{P_0}, \epsilon, \delta, \theta) = f_1(V_{P_0}, \epsilon, \delta, \tilde{\theta}, \nu),
\]

(3.3)

where $\theta = \tilde{\theta} - \nu$.

For the zero-offset reflection, the phase-velocity (slowness) vector is perpendicular to the reflector, and the phase angle with the vertical $\tilde{\theta}$ is equal to the dip $\phi_b$ (Figure 3.3a; the
subscript “b” denotes borehole data). The phase velocity for the zero-offset reflection can be computed through the known values of $\phi_b$ and $p$ as

$$V_{\phi_b} = \frac{\sin \phi_b}{p}.$$  \hfill (3.4)

Substituting equation 3.4 into equation 3.3 yields

$$f_1(V_{P0}, \epsilon, \delta, \phi_b, \nu) = \frac{\sin \phi_b}{p}. \hfill (3.5)$$

The P-wave group velocity $V_G$ in TI media can be found as a function of the phase velocity $V$ and its derivative with respect to $\theta$ (e.g., Tsvankin, 2005):

$$V_G = V \sqrt{1 + \left(\frac{1}{V} \frac{dV}{d\theta}\right)^2}. \hfill (3.6)$$

Therefore, $V_G$ represents a function (different from $f_1$) of the parameters $V_{P0}$, $\epsilon$, $\delta$, $\tilde{\theta}$, and $\nu$:

$$V_G = f_2(V_{P0}, \epsilon, \delta, \tilde{\theta}, \nu). \hfill (3.7)$$

The P-wave group angle $\psi$ with the symmetry axis is also controlled by the angle-dependent phase velocity (e.g., Tsvankin, 2005):

$$\tan \psi = \tan \theta + \frac{1}{V} \frac{dV}{d\theta}. \hfill (3.8)$$

Hence, the angle $\tilde{\psi}$ with the vertical in a TTI layer can be written as

$$\tilde{\psi} = f_3(V_{P0}, \epsilon, \delta, \tilde{\theta}, \nu). \hfill (3.9)$$

For the zero-offset reflection, the phase angle $\tilde{\theta} = \phi_b$ (Figure 3.3a), so

$$V_{G0} = f_2(V_{P0}, \epsilon, \delta, \phi_b, \nu), \hfill (3.10)$$

and the group angle with the vertical is

$$\tilde{\psi}_0 = f_3(V_{P0}, \epsilon, \delta, \phi_b, \nu). \hfill (3.11)$$

The length of the zero-offset raypath (AB in Figure 3.3a) can be calculated from the vertical thickness $z_b$ of the layer measured from the borehole and the angles $\phi_b$ and $\tilde{\psi}_0$ (Figure 3.3a). AB can also be expressed through the two-way zero-offset reflection time $t_0$ and the group
velocity given by equation 3.10:

\[
\frac{z_b \cos \phi_b}{
\cos(\tilde{\psi}_0 - \phi_b)
\} = \frac{V_{G0} t_0}{2},
\]

(3.12)

\(\tilde{\psi}_0\) is found from equation 3.11. Note that if the CMP is displaced from the well by a known distance, equation 3.12 can be modified accordingly. Hence, I have constructed two equations (3.5 and 3.12) for the four unknown parameters.

I assume that the check-shot ray is vertical (i.e., its group angle with the vertical is zero), but the corresponding phase angle \(\tilde{\theta}_{cs}\) is unknown (Figure 3.3b; the subscript “cs” denotes check-shot). Applying equations 3.7 and 3.9 to the vertical ray gives

\[
f_2(V_{P0}, \epsilon, \delta, \tilde{\theta}_{cs}, \nu) = V_{G,cs},
\]

(3.13)

\[
f_3(V_{P0}, \epsilon, \delta, \tilde{\theta}_{cs}, \nu) = 0.
\]

(3.14)

The pure-mode NMO velocity for 2D wave propagation in a vertical symmetry plane of a homogeneous layer can be obtained as the following function of the phase velocity \(V(\theta)\) and reflector dip \(\phi\) (Tsvankin, 2005):

\[
V_{nmo}(\phi) = \frac{V(\phi)}{\cos \phi} \sqrt{1 + \frac{1}{V(\phi)} \frac{d^2V}{d\theta^2} \bigg|_{\theta=\phi}}.
\]

(3.15)

In a dipping TTI layer (Figure 3.3), the phase velocity and its derivatives in equation 3.15 should be computed at the phase angle \(\theta_0 = \phi_b - \nu\) with the symmetry axis. Alternatively, it is possible to obtain \(V_{nmo}\) as a function of the known reflection slope \(p\). Therefore, equation 3.15 provides another constraint on the medium parameters:

\[
V_{nmo} = f_4(V_{P0}, \epsilon, \delta, \phi_b, \nu).
\]

(3.16)

Therefore, the input data provide five equations (3.5, 3.12, 3.13, 3.14, and 3.16) to be inverted for the four TTI parameters \((V_{P0}, \epsilon, \delta, \nu)\) and the phase angle \(\tilde{\theta}_{cs}\) corresponding to the check-shot ray:
\[ f_1(V_{P0}, \epsilon, \delta, \phi_b, \nu) = \frac{\sin \phi_b}{p}; \quad (3.17) \]
\[ f_2(V_{P0}, \epsilon, \delta, \phi_b, \nu) \cos [f_3(V_{P0}, \epsilon, \delta, \phi_b, \nu) - \phi_b] = \frac{2z_b \cos \phi_b}{t_0}; \quad (3.18) \]
\[ f_2(V_{P0}, \epsilon, \delta, \tilde{\theta}_{cs}, \nu) = V_{G,cs}; \quad (3.19) \]
\[ f_3(V_{P0}, \epsilon, \delta, \tilde{\theta}_{cs}, \nu) = 0; \quad (3.20) \]
\[ f_4(V_{P0}, \epsilon, \delta, \phi_b, \nu) = V_{nmo}. \quad (3.21) \]

For VTI media (i.e., \( \nu = 0^\circ \)), \( V_{P0} \) is obtained directly from check shots because for a vertical borehole \( V_{P0} = V_{G,cs} \); then \( \epsilon \) and \( \delta \) are found from equations 3.17 and 3.21. Even if the dip is unknown, the parameters \( \epsilon, \delta, \) and \( \phi_b \) can be estimated from equations 3.17, 3.18, and 3.21. Here, however, I concentrate on the inversion for a nonzero tilt \( \nu \).

### 3.2.1.2 Synthetic example

Although the number of equations is equal to the number of unknowns, equations 3.17–3.21 form a nonlinear system, which is not guaranteed to have a unique solution. To evaluate the feasibility of the inversion, I computed the input data (\( p, t_0, V_{nmo}, \) and \( V_{G,cs} \)) from the exact equations and contaminated them by Gaussian noise with the standard deviations equal to 1\% for \( p \) and \( t_0 \), and 2\% for \( V_{nmo} \) and \( V_{G,cs} \). The reflector dip \( \phi_b \) and depth \( z_b \) were assumed to be known exactly, and the starting model was isotropic (i.e., \( \epsilon = \delta = 0 \)). Table 3.1 shows the inversion results for typical TTI parameters using 100 realizations of the input data. Despite the borehole constraints, the inversion proves to be highly unstable, with small errors in the data producing large distortions in the estimated parameters. This instability is partially caused by the nonlinear dependence of the phase velocity \( V \) on the tilt \( \nu \) (Grechka et al., 2002a). Similar results were obtained for a wide range of model parameters.

### 3.2.2 Symmetry axis orthogonal to the reflector

If TTI symmetry is associated with dipping shale layers, the symmetry axis is typically assumed to be orthogonal to the layer boundaries (Charles et al., 2008; Isaac & Lawton,
Table 3.1: Actual and estimated parameters of a homogeneous TTI layer. The dip and depth of the reflector are assumed to be known. The input data are contaminated by Gaussian noise with the standard deviations equal to 1% for \( p \) and \( t_0 \), and 2% for \( V_{nmo} \) and \( V_{G,cs} \). The mean values and standard deviations of the inverted parameters are denoted by “mean” and “sd”, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Estimated</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>sd</td>
<td>mean</td>
<td>sd</td>
</tr>
<tr>
<td>( V_{P0} ) (km/s)</td>
<td>2.50</td>
<td>2.92</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>0.25</td>
<td>0.15</td>
<td>0.77</td>
<td></td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.10</td>
<td>-0.14</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>( \nu ) (°)</td>
<td>50</td>
<td>-20</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>Dip ( \phi_b ) (°)</td>
<td>30</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Depth ( z_b ) (km)</td>
<td>1</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

1999; Vestrum et al., 1999). Fixing the orientation of the symmetry axis helps mitigate the nonuniqueness of the inversion procedure (Behera & Tsvankin, 2009; Grechka et al., 2002a; Zhou et al., 2008).

### 3.2.2.1 Inversion methodology

If the symmetry axis is orthogonal to the reflector, the tilt \( \nu \) is equal to the reflector dip \( \phi_b \) measured in the borehole. Also, the phase-velocity vector of the zero-offset reflection is parallel to the symmetry axis, and the velocity \( V_{P0} \) can be obtained directly from surface data and the dip \( \phi_b \) (equation 3.17):\[ V_{P0} = \frac{\sin \phi_b}{p}. \] (3.22)

The NMO velocity (equation 3.21) for \( \nu = \phi_b \) is given by the isotropic cosine-of-dip relationship (Tsvankin, 2005):\[ V_{nmo} = \frac{V_{nmo}(0)}{\cos \phi_b}, \] (3.23)

where \( V_{nmo}(0) = V_{P0} \sqrt{1 + 2\delta} \). Since \( V_{P0} \) is already known, equation 3.23 constrains the parameter \( \delta \).
Because the group and phase velocities in the symmetry direction coincide, equation 3.18 includes only known quantities and can be used to check the validity of the model. Therefore, the inverse problem reduces to estimating the parameters $\epsilon$ and $\tilde{\theta}_{cs}$ from the vertical group velocity (i.e., from equations 3.19 and 3.20):

\[
\begin{align*}
    f_2(V_{P0}, \epsilon, \delta, \tilde{\theta}_{cs}, \phi_b) &= V_{G,cs} ; \\
    f_3(V_{P0}, \epsilon, \delta, \tilde{\theta}_{cs}, \phi_b) &= 0 .
\end{align*}
\]

When $\nu = \phi_b$, the inversion equations do not include $z_b$ and are independent of the CMP location. Moreover, if the check-shot ray is not exactly vertical but its inclination is known, equations 3.24 and 3.25 retain the same form with a nonzero group angle on the right-hand side of equation 3.25.

### 3.2.2.2 Synthetic examples

First, I perform a test on noise-contaminated data for a model with $\nu = \phi_b = 30^\circ$ and typical values of the Thomsen parameters (Figure 3.4a). The parameters $V_{P0}$ and $\delta$ can be estimated with high accuracy because they are well constrained by our data; the mean value of $V_{P0}$ is 2.50 km/s with the standard deviation 1%; the mean value of $\delta$ is 0.10 with the standard deviation 0.03. However, the parameter $\epsilon$ is practically unconstrained (the standard deviation is 1.37). The instability in estimating $\epsilon$ can be explained using the linearized weak-anisotropy approximation. For weak anisotropy, the magnitudes of the phase- and group-velocity vectors coincide (Thomsen, 1986), and for the vertical ray

\[
V_{G,cs} = V_{P0} (1 + \delta \sin^2 \phi_b \cos^2 \phi_b + \epsilon \sin^4 \phi_b) .
\]

For moderate dips, such as $\phi_b = 30^\circ$ used in the test, the contribution of $\epsilon$ to $V_{G,cs}$ is much smaller than that of $\delta$ because $\epsilon$ is multiplied with $\sin^4 \phi_b$. As a result, the objective function has multiple local minima for $\epsilon$ that hamper the convergence of the algorithm.

The estimates of $V_{P0}$ and $\delta$ are sufficiently accurate for a wide range of dips, with small (and practically constant) standard deviations (Table 3.2). The errors in the parameter $\epsilon$,
Figure 3.4: Inversion results (dots) for a TTI layer with the symmetry axis orthogonal to its bottom. The inversion was carried out for 1000 realizations of input data contaminated by Gaussian noise with the standard deviations equal to 1% for the reflection slope $p$ and 2% for $V_{nmo}$ and $V_{G,cs}$. Due to the large standard deviation (1.37) of $\epsilon$, the vertical axis on plot (b) is clipped. The actual parameter values are marked by the crosses. The starting model was isotropic.

Table 3.2: Inversion results for a TTI layer with the symmetry axis perpendicular to its bottom (i.e., the tilt is equal to the dip). The medium parameters are $V_{P0} = 2.50$ km/s, $\delta = 0.10$, and $\epsilon = 0.25$. The data are contaminated by Gaussian noise with the standard deviations equal to 1% for $p$, and 2% for $V_{nmo}$ and $V_{G,cs}$.

<table>
<thead>
<tr>
<th>Dip $\phi_b$ ($^\circ$)</th>
<th>$V_{P0}$ mean (km/s)</th>
<th>$\delta$ mean</th>
<th>$\epsilon$ mean sd</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sd (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2.50</td>
<td>0.10</td>
<td>0.03</td>
</tr>
<tr>
<td>30</td>
<td>2.50</td>
<td>0.10</td>
<td>0.03</td>
</tr>
<tr>
<td>50</td>
<td>2.50</td>
<td>0.10</td>
<td>0.03</td>
</tr>
<tr>
<td>70</td>
<td>2.50</td>
<td>0.10</td>
<td>0.03</td>
</tr>
</tbody>
</table>
When the symmetry axis is not orthogonal to the reflector, the algorithm based on setting $\nu = \phi_b$ produces errors in the inverted parameters. However, for typical moderate magnitudes of $|\epsilon|$ and $|\delta|$ ($|\epsilon| \leq 0.5; |\delta| \leq 0.3$), the errors in $V_{P0}$ and $\delta$ remain small, if the symmetry axis deviates from the reflector normal by less than 5° and the dip ranges from 5° to 50° (Table 3.3). For example, I computed the input data with the actual tilt $\nu = 15°$ and dip $\phi_b = 20°$, then obtained the parameters $V_{P0} = 2.5$ km/s and $\delta = 0.11$ under the assumption that $\nu = \phi_b = 20°$. The inversion results become more distorted for strong anisotropy and/or large dips because the value of $(\sin \phi_b/p)$ differs more significantly from the actual symmetry-direction velocity $V_{P0}$, and the errors are amplified in the inversion of the NMO velocity for $\delta$.

Table 3.3: Inverted values of $\delta$ for a TTI layer with the symmetry axis deviating from the reflector normal by $\pm 5°$. The parameters $V_{P0}$ and $\delta$ are obtained under the assumption that the symmetry axis is orthogonal to the reflector. The input data are contaminated by Gaussian noise with the standard deviations equal to 1% for $p$, and 2% for $V_{nmo}$ and $V_G,cs$. The mean values of $V_{P0}$ are close to 2.50 km/s, and the standard deviation to 1%.

<table>
<thead>
<tr>
<th>Dip $\phi_b$ (°)</th>
<th>$\nu$ (°)</th>
<th>$\delta$</th>
<th>mean</th>
<th>sd</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
<td>0.11</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>0.10</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>15</td>
<td>0.11</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>25</td>
<td>0.10</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
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<td>0.12</td>
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<td></td>
</tr>
<tr>
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<td>0.09</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>55</td>
<td>0.15</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>65</td>
<td>0.07</td>
<td>0.03</td>
<td></td>
</tr>
</tbody>
</table>

3.3 Inversion for layered TTI media

In the previous section I demonstrated the feasibility of 2D inversion of P-wave moveout and borehole measurements for the parameters of a single TTI layer with the symmetry axis orthogonal to its bottom. Here, I present an algorithm for interval parameter estimation in layered TTI media using reflection and borehole data.
The model is composed of homogeneous TTI layers separated by plane dipping boundaries with the same azimuth of the dip plane. The symmetry axis in each layer is perpendicular to its bottom, which makes wave propagation two-dimensional. The model vector for an $N$-layered medium contains $3N$ unknowns:

$$\tilde{m} = \{V^{(n)}_P, \epsilon^{(n)}, \delta^{(n)}\}, \quad (n = 1, 2, \ldots, N). \quad (3.27)$$

The data vector has the form

$$\tilde{d} = \{t_0(n), p(n), V_{nmo}^{(n)}, z_b^{(n)}, \phi_b^{(n)}, t_{cs}(m)\},$$

$$\quad (n = 1, 2, \ldots, N), \quad (m = 1, 2, \ldots, M), \quad (3.28)$$

where $t_0(n), p(n),$ and $V_{nmo}^{(n)}$ are the effective values for the $n$-th reflector measured from reflection data, $z_b^{(n)}$ and $\phi_b^{(n)}$ are the depth and dip of the $n$-th reflector, respectively, at the borehole location, and $t_{cs}(m)$ is the check-shot traveltime for the $m$-th receiver placed in the borehole. The tilt $\nu^{(n)}$ in each layer is equal to the dip $\phi_b^{(n)}$ of the layer’s bottom.

### 3.3.1 Inversion methodology

This algorithm generalizes the inversion scheme for a single TTI layer discussed above, and represents a modification of P-wave stacking-velocity tomography introduced for VTI media by Grechka et al. (2002b). The model geometry can be fully reconstructed from the known depths and dips of the interfaces. Then for a trial set $\tilde{m}$ of the interval parameters (equation 3.27), I trace the zero-offset ray from an arbitrary point on the $n$-th reflector up to the surface. The slowness vector at the reflection point is perpendicular to the interface. Because the model is composed of homogeneous layers separated by plane interfaces, the zero-offset rays for different CMP locations have the same slowness vector in each layer. Therefore, the calculated horizontal slowness $p^{\text{calc}}(n)$ of the zero-offset ray at the surface can be used to fit the measurements $p(n)$. Then I trace the zero-offset ray downward from the source location using the slowness components computed in each layer (but with the opposite sign), and calculate the zero-offset traveltimes $t_0^{\text{calc}}(n)$. Also, the NMO velocities $V_{nmo}^{\text{calc}}(n)$ are
obtained using the Dix-type averaging equations for piecewise–homogeneous media (Grechka et al., 2002b).

For check-shot data, I employ one receiver per layer located close to the layer’s bottom so that the interval traveltime can be estimated with sufficient accuracy. Using the trial interval parameters and the known model geometry, I trace the check-shot ray for a specific receiver, which yields the traveltime $t_{cs}^{\text{calc}}(n)$. Note that check-shot data for multilayered media do not directly constrain the interval group velocity (equation 3.24). Fitting the traveltimes $t_{cs}(n)$ is equivalent to solving equations 3.24 and 3.25 for a single layer because the phase angle $\tilde{\theta}_{cs}$ for each ray is found from the trial medium parameters.

The interval parameters $\{V_p^{(n)}, \epsilon^{(n)}, \delta^{(n)}\}$ are estimated by minimizing the objective function that contains the differences between the calculated and measured quantities:

$$
\mathcal{F}(\vec{m}) \equiv \sum_{n=1}^{N} \left( \frac{||p^{\text{calc}}(n) - p(n)||^2}{\sigma^2[p(n)]} + \frac{||t_0^{\text{calc}}(n) - t_0(n)||^2}{\sigma^2[t_0(n)]} \right) + \frac{||V_{\text{nmo}}^{\text{calc}}(n) - V_{\text{nmo}}(n)||^2}{\sigma^2[V_{\text{nmo}}(n)]} + \frac{||t_{cs}^{\text{calc}}(n) - t_{cs}(n)||^2}{\sigma^2[t_{cs}(n)]} ,
$$

where $\sigma^2$ represents the variance of each measurement. Grechka et al. (2002b) fit only P-wave NMO ellipses in their objective function for VTI media, because their input data do not include borehole information. Our algorithm operates with 2D data, so I use a single NMO velocity instead of the three parameters of the NMO ellipse. However, I assume to know the model geometry and have check-shot traveltimes.

The depths $z_b^{(n)}$ and dips $\phi_b^{(n)}$ are not included in the objective function because they are used to compute the other quantities; also, the dip $\phi_b^{(n)}$ helps constrain the tilt $\nu^{(n)}$ of the symmetry axis in each layer. For a single layer, the parameter $\epsilon$ is obtained from the group velocity (equations 3.24 and 3.25). However, for layered TTI media, the interval parameter $\epsilon$ also contributes to $p(n)$, $t_0(n)$, and $V_{\text{nmo}}(n)$ (except for $n = 1$). Therefore, although $\epsilon^{(n)}$ is not expected to be well constrained, it has to be estimated together with $V_p^{(n)}$ and $\delta^{(n)}$ using equation 3.29.
It should be emphasized that I invert for all interval parameters simultaneously without employing layer stripping. This feature of the algorithm helps mitigate error accumulation with depth and is particularly beneficial when the model includes a layer at depth that is known to be isotropic. Then, as demonstrated by Grechka et al. (2001) on physical-modeling data for a bending TTI thrust sheet, the reflection from the bottom of the isotropic layer provides valuable constraints on the parameters of the TTI overburden.

As the previous implementations of stacking-velocity tomography, our algorithm assumes each layer to be homogeneous. The influence of lateral velocity variation on the inversion results is expected to be relatively minor because the maximum offset of reflection data can be limited by the reflector depth. Ignoring the vertical velocity gradient between reflectors typically causes overestimation of the parameter \( \delta \) (Grechka & Tsvankin, 2002a). The purpose of our algorithm, however, is to provide a simple tool for building an initial velocity model that can be refined using reflection tomography or other methods operating in the migrated domain (Bakulin et al., 2010c; Woodward et al., 2008).

### 3.3.2 Synthetic example

The algorithm was tested on models with up to four TTI layers with dips ranging from \( 0^\circ \) to \( 60^\circ \) and the anisotropy parameters varying within the plausible range (from zero to 0.5 for \( \epsilon \) and from \(-0.2 \) to \( 0.3 \) for \( \delta \)). The results of a typical test for a three-layer medium (Figure 3.5 and Table 3.4) are shown in Figure 3.6. The inversion is performed for 200 realizations of noise-contaminated input data using the value of each measurement as its variance \( \sigma^2 \) in equation 3.29. Since the dips are moderate, estimation of the interval parameter \( \epsilon \) is unstable, while \( V_{P0} \) and \( \delta \) can be recovered with sufficiently high accuracy. The standard deviations of the estimated parameters are higher in the third layer (about 3\% for \( V_{P0} \) and 0.06 for \( \delta \)). This reduction in accuracy is related primarily to error accumulation with depth and the smaller contribution of the deeper layers to the effective reflection traveltimes. Our tests indicate that the thickness-to-depth ratio of a layer at the borehole location should be at least 25\% to ensure reliable estimates of the interval parameters.
Figure 3.5: Three-layer TTI model used to test the inversion algorithm. The input data for parameter estimation are computed by anisotropic ray tracing. a) The dips are $\phi_b^{(1)} = 10^\circ$, $\phi_b^{(2)} = 30^\circ$, and $\phi_b^{(3)} = 20^\circ$. The reflector depths at the borehole location are $z_b^{(1)} = 1$ km, $z_b^{(2)} = 2$ km, and $z_b^{(3)} = 3$ km. The check-shot source is located 10 m to the right of the borehole; the receivers (marked by triangles) are placed at the intersection of the borehole with each reflector. b) The check-shot (dashed) and zero-offset (dotted) rays for the third reflector.

Table 3.4: Interval parameters of the three-layer TTI model from Figure 3.5. The symmetry axis in each layer is orthogonal to its lower boundary. The input data are distorted by Gaussian noise with the standard deviations equal to 1% for $p(n)$, $t_0(n)$, and $t_{cs}(n)$, and 2% for $V_{nmo}$.

<table>
<thead>
<tr>
<th></th>
<th>Layer 1</th>
<th>Layer 2</th>
<th>Layer 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{P_0}$ (km/s)</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.10</td>
<td>0.20</td>
<td>0.25</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-0.10</td>
<td>0.10</td>
<td>0.12</td>
</tr>
<tr>
<td>$\nu$ ($^\circ$)</td>
<td>10</td>
<td>30</td>
<td>20</td>
</tr>
</tbody>
</table>
Figure 3.6: a) Interval symmetry-direction velocities $V_{P0}^{(n)}$ and b) anisotropy parameters $\delta^{(n)}$ ($n = 1, 2, 3$) estimated by our algorithm for the model from Figure 3.5 and Table 3.4. The dots mark the mean values, and the bars correspond to the $\pm$ standard deviation in each parameter.

If the maximum difference between the reflector dips exceeds $30^\circ$, the errors in the interval parameter $\delta$ rapidly increase for the deeper layers. This happens mainly because the zero-offset and check-shot traveltimes depend not only on the interval parameters $V_{P0}$ and $\delta$, but also on the values of $\epsilon$ in the overburden. This influence of the interval $\epsilon$ becomes more significant for models with a wide range of dips. Since $\epsilon$ is not well constrained by the input data, its contribution to the objective function increases errors in $\delta$.

### 3.3.3 Inversion without dip information

If accurate dip measurements are not available, it may still be possible to obtain stable estimates of $V_{P0}$ and $\delta$, as well as of the dip itself. In a single TTI layer with the symmetry axis orthogonal to its bottom, $V_{P0}$ cannot be obtained from the time slope $p$ if the dip $\phi$ (here I do not use the subscript “b”) is unknown (equation 3.22). However, the reflector depth $z_b$ measured in the borehole provides a second relationship between $V_{P0}$ and $\phi$:

$$z_b = \frac{t_0 V_{P0}}{2 \cos \phi}.$$  \hspace{1cm} (3.30)
Equation 3.30 represents a simplified form (valid for \( \nu = \phi \)) of equation 3.18. Equations 3.22 and 3.30 can be solved for \( V_{P0} \) and \( \phi \), which allows us to find \( \delta \) from the NMO velocity (equation 3.23). Synthetic tests confirm that our algorithm yields accurate values of the parameters \( V_{P0} \), \( \delta \), and \( \phi \) for a single layer.

However, estimation of the interval parameters \( V_{P0} \) and \( \delta \) for layered TTI models with unknown interface dips requires additional information. One practical option is to include walkaway VSP (vertical seismic profiling) traveltimes to increase the angle coverage of the input data. Extensive numerical testing shows that it is sufficient to add two VSP sources placed on both sides of the borehole at a distance that reaches at least 1/5 of the largest reflector depth. I reproduced the test for the model in Figure 3.5 and Table 3.4 with the input data vector (see equation 3.28) that did not include the dip information. Instead, I added the noise-contaminated VSP traveltimes (\( t_{\text{vsp}} \)) for two sources located \( \pm 0.6 \) km to the left and right of the borehole, so that the input data were comprised of \( V_{\text{nmo}} \), \( p \), \( t_0 \), \( t_{cs} \), and \( t_{\text{vsp}} \). The results are similar to those in Figure 3.6, with the standard deviation of the interval \( \delta \)-values changing by less than 0.01.

To simulate an overthrust structure typical for such areas as the Canadian Foothills, I construct a model that includes a TTI layer with parallel dipping boundaries embedded in a homogeneous, isotropic background (Figure 3.7; the dip is 50\(^\circ\)). The inversion for the interval parameters \( V_{P0} \), \( \delta \), \( \epsilon \), and \( \phi \) is carried out under the assumption that each layer is TTI with the symmetry axis orthogonal to its lower boundary. The algorithm is applied to 200 realizations of noise-contaminated input data with the standard deviations equal to 2\% for \( V_{\text{nmo}} \), and 1\% for \( p \), \( t_0 \), \( t_{cs} \) and \( t_{\text{vsp}} \). The mean values of \( V_{P0} \), \( \delta \), and \( \phi \) for the TTI layer are close to the actual parameters, while the standard deviations are 4\%, 0.04, and 0.6\(^\circ\), respectively. Since the dip is relatively large, even the interval parameter \( \epsilon \) in this model is well-constrained (the mean value is 0.18, the standard deviation is 0.05).
Figure 3.7: Dipping TTI layer with parallel boundaries embedded in isotropic host rock. The known reflector depths at the borehole location are $z_b^{(1)} = 1$ km, $z_b^{(2)} = 1.7$ km, and $z_b^{(3)} = 2.5$ km. The dips (assumed to be unknown) are $\phi^{(1)} = 50^\circ$, $\phi^{(2)} = 50^\circ$, and $\phi^{(3)} = 0^\circ$. The check-shot source is located 10 m to the right of the borehole; two additional VSP sources are ±500 m away from the borehole. The receivers (marked by triangles) are placed at the intersection of the borehole with each reflector. The parameters of the TTI layer are $V_{P0} = 2.9$ km/s, $\delta = 0.08$, $\epsilon = 0.16$, and $\nu = \phi = 50^\circ$. The velocity in the isotropic background is 2.7 km/s.
3.4 Discussion and conclusions

P-wave reflection traveltimes typically do not contain enough information for estimating the full parameter set of tilted TI models and performing depth imaging. Here, I presented a 2D inversion algorithm for TTI media that supplements P-wave NMO velocities, zero-offset traveltimes, and reflection time slopes with borehole data. It was assumed that borehole measurements include check-shot traveltimes along with the depths and dips of layer boundaries.

The inversion for a single TTI layer above a dipping reflector (the symmetry axis is confined to dip plane) was based on exact expressions for the phase, group, and NMO velocities. Although the input data allow us to construct enough equations for the symmetry-direction velocity $V_{P0}$, anisotropy parameters $\epsilon$ and $\delta$, and the tilt $\nu$ of the symmetry axis, synthetic tests proved the inversion procedure to be highly unstable. Since this problem is caused primarily by an unknown tilt $\nu$, I fixed the symmetry axis in the direction orthogonal to the reflector. This common constraint makes it possible to obtain the parameters $V_{P0}$ and $\delta$ with high accuracy for the full range of reflector dips. However, the parameter $\epsilon$ still cannot be resolved for dips (and tilts) under $60^\circ$ because its estimation requires large propagation angles with the symmetry axis. If the magnitude of anisotropy is not uncommanly large ($|\epsilon| \leq 0.5; |\delta| \leq 0.3$) and the dip does not exceed $50^\circ$, the algorithm can tolerate the deviation of the symmetry axis from the reflector normal by $\pm 5^\circ$.

To perform interval parameter estimation for a stack of TTI layers separated by plane dipping interfaces, I employed a tomographic-style algorithm that operates with conventional-spread P-wave moveout. The current implementation is limited to the 2D model, in which the vertical incidence plane coincides with the dip planes of all interfaces, and the symmetry axis in each layer is perpendicular to its bottom. The depths and dips of the reflectors are assumed to be measured in the borehole, which allows us to reconstruct the model geometry. The objective function, computed by ray tracing using trial interval parameters, includes the time slope, zero-offset traveltime, and NMO velocity of each reflection event.
along with check-shot traveltimes. By performing the inversion for all layers simultaneously, the algorithm mitigates error accumulation with depth. Testing on noise-contaminated data showed that for models with the maximum difference between reflector dips not exceeding $30^\circ$, the interval $V_{p0}$ and $\delta$ are well-resolved. If long-spread P-wave data are available, $\epsilon$ can be obtained from nonhyperbolic moveout inversion.

When accurate dip information is unavailable, it can be replaced by traveltimes for at least two VSP sources placed on both sides of the borehole. If the distance between each VSP source and the borehole reaches $1/5$ of the largest reflector depth, the algorithm produces stable estimates of $V_{p0}$, $\delta$, and the reflector dips.

Our method can be extended in a straightforward way to 3D wide-azimuth P-wave data by including the NMO ellipses in the objective function. Under the same assumption as in 2D case (i.e., the symmetry axis in each layer is orthogonal to its bottom), wide-azimuth data provide additional information for estimating the interval Thomsen parameters. As will be shown in a sequel paper, the interval parameters $V_{p0}$ and $\delta$ along with the reflector dips and azimuths can be resolved from 3D stacking-velocity inversion with only one borehole constraint – the depth of each reflector (no check-shot or walkaway VSP data are needed).

Stacking-velocity tomography, possibly supplemented with nonhyperbolic moveout inversion for $\epsilon$, represents an efficient tool for building an initial model for migration velocity analysis (MVA) and post-migration reflection tomography. After carrying out the interval parameter estimation at well locations, the $V_{p0}$- and $\delta$-fields can be computed by interpolation between the wells. An accurate initial TTI model is critically important to ensure the convergence of MVA-based algorithms.
CHAPTER 4
MOVEOUT INVERSION OF WIDE-AZIMUTH P-WAVE DATA FOR TILTED TI MEDIA

Currently TTI (transversely isotropic with a tilted symmetry axis) models are widely used for velocity analysis and imaging in many exploration areas. I develop a 3D parameter-estimation algorithm for TTI media composed of homogeneous layers separated by plane dipping interfaces. The input data include P-wave NMO ellipses and time slopes (horizontal slownesses of the zero-offset rays) combined with borehole information.

If the symmetry axis is perpendicular to the bottom of each layer, it is possible to estimate the interval symmetry-direction velocity $V_{P0}$, anisotropy parameter $\delta$, and the reflector orientation using a single constraint—the reflector depth. The algorithm can tolerate small ($\leq 5^\circ$) deviation of the symmetry axis from the reflector normal. However, as is the case for the 2D problem, the parameter $\epsilon$ can seldom be obtained without nonhyperbolic moveout inversion.

If the symmetry axis deviates from the reflector normal but is confined to the dip plane, stable parameter estimation requires specifying a relationship between the tilt and dip in each layer. When the tilt represents a free parameter, the input data have to be supplemented by wide-azimuth VSP traveltimes with the offset reaching at least $1/4$ of the maximum reflector depth. Moreover, the additional angle coverage provided by VSP data may help resolve the parameter $\epsilon$ in the upper part of the model. The developed methodology can be used to build an accurate initial anisotropic velocity model for processing of wide-azimuth surveys.

4.1 Introduction

Transversely isotropic media with a tilted symmetry axis (TTI) provide marked improvements in prestack imaging of P-wave data (Charles et al., 2008; Huang et al., 2008; Neal et al., 2009). Allowing for the symmetry-axis tilt results in more plausible velocity models
for sedimentary formations in complex geological settings including fold-and-thrust belts and subsalt plays (Bakulin et al., 2010c; Behera & Tsvankin, 2009; Vestrum et al., 1999).

P-wave velocities and traveltimes in TTI media can be expressed through the symmetry-direction velocity $V_{p0}$ and Thomsen (1986) anisotropy parameters $\epsilon$ and $\delta$ defined with respect to the symmetry axis. The symmetry-axis orientation is specified by the tilt angle $\nu$ with the vertical and the azimuth $\beta$. Although many migration algorithms have been extended to TTI media, accurate estimation of the interval anisotropy parameters and the symmetry-axis orientation remains a difficult problem.

For example, Grechka et al. (2001) discuss 2D inversion of P-wave normal-moveout (NMO) velocities and zero-offset traveltimes for the parameters of a dipping TTI layer with the symmetry axis perpendicular to the bedding. Their algorithm is based on several a priori assumptions about the model and requires reflection data from a horizontal interface beneath the TTI layer. Joint moveout inversion of wide-azimuth PP and PS (or SS) reflection data for layered TI media with arbitrary symmetry-axis orientation is developed by Grechka et al. (2002a). Despite the addition of shear-wave traveltimes, parameter estimation is well-posed only for relatively large tilts $\nu$ and reflector dips.

A review of several other velocity-analysis algorithms for TTI media can be found in chapter 3, where I develop a 2D inversion methodology for a stack of homogeneous TTI layers separated by plane dipping interfaces. P-wave NMO velocities, reflection slopes, and zero-offset traveltimes are supplemented with reflector depths measured in a borehole, as well as with check-shot and near-offset VSP traveltimes. Even for a single TTI layer, the medium parameters cannot be resolved without a priori knowledge of the tilt of the symmetry axis. Therefore, the symmetry axis is assumed to be orthogonal to the layer’s bottom, which is typical for dipping shale layers (Charles et al., 2008; Isaac & Lawton, 1999; Vestrum et al., 1999). Then the 2D algorithm with borehole constraints produces stable estimates of the interval parameters $V_{p0}$ and $\delta$ for models with a limited range of interface dips.
Here, I present a 3D extension of the inversion algorithm from chapter 3 by including the NMO ellipses and two horizontal slownesses of the zero-offset rays (reflection time slopes) in the objective function. Additional information provided by wide-azimuth data makes it possible to relax the constraints on model geometry and increase the stability of the inversion. First, I discuss 3D parameter estimation for models with the symmetry axis orthogonal to reflectors. Then I extend the method to media with arbitrary tilt and show that stable inversion requires the addition of VSP data. The accuracy and stability of estimating the interval TTI parameters for different types of input data is evaluated using synthetic tests on noise-contaminated data.

4.2 3D input data vector

As in chapter 3 (based on the paper by Wang & Tsvankin (2010)), I consider a stack of homogeneous TTI layers separated by plane, dipping, non-intersecting boundaries (Figure 4.1). However, the dip planes of model interfaces no longer have to be aligned. From 3D multi-azimuth P-wave data recorded at a common midpoint (CMP) with the coordinates $Y = [Y_1, Y_2]$, it is possible to obtain the zero-offset reflection traveltimes $t_0(Y, n)$ for all reflectors and the corresponding NMO velocities $V_{nmo}(\alpha)$ ($\alpha$ is the azimuth). Input data also include the time slopes $p(Y, n) = [p_1(Y, n), p_2(Y, n)]$ on the zero-offset (or stacked) section, where $p_1$ and $p_2$ are the horizontal slowness components of the zero-offset ray. The azimuthally-dependent NMO velocity is described by an elliptical function in the horizontal plane (Grechka & Tsvankin, 1998a):

$$V_{nmo}^{-2}(\alpha) = W_{11} \cos^2 \alpha + 2W_{12} \sin \alpha \cos \alpha + W_{22} \sin^2 \alpha,$$

(4.1)

where $W$ is a symmetric matrix,

$$W_{ij} = \tau_0 \frac{\partial^2 \tau}{\partial x_i \partial x_j} \bigg|_{x=Y}, \quad (i, j = 1, 2).$$

(4.2)

Here $\tau(x_1, x_2)$ is the one-way traveltime from the zero-offset reflection point to the location $x = \{x_1, x_2\}$ at the surface and $\tau_0$ is the one-way zero-offset traveltime. The matrices
\( W(Y, n) \) can be obtained from azimuthal velocity analysis based on the hyperbolic moveout equation parameterized by the NMO ellipse (Grechka & Tsvankin, 1999b).

Figure 4.1: Zero-offset rays and a multiazimuth CMP gather for a stack of TTI layers separated by plane dipping interfaces (after Grechka et al., 2002b).

To model the effective NMO ellipse for a stack of TTI layers, I use the Dix-type averaging procedure devised by Grechka & Tsvankin (2002c) for heterogeneous anisotropic media. They show that the exact NMO ellipse can be built by averaging the intersections of the interval NMO-velocity surfaces with the layer boundaries. All information for computing the NMO ellipse of a given reflection event is contained in the results of tracing just one (zero-offset) ray.

Because each layer is homogeneous with plane boundaries (Figure 4.1), it is sufficient to acquire the input data in a single multiazimuth CMP gather (Grechka et al., 2002b). The reflector depths \( z_b(n) \) are assumed to be measured in a borehole, which may be placed away from the CMP location (the subscript “b” denotes borehole data). Therefore, the vector of
input data for 3D inversion is as follows:

\[ d = \{ t_0(n), p_1(n), p_2(n), W_{11}(n), W_{12}(n), W_{22}(n), z_0(n) \} \]

\[ (n = 1, 2, \ldots, N) , \]  

(4.3)

where all components are the effective quantities for the \( n \)-th reflector.

4.3 Symmetry axis orthogonal to the reflector

It is common to put constraints on the symmetry-axis orientation using \textit{a priori} information (Bakulin \textit{et al.}, 2010c; Charles \textit{et al.}, 2008; Huang \textit{et al.}, 2008). If TI layers were rotated by tectonic processes after sedimentation, the symmetry axis typically remains perpendicular to the layering, which means that its tilt \( \nu \) and azimuth \( \beta \) coincide with the dip \( \phi \) and azimuth \( \psi \) of the reflector, respectively. The relative simplicity of this model significantly improves the stability of parameter estimation.

4.3.1 Inversion for a single TTI layer

First, I consider a homogeneous TTI layer with the symmetry axis orthogonal to its bottom. The dip plane of the reflector represents a vertical symmetry plane for the whole model, and therefore, includes one of the axes of the NMO ellipse. Thus, the orientation of the NMO ellipse yields the reflector azimuth \( \psi \) which coincides with the symmetry-axis azimuth \( \beta \).

The semiaxis of the NMO ellipse in the dip plane is obtained from the isotropic cosine-of-dip relationship (Tsvankin, 2005):

\[ V_{nmo}^{(1)}(\phi) = \frac{V_{nmo}(0)}{\cos \phi} , \]

(4.4)

where \( V_{nmo}(0) = V_{P0} \sqrt{1 + 2\delta} \) is the NMO velocity from a horizontal interface beneath a VTI medium with the same Thomsen parameters (i.e., the symmetry axis is rotated along with the reflector). Alternatively, the dip component of the NMO velocity can be represented
using the ray parameter $p = \sqrt{p_1^2 + p_2^2}$:

$$V_{nmo}^{(1)}(p) = \frac{V_{nmo}(0)}{\sqrt{1 - p^2 V_{P0}^2}}, \quad (4.5)$$

where $p = \sin \phi/V_{P0}$ because the phase-velocity vector of the zero-offset ray (and the ray itself) is parallel to the symmetry axis. The strike component $V_{nmo}^{(2)}$ of the NMO velocity is given by (Grechka & Tsvankin, 2000):

$$V_{nmo}^{(2)} = V_{nmo}(0) = V_{P0} \sqrt{1 + 2 \delta}. \quad (4.6)$$

Therefore, by combining the two semiaxes of the NMO ellipse (equations 4.5 and 4.6) and using the measured time slope $p$, one can find the symmetry-direction velocity $V_{P0}$. Then the dip $\phi = \nu$ is obtained from equation 4.4, and the anisotropy parameter $\delta$ from equation 4.6.

Depth information for a single layer is not needed because the reflector depth $z$ below the CMP location can be computed from the zero-offset traveltime $t_0$:

$$z = \frac{V_{P0} t_0}{2 \cos \phi}. \quad (4.7)$$

P-wave hyperbolic moveout in this model, however, is independent from the anisotropy parameter $\epsilon$. In summary, the geometry and parameters $V_{P0}$ and $\delta$ of a single TTI layer can be resolved from P-wave reflection traveltimes without using any borehole information.

### 4.3.2 Inversion for layered TTI media

Here, I present a 3D extension of the 2D stacking-velocity inversion algorithm (chapter 3) to layered TTI media. If the symmetry axis in each layer $n$ is perpendicular to its bottom $(\nu^{(n)} = \phi^{(n)}$ and $\beta^{(n)} = \psi^{(n)})$, the model vector is

$$m = \{V_{P0}^{(n)}, \epsilon^{(n)}, \delta^{(n)}, \phi^{(n)}, \psi^{(n)}\}, \quad (n = 1, 2, \ldots, N). \quad (4.8)$$

First, I assume the depths $z_0(n)$ to be known (albeit with a certain error) from borehole measurements; later on, I discuss the inversion without using depth constraints.
4.3.2.1 Inversion methodology

I specify the trial set \( m \) of the interval parameters (equation 4.8) and trace zero-offset rays through the model with the geometry partially fixed by the known reflector depths. The ray-tracing results yield the zero-offset traveltimes \( t_0^{\text{calc}}(n) \), the horizontal slowness components \( p_1^{\text{calc}}(n) \) and \( p_2^{\text{calc}}(n) \), and the Dix-type averaging procedure produces the effective NMO ellipses \( W^{\text{calc}}(n) \). The NMO velocity \( V_{\text{nmo}}^{\text{calc}}(n, \alpha) \) for any azimuth \( \alpha \) can be computed from equation 4.1.

The vector \( m \) is estimated by minimizing the following objective function (based on the L2-norm) for all \( N \) reflectors simultaneously:

\[
\mathcal{F}(m) \equiv \sum_{n=1}^{N} \left\{ \frac{\| p_1^{\text{calc}}(n) - p_1(n) \|^2}{\sigma^2[p_1(n)]} + \frac{\| p_2^{\text{calc}}(n) - p_2(n) \|^2}{\sigma^2[p_2(n)]} + \frac{\| t_0^{\text{calc}}(n) - t_0(n) \|^2}{\sigma^2[t_0(n)]} + \frac{\| V_{\text{nmo}}^{\text{calc}}(n, \alpha) - V_{\text{nmo}}(n, \alpha) \|^2}{\sigma^2[V_{\text{nmo}}(n, \alpha)]} \right\},
\]

where \( \sigma^2 \) represents the variance of each measurement, and the azimuth \( \alpha \) varies from 0° to 180°. For 2D models, the objective function also includes check-shot traveltimes, and reflector dips are assumed to be known (chapter 3). Here, wide-azimuth data provide additional information that replaces those borehole constraints.

In a single TTI layer, the parameter \( \epsilon \) cannot be found from conventional-spread P-wave moveout. However, as discussed in chapter 3, the input parameters \( p(n), t_0(n), \) and \( W(n) \) for layered TTI models are influenced by the values of \( \epsilon^{(n)} \) in the overburden (except for models with parallel interfaces). Therefore, the interval parameter \( \epsilon \) is estimated along with the other unknowns, although typically it is not expected to be well-constrained.

4.3.2.2 Synthetic examples

The algorithm was tested for a suite of layered TTI models with a wide range of the anisotropy parameters (\( 0 \leq \epsilon \leq 0.5 \) and \( -0.2 \leq \delta \leq 0.3 \)) and reflector dips between 0° and 60°. I used so-called trust-region-reflective optimization (Coleman & Li, 1996a) to solve
the nonlinear least-squares inverse problem defined by equation 4.9. Table 4.2 shows the inversion results for a three-layer medium with a wide range of interface azimuths (Table 4.1 and Figure 4.2). The results of a test for another three-layer medium with relatively close azimuths of the interfaces and large dips (Table 4.1 and Figure 4.3) are listed in Table 4.3. The inversion was performed for 200 realizations of noise-contaminated input data using the measurement values as the variances $\sigma^2$ in equation 4.9. Typically, the initial models were composed of horizontal isotropic layers with velocities sufficiently different (by 10% or more) from the exact values of $V_{p0}$. I also tested several initial models with nonzero reflector dips and found that the algorithm converged to the same global minimum.

Table 4.1: Interval parameters of a three-layer TTI model.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$V_{p0}$ (km/s)</th>
<th>$\epsilon$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
<td>0.10</td>
<td>-0.10</td>
</tr>
<tr>
<td>2</td>
<td>2.0</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
<td>0.25</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 4.2: Inversion results for the model from Table 4.1 and Figure 4.2. The input data are distorted by Gaussian noise with the standard deviations equal to 2% for $p_1(n)$, $p_2(n)$ and the NMO velocities, 1% for $t_0(n)$, and 0.2% for $z_b(n)$. The mean values and standard deviations of the inverted parameters are denoted by “mean” and “sd,” respectively.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$V_{p0}$ (km/s)</th>
<th>$\delta$</th>
<th>$\epsilon$</th>
<th>$\phi$ (°)</th>
<th>$\psi$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.50</td>
<td>-0.10</td>
<td>0.01</td>
<td>0.09</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>2.00</td>
<td>0.10</td>
<td>0.03</td>
<td>0.20</td>
<td>0.08</td>
</tr>
<tr>
<td>3</td>
<td>2.51</td>
<td>0.12</td>
<td>0.06</td>
<td>0.23</td>
<td>0.24</td>
</tr>
</tbody>
</table>

For both models, the interval parameters $V_{p0}$ and $\delta$ and the reflector dips and azimuths are recovered with sufficiently high accuracy. As expected, the standard deviations are higher in the third (deepest) layer (about 5% for $V_{p0}$ and 0.06 for $\delta$), primarily due to the smaller contribution of the deeper layers to the effective reflection traveltimes. However, in contrast to layer-stripping techniques, our tomography-style algorithm possesses the advantage of mitigating error accumulation with depth. An important factor that influences the inversion
Figure 4.2: Zero-offset P-wave rays in a three-layer TTI model with the interval parameters listed in Table 4.1. The input data are computed by anisotropic ray tracing. The symmetry axis in each layer is perpendicular to its bottom. The dips and azimuths are $\phi^{(1)} = \phi^{(2)} = \phi^{(3)} = 30^\circ$, $\psi^{(1)} = 0^\circ$, $\psi^{(2)} = 45^\circ$, and $\psi^{(3)} = 90^\circ$. The reflector depths below the CMP are $z_b^{(1)} = 1$ km, $z_b^{(2)} = 2$ km, and $z_b^{(3)} = 3$ km.

Table 4.3: Inversion results for the model from Table 4.1 and Figure 4.3. The noise level in the input data is the same as that in Table 4.2.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$V_{P0}$ (km/s)</th>
<th>$\delta$</th>
<th>$\epsilon$</th>
<th>$\phi$ (°)</th>
<th>$\psi$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>sd (%)</td>
<td>mean</td>
<td>sd</td>
<td>mean</td>
</tr>
<tr>
<td>Layer 1</td>
<td>1.50</td>
<td>1</td>
<td>-0.10</td>
<td>0.01</td>
<td>0.23</td>
</tr>
<tr>
<td>Layer 2</td>
<td>2.00</td>
<td>4</td>
<td>0.10</td>
<td>0.04</td>
<td>0.25</td>
</tr>
<tr>
<td>Layer 3</td>
<td>2.49</td>
<td>5</td>
<td>0.12</td>
<td>0.07</td>
<td>0.25</td>
</tr>
</tbody>
</table>
accuracy is the layer thickness; the thickness-to-depth ratio below the CMP location should reach at least 0.25 to ensure stable interval estimates. As expected, the standard deviations of the parameter $\epsilon$ are much larger than those of $\delta$, although $\epsilon$-estimates for the first model are not substantially biased.

Figure 4.3: Zero-offset P-wave rays in a three-layer TTI model with the interval parameters listed in Table 4.1. The symmetry axis in each layer is perpendicular to its bottom. The dips and azimuths are $\phi^{(1)} = \phi^{(2)} = 50^\circ$, $\phi^{(3)} = 20^\circ$, $\psi^{(1)} = 10^\circ$, $\psi^{(2)} = 20^\circ$, and $\psi^{(3)} = 30^\circ$. The reflector depths below the CMP (located at the origin of the coordinate system) are $z_b^{(1)} = 1 \text{ km}$, $z_b^{(2)} = 2 \text{ km}$, and $z_b^{(3)} = 3 \text{ km}$.

For plausible ranges of $\epsilon$ and $\delta$ ($|\epsilon| \leq 0.5$; $|\delta| \leq 0.3$), the errors in the interval parameters $V_{P0}$, $\delta$, $\phi$, and $\psi$ remain small if the symmetry axis deviates from the reflector normal in the dip plane by less than $5^\circ$ (i.e., $\beta = \psi$, but $\nu \neq \phi$). To evaluate the sensitivity to the difference between $\nu$ and $\phi$, I changed the tilts for the second model (Table 4.1 and Figure 4.2; all dips are equal to $30^\circ$) to $\nu^{(n)} = 25^\circ$ ($n = 1, 2, 3$), generated the input data, and applied our
algorithm assuming that \( \nu^{(n)} = \phi^{(n)} \). The slight deviation of the symmetry axis from the reflector normal causes a mild bias in the estimates of \( V_{P0} \), \( \delta \), \( \phi \), and \( \psi \), but the standard deviations are mostly controlled by the noise level, which is the same as in the previous tests.

If the reflector depths are also unknown, the trade-off between \( z(n) \) and other parameters increases errors in the inversion results. For instance, I repeated the inversion for the model from Table 4.1 and Figure 4.2 without using depth information. For the same level of noise in the input data, the standard deviation of \( V_{P0} \) in the third layer increases from 5% (see Table 4.2) to 7% and in \( \delta \) from 0.06 to 0.10. The estimated mean value of \( z(3) \) (2.98 km) is almost unbiased with the standard deviation 0.11 km.

### 4.4 Symmetry axis deviating from reflector normal

As pointed out by Bakulin et al. (2010c), the assumption of the symmetry axis being perpendicular to the reflector can become too restrictive when tectonic processes and sedimentation occur together. Also, for stress-induced anisotropy in sediments near salt bodies, the symmetry is largely controlled by the principal stress direction which is not necessarily aligned with the normal to the bedding.

To account for the deviation of the symmetry axis from the reflector normal, in some cases the tilt \( \nu \) can be expressed as a function of the dip \( \phi \) using geologic data. For example, the simultaneous influence of tectonic forces and sedimentation typically makes \( \nu \) smaller than \( \phi \) (e.g., \( \nu = \phi/2 \) or \( \nu = 3\phi/4 \)). In the next test, I used the three-layer model with the interval parameters listed in Table 4.1 and the model geometry shown in Figure 4.2, but with \( \nu \neq \phi \). The symmetry axis in each layer is confined to the dip plane (i.e., \( \beta^{(n)} = \psi^{(n)} \), \( n = 1, 2, 3 \)) with the tilt \( \nu^{(n)} = \phi^{(n)}/2 \). The known relationship between \( \nu \) and \( \phi \) was sufficient for the algorithm to produce results (Table 4.4) similar to those for the symmetry axis orthogonal to the reflector (Table 4.2).
Table 4.4: Inversion results for the model from Table 4.1 with the reflector geometry shown in Figure 4.2. The tilt of the symmetry axis in each layer is equal to one-half of the reflector dip (the symmetry axis lies in the dip plane). The noise level is the same as in the previous tests.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$V_{P0}$ (km/s)</th>
<th>$\delta$</th>
<th>$\nu$ (°)</th>
<th>$\psi$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>sd (%)</td>
<td>mean</td>
<td>sd</td>
</tr>
<tr>
<td>Layer 1</td>
<td>1.50</td>
<td>1</td>
<td>-0.10</td>
<td>0.01</td>
</tr>
<tr>
<td>Layer 2</td>
<td>2.00</td>
<td>2</td>
<td>0.10</td>
<td>0.04</td>
</tr>
<tr>
<td>Layer 3</td>
<td>2.50</td>
<td>5</td>
<td>0.12</td>
<td>0.08</td>
</tr>
</tbody>
</table>

4.4.1 Tilt as an unknown parameter

Here, I relax the assumption that the tilt $\nu$ represents a known function of the dip $\phi$. It is still assumed that the symmetry-axis azimuth $\beta$ in each layer coincides with the dip-plane azimuth $\psi$, but the parameter $\nu$ has to be found from the data. Thus, the model vector includes one more unknown:

$$\mathbf{m} = \{ V_{P0}^{(n)}, \epsilon^{(n)}, \delta^{(n)}, \nu^{(n)}, \phi^{(n)}, \psi^{(n)} \}, \quad (n = 1, 2, \ldots, N). \quad (4.10)$$

Making $\nu$ a free parameter significantly increases the nonuniqueness of the inversion. For 2D models, simultaneous estimation of $V_{P0}$, $\epsilon$, $\delta$, and $\nu$ proves to be ambiguous, even if the reflector depths and dips are measured in a borehole. Our tests indicate that 3D wide-azimuth data supplemented by the known reflector depths still cannot be used to resolve the tilt along with the other TTI parameters. Therefore, I propose to add wide-azimuth walkaway VSP (vertical seismic profiling) traveltimes $t_{VSP}$ to the vector $\mathbf{d}$ of input data:

$$\mathbf{d} = \{ t_0(n), p_1(n), p_2(n), W_{11}(n), W_{12}(n), W_{22}(n), z_b(n), t_{VSP} \}. \quad (4.11)$$

The VSP data are excited by an array of sources at the surface and recorded by one receiver per layer located close to the layer’s bottom. Similar to the zero-offset reflected rays, I trace VSP rays in a trial model and compute the difference between the modeled and observed traveltimes. Then the modified objective function takes the following form:
\[ \mathcal{F}(\mathbf{m}) \equiv \sum_{n=1}^{N} \left\{ \frac{\|p_{1}^{\text{calc}}(n) - p_{1}(n)\|^2}{\sigma^2[p_{1}(n)]} + \frac{\|p_{2}^{\text{calc}}(n) - p_{2}(n)\|^2}{\sigma^2[p_{2}(n)]} + \frac{\|t_{0}^{\text{calc}}(n) - t_{0}(n)\|^2}{\sigma^2[t_{0}(n)]} \right. \\
\left. + \frac{\|V_{\text{nmo}}^{\text{calc}}(n, \alpha) - V_{\text{nmo}}(n, \alpha)\|^2}{\sigma^2[V_{\text{nmo}}(n, \alpha)]} + \frac{\|t_{VSP}^{\text{calc}} - t_{VSP}\|^2}{\sigma^2[t_{VSP}]} \right\}. \] (4.12)

### 4.4.1.1 Synthetic examples

Numerical testing shows that it is sufficient to add one check-shot source and several walkaway VSP sources located around the borehole with the offset exceeding 1/4 of the largest reflector depth. To achieve full azimuthal coverage, typically we place eight VSP sources along a circle with a 45° increment in azimuth. With the distribution of the VSP sources in Figure 4.4, I computed the input data for a three-layer model (Table 4.1 and Figure 4.5) using anisotropic ray tracing. The inversion results for 100 realizations of the noise-contaminated data are listed in Table 4.5. I also tested another model with the same geometry (Figure 4.5) but different tilts \( \nu^{(1)} = 30°, \nu^{(2)} = 0°, \) and \( \nu^{(3)} = 45° \) (Table 4.6).

Table 4.5: Inversion results for the model from Table 4.1 and Figure 4.5 using reflection and VSP data. The positions of the check-shot and walkaway VSP sources are shown in Figure 4.4. The tilts are \( \nu^{(1)} = \nu^{(2)} = \nu^{(3)} = 20° \). The input data are distorted by Gaussian noise with the standard deviations equal to 3% for \( p_{1}(n) \) and \( p_{2}(n) \), 2% for the NMO velocities, 1% for \( t_{0}(n) \) and \( t_{\text{VSP}}^{\text{calc}} \), and 0.5% for \( z_{b}(n) \).

<table>
<thead>
<tr>
<th>Layer</th>
<th>( V_{P0} ) (km/s)</th>
<th>( \delta )</th>
<th>( \epsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>sd (%)</td>
<td>mean</td>
</tr>
<tr>
<td>Layer 1</td>
<td>1.50</td>
<td>1</td>
<td>-0.10</td>
</tr>
<tr>
<td>Layer 2</td>
<td>2.00</td>
<td>2</td>
<td>0.09</td>
</tr>
<tr>
<td>Layer 3</td>
<td>2.48</td>
<td>3</td>
<td>0.13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \nu ) (°)</th>
<th>( \phi ) (°)</th>
<th>( \psi ) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>21.1</td>
<td>3.0</td>
</tr>
<tr>
<td>Layer 2</td>
<td>20.5</td>
<td>5.5</td>
</tr>
<tr>
<td>Layer 3</td>
<td>24.9</td>
<td>11.4</td>
</tr>
</tbody>
</table>

As before, the initial models for the inversion were isotropic. However, if the tilt represents a free parameter, different initial guesses for reflector dips can lead to different inverted
Figure 4.4: Distribution of VSP sources at the surface used for the model in Figure 4.5. The VSP lines are separated by $45^\circ$, and the offset of each VSP source is 1 km. The check-shot source is located close to the borehole ($x_1 = 0.01, x_2 = 0$).

Table 4.6: Inversion results using reflection and VSP data. The model is the same as in Table 4.5, except for the tilts $\nu^{(1)} = 30^\circ$, $\nu^{(2)} = 0^\circ$, and $\nu^{(3)} = 45^\circ$.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$V_{P0}$ (km/s)</th>
<th>$\delta$</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>sd (%)</td>
<td>mean</td>
</tr>
<tr>
<td>Layer 1</td>
<td>1.50</td>
<td>1</td>
<td>-0.10</td>
</tr>
<tr>
<td>Layer 2</td>
<td>1.99</td>
<td>1</td>
<td>0.11</td>
</tr>
<tr>
<td>Layer 3</td>
<td>2.52</td>
<td>3</td>
<td>0.11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Layer</th>
<th>$\nu$ ($^\circ$)</th>
<th>$\phi$ ($^\circ$)</th>
<th>$\psi$ ($^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>sd</td>
<td>mean</td>
</tr>
<tr>
<td>Layer 1</td>
<td>29.7</td>
<td>2.6</td>
<td>29.9</td>
</tr>
<tr>
<td>Layer 2</td>
<td>1.7</td>
<td>2.4</td>
<td>30.1</td>
</tr>
<tr>
<td>Layer 3</td>
<td>40.7</td>
<td>11.5</td>
<td>29.9</td>
</tr>
</tbody>
</table>
models corresponding to local minima of the objective function. Therefore, I ran the algorithm for several initial models with different interface dips and selected the result that provided the smallest data misfit. In practice, approximate dips picked on a depth image can serve as the initial guesses.

Figure 4.5: VSP rays for a receiver located at the bottom of a three-layer TTI model with the interval parameters listed in Table 4.1. The symmetry axis in each layer is confined to the dip plane. The tilts, dips, and azimuths are $\nu(1) = \nu(2) = \nu(3) = 20^\circ$, $\phi(1) = \phi(2) = \phi(3) = 30^\circ$, $\psi(1) = 0^\circ$, $\psi(2) = 45^\circ$, and $\psi(3) = 90^\circ$. The vertical borehole is below the coordinate origin, and the reflector depths at the borehole location are $z_b(1) = 1$ km, $z_b(2) = 2$ km, and $z_b(3) = 3$ km.

Despite the additional constraints provided by VSP data, the standard deviations in the tilt increase rapidly with depth because the inversion for $\nu$ is still ill-conditioned. However, the interval parameters $V_{P0}$ and $\delta$ and the reflector orientation can be well resolved despite
a higher level of noise than that in the previous tests. Also, the VSP data help constrain the
parameter $\epsilon$ in the top two layers, while estimation of $\epsilon$ in the bottom layer is ambiguous
because of the poor angle coverage of the VSP rays at depth. To resolve the parameter $\epsilon$ in
piecewise-homogeneous TTI models, it is necessary to use long-offset VSP or reflection data
(Behera and Tsvankin, 2009).

It should be mentioned that the deviation of the symmetry axis from the reflector normal
reduces the stability of parameter estimation. When the difference between $\nu$ and $\phi$ is large,
small errors in the input data can be significantly amplified by the inversion algorithm. Our
numerical results indicate that the deviation of the symmetry axis from the reflector normal
should not exceed $30^\circ$, which is typical for most TTI formations.

4.5 Conclusions

The tilt of the symmetry axis makes the medium azimuthally anisotropic, and wide-
azimuth P-wave data provide valuable constraints on the TTI parameters. If the symmetry
axis is perpendicular to the reflector, the P-wave NMO ellipse is sufficient for estimating the
parameters $V_{P0}$ and $\delta$ of a single dipping TTI layer. Conventional-spread P-wave data also
yield the depth and orientation of the reflector, but the parameter $\epsilon$ remains unconstrained
without using long-offset (nonhyperbolic) moveout.

For homogeneous TTI layers separated by plane dipping interfaces, the input data include
the effective NMO ellipses, zero-offset traveltimes, and reflection slopes supplemented by
the reflector depths (presumably measured in a borehole). The interval parameters are
estimated by a 3D tomography-style algorithm that represents an extension of the 2D method
introduced in our previous publication. As long as the symmetry axis in each layer is
kept orthogonal to its bottom, the interval parameters $V_{P0}$ and $\delta$ and the reflector dips
$\phi$ and azimuths $\psi$ (and, therefore, the symmetry-axis orientation) are well-resolved. For
models with common moderate values of the anisotropy parameters ($|\epsilon| \leq 0.5; |\delta| \leq 0.3$),
small deviations of the symmetry axis from the reflector normal ($\leq 5^\circ$) do not substantially
distort the inversion results. If depth information is not available, the model parameters are
estimated with larger bias and standard deviation.

I also examined the possibility of extending the inversion to models in which the symmetry axis deviates from the reflector normal but is confined to the dip plane. If the relationship between the symmetry-axis tilt and reflector dip in each layer is known, the algorithm can still resolve the interval parameters $V_{P0}$ and $\delta$ along with the interface orientation. However, when tilt represents a free parameter, stable inversion requires additional input data, such as check-shot and walkaway VSP traveltimes. VSP data should have full azimuthal coverage, and the distance between the VSP sources and the borehole has to reach 1/4 of the largest reflector depth. Another essential requirement is for the angle between the symmetry axis and reflector normal not to exceed 30°. Depending on the offset coverage of VSP data, it may be possible to constrain the parameter $\epsilon$ in the shallow part of the section. The addition of walkaway VSP data can also increase the accuracy of the inversion for models with the symmetry axis orthogonal to the reflector.
Transversely isotropic models with a tilted symmetry axis (TTI media) are widely used in depth imaging of complex geologic structures. Here, I develop a 2D P-wave tomographic algorithm for heterogeneous transversely isotropic media with a tilted symmetry axis (TTI). The model is divided into rectangular cells with the symmetry-direction velocity $V_{P0}$, the anisotropy parameters $\epsilon$ and $\delta$, and the tilt $\nu$ of the symmetry axis defined at each grid point. To increase the stability of the inversion, the symmetry axis is set orthogonal to the imaged reflectors, with the tilt interpolated inside each layer.

The iterative migration velocity analysis involves efficient linearized parameter updating designed to minimize the residual moveout in image gathers for all available reflection events. The moveout equation in the depth-migrated domain includes a nonhyperbolic term that describes long-offset data, which are particularly sensitive to $\epsilon$. To ensure stable reconstruction of the TTI parameter fields, reflection data are combined with walkaway VSP (vertical seismic profiling) traveltimes in joint tomographic inversion. I also incorporate geologic constraints into tomography by designing regularization terms that penalize parameter variations in the direction parallel to the interfaces. To improve the convergence of the inversion algorithm, I propose a three-stage model-updating procedure that gradually relaxes the constraints on the spatial variations of the anisotropy parameters. Only at the final stage of the inversion the parameters $V_{P0}$, $\epsilon$, and $\delta$ are updated on the same grid.

5.1 Introduction

Prestack depth migration, or PSDM (e.g., Berkhout, 1982; Etgen, 1988; Lumley, 1989) has become the most widely used imaging technique in seismic exploration because of its high accuracy for complex subsurface structures. Velocity models for depth imaging are
usually built by migration velocity analysis (MVA), which operates in the migrated domain (Deregowski, 1990; Etgen, 1990; Fowler, 1988; Liu, 1997; van Trier, 1990).

The goal of MVA is to remove residual moveout of reflection events in common-image gathers (CIGs), obtained by computing migrated depth as a function of offset. Due to the high sensitivity of CIGs to medium parameters, quantitative analysis of the relationship between the residual moveout and velocity field helps refine the model, usually in iterative fashion. However, the flatness of CIGs is a necessary, but not a sufficient condition for resolving the medium parameters. Therefore, velocity model-building using CIGs typically requires additional constraints (e.g., well measurements) to reduce the nonuniqueness of the inverse problem (Bakulin et al., 2010c; Morice et al., 2004; Tsvankin, 2005).

Since most subsurface formations are anisotropic, ignoring anisotropy in P-wave processing leads to image distortions and interpretation errors (e.g., Alkhalifah & Larner, 1994; Alkhalifah et al., 1996; Vestrum et al., 1999). For example, in complex geologic settings including fold-and-thrust belts and subsalt plays, sedimentary formations are often described by transversely isotropic models with a vertical (VTI) or tilted (TTI) symmetry axis (Bakulin et al., 2010a; Douma et al., 2009; Neal et al., 2009). To ensure stable estimation of the symmetry-direction P-wave velocity $V_{P0}$ and anisotropy parameters $\epsilon$ and $\delta$, the orientation of the symmetry axis is commonly assumed to be known from structural information (Audebert et al., 2006; Bakulin et al., 2010c).

Behera & Tsvankin (2009) develop a 2D MVA algorithm for heterogeneous TTI media based on the approach suggested by Sarkar & Tsvankin (2004) for vertical transverse isotropy. To reduce the number of unknown parameters, they divide the model into “quasi-factorized” TTI blocks. Within each block, the parameters $\epsilon$ and $\delta$ are constant, and the symmetry-direction velocity $V_{P0}$ varies linearly according to the vertical ($k_z$) and lateral ($k_x$) gradients. Behera & Tsvankin (2009) also adopt the widely used assumption that the symmetry axis is perpendicular to the bottom of TTI layers (i.e., the tilt $\nu$ of the symmetry axis is equal to the reflector dip). They show that the gradients $k_z$ and $k_x$ and the parameters $\epsilon$ and
\(\delta\) can be accurately resolved, if \(V_{P0}\) is specified at a single point in each block. Note that stable estimation of \(\epsilon\) in TTI media requires long-spread (nonhyperbolic) moveout with the maximum offset reaching at least two reflector depths. Despite the efficiency of Behera and Tsvankin’s (2009) algorithm, their relatively simple model representation may be inadequate for complex subsurface structures with nonlinear spatial parameter variations.

To handle realistic subsurface geology, the model can be divided into relatively small cells, and the parameters at each grid point are often estimated using ray-based postmigration tomography (Campbell et al., 2006; Stork, 1992; Woodward et al., 2008). Most current applications of gridded tomography to TTI media simplify the inversion by keeping \(\epsilon\) and \(\delta\) fixed and updating only the symmetry-direction velocity \(V_{P0}\) (Charles et al., 2008; Huang et al., 2008). This procedure, however, does not adequately describe anisotropic velocity fields and may distort NMO velocities for both horizontal and dipping events. Zhou et al. (2011) develop multiparameter tomography for TTI media and apply it to field data. They find that simultaneous estimation of all three relevant parameters \((V_{P0}, \epsilon, \text{and } \delta)\) provides a better data fit than single-parameter (only velocity) inversion. Zhou et al. (2011) also conclude that trade-offs between the TTI parameters cannot be eliminated using only P-wave reflections, and point out the importance of additional constraints from well data. However, they do not carry out joint inversion of reflection and borehole data including, for example, VSP (vertical seismic profiling) traveltimes.

Bakulin et al. (2010c) develop localized gridded anisotropic tomography, which combines surface seismic data with borehole measurements (acoustic logs or check-shot surveys). Despite the additional constraints, they still obtain a wide range of TTI models that flatten the CIGs and fit the borehole data. The nonuniqueness may be partially caused by the limited angle coverage of check-shot rays. Bakulin et al. (2010c) also use a constant \((45^\circ)\) tilt of the symmetry axis for their synthetic model, not a practical assumption for field data.

Nonuniqueness of the inversion of reflection data can be mitigated by regularization which imposes a priori constraints on the estimated model (Engl et al., 1996). Fomel (2007)
develops so-called “shaping regularization” designed to steer the velocity variations along geologic structures (e.g., layers). His mapping (shaping) operator is integrated into the conjugate-gradient iterative solver. Using the steering-filter preconditioner (Clapp et al., 2004) similar to shaping regularization, Bakulin et al. (2010b) perform joint tomographic inversion of P-wave reflection data (from horizontal and dipping reflectors) and check-shot traveltimes for VTI media. They conclude that in the vicinity of the well it is possible to resolve the vertical variation of all three relevant parameters ($V_{P0}$, $\epsilon$, and $\delta$).

Here, I present a 2D ray-based tomographic algorithm designed to iteratively update TTI parameters defined on rectangular (in some cases square) grids. The symmetry axis is set perpendicular to the interfaces that may be dipping or curved. To construct the Fréchet matrix, which links the model update and the data misfit, the traveltime derivatives with respect to the parameters at each grid point are computed numerically along the ray trajectory. The regularization is designed to smooth the parameters in the direction parallel to the interfaces, while allowing for more pronounced variations in the orthogonal direction.

5.2 Methodology

I start from an initial model, which can be built using conventional-spread P-wave reflection data combined with borehole information, such as check-shot traveltimes and reflector depths (chapter 3). Then, the entire P-wave data volume is used to update the subsurface velocity field by flattening long-spread image gathers and minimizing VSP traveltme misfit. This linearized parameter update is implemented using the 2D ray-based gridded tomographic technique described below.

5.2.1 Input data

Images at each step of parameter updating are generated by 2D prestack Kirchhoff depth migration (Seismic Unix program ‘sukdmig2d’). The moveout of migrated reflection events in CIGs serves as one input to the tomographic algorithm. To avoid manual moveout picking, MVA typically employs semblance analysis with an appropriate analytic representation of
moveout as a function of offset. Conventionally, moveout in CIGs is described by the best-fit hyperbola with a single parameter (equivalent to NMO velocity) responsible for the term quadratic in the half-offset \( h \). To constrain the TI anellipticity parameter \( \eta \) (and, therefore, \( \epsilon \)), the hyperbolic approximation can be replaced by a more general nonhyperbolic equation (Sarkar & Tsvankin, 2004):

\[
z^2(h) \approx z^2(0) + R h^2 + S \frac{h^4}{h^2 + z^2(0)},
\]

where \( z \) is migrated depth, and \( R \) and \( S \) are dimensionless coefficients used to estimate the magnitude of residual moveout. Alternatively, the residual moveout can be evaluated using nonparametric methods (e.g., Murphy & Gray, 1999). If walkaway VSP surveys (check shots represent zero-offset VSP data) are available, VSP traveltimes are computed for each trial model and included in the tomographic inversion.

### 5.2.2 Model representation

The model is divided into rectangular (sometimes square) cells (Figure 5.1), with the symmetry-direction velocity \( V_P^0 \) and the anisotropy parameters \( \epsilon \) and \( \delta \) defined at each vertex of the grid. The spatial variation of the model parameters inside each cell is obtained by 2D interpolation. The grid size is determined by the expected resolution, acquisition geometry, and subsurface structure. If the grid size is too large compared to the dimensions of geologic units, the property variation is oversimplified. On the other hand, if the grid is too fine, the parameters may be poorly constrained and reside in the null space. Also, the grid size determines the number of unknown parameters and, therefore, influences the demands on computer memory.

In 2D, the normal direction of a reflector is defined by the dip angle with the vertical. When an interface crosses a cell, the symmetry axis at the four vertices of the cell is assumed to be orthogonal to the corresponding interface segment (Figure 5.1). Therefore, the tilt \( \nu \) of the symmetry axis is taken equal to the dip \( \phi \) which can be computed from the depth image using Madagascar program “sfdip.”
5.2.3 Model updating

To update the velocity model, I extend to gridded TTI media the MVA algorithm of Sarkar & Tsvankin (2004) designed for piecewise-factorized VTI models. Because the number of unknowns in our model is much larger, the partial derivatives of traveltime with respect to the medium parameters cannot be obtained simply from traveltime differences caused by certain parameter perturbations (Sarkar & Tsvankin, 2004). Instead, the traveltime derivatives are found numerically along computed raypaths, as described below.

Suppose the number of grid points in the model is $W$, and there are $N$ parameters defined at each grid point. Then I iteratively update the parameter vector $\lambda$, which contains $W \times N$ elements, using the inversion algorithm introduced in Appendix. Since the number of unknowns can be very large and the coverage of seismic rays for each cell is uneven, the tomographic inversion is generally ill-conditioned. Therefore, the inverse problem (equation A-4) should be constrained using regularization terms. To obtain the vector of model updates $\Delta \lambda$, I minimize the following objective function (based on the $L^2$-norm):

$$F(\Delta \lambda) = \| A \Delta \lambda - b \|^2 + \zeta_{\text{VSP}}^2 \| E \Delta \lambda - d \|^2 + R(\Delta \lambda),$$

(5.2)
where \( A \) is a matrix with \( M \times P \) rows (\( M \) is the number of offsets in each CIG and \( P \) is the number of CIGs) and \( W \times N \) columns whose elements are the derivatives of migrated depth with respect to the medium parameters, and \( b \) is a vector which contains the residual moveout in CIGs (see Appendix); the matrix \( E \) is composed of VSP traveltime derivatives, the vector \( d \) is the difference between the observed and calculated VSP traveltimes for each source-receiver pair, and \( \zeta_{\text{VSP}} \) is the weight for the term of VSP data misfit. The regularization term \( R \) in equation 5.2 will be discussed later.

Defining the TTI parameters on a relatively small grid results in a large number of unknowns, and the Fréchet matrices \( A \) and \( E \) in equation 5.2 are sparse (i.e., only nonzero or large elements are stored due to the limited computer memory). To solve the large sparse linear system of equations in an efficient way, I employ the parallel direct sparse solver (PARDISO) from Intel Math Kernel Library.

5.2.3.1 Computation of traveltime derivatives

The matrices \( A \) and \( E \) contain the traveltime derivatives with respect to the medium parameters defined on grids (equation A-5). These derivatives can be computed numerically along each raypath (Jech & Pšenčík, 1992; Zhou & Greenhalgh, 2008; Zhou et al., 2004) using the first-order traveltime perturbation theory (Červený, 2001; Červený & Jech, 1982). The location of the imaged reflection point for each source-receiver pair (equation A-1) is obtained from the results of semblance analysis using equation 5.1. Then, starting from that reflection point, we use the initial-value ray-tracing algorithm of Alkhalifah (1995) for 2D TI media to search for the incidence and reflected ray trajectories that satisfy Snell’s law and match the corresponding offset. The perturbation of the medium parameter \( \delta \lambda_{is} \) on any raypath \( R \) causes the traveltime change \( \delta \tau \) expressed to the first order by

\[
\delta \tau = - \int_{R} \frac{1}{VV_{G}} \left( \frac{\partial V}{\partial \lambda_{is}} \right) \delta \lambda_{is} \, ds ,
\]

(5.3)

where \( V \) and \( V_{G} \) are P-wave phase and group velocities, respectively, and \( ds \) is a small segment of \( R \) (denoted by the subscript \( s \)). Therefore, the traveltime derivative with respect
to the parameter $\lambda_i$ at a specific ray step is approximately given by

$$\frac{\partial \tau}{\partial \lambda_{is}} = -\frac{\delta t}{V} \left( \frac{\partial V}{\partial \lambda_{is}} \right), \quad (5.4)$$

where $\delta t$ is the time sample in ray tracing.

Next, I need to convert $\partial \tau/\partial \lambda_{is}$ into the derivative with respect to the parameter $\lambda_{ic}$ at a grid point ($\partial \tau/\partial \lambda_{ic}$). If there are several time samples on a seismic ray inside one cell (Figure 5.2), the parameter perturbation for a specific time sample ($\delta \lambda_{is}$) can be obtained by Lagrange interpolation of the parameter perturbation at one vertex $c$ (assuming the perturbations at other three vertices $c+1$, $c+2$, and $c+3$ are all zero):

$$\delta \lambda_{is} = \delta \lambda_{ic} \prod_{d=1}^{3} \frac{\| \mathbf{x}_{s} - \mathbf{x}_{c+d} \|}{\| \mathbf{x}_{c} - \mathbf{x}_{c+d} \|}, \quad (5.5)$$

where $\mathbf{x}$ is the coordinate vector, and $\| \ldots \|$ represents the distance between two points. Since the parameter perturbation at a grid point only influences the ray segments in the cells that have that grid point as a vertex, the traveltime derivative with respect to the parameter at the vertex $c$ (in equation A-5) is

$$\frac{\partial \tau}{\partial \lambda_{ic}} = -\sum_{s=m}^{n} \left[ \delta t \left( \frac{\partial V}{\partial \lambda_{is}} \right) \prod_{d=1}^{3} \frac{\| \mathbf{x}_{s} - \mathbf{x}_{c+d} \|}{\| \mathbf{x}_{c} - \mathbf{x}_{c+d} \|} \right]. \quad (5.6)$$

Here, the time samples along the ray passing through the cells that share the vertex $c$ vary from $m$ to $n$ (Figure 5.2).

Finally, the problem reduces to the computation of the derivatives of the P-wave phase velocity $V$ with respect to the medium parameters on the raypath. Using the P-wave phase-velocity function in TI media, the derivative $\partial V/\partial \lambda_{is}$ can be obtained analytically. The exact P-wave phase velocity in terms of the Thomsen parameters is given by Tsvankin (1996, 2005):

$$\frac{V^2}{V_{P0}^2} = 1 + \epsilon \sin^2 \theta - \frac{f}{2} + \frac{f}{2} \sqrt{\left( 1 + \frac{2\epsilon \sin^2 \theta}{f} \right)^2 - \frac{2(\epsilon - \delta) \sin^2 2\theta}{f}}, \quad (5.7)$$

where $\theta$ is the phase angle with the symmetry axis, $V_{P0}$ is the symmetry-direction velocity, and $f \equiv 1-V_{S0}^2/V_{P0}^2$; $V_{S0}$ is the symmetry-direction velocity of S-waves. Because the influence of $V_{S0}$ on P-wave kinematics is negligible, the value of $f$ can be set to a constant using a
Figure 5.2: (a) Diagram of seismic ray $R$ passing through three cells that share a grid point (in red); time samples along the ray change from $m$ to $n$. (b) Enlarged cell with each time sample (ray step) marked in blue. The four vertices of the grid are defined by the coordinate vectors $\mathbf{x}_c$, $\mathbf{x}_{c+1}$, $\mathbf{x}_{c+2}$, and $\mathbf{x}_{c+3}$. The coordinate vector of a specific ray step is $\mathbf{x}_s$.

A typical $V_{P0}/V_{S0}$ ratio (e.g., $V_{P0}/V_{S0} = 2$).

The P-wave phase velocity can also be obtained from an approximation suggested by Fowler (2003) for VTI media:

$$2V^2 \approx V_h^2 \sin^2 \theta + V_{P0}^2 \cos^2 \theta + \sqrt{(V_h^2 \sin^2 \theta + V_{P0}^2 \cos^2 \theta)^2 + V_{P0}^2 (V_{nmo}^2 - V_h^2) \sin^2 2\theta},$$  \hspace{1cm} (5.8)$$

where $V_h = V_{P0}\sqrt{1 + 2\epsilon}$ is the velocity in the direction (horizontal in VTI media) perpendicular to the symmetry axis, and $V_{nmo} = V_{P0}\sqrt{1 + 2\delta}$ is the zero-dip NMO velocity. The advantage of equation 5.8 is that three velocity variables have the same units and similar magnitudes; therefore, all elements of $A$ (equation 5.2) have the units of time (the elements of $E$ have the same unit too). Moreover, equation 5.8 helps in constructing the regularization terms in equation 5.2 (no weighting issues for different parameters). If the changes in $\epsilon$ and $\delta$ are small (e.g., $|\Delta \epsilon| \leq 0.02$; $|\Delta \delta| \leq 0.02$), the updates of $V_h$ and $V_{nmo}$ can be converted
into the updates of the Thomsen parameters via the relationships
\[ \Delta V_h \approx V_{P0} \Delta \epsilon; \quad \Delta V_{nmo} \approx V_{P0} \Delta \delta. \] (5.9)

### 5.2.3.2 Regularization

Conventionally, Tikhonov (1963) regularization is used to constrain the estimated model with the regularization terms in equation 5.2 expressed by
\[ R(\Delta \lambda) = \zeta \| \Delta \lambda \|^2 + \zeta_2^2 \| L(\Delta \lambda + \lambda^0) \|^2, \] (5.10)
where the matrix \( L \) is a finite-difference approximation of the Laplacian operator, which penalizes solutions that are rough in a second-derivative sense (\( \lambda^0 \) is the model obtained at the previous iteration), and \( \zeta_2 \) is a regularization parameter that controls the trade-off between minimizing the data misfit and the norm of the model parameters scaled by \( L \). Also, the magnitudes of the parameter updates corresponding to small derivatives in the matrix \( A \) (e.g., in areas with poor ray coverage) should be restricted by including the norm of \( \Delta \lambda \) with the weight \( \zeta \).

Because the operator \( L \) in equation 5.10 smoothes the parameter field equally along horizontal and vertical directions, inversion with such regularization may result in geologically inappropriate models. Next, geologic constraints are incorporated into the inversion by designing so called “structure-guided” regularization with the form:
\[ R(\Delta \lambda) = \zeta \| \Delta \lambda \|^2 + \zeta_1^2 \| L_1(\Delta \lambda + \lambda^0) \|^2 + \zeta_2^2 \| L_2(\Delta \lambda + \lambda^0) \|^2, \] (5.11)
where the operators \( L_1 \) and \( L_2 \) are designed to make parameter variations more pronounced in the direction normal to the interfaces, and \( \zeta, \zeta_1, \) and \( \zeta_2 \) are the regularization coefficients/weights.

To construct the matrix \( L_1 \), I first compute two components of the gradient vector \( \nabla \lambda \) from the following finite-difference approximation:
\[
\lambda'(x) = \frac{\lambda(x + dx) - \lambda(x - dx)}{2dx} + O[(dx)^2], \quad (5.12)
\]
\[
\lambda'(z) = \frac{\lambda(z + dz) - \lambda(z - dz)}{2dz} + O[(dz)^2], \quad (5.13)
\]

where \( \lambda \) is the parameter \((V_{P0}, \epsilon, \delta)\) at the grid point with the coordinates \(x\) and \(z\), and \(dx\) and \(dz\) are the cell dimensions. Since the dip field yields the vector \(n\) orthogonal to reflectors, I minimize the norm of the cross-product \(\|n \times \nabla \lambda\|\) at all grid points, which is equivalent to aligning the direction of the largest parameter variation with \(n\) and restricting the variations along interfaces. To include more cells, we can use a higher-order finite-difference approximation:

\[
\lambda'(x) = \frac{[-\lambda(x + 2dx) + 8\lambda(x + dx) - 8\lambda(x - dx) + \lambda(x - 2dx)]}{(12dx)} + O[(dx)^4], \quad (5.14)
\]
\[
\lambda'(z) = \frac{[-\lambda(z + 2dz) + 8\lambda(z + dz) - 8\lambda(z - dz) + \lambda(z - 2dz)]}{(12dz)} + O[(dz)^4]. \quad (5.15)
\]

Similarly, the operator \( L_2 \) is built from the cross-product between \(n\) and the vector with two components (second-order derivatives):

\[
\lambda''(x) = \frac{\lambda(x + dx) - 2\lambda(x) + \lambda(x - dx)}{(dx)^2}, \quad (5.16)
\]
\[
\lambda''(z) = \frac{\lambda(z + dz) - 2\lambda(z) + \lambda(z - dz)}{(dz)^2}. \quad (5.17)
\]

Then we minimize the norm of the cross-product to smooth the parameter variations in the direction parallel to the interfaces.

### 5.2.3.3 Three-stage parameter-estimation procedure

In summary, the anisotropic velocity field is iteratively updated starting from an initial model that may be obtained from stacking-velocity tomography at borehole locations (Wang & Tsvankin, 2010). In the first few iterations, the symmetry-direction velocity \(V_{P0}\) is typically inaccurate and simultaneous inversion for all TTI parameters may result in unacceptably
large updates for $\epsilon$ and $\delta$. If the anisotropy parameters are moderate (e.g., $\epsilon < 0.25$ and $\delta < 0.15$ in the following tests), it is convenient to fix them temporarily at the initial values (typically small) and limit the updates to the velocity $V_{P0}$.

At the second stage of parameter updating, the model is divided into several layers based on the picked reflectors, and the anisotropy parameters are assumed to be spatially invariant within each layer. The velocity $V_{P0}$ is then updated on a grid, while $\epsilon$ and $\delta$ change in each layer (i.e., the inversion for $\epsilon$ and $\delta$ is layer-based). Such a “quasi-factorized” assumption is equivalent to strong smoothing of $\epsilon$ and $\delta$ and may help resolve all TTI parameters if $V_{P0}$ is a linear function of the spatial coordinates and at least two distinct dips are available (Behera & Tsvankin, 2009).

At the third and last stage of velocity analysis, the anisotropy parameters are updated on the same grid as that for $V_{P0}$ to allow for more realistic treatment of heterogeneity. Still, because P-wave kinematics are less sensitive to anisotropy than to the symmetry-direction velocity, the $\epsilon$- and $\delta$-fields in equation 5.11 are regularized with larger weights.
CHAPTER 6
SYNTHETIC TESTS OF TTI TOMOGRAPHY

The 2D P-wave tomographic algorithm described in chapter 5 is first tested on two relatively simple models containing a “quasi-factorized” TTI syncline (i.e., $\epsilon$ and $\delta$ are constant inside the TTI layer) and a TTI thrust sheet. Stable parameter estimation requires either strong smoothness constraints or additional information from walkaway VSP (vertical seismic profiling) traveltimes. If the model is quasi-factorized with a linear spatial variation of $V_{P0}$, it may be possible to obtain the interval TTI parameters just from long-spread reflection data.

Then I examine the performance of the regularized joint tomography of reflection and VSP data for two sections of the BP TTI model that contain an anticline and a salt dome. The TTI parameters in the shallow part of both sections (down to 5 km) are well-resolved by the three-stage model-updating process. Due to the limited constraints from reflection events and sparse coverage of VSP rays at depth, the velocity field in the deeper part of the section is estimated with larger errors. These results provide useful guidance for building accurate TTI models for prestack depth imaging.

6.1 Syncline model

Using a 2D finite difference program (‘suea2df’ in Seismic Unix), I generate P-wave reflection data for a medium similar to the syncline model (Figure 6.1) of Behera & Tsvankin (2009). A “quasi-factorized” TTI layer, with the boundaries dipping at angles up to 35° and the symmetry axis perpendicular to its bottom, is embedded between isotropic media. Each layer includes an additional reflector that helps estimate the vertical velocity gradient. The data are computed with shot and receiver intervals of 0.05 km; the maximum offset is 7 km. A depth image produced by Kirchhoff migration with the correct velocity model is shown in Figure 6.2; the artifacts are caused by noise in the synthetic data.
Figure 6.1: Model with a quasi-factorized TTI layer. (a) The layer boundaries are marked by bold black lines; within each layer, there is one more reflector (thin lines). The TTI layer in the middle is embedded between two isotropic layers. The symmetry-direction velocities at the top of each layer (at locations marked by three black dots in the middle of the model) are $V_{P0}^{(1)} = 1.62$ km/s, $V_{P0}^{(2)} = 2.66$ km/s, and $V_{P0}^{(3)} = 3.44$ km/s (from top to bottom). The velocity $V_{P0}$ varies linearly in each layer according to the lateral gradients $k_x^{(1)} = 0.03$ s$^{-1}$, $k_x^{(2)} = 0.05$ s$^{-1}$, and $k_x^{(3)} = 0.05$ s$^{-1}$, and the vertical gradients $k_z^{(1)} = 0.5$ s$^{-1}$, $k_z^{(2)} = 0.4$ s$^{-1}$, and $k_z^{(3)} = 0.3$ s$^{-1}$. The symmetry axis (black arrows) is perpendicular to the boundaries of the TTI layer. (b) The field of the tilt angle $\nu$ of the symmetry axis with the vertical. (c), (d) The anisotropy parameters $\epsilon$ and $\delta$, which are constant in each layer ($\epsilon^{(1)} = \delta^{(1)} = \epsilon^{(3)} = \delta^{(3)} = 0$, $\epsilon^{(2)} = 0.1$, and $\delta^{(2)} = -0.1$).
6.1.1 Test 1

In the first test, I assume that the first layer is known to be isotropic, while the other two layers are treated as quasi-factorized TTI. In each layer, the anisotropy parameters $\epsilon$ and $\delta$ are constant, and the symmetry-direction velocity $V_{P0}$ is defined as

$$V_{P0}(x,z) = V_{P0}(x_0,z_0) + k_x(x - x_0) + k_z(z - z_0),$$

(6.1)

where $V_{P0}(x_0,z_0)$ is the value at a specific point $(x_0,z_0)$; $k_x$ and $k_z$ are the lateral and vertical gradients, respectively. Thus, the model vector becomes

$$\lambda_f = \{V_{P0}^{(n)}, k_x^{(n)}, k_z^{(n)}, \epsilon^{(n)}, \delta^{(n)}\}, \quad (n = 1, 2, 3),$$

(6.2)

where $V_{P0}^{(n)}$ is defined on top of each layer at the point with lateral coordinate $x_0 = 4$ km (Figure 6.1a); the depth $z_0^{(n)}$ of that point may change with the interface position after each iteration of MVA. Behera & Tsvankin (2009) assume that $V_{P0}^{(n)}$ is known at one point in each layer (e.g., from check shots) and demonstrate that the other elements of the model vector (equation 6.2) can be resolved by flattening CIGs of long-spread P-wave reflections.
The available range of dips in the TTI syncline, however, helps resolve the parameters $V_p$ and $\delta$ (Tsvankin & Grechka, 2011, section 2.4.1) in the second and third layer using just reflection traveltimes. Therefore, I relax the requirement of specifying the correct velocity value at a single location in each layer, and invert for $V_p^{(n)}$. However, because the first layer is quasi-horizontal, $V_p^{(1)}$ cannot be constrained without assuming isotropy (Behera & Tsvankin, 2009). Therefore, $\epsilon^{(1)}$ and $\delta^{(1)}$ in the top layer are set to zero, with only $V_p^{(1)}$, $k_x^{(1)}$, and $k_z^{(1)}$ estimated by MVA. The symmetry axis is assumed (correctly) to be perpendicular to the interfaces (see above), with the spatial distribution of the tilt $\nu$ between the interfaces obtained by linear interpolation.

The “quasi-factorized” assumption (which is valid for the model at hand) is equivalent to applying a strong smoothing constraint (described by the operator $L$ in equation 5.10) to the anisotropic velocity field. Also, the number of unknown parameters (equation 6.2 with $\epsilon^{(1)} = \delta^{(1)} = 0$) is significantly reduced.

The traveltime derivatives, however, are still calculated at the vertices of relatively small grids (equations 5.6 and 5.8). Therefore, I need to construct a mapping matrix $C$ using the picked boundaries to convert the model update $\Delta \lambda_f$ into the parameter perturbations $\Delta V_p$, $\Delta V_h$, and $\Delta V_{nmo}$ at each grid point (equation 5.9; the grid size is 100 m $\times$ 100 m). Also, the updates at each iteration are constrained by $|\Delta V_p^{(n)}| \leq 0.05$ km/s, $\epsilon^{(n)} \geq 0$, $|\Delta \epsilon^{(n)}| \leq 0.02$, and $|\Delta \delta^{(n)}| \leq 0.02$, which corresponds to restricting the norm of model updates in equation 5.10. Therefore, without a regularization term, the inverse problem reduces to minimizing the function

$$\mathcal{F}(\Delta \lambda_f) = \|AC\Delta \lambda_f - b\|^2,$$

which can be accomplished by applying a linear least-squares algorithm (Coleman & Li, 1996b; Gill et al., 1981).

Tomographic inversion is performed for 31 image gathers uniformly distributed between the horizontal coordinates 1 km and 7 km. The initial model used in the first iteration of MVA is composed of horizontal isotropic layers (Figure 6.3a). Because the velocity field
Figure 6.3: (a) Horizontally layered isotropic model used in the first iteration of velocity analysis. (b) CIGs (displayed every 0.5 km from 1 km to 7 km) after migration with the initial model. (c) The corresponding depth image. The bottom of the model is shifted up due to the wrong velocity field.
is strongly distorted, the events exhibit significant residual moveout (Figure 6.3b) and the depth image is inaccurate (Figure 6.3c).

The inverted parameters after 20 iterations are listed in Table 6.1. As expected, the errors in the parameters of the third (deepest) layer are largest, primarily due to its smaller contribution to the effective reflection traveltimes. Also, because the two bottom reflectors have mild dips and the maximum offset-to-depth ratio for the deepest reflector is less than 1.5, the anisotropy parameters in the third layer are not well constrained. After PSDM, all CIGs become sufficiently flat (Figure 6.4a) with the maximum error in the migrated depth reaching 80 m for the bottom of the model (Figure 6.4b). If the correct values of $V_{P0}^{(n)}$ on top of each layer are used (as Behera & Tsvankin (2009) did in their inversion), the velocity gradients and anisotropy parameters can be recovered with higher accuracy.

Table 6.1: Inversion results for test 1.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$V_{P0}$ (km/s)</th>
<th>$k_x$ (s$^{-1}$)</th>
<th>$k_z$ (s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>inverted</td>
<td>error (%)</td>
<td>inverted</td>
</tr>
<tr>
<td>Layer 1</td>
<td>1.63</td>
<td>1</td>
<td>0.030</td>
</tr>
<tr>
<td>Layer 2</td>
<td>2.63</td>
<td>-1</td>
<td>0.048</td>
</tr>
<tr>
<td>Layer 3</td>
<td>3.29</td>
<td>-4</td>
<td>0.046</td>
</tr>
</tbody>
</table>

| Layer 1 | $\epsilon$ | error (%) | $\delta$ |
|-------|-------------|-----------|
|       | inverted | error (%) | inverted | error |
| Layer 1 | –     | –         | –        | –     |
| Layer 2 | 0.10  | 0         | -0.09    | 0.01  |
| Layer 3 | 0.05  | 0.05      | 0.05     | 0.05  |

6.1.2 Test 2

This time, I relax the constraint on the spatial velocity variation, and update the symmetry-direction velocity $V_{P0}$ at each grid point (the grid size is the same – 100 m $\times$ 100 m). The parameters $\epsilon$ and $\delta$ are still taken constant within each layer, as they are in the actual model. Therefore, the model-update vector becomes

$$\Delta \lambda = \{\Delta V_{P0}^{(c)}, \Delta \epsilon^{(n)}, \Delta \delta^{(n)}\} \quad (c = 1, 2, \ldots, W), \ (n = 2, 3), \quad (6.4)$$
Figure 6.4: (a) CIGs after PSDM with the inverted parameters (Table 6.1), and (b) the corresponding depth image.
where \( W \) is the number of grid points in the model (\( W = 81 \times 51 \)); \( \epsilon^{(1)} \) and \( \delta^{(1)} \) in the top layer are still fixed at zero.

To solve the tomographic inverse problem, equation 5.2 is modified as

\[
\mathcal{F}(\Delta \lambda) = \| AC \Delta \lambda - b \|^2 + R(\Delta \lambda),
\]

(6.5)

where \( C \) is a mapping matrix similar to the one in equation 6.3, and the Tikhonov (1963) regularization (equation 5.10) is applied to smooth the velocity field in the process of flattening CIGs. Here, because the matrix \( L \) in equation 5.10 is a finite-difference approximation of the Laplacian operator, only parameter variations between adjacent grid points are restricted. Therefore, equation 6.5 can be used to recover nonlinear velocity fields, while in test 1 the spatial variation of \( V_{P0} \) was held linear. The parameter \( \epsilon^{(n)} \) is kept nonnegative, which is a plausible constraint for sedimentary rocks.

To build an initial isotropic model, I use the Dix-derived values of \( V_{nmo} \) from a common-midpoint (CMP) gather in the area with relatively flat reflectors (close to the left edge of the model). Then, the initial velocity field is obtained by image-guided interpolation (Hale, 2009) (Figure 6.5a). After 10 iterations, the residual moveout in the CIGs is largely removed (Figure 6.6a). On the final image (Figure 6.6b), the middle sections of the reflectors in the top two layers are well positioned (errors are up to \( \pm 80 \) m), but the two bottom reflectors and the interface segments near the model edges are somewhat distorted.

Because the symmetry-direction velocity \( V_{P0} \) is estimated on the grid, specifying it at any single point does not help in the inversion. Without a strong smoothing constraint (as the one in test 1), the velocity \( V_{P0} \) is obtained with substantial errors. In some areas of the top two layers, flattening the CIGs yields \( V_{P0} \) with accuracy higher than 5% (Figure 6.7a and Figure 6.8a). The errors in \( V_{P0} \), however, are much larger in the top left part of the model and in the section below the syncline. Moreover, because the velocity across the layer boundaries is discontinuous (Figure 6.1a), whereas the inversion operates with a smoothed model (equation 6.5), \( V_{P0} \) is distorted near the interfaces.
Figure 6.5: (a) Isotropic model for the first iteration of velocity analysis in test 2. The velocity is defined at each vertex of the 100 m × 100 m grid. There is substantial residual moveout in the CIGs which are displayed every 0.5 km from 1 km to 7 km on plot (b), and the depth image (c) is distorted.
Figure 6.6: (a) CIGs after 10 iterations and (b) the corresponding depth image in test 2.
Figure 6.7: (a) Symmetry-direction velocity $V_{P0}$ estimated at each grid point in test 2. The inverted interval parameters (b) $\epsilon$ and (c) $\delta$. Vertical smoothing over a distance of 100 m was applied to the $\epsilon$- and $\delta$-fields.
Figure 6.8: Percentage velocity errors in test 2 for $1 \text{ km} \leq x \leq 7 \text{ km}$ and $0.3 \text{ km} \leq z \leq 3.9 \text{ km}$. (a) $V_{P0}$, (b) $V_h$, and (c) $V_{nmo}$.
Because $\epsilon$ and $\delta$ were correctly assumed to be constant in each layer, both parameters are well-resolved (Figure 6.7b and Figure 6.7c). To avoid inconsistent updates of $\epsilon$ and $\delta$ caused by interface movement at each iteration, I applied vertical smoothing over a distance of 100 m. As a result, there are no jumps in the inverted values of $\epsilon$ and $\delta$ across the boundaries. The velocities $V_h = V_{P0}\sqrt{1+2\epsilon}$ and $V_{\text{nmo}} = V_{P0}\sqrt{1+2\delta}$, computed with the estimated parameters $V_{P0}$, $\epsilon$, and $\delta$, are distorted by less than 5% in the top two layers, except for the vicinity of the layer boundaries (Figure 6.8b and Figure 6.8c). However, $V_h$ and $V_{\text{nmo}}$ below the syncline are poorly constrained, with errors up to 8%.

6.1.3 Test 3

The nonuniqueness of parameter estimation in anisotropic media is often reduced by including additional information, such as borehole data. Therefore, in the next test I combine the surface reflection data with P-wave walkaway VSP traveltimes.

The parameters $\epsilon$ and $\delta$ are still assumed to be constant in each layer, but the number of layers is doubled to increase the complexity of the trial model and make it more realistic (i.e., each of the three layers is divided into two by an additional interface). Therefore, the model-update vector becomes

$$\Delta \lambda = \{ \Delta V_{P0}^c, \Delta \epsilon^{(n)}, \Delta \delta^{(n)} \}, \quad (c = 1, 2, \ldots, W), \quad (n = 1, 2, \ldots, 6),$$

(6.6)

where $W = 81 \times 51$ is the same as in test 2. Because the constraints from walkaway VSP data allow us to resolve all three TTI parameters, each layer is treated as anisotropic with the parameters $V_{P0}^c$, $\epsilon^{(n)}$, and $\delta^{(n)}$ updated simultaneously.

To generate synthetic VSP data, a vertical “well” is placed at location $x_{\text{VSP}} = 3$ km, with a string of 46 receivers spanning the depth interval from 0.09 km to 4.59 km every 100 m. The sources are located at the surface between $x = 0.05$ km and $x = 5.95$ km, also with an interval of 100 m. The traveltime $t_{\text{VSP}}$ for each source-receiver pair is obtained by ray tracing in the actual model (Figure 6.1).
During the iterative inversion, VSP traveltimes $t_{VSP}^{\text{calc}}$ are computed by anisotropic ray tracing for each trial model. The difference between the observed and calculated traveltimes $d = t_{VSP} - t_{VSP}^{\text{calc}}$ can serve as the input to the tomographic inversion with the objection function 5.2. The same Tikhonov (1963) regularization (equation 5.10) as in test 2 is used.

If borehole information is available, it is possible to build an initial TTI velocity model using, for example, stacking-velocity tomography described in chapter 3. Here, to ensure consistency, I use the same isotropic initial model as that in test 2. The VSP data are directly used in the iterative tomographic algorithm to constrain the anisotropic velocity field.

After 15 iterations, the tomographic inversion significantly reduced the misfit of VSP traveltimes (Figure 6.9b). Also, CIGs are flat throughout much of the model (Figure 6.9a and Figure 6.10a), although several gathers close to the right edge still exhibit apparent RMO because VSP data do not provide constraints on the right side of the section. The reflectors on the final image (Figure 6.10b) are accurately positioned (errors are up to ±50 m), with somewhat larger distortions near the edges due to errors in the velocity field.

With the constraints from VSP data, the spatially varying symmetry-direction velocity $V_{P0}$ is well-recovered (Figure 6.11a), with absolute errors in most areas smaller than 2% (Figure 6.12a). As in test 2, the errors in $V_{P0}$ are higher near the layer boundaries. Due to the insufficient angle coverage of both reflection and VSP data, the errors in $\epsilon$ and $\delta$ in the bottom layer increase up to 0.05 (Figure 6.11b and Figure 6.11c). The velocities $V_h$ and $V_{nmo}$ are generally distorted by less than 3% (Figure 6.12b and Figure 6.12c), although the errors at grid points near the layer boundaries are somewhat larger.

6.2 TTI thrust sheet

Next, I test the algorithm on the synthetic data of Zhu et al. (2007), whose model (simulating typical structures in the Canadian Foothills) includes a TTI thrust sheet embedded in an otherwise isotropic, homogeneous medium (Figure 6.13). P-wave reflection data are generated by anisotropic finite-difference modeling. The sources and receivers used in our
Figure 6.9: (a) $L^2$-norm of the residual moveout in CIGs at each iteration for joint inversion of reflection and VSP data (test 3, equation 5.2). (b) The $L^2$-norm of the VSP traveltime misfit in equation 5.2.
Figure 6.10: (a) CIGs after 15 iterations and (b) the corresponding depth image in test 3. There is residual moveout ("hockey sticks") at long offsets for events near both edges of the model.
Figure 6.11: (a) Symmetry-direction velocity $V_{P0}$ estimated at each grid point in test 3. (b), (c) The inverted layer-based parameters $\epsilon$ and $\delta$ (the model includes six layers instead of three in test 2).
Figure 6.12: Percentage velocity errors in test 3 for \(1 \text{ km} \leq x \leq 7 \text{ km}\) and \(0.2 \text{ km} \leq z \leq 4 \text{ km}\). (a) \(V_{P0}\), (b) \(V_h\), and (c) \(V_{nmo}\).
test are placed every 60 m with the maximum offset reaching 1980 m. Because the exact model geometry (i.e., interface positions) is unavailable, I cannot provide comparisons of our migration results with the actual model.

Figure 6.13: Schematic plot of the synthetic model from Zhu et al. (2007) (exact model geometry is unknown). The bending thrust sheet is TTI with the symmetry axis perpendicular to the boundaries. The dips range from 0° to 61°, and the interval parameters of the sheet are \( V_{P0} = 2925 \text{ m/s}, \epsilon = 0.16, \) and \( \delta = 0.08. \) The P-wave velocity in the isotropic background medium is 2740 m/s.

The initial model for MVA includes two horizontal isotropic layers (Figure 6.14a). Although the P-wave velocity in the isotropic background is set to the correct value, ignoring transverse isotropy causes noticeable RMO in CIGs (Figure 6.14b) and a strong distortion of the imaged reflector beneath the thrust sheet (Figure 6.14c).

In the model-updating process, the velocity \( V_{P0} \) is defined on a square \((100 \text{ m} \times 100 \text{ m})\) grid. Based on the picked reflectors, the model is divided into two isotropic blocks and a TTI layer sandwiched between them. Within each layer/block, the anisotropy parameters \( \epsilon \) and \( \delta \) are assumed to be constant. The symmetry axis is taken perpendicular to the reflectors, with the tilt changing during the updates. Because there is only a single deep horizontal reflector on the right side of the model (Figure 6.13), the parameters \( \epsilon \) and \( \delta \) cannot be resolved solely from P-wave reflection data. Therefore, both \( \epsilon \) and \( \delta \) in the block to the right of the TTI layer are set to zero.
Figure 6.14: (a) Initial isotropic model used in velocity analysis for the model from Figure 6.13. The P-wave velocities in the top and bottom layers are 2740 m/s and 3200 m/s, respectively. (b) The CIGs computed with the initial model and displayed every 0.6 km. (c) The corresponding depth image.
After 12 iterations using the objective function in equation 6.5, the velocity in the TTI layer is partially recovered (Figure 6.15a). Because $V_{P0}$ is updated on a relatively fine grid, flattening the CIGs along three reflectors (only one reflector for the block on the right side) with general smoothing regularization (equation 6.5) is insufficient to recover the velocity field. For example, there is noticeable heterogeneity in each block that does not exist in the actual model. With the constraints provided by a wide range of dips in the TTI thrust sheet and the correct assumption about the spatial variation of $\epsilon$ and $\delta$, both parameters inside the sheet are well resolved (Figure 6.15b and Figure 6.15c). The error in $\epsilon$ in the TTI layer (0.03) is somewhat larger than that in $\delta$ (-0.01) because of the small offset-to-depth ratio, which is close to unity for the bottom reflector. Despite remaining distortions in $V_{P0}$, the obtained anisotropic velocity field largely removes the residual moveout in the CIGs (Figure 6.16a) and improves the depth image, especially that of the bottom horizontal reflector (Figure 6.16b). Additional reflectors or walkaway VSP data would allow us to improve reconstruction of the velocity field.

### 6.3 BP anticline model

Next, the joint tomography of P-wave long-spread reflection data and walkaway VSP traveltimes is tested on a section of the TTI model produced by BP. That section contains an anticline structure surrounded by gently dipping anisotropic layers. The velocity $V_{P0}$ in the actual model is smoothly varying (Figure 6.17a), except for a small jump at the water bottom. The symmetry axis is set perpendicular to the interfaces (Figure 6.17b), and the anisotropy parameters $\epsilon$ and $\delta$ change from layer to layer with relatively weak lateral variations compared to those of $V_{P0}$ (Figure 6.17c and Figure 6.17d). The depth image produced by Kirchhoff prestack depth migration with the correct velocity model is shown in Figure 6.18.

The tomographic inversion with “structure-guided” regularization (chapter 5) is applied to CIGs from $x = 40$ km to 61 km with an interval of 150 m (the maximum offset is 10 km). Synthetic VSP data were generated in a vertical “well” placed at location $x_{\text{VSP}} = 51.4$ km.
Figure 6.15: (a) Symmetry-direction velocity $V_{P0}$ estimated at each grid point. The inverted block-based parameters (b) $\epsilon$ and (c) $\delta$. In the TTI layer, the estimated $\epsilon = 0.19$ and $\delta = 0.07$. The parameters $\epsilon$ and $\delta$ in the block to the right of the TTI layer are set to zero.
Figure 6.16: (a) CIGs after 12 iterations and (b) the corresponding depth image. Note that the reflector beneath the thrust sheet has been flattened.
Figure 6.17: Section of the BP TTI model that includes an anticline (the grid size is 6.25 m × 6.25 m). The top water layer is isotropic with velocity 1492 m/s. (a) The symmetry-direction velocity $V_{P0}$. The black line marks a vertical “well” at $x = 51.4$ km. (b) The symmetry-axis tilt $\nu$. The anisotropy parameters (c) $\epsilon$ and (d) $\delta$. 
Figure 6.18: Depth image produced by prestack migration with the actual parameters from Figure 6.17. Imaging was performed using sources and receivers placed every 50 m. with 24 receivers spanning the interval from \( z = 4756.25 \) m to 5043.75 m every 12.5 m and 24 more receivers evenly placed between 7837.5 m and 8125 m. The VSP sources were located at the surface every 50 m from \( x = 41.4 \) km to 61.4 km (the maximum offset is also 10 km). In addition, check-shot traveltimes were recorded every 50 m from \( z = 943.5 \) m to 9493.75 m.

To build an initial model, I compute a 1D profile of \( V_{P0} \) from the check-shot traveltimes and then obtain the 2D velocity field (Figure 6.19) by extrapolation that conforms to the picked interfaces. The exact position of the water bottom is assumed to be known, and the velocity of the water layer is fixed at the correct value. The noticeable residual moveout in the CIGs (Figure 6.20a) and the distorted depth image (Figure 6.20b) indicate that the velocity field contains significant errors.

Following the three-stage parameter-estimation procedure described in chapter 5, first I update only the velocity \( V_{P0} \) defined on a rectangular 200 m \( \times \) 100 m grid, while keeping
the anisotropy parameters $\epsilon$ and $\delta$ set to zero. Then $\epsilon$ and $\delta$ are taken constant in each layer (delineated by the interfaces picked on the image), but updated simultaneously with the velocity. With this “quasi-factorized” TTI assumption, the inverted model (Figure 6.21) improves the positioning of the reflectors (Figure 6.22b) and reduces the residual moveout in CIGs (Figure 6.22a) and the VSP traveltime misfit.

Figure 6.19: Initial isotropic model with the velocity $V_{P0}$ defined on a 200 m $\times$ 100 m grid. The velocity in the water is set to the correct value.

However, the residual moveout in Figure 6.22a is not completely removed, mainly because the assumption about the anisotropy parameters does not conform to the actual $\epsilon$- and $\delta$-fields (Figure 6.17c and Figure 6.17d). To allow for more realistic spatial variations, at the last stage of parameter updating I estimate the parameters $\epsilon$ and $\delta$ on the same grid as the one used for the velocity $V_{P0}$. However, because the trade-offs between the parameters may cause large errors in $\epsilon$ and $\delta$ defined on small grids, the anisotropy parameters should be more tightly constrained, so the corresponding regularization coefficients should be larger than those for $V_{P0}$.

After five more iterations with all three parameters updated on grids, the velocity $V_{P0}$ above $z = 7$ km is relatively well-recovered with percentage errors in most areas smaller than 4% (Figure 6.23a). The spatial variations of $\epsilon$ and $\delta$ are partially resolved from the water bottom down to $z = 5$ km (Figure 6.23c and Figure 6.23d). The coverage of VSP rays, however, becomes more sparse with depth. Also, the offset-to-depth ratio of P-wave reflections is insufficient to constrain the parameter $\epsilon$ below 7 km, although the maximum offset reaches 10 km. Therefore, the accuracy in $\epsilon$ and $\delta$ decreases in the deep part of the
Figure 6.20: (a) CIGs (displayed every 3 km from 41 km to 62 km) and (b) the migrated section computed with the initial model from Figure 6.19.
model. The final inverted model (Figure 6.23) practically removes the residual moveout in CIGs (Figure 6.24a) (except for the locations close to the left and right edges due to poor ray coverage), and the reflections are better focused (Figure 6.24b), especially above \( z = 7 \) km.

An important parameter that influences the accuracy of the reconstructed velocity model is the symmetry-axis tilt \( \nu \), which is computed directly from the depth image. Poorly constrained TTI parameters in the deep part of the model yield a strongly distorted image, which produces large errors in the estimated values of \( \nu \). The obtained tilt field is used for the next iteration of MVA, which further distorts the other estimated TTI parameters. Therefore, without sufficient constraints from deep reflection events and VSP rays, the trade-offs between the tilt \( \nu \) and the other TTI parameters increase the uncertainty in velocity analysis at depth.

![Figure 6.21: Anticline model from Figure 6.17 updated using the “quasi-factorized” TTI assumption. (a) The symmetry-direction velocity \( V_{P0} \) estimated on a 200 m \( \times \) 100 m grid. (b) The tilt \( \nu \) obtained by setting the symmetry axis perpendicular to the reflectors. The inverted interval parameters (c) \( \epsilon \) and (d) \( \delta \), which are constant within each layer.](image-url)
Figure 6.22: (a) CIGs and (b) the migrated section obtained with the “quasi-factorized” TTI model from Figure 6.21.
Figure 6.23: Inverted TTI parameters (a) $V_{P0}$, (c) $\epsilon$, and (d) $\delta$ after the final iteration of MVA. All three parameters are estimated on a $200 \text{ m} \times 100 \text{ m}$ grid. (b) The symmetry-axis tilt $\nu$ computed from the depth image obtained before the final iteration.
Figure 6.24: (a) CIGs and (b) the migrated section computed with the final inverted model in Figure 6.23.
6.4 BP salt model

The last test is performed for another section of the BP TTI model that includes a salt dome (Figure 6.25). The strong reflections from the top of the salt dome and the flanks right beneath it are clearly imaged (Figure 6.26) by Kirchhoff depth migration, but the deeper segments of the flanks are blurred even when the correct model is used. The image quality can be improved with a wavefield-based imaging algorithm, such as reverse time migration (RTM).

![Images of Vp0, ν, ε, and δ](image1)

Figure 6.25: Section of the BP TTI model with a salt dome (the grid size is 6.25 m × 6.25 m). The top water layer and the salt body are isotropic with the P-wave velocity equal to 1492 m/s and 4350 m/s, respectively. (a) The symmetry-direction velocity $V_{p0}$. The vertical “well” at $x = 29.9$ km is marked by a black line. (b) The tilt of the symmetry axis, which is set orthogonal to the interfaces. The anisotropy parameters (c) $\epsilon$ and (d) $\delta$.

The maximum offset (10 km) and the source and receiver intervals (50 m) are the same as in the previous test. The CIGs used for MVA are computed every 150 m from $x = 16$ km to 46 km. The dataset contains a vertical “well” at location $x_{VSP} = 29.9$ km to the left of the salt body (Figure 6.25). Two sets of 24 receivers (one between $z = 5275$ m and 5562.5
m and the other between \( z = 8400 \) m and \( 8687.5 \) m) were placed at even intervals in the well to record a walkaway VSP survey. The maximum offset for the VSP data is 10 km as well with a source interval of 50 m. The input data also include the check-shot traveltimes obtained every 50 m from \( z = 1743.75 \) m to \( 9093.75 \) m.

During the inversion, the water layer and salt body are kept isotropic with the velocities fixed at the correct values. Also, the actual positions of the top and flanks of the salt dome are assumed to be known, and the update is performed only for the sedimentary formations around the salt body. Since ray tracing becomes unstable in the presence of sharp velocity contrasts, I apply 2D smoothing to the velocity model to find the raypaths crossing the salt and then calculate the traveltimes and their derivatives in the original (unsmoothed) model.

Similar to the previous test, an initial isotropic model (Figure 6.27) is built from check-shot traveltimes. Because the TTI parameters are different on both sides of the salt body, the residual moveout in the CIGs (Figure 6.28a) is larger to the right of the salt (i.e., further away from the well). Due to the large velocity errors in the initial model, most reflectors are

Figure 6.26: Depth image produced with the actual parameters from Figure 6.25.
misplaced (Figure 6.28b).

Figure 6.27: Initial isotropic model with $V_{P0}$ defined on a 200 m $\times$ 100 m grid. The velocity in the top water layer is set to the correct value.

After two iterations of velocity ($V_{P0}$) updating with fixed $\epsilon$ and $\delta$ and two more iterations with the “quasi-factorized” TTI model assumption (see above), the estimated parameter fields (Figure 6.29) produce relatively flat CIGs (Figure 6.30a) and an improved image (Figure 6.30b). For the VSP sources placed to the left of the well, the corresponding rays pass through the relatively simple sedimentary section. The VSP rays originated to the right of the well, however, cross the high-velocity salt body, and errors in the velocity field near the salt boundaries may cause large perturbations of the ray trajectories. Therefore, I assign smaller weights in the objective function to the VSP traveltimes for the sources to the right of the well. Hence, the anisotropic velocity field on the right side of the salt dome has to be determined mostly from the P-wave reflection data, which leads to larger uncertainty in the TTI parameters.

To further reduce the residual moveout in CIGs and the VSP traveltime misfit, the anisotropy parameters are estimated on the same grid as that for $V_{P0}$. After three more iterations, the velocity (Figure 6.31a) above $z = 7$ km on the left side of the salt body is relatively well-resolved (errors in most areas do not exceed 3%); however, the errors in $V_{P0}$ on the right side are higher because of the limited constraints from VSP data, as described above. The spatial variations of $\epsilon$ and $\delta$ are partially recovered from the water bottom down to $z = 5$ km (Figure 6.31c and Figure 6.31d). Because of the limited offset-to-depth ratio and poor coverage of VSP rays (especially for the right part of the model) at depth,
Figure 6.28: (a) CIGs (displayed every 3.25 km from 18 km to 44 km) and (b) the migrated section computed with the initial model in Figure 6.27.
the anisotropy parameters for grid points below 5 km could not be updated after the first iteration. Using the final model, the residual moveout in CIGs (Figure 6.32a) and the VSP traveltime misfit are largely reduced, which produces a more accurate image (Figure 6.32b).

Figure 6.29: Salt model from Figure 6.25 updated using the “quasi-factorized” TTI assumption. (a) The symmetry-direction velocity $V_{P0}$ estimated on a 200 m $\times$ 100 m grid. (b) The tilt $\nu$ obtained from the image. The inverted interval parameters (c) $\epsilon$ and (d) $\delta$, which are constant within each block.

### 6.5 Conclusions

Currently TTI models are often used to improve imaging results in complex geologic environments including subsalt plays and active tectonic areas (e.g., the Canadian Foothills). However, allowing for the tilt of the symmetry axis introduces additional uncertainty into estimation of the interval TTI parameters, even if the symmetry-axis orientation is fixed using a priori information. In chapter 5, I developed an efficient 2D tomographic algorithm for TTI models, with the parameters $V_{P0}$, $\epsilon$, $\delta$, and the symmetry-axis tilt $\nu$ defined on a rectangular grid.
Figure 6.30: (a) CIGs and (b) the migrated section obtained with the “quasi-factorized” TTI model from Figure 6.29.
Figure 6.31: Inverted TTI parameters (a) $V_{P0}$, (c) $\epsilon$, and (d) $\delta$ after the final iteration of joint tomography. All three parameters are estimated on a 200 m × 100 m grid. (b) The symmetry-axis tilt $\nu$ computed from the depth image obtained before the final iteration.
Figure 6.32: (a) CIGs and (b) the migrated section computed with the final inverted model in Figure 6.31.
Model updating is performed by iterative linearized inversion for $V_{P0}$, $\epsilon$, and $\delta$, while the spatial distribution of $\nu$ is obtained by setting the symmetry axis orthogonal to the reflectors. The Fréchet matrix at each iteration is constructed by approximately evaluating the traveltime derivatives with respect to the TTI parameters at all grid points near the reflection raypaths. The devised algorithm is first used to explore the influence of different assumptions about the spatial variation of TTI parameters on the accuracy of the inverted model.

Numerical testing is first performed for a three-layer medium with a “quasi-factorized” TTI syncline (in which $\epsilon$ and $\delta$ are constant) embedded between isotropic layers with linear velocity variation. The symmetry-direction velocity $V_{P0}$ in the TTI layer also varies linearly, while the symmetry axis is orthogonal to the layer boundaries. With several correct model assumptions (i.e., if $\epsilon$ and $\delta$ are known to be constant in each layer, the interval velocity is defined by the vertical and lateral gradients, and the first layer is treated as isotropic), the algorithm accurately reconstructs the velocity field with no other a priori information. If $V_{P0}$ is updated separately at each grid point while the anisotropy parameters are still assumed to be constant in each layer, flattening the CIGs along with Tikhonov regularization helps recover the velocities $V_h = V_{P0} \sqrt{1 + 2\epsilon}$ and $V_{nmo} = V_{P0} \sqrt{1 + 2\delta}$ (and, therefore, $\epsilon$ and $\delta$) in the first two layers with errors reaching 5%. The accuracy of the inverted velocities is lower in the layer below the TTI syncline.

In another test for the same model, reflection data are combined with P-wave walkaway VSP traveltimes. While $V_{P0}$ is updated at each grid point, $\epsilon$ and $\delta$ are taken constant in six “sublayers” (each layer was divided into two). Because of the additional constraints from VSP data, the velocities $V_{P0}$, $V_h$, and $V_{nmo}$ are well-resolved (errors are smaller than 3%) for most of the model, and the errors in $\epsilon$ and $\delta$ do not exceed 0.02 (except for the bottom layer).

The second synthetic model includes a bending TTI thrust sheet with constant parameters $\epsilon$ and $\delta$. The velocity $V_{P0}$ is updated on a grid, while the “quasi-factorized” assumption
proves sufficient to recover both $\epsilon$ and $\delta$ due to a wide range of reflector dips. Without walkaway VSP traveltimes, however, the velocity $V_{P0}$ in each layer cannot be resolved just by flattening the CIGs for the available three reflectors, despite application of general smoothing regularization.

To handle more complex and realistic data such as those generated from BP TTI model, in chapter 5 I design so called “structure-guided” regularization that incorporates useful geologic constraints into the tomographic algorithm. The regularization terms in the objective function allow for parameter variations across layers, but suppress them in the direction parallel to boundaries. In the iterative inversion, such structure-guided regularization also helps propagate along interfaces the most reliable updates corresponding to large derivatives in the Fréchet matrix (e.g., those in the cells crossed by dense VSP rays).

To improve the convergence of the algorithm, I propose a three-stage parameter-updating procedure (chapter 5). In the first several iterations, only the velocity $V_{P0}$ is updated on a grid, while the anisotropy parameters $\epsilon$ and $\delta$ are fixed at their initial values. This operation eliminates potentially large distortions in $\epsilon$ and $\delta$ caused by the parameter trade-offs. At the second stage of the inversion, $\epsilon$ and $\delta$ are taken constant in each layer and updated together with the grid-based velocity $V_{P0}$. Finally, all three TTI parameters are estimated simultaneously on the grid with the constraints provided by the regularization terms described above.

The joint tomography of reflection and VSP data with structure-guided regularization was successfully applied to two sections of the BP TTI model that include an anticline and a salt dome. In both tests, a purely isotropic velocity field, which was obtained from check-shot traveltimes and extrapolated along the horizons, served as the initial model. With constraints from P-wave reflection and VSP data, the TTI parameters in the shallow part (above 5 km) of both sections are well-resolved. However, the errors in the anisotropy parameters $\epsilon$ and $\delta$ increase with depth due to the small offset-to-depth ratio and poor coverage of VSP rays. For the model with the salt dome, the anisotropic velocity field is recovered with higher
accuracy to the left of the salt, where the inversion was tightly constrained by VSP data from a nearby well.

6.6 Acknowledgments

I am grateful to Hemang Shah of BP for creating the TTI model and BP Exploration Operation Company Limited for generating the synthetic data set.
CHAPTER 7
APPLICATION OF TTI TOMOGRAPHY TO VOLVE OBS DATA

The gridded TTI tomography was introduced in chapter 5 and tested on synthetic data in chapter 6. Here, the joint tomographic algorithm is applied to a 2D section from 3D OBS (ocean bottom seismic) data acquired at Volve field in the North Sea. Starting from an initial model built using check-shot traveltimes and nonhyperbolic moveout inversion, the parameters $V_p$, $\epsilon$, and $\delta$ are iteratively updated by minimizing the residual moveout in common-image gathers along with check-shot travelt ime misfit. The inverted TTI model produces well-focused reflectors throughout the section as well as accurate interface positioning, which is confirmed by the available well markers.

7.1 The Volve field and OBS survey

The Volve field is located offshore Norway in the gas/condensate-rich Sleipner area of the North Sea (Figure 7.1a). It is a small oil field with a dome-shaped structure formed by the collapse of adjacent salt ridges during the Jurassic period (Szydlik et al., 2007). The reservoir in the Middle Jurassic Hugin sandstone formation is a structural trap bounded by faults which are mainly associated with salt tectonics (Figure 7.1b).

In 2002, a 3D ocean-bottom seismic (OBS) survey was acquired over a 12.3 km $\times$ 6.8 km area of the field (Figure 7.2a). Six swaths of four-component (4C) data were recorded using inline shooting geometry. Each swath includes two 6 km-long cables placed on the seafloor (about 92 m below the water surface) with 400 m spacing (Figure 7.2a), and a cable contains 240 receivers with an interval of 25 m. In each swath, flip-flop shooting was conducted along 25 dual source sail lines with 100 m separation (Figure 7.2b). The sail line is 12 km long with a shot interval of 25 m (50 m between the flips).
Figure 7.1: (a) Location of Volve field in the North Sea (figure courtesy of Statoil) and (b) a representative geologic cross-section of Volve field (after Akalin et al., 2010).
Figure 7.2: (a) Geometry of the Volve 3D OBS survey. Two cables (black lines) are placed on the seafloor, and 25 sail lines parallel to the cables are shot with flip-flop sources in a 12 km × 2.4 km rectangular area (gray). After one swath of data is recorded, the cables and source lines are moved 800 m along the inline (y) direction. (b) Diagram of the source lines with the shots marked by stars. Each vessel tows two lines with a shot interval of 50 m, and the sources are staggered by 25 m between the lines. The sail-line spacing (dotted lines) is 100 m.
7.2 Anisotropic velocity analysis

The acquired 3D PP and PS data were preprocessed by Statoil. Preprocessing included noise suppression, multiple attenuation, and other standard steps described by Szydlik et al. (2007). Only traces with offsets less than 5 km were kept, and a layer-stripping technique was employed to construct a 3D VTI model for prestack depth imaging. The P- and S-wave vertical velocities \( V_P \) and \( V_S \) and the anisotropy parameters \( \epsilon \) and \( \delta \) in each layer (Figure 7.3; \( V_S \) is not shown) were updated by flattening common-image gathers (Figure 7.4a), minimizing mismatches between seismic and well data, codepthing the key horizons on PP (Figure 7.4b) and PS migrated sections, and incorporating compressional sonic logs (Szydlik et al., 2007). However, complete information about the VTI model-building process is not available to us.

Here, we use a 2D section from the 3D P-wave (vertical component) data recorded by the cable laid along \( y = 2.8 \text{ km} \) (Figure 7.2a). Two adjacent source lines \( (y = 2.8 \pm 0.025 \text{ km}) \) provided 481 shots with a shot interval of 25 m (Figure 7.2b). The tomographic MVA algorithm described in chapter 5 is applied to the CIGs from \( x = 2.7 \text{ km} \) to 9.5 km with an interval of 50 m, under the assumption that out-of-plane propagation can be ignored. As before, the symmetry axis is assumed to be perpendicular to the interfaces; the parameters \( V_{P0} \), \( \epsilon \), and \( \delta \) are defined on a 100 m \( \times \) 50 m grid.

There are two deviated wells (Figure 7.5) in the vicinity of the chosen line, and P-wave reflections are combined in the joint inversion with vertical check-shot (normal-incidence VSP) data recorded in the wells. Although the wells lie outside the vertical plane \( (y = 2.8 \text{ km}) \), only the \( x \) and \( z \) coordinates of the check shots are used to place each check-shot measurement. The provided data set also includes the well markers (i.e., depth measurements in the well) of several key horizons (e.g., the top and base of Utsira formation, top of Shetland Group, and base of Cretaceous). To evaluate the accuracy of velocity analysis, these well markers can be compared with the migrated reflector depths.

To build an initial model, the section is divided into eight layers based on key geologic horizons. The interval velocity \( V_{P0} \) in each layer is computed from the check-shot traveltimes
Figure 7.3: Cross-sections along the line with $y = 2.8$ km (Figure 7.2a) of the 3D VTI model built by Statoil.  
(a) The P-wave vertical velocity $V_{P0}$ and the anisotropy parameters (b) $\epsilon$ and (c) $\delta$.  

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Figure 7.4: (a) CIGs (displayed every 0.6 km from 3 km to 9 km) and (b) the depth image produced by Kirchhoff prestack migration with the parameters from Figure 7.3.
Figure 7.5: (a) Trajectories of two deviated wells near the line ($y = 2.8$ km, dashed) used for velocity analysis. (b) Well projections onto the water surface. The maximum deviations ($\Delta y$) of the two wells from the vertical plane ($y = 2.8$ km) are 477 m and 603 m, respectively.
in well 1, whose surface projection is closer to the line used for MVA than the projection of well 2 (Figure 7.5). Assuming the check-shot raypaths to be vertical, the velocity is found as \( V_{P0} = \Delta z / \Delta t \), where \( \Delta z \) is the depth interval, and \( \Delta t \) is the check-shot traveltime difference corresponding to \( \Delta z \). The interval NMO velocity and parameter \( \eta \) in each layer are estimated using nonhyperbolic moveout (Tsvankin, 2005) inversion applied to a CMP gather near the well. In combination with the velocity \( V_{P0} \) estimated along the well, \( V_{nmo} \) and \( \eta \) yield initial guesses for the parameters \( \epsilon \) and \( \delta \) at the well location.

Because of the limited offset-to-depth ratio (smaller than 1.2 for \( z > 3 \) km), the accuracy of the effective parameters \( \eta \) and \( \epsilon \) estimated from nonhyperbolic moveout analysis decreases with depth. Also, the interval values of \( \epsilon \) and \( \delta \) may be strongly distorted by layer stripping in the bottom part of the section. Therefore, we have to assume the deeper layers to be isotropic and set \( \epsilon \) and \( \delta \) below 3 km to zero (Figure 7.6b and Figure 7.6c). Then the initial TTI model (Figure 7.6) is obtained by extrapolating the 1D profiles of \( V_{P0} \), \( \epsilon \), and \( \delta \) at well 1 along the picked interfaces. As expected, the CIGs obtained after Kirchhoff migration with the initial model exhibit noticeable residual moveout (Figure 7.7a).

Using the three-stage parameter-estimation procedure described in chapter 5, we relax the constraints on the spatial variation of \( \epsilon \) and \( \delta \), while updating \( V_{P0} \) at each grid point in all iterations. At the first stage, the parameters \( \epsilon \) and \( \delta \) are fixed at the initial layer-based values. Then, using the same eight layers as in the initial model, \( \epsilon \) and \( \delta \) are updated in each layer (no spatial variations of \( \epsilon \) and \( \delta \) inside a layer are permitted). Finally, in the last several iterations, all three parameters (\( V_{P0} \), \( \epsilon \), and \( \delta \)) are updated on the same grid. Each iteration aims to simultaneously minimize the residual moveout in the CIGs and the check-shot traveltime misfit in both wells. In addition, the anisotropic velocity field is regularized by the structure-guided operators defined in chapter 5 (equation 5.11).

The TTI model (Figure 7.8) obtained after 10 iterations of tomography yields relatively flat CIGs (Figure 7.9a) and a better focused image (Figure 7.9b) than that computed with the initial model (Figure 7.7b), especially at depth. The remaining residual moveout in CIGs
Figure 7.6: Initial TTI model for line $y = 2.8\text{ km}$. (a) The symmetry-direction velocity $V_{P0}$ and the anisotropy parameters (b) $\epsilon$ and (c) $\delta$. All three parameters are defined on a $100 \text{ m} \times 50 \text{ m}$ grid.
Figure 7.7: (a) CIGs and (b) the depth-migrated section computed with the initial TTI model from Figure 7.6.
Figure 7.8: Inverted parameters (a) $V_{P0}$, (c) $\epsilon$, and (d) $\delta$ after the final iteration of joint tomography.
Figure 7.9: (a) CIGs and (b) the migrated section computed with the final inverted TTI model from Figure 7.8.
below $z = 3\text{ km}$ is partially due to internal multiples that were not completely removed in the preprocessing.

The average velocity $V_P$ (Figure 7.8a) in the Cretaceous Unit (approximately between $z = 2.5\text{ km}$ and $3\text{ km}$) is reduced after MVA to about $4300\text{ m/s}$ from its initial value $4500\text{ m/s}$ obtained from check shots. In the layer-based VTI model provided by Statoil (Figure 7.3a), $V_P$ in the Cretaceous Unit is larger, reaching $4900\text{ m/s}$ in some areas. Although the joint tomography used here minimizes the check-shot traveltime misfit in both wells and the structure-guided regularization helps propagate the well constraints along the interfaces, the depth scale of the section may not be sufficiently accurate away from the wells. It is possible that the high velocity in the Cretaceous Unit obtained by Statoil came from sonic logs or other information not available to us.

In the second stage of the parameter-estimation procedure, the anisotropy parameters $\epsilon$ and $\delta$ are assumed to be constant in each layer. This quasi-factorized assumption helps resolve $\epsilon$ and $\delta$ when the velocity $V_P$ varies linearly in each layer and is constrained by the check-shot data. In the last iterations, the anisotropy parameters are updated on the same grid as that for $V_P$, but the $\epsilon$- and $\delta$-fields (Figure 7.8b and Figure 7.8c) are regularized with larger weights than $V_P$ to maintain a higher degree of smoothness for both anisotropy coefficients. The deeper part of the section ($z > 3\text{ km}$) is kept isotropic because of the limited constraints from reflection events and small offset-to-depth ratios.

The inverted model helps image several reflectors in the Cretaceous Unit (Figure 7.10a), which are difficult to identify on the sections computed with both the initial model (Figure 7.7b) and Statoil’s VTI model (Figure 7.10b). Also, some reflectors beneath the high-velocity Cretaceous Unit look more coherent (compare Figure 7.10a and Figure 7.10b), which is important for structural interpretation of the reservoir. The key horizons on the final image (Figure 7.9b) are close to the well markers, which confirms that the migrated section has an accurate depth scale near the well locations.
Figure 7.10: Segment of the migrated section between $z = 2$ km and 4 km computed with (a) the inverted model from Figure 7.8 and (b) the VTI model provided by Statoil (Figure 7.3).
7.3 Discussion and Conclusions

The 2D tomographic algorithm introduced in chapter 5 was applied here to OBS data from Volve field. The parameters $V_{P0}$, $\epsilon$, $\delta$, and the symmetry-axis tilt $\nu$ were defined on a 100 m $\times$ 50 m grid, with the symmetry axis kept orthogonal to the reflectors. Iterative updating of $V_{P0}$, $\epsilon$, and $\delta$ was performed by the joint tomography of P-wave reflection and check-shot data with the structure-guided regularization. An initial anisotropic model was built by combining check-shot traveltimes with nonhyperbolic moveout analysis. Then, the three-stage parameter-estimation procedure discussed in chapter 5 was employed to produce the final gridded TTI model.

The TTI model used here allows the symmetry-axis direction to vary spatially according to the reflector dips. Although the dips are gentle, the employed model is more geologically plausible than VTI and should produce more accurate anisotropy parameters. In addition, the joint tomography simultaneously inverts for the TTI parameters defined at all grid points, which helps avoid error accumulation typical for layer stripping. The inverted TTI model made it possible to focus reflectors within and below the Cretaceous Unit and accurately position several key horizons in depth.

As any other 2D method, the algorithm applied here is based on the assumption that waves propagate in the vertical incidence plane. However, recorded events may correspond to out-of-plane reflection points; also, the check-shot raypaths in the nearby deviated wells lie outside the incidence plane. Moreover, the symmetry axis, even if it is orthogonal to dipping reflectors, may not be confined to the incidence plane. These 3D phenomena likely explain some remaining residual moveout in CIGs after application of the 2D TTI inversion. To construct a more robust TTI model and make use of the whole OBS data set, the tomographic algorithm should be generalized for 3D and, preferably, wide-azimuth data.
7.4 Acknowledgments

I would like to thank Statoil ASA and the Volve license partners ExxonMobil E&P Norway and Bayerngas Norge for the release of the Volve data. Special thanks to Marianne Houbiers of Statoil for numerous helpful discussions of the Volve case study. The view-points about the Volve data in this thesis are of the author and do not necessarily reflect the views of Statoil ASA and the Volve field license partners.
CHAPTER 8  
DISCUSSION AND CONCLUSIONS

Transversely isotropic velocity models have provided marked improvements in prestack imaging of P-wave data. The presence of anisotropy, however, increases the uncertainty in velocity analysis and typically requires a priori constraints on the model parameters. In this thesis, I developed time- and depth-domain algorithms for robust anisotropic parameter estimation using P-wave reflection and VSP (vertical seismic profiling) data.

Chapter 2 is devoted to application of the velocity-independent layer-stripping method (VILS) of Dewangan & Tsvankin (2006c) to nonhyperbolic moveout inversion for layered VTI and orthorhombic media. If the overburden is laterally homogeneous and has a horizontal symmetry plane, VILS produces the exact interval traveltimes from effective reflection moveout without any information about the velocity field. Effective traveltimes are much better constrained by reflection data than the corresponding effective moveout parameters employed in Dix-type algorithms. Therefore, VILS substantially increases the accuracy and stability of nonhyperbolic moveout inversion for the interval time-processing parameter $\eta$ in VTI media, which is confirmed by our synthetic tests on noise-contaminated data.

In chapter 2 VILS was also extended to 3D wide-azimuth P-wave data from orthorhombic media. To identify the target and overburden reflections that share the same ray segments, the horizontal slowness components are computed from the best-fit effective moveout parameters, which helps avoid direct differentiation of traveltimes. Although wide azimuthal coverage increases the stability of $\eta$-estimation using 3D Dix-type layer stripping, VILS produces more accurate interval parameters for typical orthorhombic models, including those with the depth-varying azimuths of the vertical symmetry planes. The method was also successfully used to estimate the interval anellipticity parameters from wide-azimuth data acquired above a fractured reservoir at Rulison field in Colorado.
Chapter 3 introduced a 2D parameter-estimation algorithm for tilted transverse isotropy (TTI) that operates with conventional-spread P-wave moveout and borehole data. The semianalytic inversion procedure for a single TTI layer showed that the parameters $V_{P0}$, $\epsilon$, and $\delta$ cannot be resolved without assuming the symmetry axis to be orthogonal to the reflector. The interval parameters of a stack of TTI layers separated by plane dipping interfaces (the dip planes in all layers are aligned) were estimated using stacking-velocity tomography combined with check-shot traveltimes and reflector depths and dips measured in a borehole. If walkaway VSP data are available, information about reflector dips can be replaced with near-offset VSP traveltimes. By performing the inversion for all layers simultaneously, the algorithm mitigates error accumulation with depth and produces stable results for TTI formations overlying isotropic strata. Tests on noise-contaminated data demonstrate that the interval parameters $V_{P0}$ and $\delta$ are constrained if the range of dips in the model does not exceed $30^\circ$, while the inversion for $\epsilon$ requires the dip to reach at least $60^\circ$.

In chapter 4, the 2D methodology presented in chapter 3 was extended to 3D TTI models composed of homogeneous layers separated by plane dipping interfaces with arbitrary orientation. The algorithm operates with the effective NMO ellipses, zero-offset traveltimes, and reflection slopes supplemented by the reflector depths assumed to be measured in a borehole. As long as the symmetry axis in each layer is kept orthogonal to its bottom, the interval parameters $V_{P0}$ and $\delta$ and the reflector dip and azimuths are well-resolved. However, as is the case for the 2D problem, the parameter $\epsilon$ can seldom be obtained without nonhyperbolic moveout inversion. When the symmetry-axis tilt represents a free parameter, the input data have to be supplemented by wide-azimuth VSP traveltimes with the offset reaching at least $1/4$ of the maximum reflector depth. Moreover, the additional angle coverage provided by VSP data may help resolve the parameter $\epsilon$ in the upper part of the model.

The algorithms described in chapters 3 and 4, possibly combined with nonhyperbolic moveout inversion for $\epsilon$, can be used to build an initial anisotropic model. After carrying
out interval parameter estimation at borehole locations, the anisotropic velocity field can be computed by interpolation between wells or extrapolation away from a well. An accurate initial model improves the convergence of velocity-analysis algorithms that operate in the migrated domain (e.g., reflection tomography discussed in chapters 5–7).

Heterogeneous TTI models needed for depth imaging in complex geological settings can be constructed by reflection tomography in the migrated domain. Such a tomographic algorithm is developed in chapter 5, with the parameters $V_{P0}$, $\epsilon$, $\delta$, and the symmetry-axis tilt $\nu$ defined on a rectangular grid. For stable parameter estimation, P-wave reflection data are combined with VSP traveltimes, and model updating is performed by iterative linearized inversion for $V_{P0}$, $\epsilon$, and $\delta$. The spatial distribution of $\nu$ is obtained by fixing the symmetry axis orthogonal to the reflectors and interpolating the tilt between layer boundaries. The Fréchet matrix at each iteration is constructed by approximately evaluating the traveltime derivatives with respect to the TTI parameters at all grid points near the reflection raypaths. The convergence of the inversion algorithm is improved by a three-stage model-updating procedure that gradually relaxes the constraints on the spatial variations of the anisotropy parameters $\epsilon$ and $\delta$. Geologic information is incorporated into the TTI tomography by means of structure-guided regularization that penalizes parameter variations in the direction parallel to the interfaces.

In chapter 6, the tomographic algorithm from chapter 5 is first tested on P-wave reflection data from a model with a “quasi-factorized” TTI syncline (i.e., $\epsilon$ and $\delta$ are constant inside the TTI layer). The results demonstrate that convergence toward the correct velocity model requires either strong smoothness constraints or additional information from walkaway VSP traveltimes. Another synthetic model examined in chapter 6 includes a bending TTI thrust sheet with constant values of $\epsilon$ and $\delta$. The velocity $V_{P0}$ defined at each grid point cannot be constrained solely by P-wave reflection data from the available three interfaces, although the “quasi-factorized” assumption is sufficient to recover the layer-based parameters $\epsilon$ and $\delta$ due to a wide range of reflector dips. Finally, the joint tomography regularized by the structure-
guided terms was applied to two sections of the BP TTI model that include an anticline and a salt dome. With constraints from P-wave reflections and walkaway VSP traveltimes, the three-stage model-updating procedure produced well-resolved TTI parameters in the shallow part (above 5 km) of the section. However, errors in $\epsilon$ and $\delta$ increase with depth due to the small offset-to-depth ratio and poor coverage of VSP rays.

Chapter 7 describes application of the joint tomography with structure-guided regularization to a 2D line from OBS data acquired at Volve field in the North Sea. The parameters $V_p$, $\epsilon$, $\delta$, and the symmetry-axis tilt $\nu$ are defined on a 100 m $\times$ 50 m grid, with the symmetry axis kept orthogonal to the interfaces. P-wave reflection data are combined with check-shot traveltimes from two nearby wells. The TTI velocity field, obtained from the three-stage model-updating procedure described in chapter 5, helps generate an image with well-focused reflectors and accurate positioning of several key horizons.

8.1 Discussion and recommendations for future work

It is well known that kinematic inversion methods do not always provide sufficiently high spatial resolution. In areas with sufficient ray coverage, higher resolution can be achieved with smaller grids. In general, resolution depends on many factors, such as acquisition geometry, frequency range, and the complexity of subsurface structures (Woodward et al., 2008). For example, spatial resolution typically decreases with depth because of a larger Fresnel zone, a smaller contribution of the interval traveltime to the effective moveout, and insufficient constraints provided by deep reflection events. Below salt bodies, resolution is also reduced because of illumination problems. Therefore, uniform rectangular/square grids used in my tomographic algorithm (chapters 5, 6 and 7) may be not optimal in practice. To improve the velocity model, grid points can be distributed according to the ray density, size of the Fresnel zone, and geologic information. Furthermore, it may be convenient to divide the model into triangular (2D) or tetrahedral (3D) cells of different size (Fomel & Guitton, 2006; Lelièvre et al., 2012).
To increase the stability of tomographic velocity analysis, the symmetry axis of TTI media was assumed to be perpendicular to the reflectors. This common assumption also simplifies ray tracing because the incidence and reflection angles at all reflectors are equal. In reality, however, the symmetry axis may not be aligned with the reflector normal if, for example, sedimentation and tectonic processes occur simultaneously (see chapter 4). Also, in sediments near salt bodies, stress-induced anisotropy may cause deviation of the symmetry axis from the normal to the bedding. The symmetry-axis orientation may sometimes be constrained using a priori information (e.g., geologic data), which may help express the tilt \( \nu \) as a function of reflector dip. Otherwise, an unknown direction of the symmetry axis significantly increases the nonuniqueness of the inversion. Note that, except for the need to compute the reflection angle from anisotropic Snell’s law, deviation of the symmetry axis from the reflector normal does not require substantial changes in the tomographic algorithm.

Even if the symmetry axis is kept orthogonal to the reflector, 2D P-wave reflection data are insufficient to constrain the parameters \( V_{P0} \) and \( \delta \) of a homogeneous TTI layer (chapter 3). Conventional-spread wide-azimuth data (i.e., the NMO ellipse), however, provide enough information for resolving both \( V_{P0} \) and \( \delta \). As shown in chapter 4, for a stack of TTI layers separated by plane dipping interfaces, the interval parameters \( V_{P0} \) and \( \delta \) and the reflector orientation are tightly constrained by wide-azimuth P-wave data combined with known reflector depths. Because of the valuable constraints provided by wide azimuthal coverage, it is would be highly beneficial to extend the developed TTI tomographic algorithm to 3D wide-azimuth data.

Handling large data volumes encountered in 3D problems requires efficient software for ray tracing, PSDM, and semblance analysis. The most time-consuming part of the 2D algorithm is construction of the Fréchet matrices in the objective function, which requires ray tracing for every reflection and VSP event. Therefore, 3D tomography should be run on a cluster using parallel computing. The proposed structure-guided regularization can also be extended to 3D to provide useful geologic constraints and speed up the convergence of
tomographic velocity analysis.
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APPENDIX - PARAMETER-UPDATING METHODOLOGY OF GRIDDED TTI TOMOGRAPHY

Here, the MVA algorithm for factorized VTI media introduced by Sarkar & Tsvankin (2004) is extended to gridded TTI models. To linearize the velocity-analysis problem, we employ an iterative technique based on the following updating procedure.

Suppose that after the \((l-1)\)th iteration of MVA, PSDM produces the migrated depths \(z_0(x_j, h_k)\) (\(x_j\) is the midpoint of the \(j\)th image gather, and \(h_k\) is the half-offset). The migrated depths \(z(x_j, h_k)\) after the \(l\)th iteration can be expressed as a linear perturbation of \(z_0(x_j, h_k)\):

\[
z(x_j, h_k) = z_0(x_j, h_k) + \sum_{c=1}^{W} \sum_{i=1}^{N} \frac{\partial z_0(x_j, h_k)}{\partial \lambda_{ic}} \Delta \lambda_{ic},
\]

where \(W\) is the number of grid points \((c = 1, 2, \ldots, W)\), \(\partial z_0(x_j, h_k)/\partial \lambda_{ic}\) are the derivatives of the migrated depths with respect to the medium parameters \(\lambda_i\) \((i = 1, 2, \ldots, N)\) at the vertex \(c\), and \(\Delta \lambda_{ic} = \lambda_{ic} - \lambda_{ic}^0\) are the desired parameter updates. Note that \(\partial z_0(x_j, h_k)/\partial \lambda_{ic} = 0\), if a specific ray does not cross any cell with the vertex \(c\) (a vertex is shared by four square cells). After obtaining the update \(\Delta \lambda_{ic}\), we can find the parameters \(\lambda_{ic}\) for next \((l)\)th iteration of PSDM.

Following the MVA algorithm of Liu (1997), Sarkar & Tsvankin (2004) define the variance of the migrated depths for all offsets and image gathers as

\[
Var = \sum_{j=1}^{P} \sum_{k=1}^{M} [z(x_j, h_k) - \hat{z}(x_j)]^2,
\]

where \(\hat{z}(x_j) = (1/M) \sum_{k=1}^{M} z(x_j, h_k)\) is the average migrated depth of a reflection event at midpoint \(x_j\), \(P\) is the number of image gathers, and \(M\) is the number of offsets in each image gather. At each iteration, the goal is to find the parameter updates that make the derivative of \(Var\) with respect to \(\Delta \lambda_{rs}\) \((r = 1, 2, \ldots, N,\) and \(s = 1, 2, \ldots, W)\) vanish, which helps minimize the RMO in all CIGs. Using equations A-1 and A-2, we can differentiate
the variance $\text{Var}$ with respect to the updates and set $\partial \text{Var}/\partial (\Delta \lambda_{rs}) = 0$, which yields an equation for $\Delta \lambda_{ic}$:

$$\sum_{j=1}^{P} \sum_{k=1}^{M} \sum_{c=1}^{W} \sum_{i=1}^{N} (g_{jk,ic} - \hat{g}_{jk,ic})(g_{jk,rs} - \hat{g}_{jk,rs}) \Delta \lambda_{ic} = - \sum_{j=1}^{P} \sum_{k=1}^{M} \left[ z_0(x_j, h_k) - \hat{z}_0(x_j) \right] (g_{jk,rs} - \hat{g}_{jk,rs}),$$

(A-3)

where $g_{jk,ic} \equiv \partial z_0(x_j, h_k)/\partial \lambda_{ic}$ (the subscripts $r$ and $s$ correspond to $i$ and $c$, respectively), and $\hat{g}_{j,ic} = (1/M) \sum_{k=1}^{M} g_{jk,ic}$. Equation A-3 can be rewritten in matrix form as

$$\mathbf{A}^T \mathbf{A} \Delta \lambda = -\mathbf{A}^T \mathbf{b},$$

(A-4)

where the matrix $\mathbf{A}$ has $M \times P$ rows and $W \times N$ columns (its elements are $g_{jk,ic} - \hat{g}_{jk,ic}$); the superscript $\mathbf{T}$ denotes the transpose, and $\mathbf{b}$ is a vector with $M \times P$ elements defined as $z_0(x_j, h_k) - \hat{z}_0(x_j)$.

To evaluate the derivatives of the migrated depths $z(x_j, h_k)$ for a gridded model, we modify the function given by Sarkar & Tsvankin (2004) as

$$\frac{dz}{d\lambda_{ic}} = - \left[ \frac{\partial \tau_s}{\partial \lambda_{ic}} + \frac{\partial \tau_r}{\partial \lambda_{ic}} \right] \frac{1}{q_s + q_r},$$

(A-5)

where $\tau_s$ is the traveltime from the source to the reflector obtained after PSDM with the medium parameters $\lambda_{ic}^0$, $\tau_r$ is the traveltime from the reflector to the receiver, and $q_s = \partial \tau_s/\partial z$ and $q_r = \partial \tau_r/\partial z$ are the vertical slownesses at the reflector for the rays connecting the reflection point with the source and receiver, respectively. Equation A-5 expresses $\partial z(x_j, h_k)/\partial \lambda_{ic}$ through the traveltime derivatives and vertical slownesses – quantities that can be computed from anisotropic ray tracing.