Automatic simultaneous multiple-well ties

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AUTOMATIC SIMULTANEOUS MULTIPLE-WELL TIES

by

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ABSTRACT

Well logs, measured in depth, must be tied to seismograms, processed in time, using a time-depth function. Well ties are commonly computed using manual techniques, and are therefore prone to human error. I first introduce an automatic single-well tie method that uses smooth dynamic time warping to compute time shifts that align a synthetic seismogram with a seismic trace. These time shifts are constrained to be smoothly varying. I also show that these well ties, in my example, are insensitive to the complexity of my synthetic seismogram modeling.

Tying multiple wells compounds errors in single well ties, and maintaining consistency among multiple single well ties is difficult. I introduce an automatic approach to tying multiple wells that improves consistency among well ties. I first model synthetic seismograms for each well. I then create a synthetic image by interpolating the synthetic seismograms between the wells and along seismic image structure. I use smooth dynamic image warping to align the synthetic image to the seismic image and compute updated time-depth functions for each well. I then interpolate the updated time-depth functions between the wells, and map the time-migrated seismic image to depth.
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Tying well logs to seismic images is one of the most important tasks in seismic interpretation (White and Simm, 2003). Well ties are important because seismic data are commonly interpreted in vertical two-way time, while well logs are recorded in depth. A well tie yields a time-depth function that enables correlation of properties measured in well logs with reflectors in seismic images. Given an approximate time-depth function and other information, such as the seismic wavelet, and accounting for errors in both logs and seismic imaging, an experienced interpreter can make an accurate well tie.

1.1 Single well tie methods

Many manual procedures for tying well logs to seismic data exist (e.g., Duchesne and Gaillot (2011); Edgar and van der Baan (2011); Walden and White (1984); White and Hu (1998); White and Simm (2003)). Also, many commercial software packages enable seismic interpreters to generate well ties from well logs and seismic traces. They provide graphical interfaces that allow the user to interactively shift, stretch, and squeeze synthetic seismograms to match time-migrated seismic traces. While the user manually matches synthetic seismograms to seismic traces, the software automatically updates a time-depth function. However, this process is tedious and its accuracy depends on the skill and experience of the user.

I introduce an automatic method for single well ties that reduces human error. This method follows steps common to most well tie processing:

(i) compute a time-depth function from a sonic log
(ii) compute reflectivity from the sonic and density logs
(iii) model a synthetic seismogram using the reflectivity
(iv) align the synthetic seismogram with a nearby seismic trace
(v) update the time-depth function

Results from each of these steps must be individually checked during the well tie process to ensure accurate well ties. Each step can also be modified to improve the accuracy of well ties. For instance, check shots improve the accuracy of the initial time-depth function. Error-free well logs improve the quality of well ties, as demonstrated by White and Hu (1998). Realistic modeling methods for synthetic seismograms might improve matching with seismic traces in some examples (White and Hu, 1998; White and Simm, 2003). White and Simm (2003) emphasize accurate wavelet estimation for improving synthetic seismogram modeling, and the importance of high-quality seismic data for well ties.

I make assumptions about these outlined steps and data used to compute well ties. I first assume that velocity and density logs are error-free, otherwise, a synthetic seismogram computed using these well logs will contain errors in amplitude that affect the well tie. Also, I assume that these velocity and density logs are representative of seismic velocities and densities. I also must assume that the seismic data is time-migrated such that reflectors are correctly located. Even though the modeling used to compute a synthetic seismogram is simple compared to the processing used to compute the seismic image, I assume that my modeled synthetic seismogram contains reflection events and amplitudes that are apparent in the processed seismic image.

In previous work, I used an automatic method for computing single well ties (Munoz and Hale, 2012). We used an algorithm commonly used in speech recognition called dynamic time warping (DTW) (Mueller, 2007) to quantitatively compute well ties and time-depth functions. Independently, Herrera and van der Baan (2012a,b,c) used DTW to guide well ties for single wells. Both of these approaches to automatic well ties used simple models of synthetic seismograms and unrealistic constraints on time shifts, which resulted in inaccurate well ties. Also, neither approach considered the effect of these unrealistic time shifts on updated interval velocities.
1.2 Multiple well tie methods

Interpreters must often tie many wells to the same seismic image. The task of keeping all well ties consistent is both error-prone and time-consuming. Few methods have been published for consistently tying multiple wells. Lallier et al. (2009) use DTW to find correlations among multiple logs. Ziolkowski et al. (1998) manually correlate wells using the seismic image to find stratigraphic traps and other geologic features. They find well correlations by processing the seismic image to be laterally consistent along lithostratigraphic features and manually following reflection events between tied synthetic seismograms.

I introduce an automatic multi-well process that yields laterally consistent well ties because I impose both vertical and lateral (structure-oriented) constraints on the shifting, stretching, and squeezing used to align synthetic seismograms with seismic images. Seismic-to-well ties are typically computed by tying synthetic seismograms to seismic traces. I instead create a synthetic image from multiple synthetic seismograms and tie that synthetic image to a seismic image using smooth dynamic image warping. For each well, I compute updated time-depth functions, which I use to compute a 3D time-depth function. I then convert the seismic image to depth using this 3D time-depth function.

1.3 Thesis overview

In this thesis, I first show improvements from previous automatic single well tie methods first introduced by Munoz and Hale (2012) and Herrera and van der Baan (2012a,b,c) through more realistic synthetic seismogram modeling and better constraints on time shifts. I then compare three new methods for computing automatic multiple-well ties: (1) multiple automatic single well ties; (2) multiple automatic single well ties refined with an automatic simultaneous multiple-well tie method; (3) an automatic simultaneous multiple-well tie method. The third approach is the most laterally consistent of these methods. Finally, I apply the results of my method to the depth-conversion of a time-migrated seismic image. I demonstrate my new simultaneous multiple-well method using the 3D seismic image and
well logs from the freely available Teapot Dome dataset (Anderson, 2009).
CHAPTER 2
TYING ONE WELL

The goal in tying one well is to compute an updated time-depth function. Given an initial time-depth function, I first model a synthetic seismogram. I then automatically tie the synthetic seismogram to nearby seismic traces using smooth dynamic time warping (Hale and Compton, 2013) and update the initial time-depth function. I compute well ties using the Teapot Dome well logs and time-migrated seismic image (Anderson, 2009).

2.1 Making a synthetic seismogram

I model synthetic seismograms individually for each well. An example is the well with Unique Well Identifier (UWI) 490251091600, which has a P-wave velocity log and a density log. Velocities \(v_0(z)\) and densities \(\rho(z)\) are uniformly sampled with interval \(\Delta z = 6\) in for depths \(z\) between \(z_{\text{min}} = 0.28\) km and \(z_{\text{max}} = 1.85\) km. I compute reflectivity as:

\[
r(z) = \frac{v_0(z + \Delta z)\rho(z + \Delta z) - v_0(z)\rho(z)}{v_0(z + \Delta z)\rho(z + \Delta z) + v_0(z)\rho(z)},
\]

(2.1)

the acoustic reflection coefficient for normal-incidence P-waves. Figure 2.1 shows the velocity, density, and computed reflectivity for this well.

Next, I compute an initial time-depth function at the well location. This can be obtained in several ways. The most accurate way to get a time-depth function is by acquiring a vertical seismic profile (VSP) or checkshot survey at the well site because these methods measure seismic traveltimes to known depths. However, because I have neither a VSP nor a checkshot survey at this well location, I must estimate an initial time-depth function using only the velocity log. These log velocities are not equivalent to seismic velocities, so this initial time-depth function does not represent seismic traveltimes at known depths.

I compute the initial time-depth function by integrating the slowness from the velocity log:
Acoustic reflectivity in the rightmost panel is computed using equation 2.1.

\[
\tau_0(z) = \tau_{\text{min}} + 2 \int_{\tau_{\text{min}}}^{z} \frac{d\xi}{v_0(\xi)},
\]

(2.2)

where \(\tau_{\text{min}}\) is an estimate of vertical two-way time to depth \(z_{\text{min}}\), the shallowest depth measured for the velocity log. To compute \(\tau_{\text{min}}\), I first estimate the overburden slowness between depths \(z = 0\) and \(z = z_{\text{min}}\) by averaging a median slowness for the shallowest depths in the log with a static replacement slowness used in the processing of the Teapot Dome seismic data; I then multiply this estimated overburden slowness by \(2z_{\text{min}}\). Figure 2.2c shows an initial time-depth function in black.

To model synthetic seismograms, I use the computed reflectivity and initial time-depth function. I first compute a synthetic seismogram with a commonly used simple model; Munoz and Hale (2012), Herrera and van der Baan (2012a,b,c), and White and Simm (2003) com-
Figure 2.2: Normalized initial simple (a) and more realistic (b) synthetic seismograms (red) are aligned to an adjacent normalized seismic trace (black) using initial time-depth functions (black) (c, h). Updated time-depth functions (red) (c, h) are used to tie the synthetic seismograms (red) (b, g) to the seismic trace using the same smooth DTW constraints on time shifts. The initial (black) and updated (red) velocities (d, i) are used to compute percentage difference in velocity (e, j), which are constrained to ±10%.

To compute similar simple synthetic seismograms. Using $\tau_0(z)$ (and ignoring attenuation, multiple reflections, and other effects), I compute a synthetic seismogram $f(\tau)$ as a simple superposition of seismic wavelets $w(\tau)$, each delayed by a reflection time $\tau_0(z)$ and weighted by a corresponding reflectivity $r(z)$ from equation 2.1:

$$f(\tau) = \int_{z_{\min}}^{z_{\max}} r(z) w[\tau - \tau_0(z)] \, dz,$$

which I sample uniformly at times $\tau = \tau_{\min}, \tau_{\min} + \Delta t, \ldots, \tau_{\min} + (N_\tau - 1)\Delta t$ for $\Delta t = 2$ ms. The wavelet $w(\tau)$ is a Ricker wavelet with a peak frequency of 35 Hz and constant phase. I discuss how to compute an optimal constant phase below.
Figure 2.3: Simple synthetic seismograms computed using equation 2.3 (a) and using the propagator matrix method without multiples or attenuation (b). The propagator matrix method is also used to compute synthetic seismograms with multiples (c) and with multiples and attenuation (d).

For comparison, I use the reflectivity shown in Figure 2.1 to model more realistic synthetic seismograms with multiples, attenuation, and dispersion using the propagator matrix method for vertically propagating plane waves in stratified media (Gilbert and Backus, 1966; Haskell, 1953; Kennett, 1983; Thomson, 1950) and a constant-Q model (Kjartansson, 1979). The receiver and source are placed at the surface $z = 0$, and I assume a constant-impedance layer (with no reflectors) between the surface and the first logged velocity and density at $z = z_{\text{min}}$. Figures 2.3a and 2.3b show that the simple synthetic seismogram and the propagator matrix method seismogram, without multiples or attenuation, are the same, as expected. Figures 2.3c and 2.3d show synthetic seismograms also computed using the propagator matrix method with multiples and then with multiples and attenuation ($Q = 100$).
In my example, the primary differences between the simple synthetic seismogram and the more realistic synthetic seismogram, shown in Figures 2.3a and 2.3d, are due to modeling with attenuation. When I apply a time-varying amplitude normalization (described below) to both the simple and more realistic synthetic seismograms, as shown in Figures 2.4a and 2.4b, these normalized synthetic seismograms are remarkably similar. Therefore it is not surprising that single well ties, shown in Figure 2.2, computed using the normalized simple and realistic synthetic seismograms contain few differences. For the rest of this thesis, I use the more realistic synthetic seismograms for computing single and multiple well ties.

I normalize amplitudes in both synthetic seismograms and seismic images because simplifying assumptions in modeling and various steps in seismic data processing cause significant amplitude differences between synthetic seismograms and seismic traces, and to align these two sequences, we must first ensure that their amplitudes are comparable. This normaliz-
tion is time varying; I divide the amplitude at each sample by the rms amplitude computed in seamlessly overlapping time windows. This is equivalent to applying an automatic gain control, which is commonly used to normalize amplitudes in seismic processing.

Additionally, seismic processing workflows use various techniques to correct data to zero-phase or minimum-phase. Unless I know the exact phase corrections applied to the seismic data, I must estimate a phase correction for the zero-phase synthetic seismogram. I find an optimal constant-phase rotation for the synthetic seismogram in the next section.

2.2 Tying a synthetic seismogram

To tie a synthetic seismogram \( f(\tau) \) to a nearby seismic trace \( g(t) \), I use smooth DTW to find a function \( \tau(t) \) such that:

\[
f(\tau(t)) \approx g(t).
\]  

As described in detail by Hale and Compton (2013), smooth DTW computes coarsely sampled (smooth) vertical time shifts \( u(\tau) \). Here, these shifts align events in a synthetic seismogram with events in a seismic trace. These time shifts are a globally optimal solution to a minimization problem with many local minima. The shifts \( u(\tau) \) are constrained to satisfy specified bounds on \( du(\tau)/d\tau \). In my experience, smoothly varying time shifts are more accurate than time shifts computed using the DTW method described by Mueller (2007), which is used by Herrera and van der Baan (2012a,b,c) and Munoz and Hale (2012) to compute well ties.

Using time shifts \( u(\tau) \) computed from smooth DTW, I first compute

\[
t(\tau) = \tau + u(\tau),
\]  

such that \( f(\tau) \approx g(t(\tau)) \). I do this because the synthetic seismogram is shorter than the seismic trace, so I can only measure time shifts \( u(\tau) \) for the times \( \tau \) in these synthetic seismograms.

The mapping function \( t(\tau) \) could be used to warp the seismic trace to match the synthetic seismogram, but I want to warp the synthetic seismogram to match the seismic trace, as
2.2.1 Optimal constant phase

The phase of the synthetic seismogram should match the phase of the seismic data, but I do not know the seismic wavelet. I approximated the wavelet’s amplitude spectrum by estimating the peak frequency from the amplitude spectrum of the seismic trace. I then approximate the phase with a constant phase rotation that is frequency-independent.

I first apply a constant-phase rotation to the synthetic seismogram. I then use smooth DTW to align the phase-rotated synthetic seismogram to the seismic trace subject to constraints on time shifts. This process yields an alignment error that corresponds to the globally optimal alignment (Hale and Compton, 2013); the smaller this value, the more optimal the alignment.

I compute an optimal constant phase by applying the steps listed above for a range of constant phases and retain the alignment error corresponding to each phase. Figure 2.5 shows the range of phases and the alignment error values obtained for each phase. For this synthetic seismogram, the minimum alignment error is obtained for a phase of 100 degrees.
2.3 Updating a time-depth function

Given the initial time-depth function \( \tau_0(z) \) computed from equation 2.2, and the mapping function \( \tau(t) \) computed using smooth DTW, I easily compute an updated time-depth function:

\[
\tau_1(z) = t(\tau = \tau_0(z)) = \tau_0(z) + u(\tau_0(z)).
\] (2.6)

If \( t(\tau_0(z)) = \tau_0(z) \), implying that no warping was necessary, then I obtain \( \tau_1(z) = \tau_0(z) \), which in turn implies that the initial time-depth function was correct.

Time shifts \( u(\tau) \), which vary with \( \tau \), correspond to stretching and squeezing of the synthetic seismogram to match the seismic trace. Such stretching and squeezing in well ties can be excessive and is not typically recommended, as noted by White and Simm (2003). To prevent excessive stretching and squeezing, I constrain the time-derivative \( \frac{du(\tau)}{d\tau} \) of the shifts \( u(\tau) \).

I can relate this derivative \( \frac{du(\tau)}{d\tau} \) to an updated velocity function:

\[
v_1(z) = 2 \left( \frac{d\tau_1}{dz} \right)^{-1}
\] (2.7)

for \( z_{\text{min}} \leq z \leq z_{\text{max}} \), where again \( z_{\text{min}} \) and \( z_{\text{max}} \) are the first and last depths logged. From equation 2.6:

\[
\frac{d\tau_1}{dz} = \frac{dt}{d\tau} \frac{d\tau_0}{dz},
\]

such that

\[
\frac{dt}{d\tau} = \frac{d\tau_1}{dz} \left/ \frac{d\tau_0}{dz} \right.,
\]

and
\[
\frac{d\tau}{d\tau} = \frac{v_0(\tau)}{v_1(\tau)}.
\] (2.8)

Using equation 2.5, I obtain:

\[
\frac{du}{d\tau} = \frac{dt}{d\tau} - 1,
\]

or, equivalently,

\[
\frac{du}{d\tau} = \frac{v_0(\tau)}{v_1(\tau)} - 1.
\] (2.9)

I then use specified bounds on \(v_0(\tau)/v_1(\tau)\) to obtain equivalent bounds on \(du(\tau)/d\tau\):

\[
\left(\frac{v_0(\tau)}{v_1(\tau)}\right)_{\text{min}} - 1 \leq \frac{du}{d\tau} \leq \left(\frac{v_0(\tau)}{v_1(\tau)}\right)_{\text{max}} - 1.
\] (2.10)

Choosing these bounds has a significant impact on the updated time-depth functions and updated velocities. Figure 2.6 compares well ties, of the same well, computed using three different bounds on \(du(\tau)/d\tau\). The top well tie bounds \(du(\tau)/d\tau\) between \(\pm 50\%\); this is equivalent to computing a well tie with DTW constraints shown by Munoz and Hale (2012). The middle well tie uses smooth DTW to bound \(du(\tau)/d\tau\) between \(\pm 10\%\). The bottom well tie uses smooth DTW to also bound \(du(\tau)/d\tau\) between \(\pm 10\%\), but now I instead increase the coarseness of time shift sampling; this makes \(du(\tau)/d\tau\) smoother and the time shifts more realistic. Also, notice that in Figures 2.6a and 2.6b the percentage differences in velocity are larger than the specified bounds on \(du(\tau)/d\tau\). This is error due to the cubic spline interpolation I use to interpolate my sparsely sampled time shifts, which is not guaranteed to honor these bounds. Hale and Compton (2013) compare the cubic spline interpolation method to examples of piecewise interpolation methods that honor these bounds on \(du(\tau)/d\tau\) but result in discontinuous updated velocities.

Figure 2.2 illustrates an example of an automatic single well tie with these realistic constraints on time shifts. The tied synthetic seismogram and the seismic trace shown in
Figure 2.2b are roughly aligned. In Figure 2.2c, the updated time-depth function, shown in red, has a large constant shift relative to the initial time-depth function, shown in black. This implies that my initial estimate of the overburden velocity was too high, which may be partly caused by sonic log velocities that are higher than reflection seismic velocities. Notice also the differences in slope between the initial time-depth function and the updated-time depth function. These differences in slope can be more easily seen by comparing the velocity log to the updated velocity function, computed using equation 2.7, shown in Figure 2.2d.

Figure 2.2e shows the percentage difference in velocities, which, as previously mentioned, correspond to stretching and squeezing of the synthetic seismogram. Note that percentage difference in velocities is bounded between ±10% because I used equation 2.10 bound $\frac{du(\tau)}{d\tau}$ to be within ±10%. I do not know these geophysical bounds in advance, so I determine them experimentally by looking at the resulting updated velocities.
Figure 2.6: A comparison of smooth DTW constraints used in well ties. There are noticeable differences between well ties computed using time shifts with derivatives bounded between $\pm 50\%$ (a), $\pm 10\%$ (b), and $\pm 10\%$ with coarse shift sampling (c).
To demonstrate my method for multiple well ties, I use eight wells and the Teapot Dome 3D seismic image shown in Figure 3.1. I first compute single well ties, as described in the previous section, by modeling synthetic seismograms for each well and then independently tying each well to nearby seismic traces using smooth DTW. The tied synthetic seismograms all roughly align with nearby seismic traces. Figure 2.2b shows one of these tied synthetics. I observe the lateral consistency of these well ties using velocity logs, which, when displayed in depth, are laterally consistent with other nearby velocity logs.

I do not have a quantitative measure of lateral consistency. Therefore, to qualitatively assess the lateral consistency of these single well ties, I first convert the velocity logs from depth to time, as shown in Figure 3.2c, using updated time-depth functions. I then interpolate these velocity logs between the wells using image-guided blended neighbor interpolation (Hale, 2009), as shown in Figure 3.2d. If I assume that log velocities in Teapot Dome are laterally consistent, then velocity logs tied to the seismic image using the correct time-depth functions should correlate laterally along lithostratigraphic boundaries, which likely align with reflectors in the seismic image. This assumption implies that laterally consistent velocities correspond to laterally consistent time-depth functions.

In Figure 3.2d, the velocities, especially for distances between 2.0 km and 2.5 km at 0.9 s, are not laterally continuous. These inconsistencies are apparent when comparing the interpolated velocities in Figure 3.2d to those in Figure 3.2b, which were computed using initial time-depth functions. Notice in Figure 3.2b, that there are apparent velocity layers not initially aligned with the background seismic reflectors. Using the updated time-depth functions, these velocity layers should roughly align with seismic reflectors. However, the poor lateral consistency shown in Figure 3.2d suggests that one or more of these single well
Inaccuracy in this example is caused primarily by logs with limited depth ranges, which result in short synthetic seismograms. The optimal alignment of individual short synthetic seismograms might not coincide with the correct well tie because of errors in synthetic seismogram modeling, which are most influential for short logs. To reduce lateral inconsistencies, I compute multiple well ties simultaneously.

To do this, I construct a *synthetic image* that I tie to the seismic image using smooth dynamic image warping (DIW) (Hale and Compton, 2013). This process finds an optimal alignment between two images, subject to both vertical and *lateral* constraints on estimated time shifts.

### 3.1 A synthetic image

To compute a synthetic image, I first independently model synthetic seismograms for each well. Figures 3.3a and 3.3b show eight initial synthetic seismograms, which are aligned to the 3D seismic image using initial time-depth functions computed from equation 2.2. Notice that the initial alignments between the synthetic seismograms and nearby seismic traces are poor.

I then compute a structure-tensor field that represents the coherence and orientations of reflectors in the seismic image (Hale, 2009). A subset of the tensors computed from the
seismic image is shown in Figure 3.4, in which the tensors are represented by blue ellipsoids. Ellipsoids are elongated in directions in which seismic reflectors are most coherent.
Figure 3.2: Vertical sections extracted from a 3D seismic image intersect five wells, as indicated by the solid curve in Figure 3.1. The velocity logs are plotted in time using initial time-depth functions (a), updated time-depth functions for single well ties (c), and updated time-depth functions for simultaneous multiple-well ties computed after single well ties (e). The velocity logs are interpolated in 3D (b, d, f) between the wells using image-guided blended-neighbor interpolation.
Figure 3.3: Vertical sections extracted from a 3D seismic image intersect five wells (solid) and four wells (dashed) as shown in Figure 3.1. The initial synthetic seismograms (a, b) are interpolated between wells to make an initial 3D synthetic image (c, d). Then, the initial 3D synthetic image is tied to the corresponding 3D seismic image to make a warped 3D synthetic image (e, f). Warped synthetic seismograms are extracted at each well location (g, h) from the warped 3D synthetic image.
With that structure-tensor field, I interpolate synthetic seismograms between wells using image-guided blended-neighbor interpolation (Hale, 2009), and thereby create an initial synthetic image. Notice that the initial synthetic image, shown in Figures 3.3c and 3.3d, exhibits discontinuities at locations between the wells, which result from misalignments of the initial synthetic seismograms.

Notice also that the seismic amplitudes of the same synthetic image are nearly zero at depths where no velocities and densities are logged. Interpolated amplitudes also decrease away from the wells due to destructive interference of positive and negative amplitudes during interpolation. Because smooth DIW is influenced more by large amplitudes than by small amplitudes, the simultaneous multiple-well ties are influenced most by seismic amplitudes near wells.

3.2 Tying a synthetic image

To simultaneously tie multiple wells to a 3D time-migrated seismic image, I align an initial synthetic image $f(x, y, \tau)$ to a seismic image $g(x, y, t)$ and use smooth DIW to find a function $\tau(x, y, t)$ such that:
\[ f(x, y, \tau(x, y, t)) \approx g(x, y, t). \]  

(3.1)

Using the time shifts \( u(x, y, \tau) \) computed from smooth DIW, I actually compute

\[ t(x, y, \tau) = \tau + u(x, y, \tau), \]  

(3.2)

such that \( f(x, y, \tau) \approx g(x, y, t(x, y, \tau)) \). I then follow the same process I used for single well ties to obtain \( \tau(x, y, t) \) from \( t(x, y, \tau) \) via inverse linear interpolation.

The shifts \( u(x, y, \tau) \) satisfy specified bounds on \( \partial u(x, y, \tau)/\partial \tau \), \( \partial u(x, y, \tau)/\partial x \), and \( \partial u(x, y, \tau)/\partial y \). I bound \( \partial u(x, y, \tau)/\partial \tau \) between \( \pm 10\% \) according to equation 2.10, as I did for single well ties. The bounds on the rate of lateral changes in time shifts \( \partial u(x, y, \tau)/\partial x \) and \( \partial u(x, y, \tau)/\partial y \) can be related to the lateral percentage change in velocities, but in practice, I do not know these values in advance. For these bounds, I experimentally determined that values of \( \pm 0.5 \) s/km best maximize the lateral consistency between well ties.

I show sections corresponding to the resulting warped synthetic image \( f(x, y, \tau(x, y, t)) \) in Figures 3.3e and 3.3f. Time shifts between the initial synthetic image and the warped synthetic image imply misalignment between the seismic image and the initial synthetic image. In Figures 3.3g and 3.3h, I show warped synthetic seismograms extracted at the wells from the warped synthetic image; these align well with the seismic image.

As discussed above, I only qualitatively assess the accuracy of my simultaneous multiple-well tie method. I first compute updated time-depth functions with equation 2.6 using time shifts \( u(\tau) \) extracted at the well locations from \( u(x, y, \tau) \). I then use these updated time-depth functions to convert the velocity logs from depth to time, as shown in Figure 3.5c, and interpolate them between the wells with image-guided blended-neighbor interpolation, as shown in Figure 3.5d. These interpolated velocities are more laterally consistent than the interpolated velocities shown in Figure 3.5b. Notice the horizontal velocity layers in Figure 3.5d, and in Figure 3.5c the sharp velocity contrasts in the logs correspond to high-amplitude reflectors, as expected. These observations of laterally consistent velocities imply laterally consistent well ties. This result suggests that my simultaneous multiple-well tie
method is more accurate than my single well tie method.

I could also attempt to remove the lateral inconsistencies from multiple single well ties using my simultaneous multiple-well tie method. I first compute multiple single well ties, create a new synthetic image by interpolating the corresponding tied synthetic seismograms, tie that synthetic image to the seismic image using smooth DIW, and obtain updated time-depth functions for each well. Again, to qualitatively assess the lateral consistency of these well ties, I convert the velocity logs to time, as shown in Figure 3.2e, with the new updated time-depth functions and interpolate them to obtain the velocities shown in Figure 3.2f. Notice that there are still lateral inconsistencies in velocity, especially at distances between 2.0 km and 2.5 km in Figure 3.2f. As I previously showed, automatic single well ties are not constrained to be laterally consistent, as seen by interpolated velocities shown in Fig-
ure 3.2d, and my simultaneous multiple-well tie method does not necessarily remove the lateral inconsistencies caused by multiple single well ties.

3.3 Seismic depth conversion

I use updated time-depth functions at eight wells, computed from simultaneous multiple-well ties, to make a new time-depth function that converts the 3D time-migrated seismic image to depth.

I first resample these updated time-depth functions to a uniform depth sampling interval of 2 m. I then want to interpolate these functions between wells, but different wells are sampled at different depths. Since these functions overlap in depth, I must extrapolate them to ensure that, after interpolation, time monotonically increases with depth.

If I linearly extrapolate these time-depth functions, I assume a constant average velocity for depths above and below the known samples in the logs. A more realistic assumption is to instead linearly extrapolate average velocity. This ensures monotonicity of time with depth and also provides a more reasonable assumption about these extrapolated velocities. I therefore convert these time-depth functions to average-velocity functions and then linearly extrapolate the latter.

For each well, using an updated time-depth function $\tau_1(z)$, I compute an average-velocity function:

$$\bar{v}(z) = \frac{2z}{\tau_1(z)}$$ (3.3)

for depths $z_{\text{min}} \leq z \leq z_{\text{max}}$, where again $z = z_{\text{min}}$ and $z = z_{\text{max}}$ are the first and last depths logged for a single well.

For each well, I then linearly extrapolate the computed average velocity below and above known log samples. I then compute a new 3D average-velocity function $\bar{v}(x, y, z)$ by interpolating the average-velocity functions between the wells. Using $\bar{v}(x, y, z)$, I compute a new time-depth function:
Figure 3.6: A time-depth function computed using equation 3.4 and updated time-depth functions obtained from simultaneous multiple-well ties.

\[ \tau(x, y, z) = \frac{2z}{\bar{v}(x, y, z)}. \]  \hspace{1cm} (3.4)

Figure 3.6 shows this new time-depth function \( \tau(x, y, z) \), which I use to convert the seismic image \( g(x, y, t) \) from time to depth:

\[ g_z(x, y, z) = g(x, y, t = \tau(x, y, z)). \]  \hspace{1cm} (3.5)

Figure 3.7 shows the time-migrated seismic image before and after this depth conversion. I cannot know the correct depths for the seismic image, so to infer the accuracy of my depth conversion, I must use assumptions about velocity variation in the subsurface, which might be obtained from external geologic data. If I assume that the velocity is laterally smooth over short distances, I might infer that any suspicious structures observed in the depth-converted image that are not observed in the time-migrated image are due to errors in my updated time-depth functions. To further test the accuracy of my depth-conversion method, I could compare my depth-converted seismic image to a depth-migrated seismic image.
Figure 3.7: The time-migrated seismic image (a) is converted to depth (b) using equation 3.5 and the new time-depth function in Figure 3.6, which is computed using updated time-depth functions from simultaneous multiple-well ties.
CHAPTER 4
CONCLUSIONS

I first presented an automatic single well tie method. I computed these well ties automatically using smooth DTW, which optimally aligns a synthetic seismogram to a seismic trace, subject to constraints. I then showed that these constraints are related to bounds on a percentage difference in velocities, and that they determine the shape of the updated time-depth function. I also showed, in this example, that these well ties are insensitive to the complexity of my synthetic seismogram modeling.

I have also presented an automatic simultaneous multiple-well tie method. My method is preferable to automatic single well tie methods because I vertically and laterally constrain multiple well ties by simultaneously aligning synthetic seismograms with the seismic image.

To simultaneously tie multiple wells, I first independently compute multiple synthetic seismograms. I then compute a synthetic image by interpolating synthetic seismograms along the seismic image structure. Using smooth DIW, I align the synthetic image with the seismic image to obtain a warped synthetic image. From this warped image, I extract warped synthetic seismograms at well locations, and compute updated time-depth functions for each well. To compute a 3D time-depth function, I extrapolate and interpolate average velocities computed from updated time-depth functions. Finally, using this 3D time-depth function, I convert the 3D seismic image from time to depth.

4.1 Shortcomings

Errors in single well ties are often caused by invalid assumptions. For example, I might incorrectly assume that wells are nearly vertical, that seismic time-migration has correctly positioned complex structures, or that there are no rapid lateral variations in velocity (White and Hu, 1998). My method is not immune to such assumptions, but might reduce the effects of human error introduced by manually (and tediously) tying wells.
Such errors are compounded when tying multiple wells. When each well is tied independently, I focus primarily on a small number of seismic traces near the well. With multiple wells, I instead simultaneously consider all traces, both near and between wells. This consideration is not automatic; the method does not maximize every measure of consistency. In tying multiple wells, I make additional assumptions about the consistency of subsurface parameters at locations between wells. Again, my simultaneous multiple-well tie method is not immune to errors in these assumptions.

I make further assumptions when choosing the smooth DIW constraints that bound the vertical and lateral changes in time shifts. The bounds on the vertical variation in time shifts are related to a percentage difference in velocities, but the bounds on the lateral variation in time shifts must be determined empirically. I have qualitatively observed that these constraints on the lateral variation in time shifts affect the lateral consistency of updated time-depth functions, however, because I do not currently use a quantitative measure of lateral consistency, I cannot objectively measure this effect.

4.2 Future work

A quantitative measure of lateral consistency might help improve both my understanding of the constraints on the lateral variation in time shifts and the lateral consistency between wells. I would first compute simultaneous multiple-well ties with large bounds on the lateral variation in the time shifts, and then quantitatively compute the lateral consistency of the resulting updated time-depth functions. I would decrease the bounds and repeat until the lateral consistency is maximized. Unfortunately, the image-guided interpolation I use to compute synthetic images and interpolate velocities is computationally expensive, which limits the number of iterations that can be practically computed.

Errors in individual synthetic seismograms, caused by errors in logs or inaccurate modeling, might also affect the accuracy of simultaneous multiple-well ties. To better understand the sensitivity of my simultaneous multiple-well tie method to errors in individual synthetic seismograms, I would use leave-one-out cross-validation. I would first compute simultaneous
multiple-well ties using every well. I then would remove one validation well, compute well ties again, compare predicted and previously computed updated time-depth functions of the validation well, and repeat for all wells. Again, though, I am limited by the computational cost of the image-guided interpolation I use to compute synthetic images.

I might also reconsider the need to interpolate synthetic seismograms to simultaneously tie multiple wells. Using image-guided interpolation is computationally expensive, and because wells are not uniformly spaced, synthetic seismograms used to compute the synthetic image do not equally influence the resulting simultaneous multiple-well ties. The computational cost of image-guided interpolation increases linearly with the size of the image, assuming that the wells are unchanged. Increasing the number of wells will linearly decrease the cost of the interpolation because the number of interpolated samples decreases.

I could also improve simultaneous multiple-well ties using horizons and formation tops. Horizons are commonly interpreted manually or automatically from the seismic image, while corresponding formation tops are interpreted from well logs. I could use the intersection points of these horizons and corresponding formation tops in each well to further constrain time shifts computed using smooth DIW.

Finally, I could potentially use updated velocities to estimate seismic velocity anisotropy. These velocities might correlate to modeling error in the synthetic seismograms or migration error in the seismic image caused by velocity anisotropy, and therefore be used to estimate parameters that describe anisotropy in seismic processing. However, to use updated velocities for this parameter estimation, I must ensure that their differences from log velocities are primarily caused by anisotropy.
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