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CWP Research Report

February 8, 2016

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http://cwp.mines.edu
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(February 9, 2016)

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ABSTRACT

By solving the Marchenko equations, the Green’s function can be retrieved between a virtual receiver in the subsurface to points at the surface (no physical receiver is required at the virtual location). We extend the idea of these equations to retrieve the Green’s function between any two points in the subsurface; i.e, between a virtual source and a virtual receiver (no physical source or physical receiver is required at either of these locations). This Green’s function is called the virtual Green’s function and includes all the primaries, internal and free-surface multiples. Similar to the Marchenko Green’s function, we require the reflection response at the surface (single-sided illumination) and an estimate of the first arrival travel time from the virtual location to the surface.
INTRODUCTION

In this paper, we retrieve the Green’s function between two points in the subsurface of the Earth. We call these two points a virtual source and a virtual receiver pair. To retrieve the Green’s function at a virtual receiver for a virtual source we require neither a physical source nor a physical receiver at the virtual source and receiver. The requirements for the retrieval of this Green’s function is the reflection response for physical sources and physical receivers at the surface (single sided-illumination) and a smooth version of the velocity model (no small-scale details of the model are necessary). For brevity we define this Green’s function i.e., the response of a virtual source recorded at a virtual receiver, as the Virtual Green’s function. We label the method of retrieving the Virtual Green’s function as the modified Marchenko method.

Similar ideas of retrieving the Green’s function between two points have been proposed, notably, in seismic interferometry (Wapenaar, 2004; Curtis et al., 2006; Snieder et al., 2007; Bakulin and Calvert, 2006; van Manen et al., 2006; Curtis et al., 2009; Curtis and Halliday, 2010) and in the Marchenko method (Broggini et al., 2012; Broggini and Snieder, 2012; Wapenaar et al., 2013; Slob et al., 2014; Wapenaar et al., 2014; Singh et al., 2015, 2016). However, these methods (interferometry and Marchenko method) have more restrictions in the source-receiver geometry, as discussed later, for the accurate retrieval of the Green’s function than our proposed method (modified Marchenko method).

In seismic interferometry, we create virtual sources at locations where there are physical receivers. We also require a closed surface of sources to adequately retrieve the Green’s function. We can, in practice, relax the requirement for a closed-surface of sources to a source distribution in the stationary-phase region of the source integral since the dominant
contributions of our Green’s function come from this region, while the other contributions usually stack out to zero (Snieder, 2004; Snieder et al., 2006).

Unlike interferometry, a physical receiver or physical source is not needed by our modified Marchenko method to create either a virtual source or a virtual receiver and we only require single-sided illumination (a closed surface of sources not needed). The Green’s function retrieved by the Marchenko equations is the response of virtual source in the subsurface recorded at physical receivers at the surface (Broggini et al., 2012; Broggini and Snieder, 2012; Wapenaar et al., 2013; Slob et al., 2014; Wapenaar et al., 2014; Singh et al., 2015, 2016). The Marchenko retrieved Green’s function requires neither a physical source nor a physical receiver at the virtual source location in the subsurface.

Our algorithm retrieves the Green’s function (both up- and down-going at the receiver) for virtual sources and virtual receivers. The Marchenko-retrieved Green’s functions are limited to virtual sources in the subsurface recorded at the surface but the Modified Marchenko method (our Work) is not restricted to recording on the surface for each virtual source. In our method, the response of the virtual source can be retrieved for a virtual receiver anywhere in the subsurface.

Wapenaar et al. (2016) has proposed similar work to ours, but their approach retrieves (1) the Green’s function without free-surface multiples while our algorithm can include these multiples (2) the two-way virtual Green’s function while our work retrieves the up- and down-going (one-way) virtual Green’s function, the summation of these one-way Green’s function gives the two-way Green’s function.

We discuss in this paper the theory of retrieving the virtual Green’s function. Our numerical examples are split into three sections (1) A verification of our algorithm to demon-
strate that we retrieve the up- and down-going virtual Green’s function (using a 1D example for simplicity) (2) A complicated 1D example illustrating our algorithm accurately retrieves the Green’s function with the free-surface multiples and without the free-surface multiples (3) A 2D numerical example of the virtual Green’s function constructed in such a way that we create a wavefield with all the reflections and first arrivals from a virtual source. This last numerical example is complicated since the discontinuities in the density and the velocity are at different locations.

**THEORY**

To retrieve the Green’s function from a virtual receiver in the subsurface for sources on the surface, one solves the Marchenko equations. The retrieval only requires the reflection response at the surface and an estimate of the first arrival travel-time from the virtual receiver to the surface. The retrieved Green’s function can either include free-surface multiples (Singh et al., 2015, 2016) or exclude these multiples (Broggini et al., 2012; Broggin and Snieder, 2012; Wapenaar et al., 2013; Slob et al., 2014; Wapenaar et al., 2014).

In addition to the retrieved Green’s function, the Marchenko equations also gives us the one-way focusing functions. These functions are outputs from the Marchenko equations that exist at the acquisition level ∂D₀ (acquisition surface) and focus on an arbitrary depth level ∂Dᵢ at t = 0 (time equal zero).

The focusing functions are auxiliary wavefields that reside in a truncated medium that has the same material properties as the actual inhomogeneous medium between ∂D₀ and ∂Dᵢ and that is homogeneous above ∂D₀ and reflection-free below ∂Dᵢ (Slob et al., 2014). Therefore, the boundary conditions on ∂D₀ and ∂Dᵢ in the truncated medium, where the
focusing function exists, are reflection-free, see Figure 1. Our algorithm moves the sources of
the Green’s function retrieved by Marchenko equations from the surface into the subsurface
at a virtual point with the help of the focusing function.

In this paper, the spatial coordinates are defined by their horizontal and depth compo-
nents; for instance $x_0 = (x_{H,0}, x_{3,0})$, where $x_{H,0}$ are the horizontal coordinates at a depth
$x_{3,0}$. Superscript $(+)$ refers to down-going waves and $(-)$ to up-going waves at the obser-
vation point $x$. Additionally, any variable with a subscript $0$ (e.g., $R_0$) indicates that no
free-surface is present.

One-way reciprocity theorems of the convolution and correlation type, equations 1 and
2 respectively, are used to relate up- and down-going fields at arbitrary depth levels to each
other in different wave states (Wapenaar and Grimbergen, 1996). The one-way reciprocity
theorems of the convolution and correlation type are

$$\int_{\partial D_0} [p_B^+ p_A - p_B p_A^+] dx_0 = \int_{\partial D_i} [p_B^+ p_A - p_B p_A^+] dx_i, \quad (1)$$

$$\int_{\partial D_0} [(p_B^+)^* p_A^+ - (p_B^+)^* p_A] dx_0 = \int_{\partial D_i} [(p_B^+)^* p_A^+ - (p_B^+)^* p_A] dx_i, \quad (2)$$

where the asterisk * denotes complex conjugation, the subscripts A and B are two wave
states, and $\partial D_0$ and $\partial D_i$ are arbitrary depth levels.

The correlation reciprocity theorem, equation 2, is based on time reversal invariance of
our wavefields, which implicitly assumes that the medium is lossless. Since we assume the
wavefields can be decomposed into up- and down-going waves, we ignore evanescent waves;
hence all our results obtained from equations 1 and 2 are spatially band-limited (Wapenaar
et al., 2004a). More details on one-way reciprocity can be obtained in Wapenaar (1998),
Wapenaar et al. (2001), and Wapenaar et al. (2004b).
Figure 1: Up- and down-going focusing function $f_1^{\pm}$ that focuses at $x_i'$ in the truncated medium. This medium is homogeneous above $\partial D_0$ and below $\partial D_i$ and is equal to the real medium between $\partial D_0$ and $\partial D_i$.

Wave state A is defined for the truncated medium and $P_A$ is the focusing function. The one-way wavefields for wave state A for a source at $x_i'$ are given in Table 1 and Figure 1.

The Green’s functions in the actual medium is defined as wave state B. The one-way wavefields for wave state B, the actual medium, for a source at $x_j''$ are given in Table 1 and Figure 2. These Green’s functions are retrieved using the Marchenko equations and include the primary, internal, and free-surface multiple reflections of the actual medium (Singh et al., 2015, 2016).

We substitute the one-way wavefields described in Table 1 into equations 1 and 2 and
On $\partial D_0$: \[ p_A^+ = f_1^+(x_0, x_i', \omega) \quad p_A^- = f_1^-(x_0, x_i', \omega) \]
\[ p_B^+ = rG(x_0, x_j'', \omega) \quad p_B^- = G(x_0, x_j'', \omega) \]

On $\partial D_i$: \[ p_A^+ = f_1^+(x_i, x_i', \omega) = \delta(x_H - x_H') \]
\[ p_A^- = f_1^-(x_i, x_i', \omega) = 0 \]
\[ p_B^+ = G^+(x_i, x_j'', \omega) \]
\[ p_B^- = G^-(x_i, x_j'', \omega) \]

Table 1: The wavefields of the focusing function $f_1$ and Green's functions at the acquisition surface $\partial D_0$ and the level $\partial D_i$. $p_A^\pm$ symbolizes one-way wavefields in the frequency domain for wave state A, at arbitrary depth levels in the reference medium, see Figure 1 while $p_B^\pm$ symbolizes one-way wavefields at arbitrary depth levels in the inhomogeneous medium in wave state B, where $r$ is the reflection coefficient of the free surface, see Figure 2.
Figure 2: The Green’s functions in the actual inhomogeneous medium in the presence of a free surface at the acquisition surface $\partial D_0$ and the arbitrary surface $\partial D_i$. The tree indicates the presence of the free surface.
also use the sifting property of the delta function to yield

$$G^{-}(x', x''_j, \omega) = \int_{-\infty}^{\infty} [G^{-}(x_0, x''_j, \omega)f^+(x_0, x', \omega) - rG^{-}(x_0, x''_j, \omega)f^-(x_0, x', \omega)] dx_0,$$  \hspace{1cm} (3)$$

$$G^{+}(x', x''_j, \omega)^* = \int_{-\infty}^{\infty} [rG^{-}(x_0, x''_j, \omega)^*f^+(x_0, x', \omega) - G^{-}(x_0, x''_j, \omega)^*f^-(x_0, x', \omega)] dx_0,$$  \hspace{1cm} (4)$$

where $r$ denotes the reflection coefficient of the free surface (in the examples shown in this paper $r = -1$.)

Equations 3 and 4 yield the up- and down-going virtual Green’s functions, respectively, for a virtual receiver at $x'_i$ and a virtual source at $x''_j$ in the subsurface. Note, we are not limited to the source $x''_j$ being below the receiver $x'_i$ since by reciprocity, $G^{-}(x'_i, x''_j, t) = G^+(x''_j, x'_i, t)$ and $G^{+}(x'_i, x''_j, t) = G^{-}(x''_j, x'_i, t)$.

To compute the up- and down-going virtual Green’s function in equations 3 and 4, we require 1) the Green’s function $G^{-}(x_0, x''_j, \omega)$ at the surface $x_0$ for a virtual source at $x''_j$ and 2) the focusing function $f^{\pm}(x_0, x'_i, \omega)$ at the surface $x_0$ for a virtual source at $x'_i$. We retrieve both these functions by solving the Marchenko equations.

We can also retrieve the virtual Green’s function which does not include free-surface multiples by simply setting the reflection coefficient at the free-surface $r$ to zero in equations 3 and 4. Thus, the equation to retrieve the virtual Green’s function without the presence of a free surface is

$$G^{-}_0(x'_i, x''_j, \omega) = \int_{-\infty}^{\infty} G^{-}_0(x_0, x''_j, \omega)f^+(x_0, x'_i, \omega) dx_0,$$  \hspace{1cm} (5)$$

$$G^{+}_0(x'_i, x''_j, \omega)^* = -\int_{-\infty}^{\infty} G^{-}_0(x_0, x''_j, \omega)^*f^-(x_0, x'_i, \omega) dx_0,$$  \hspace{1cm} (6)$$
where $G^±_0(x'_i, x''_j, \omega)$ is the up- and down-going Green’s function without free-surface multiples for a virtual receiver at $x'_i$ and virtual source at $x''_j$.

**NUMERICAL EXAMPLES**

To show a proof of concept for our algorithm, we begin with a simple 1D numerical example. Our model consists of two homogeneous layers separated by an interface at 0.5 km with constant density and velocity in each layer and a free surface. At the interface, the velocity changes from 2 km to 2.8 km, while the density changes from 1 g/cm$^3$ to 2.5 g/cm$^3$.

Our objective is to retrieve the virtual up- and down-going Green’s function $G^±(x'_i, x''_j, \omega)$ at the virtual receiver $x'_i = 0.25$ km for a virtual source at $x''_j = 1.75$ km, a schematic of these waves is shown in Figure 3. To retrieve the virtual Green’s function, we solve equations 3 and 4. These equations (3 and 4) require at the surface $x_0$: 1) the focusing function $f^±(x_0, x'_i, t)$ and 2) the Green’s function $G(x_0, x''_j, t)$; with their virtual source locations are at $x'_i$ and $x''_j$, respectively. The focusing functions are up- and down-going at the surface $\partial D_0$ and are shaped to focus at the virtual receiver $x'_i = 0.25$ km for $t = 0$.

We retrieve the one-way focusing functions at the surface by solving the Marchenko equations (Singh et al., 2015, 2016). The down-going focusing function is shown in Figure 4 in blue (normalized by its maximum amplitude). Since the medium between the surface and the focusing point is homogeneous, the up-going focusing function vanishes as it suffices to only have a down-going focusing function to create a focus at $x'_i = 0.25$ km (see Figure 4).

The Green’s function $G(x_0, x''_j, t)$ (in the right-hand side of equations 3 and 4) needed for our virtual Green’s function (in the left-hand side of equations 3 and 4) is retrieved
by the Marchenko equations as well. $G(x_0, x''_j, t)$ includes free-surface multiples, internal multiples, and primaries. $G(x_0, x''_j, t)$ is the response of a virtual source at $x''_j = 1.75 \text{ km}$ recorded at the surface $\partial D_0$, see Figure 4.

In order to solve the Marchenko equation, to obtain the focusing functions $f^\pm(x_0, x'_i, t)$ and the Green’s function $G(x_0, x''_j, t)$; and consequently the virtual Green’s function from equations 3 and 4, we require the reflection response at the acquisition level (which includes all multiples and primaries) and an estimate of the travel-time from each of the virtual points to the acquisition level.

By substituting $G(x_0, x''_j, t)$ and $f^\pm(x_0, x'_i, t)$ (shown in Figure 4) into equations 3 and 4 and adding the corresponding up- and down-going virtual Green’s function, we obtain the two-way virtual Green’s function $G(x'_i, x''_j, t)$ recorded at a virtual receiver located at $x'_i = 0.25 \text{ km}$ and a virtual source at $x''_j = 1.75 \text{ km}$, see Figure 5. This Figure illustrates that we can accurately retrieve the virtual Green’s function as there is minimal mismatch with the modeled Green’s function (in black) (source at 1.75 km and receiver at 0.25 km). The modeled Green’s function is computed using finite differences with the exact velocity and density.

Figure 3 shows a schematic of a few of the up- (solid lines) and down-going (dashed lines) events at the virtual receiver for the virtual Green’s function in our one-interface model. These events (numbered) in Figure 3 corresponds to the retrieved virtual up- and down-going Green’s functions in Figure 6, at the correct travel-times; hence confirming the proof of concept of our one-way equations in 3 and 4.

The second example illustrates the retrieval of the virtual Green’s function without the free-surface reflections (Figure 7) for the model given in Figure 8 with the virtual source and
Figure 3: Schematic of the up- and down-going events at the receiver position for the simple 2 layer model. Solid lines represent up-going while dotted events represent down-going events.
Figure 4: The input wavefields required for equations 4 and 3: The two-way Green’s functions $G(x_0, x_j''', t)$ (in red) retrieved by the Marchenko equations in the presence of a free surface for $x''_j = 1.75 \text{ km}$ and the down-going focusing function $f^+(x_0, x_i', t)$ (in blue) also retrieved by the Marchenko equations for $x_i' = 0.25 \text{ km}$. 
Figure 5: Green’s function with virtual source $x_j'$ at depth 1.75 km and recording at the virtual receiver $x_i'$ at depth 0.25 km. The black thicker line is the modeled Green’s function, superimposed on it is the retrieved Green’s function. Each trace is divided by its maximum amplitude hence the y-axis label is called normalized amplitude. The plot limits are chosen between 0.5 to -0.5 normalized amplitude to visualize the smaller amplitude events better.
Figure 6: The up- (blue line) and the down-going (red line) Green's function with the numbers in Figure 3 corresponding to the appropriate event.
Figure 7: Virtual Green’s function (white line) with virtual source $x_j''$ at depth 1.75 km and recording at the virtual receiver $x_i'$ at depth 0.75 km superimposed on it the modeled Green’s function (black line). Both Green’s functions do not include free-surface multiples. The receiver shown by the red and blue dots, respectively. This example also contains variable density, with discontinuities at the same depth as the velocity model, with densities ranging from $1 \text{ gcm}^{-3}$ to $3 \text{ gcm}^{-3}$. As shown in Figure 7, there is an almost perfect match between the modeled Green’s function and the retrieved virtual Green’s function.

Figure 9 is the virtual Green’s function using the same model in Figure 8 but with free-surface multiples. Note the increased reflections and complexity the free surface introduces (Figure 9) compared to the case without the free surface (Figure 7). The match between the modeled Green’s function and the retrieved Green’s function using our algorithm is almost
Figure 8: 1D velocity model without a free surface. The blue dot at 0.75 km is the location of the virtual receiver while the red dot at 1.75 km is the position of the virtual source for the retrieved virtual Green’s function.

exact, see Figure 9.

The 1D numerical examples have perfect aperture, hence, the 1D examples almost perfectly match the retrieved virtual Green’s function to the modeled Green’s function. The equations to obtain the virtual Green’s function are multidimensional. We next show a 2D numerical example of the virtual Green’s function in a velocity and density model shown in Figures 10 and 11, respectively. Notice that the discontinuities and the dip of the interfaces in the velocity are different from those in the density.

Our algorithm allows us to place virtual receivers and virtual sources in any target location in the subsurface. For our numerical example, we retrieve the virtual Green’s function $G(x'_i, x''_j, t)$, Figure 12, where $x'_i$ are the virtual receivers populating the target location at every 32 m (black box in Figure 10) and $x''_j = (0, 0.7) \text{ km}$ is the virtual source.
Figure 9: Virtual Green’s function with free-surface multiples (white line) with virtual source $x''_j$ at depth 1.75 km and recording at the virtual receiver $x'_i$ at depth 0.75 km for the model in Figure 8 with a free surface. The modeled Green’s function is superimposed on it which also includes the free-surface multiples (black line).
Figure 10: Two-interface velocity model with velocities ranging from 2.0 to 2.4 $km/s$. The dot shows the position of the virtual source for the virtual Green’s function and the black box is the target zone where we place virtual receivers.
Figure 11: One-interface density model with densities ranging from 2.0 to 3.0 g/cm$^3$. The dot shows the position of the virtual source for the virtual Green’s function and the black box is the target zone where we place virtual receivers.
(black dot in Figure 10). In Figure 12 notice:

1. In panel b, the first arrival from the virtual source \( x_j'' = (0, 0.7) \) km and the reflection from the bottom velocity layer.

2. In panels c and d, the inability of our algorithm to handle the horizontal propagating energy of the first arrival from the virtual source, hence the dimming on the sides of the first arrival of the virtual Green’s function. To retrieve near-horizontally propagating events (in this case, these waves are not evanescent) especially in the first arrival of the virtual Green’s function, we also require a much larger aperture than is used in this example. Note that the later arriving up- and down-ward propagating waves are retrieved accurately at the depth of the virtual source \( x_j'' = (0, 0.7) \) km in Figure 12, panel d and e, since the reflections are purely up- and down-going.

3. In panels c and d, we do however, retrieve the reflections from the density layer (pink line in Figure 12) although we did not use any information of the density model in our numerical retrieval of the virtual Green’s function.

4. In panel f, the free-surface multiple is present. As expected, there is a polarity change of the free surface multiple compared to the incident wave at the top of panel e due to the interaction of this wave in panel e with the free surface.

5. In panel h, we obtain the up-going reflections caused by the free-surface multiple interacting with the velocity and density layer.

We compare in Figure 13 a trace of the virtual Green’s function \( G(x_i', x_j'', t) \) at virtual location \( x_i' = (0, 0.17) \) km and \( x_j'' = (0, 0.7) \) km to the modeled Green’s function with
Figure 12: Snapshots of the virtual Green’s function $G(x_i', x_j'', t)$ with virtual sources $x_j'' = (0, 0.7) \ km$ and virtual receivers $x_i'$ populating the target box in Figure 10.
Figure 13: Virtual Green’s function (white line) with virtual source $x''_j = (0, 0.7) \ km$ and recording at the virtual receiver $x'_i = (0, 0.17) \ km$ for the 2D model in Figure 10. The black thicker line is the modeled Green’s function, superimposed on it is the retrieved Green’s function.

physical receiver and physical source at the virtual receiver and virtual source location $x'_i$ and $x''_j$, respectively. The modeled Green’s function in Figure 13 is generated by finite differences using the exact velocity and density. The trace of the virtual Green’s function is comparable to the modeled Green’s function as shown in Figure 13.

In our algorithm, we evaluate an integral over space using a sampling interval $dx$, for example, in equations 3 and 4. These integrals over space, which yields the stationary phase contribution, also generate artifacts due to end point contributions. Similar to interferome-
try, these artifacts can be mitigated through tapering at the edges of the integration interval (Mehta et al., 2008; van der Neut et al., 2009). In our 2D model these artifacts that arise from the integrals over space are also present. We remove these artifacts by muting the wavefield before the first arrival of the virtual source $x_j''$, and estimate the travel time of the first arrival using the smooth velocity model.

**DISCUSSION**

The theory of the virtual Green’s function is built on Marchenko equations and uses the Marchenko solutions as well; hence, the virtual Green’s function also suffers from the shortcomings and requirements of the Marchenko retrieved Green’s function that are described elsewhere (Broggini et al., 2012; Broggin and Snieder, 2012; Wapenaar et al., 2013; Slob et al., 2014; Wapenaar et al., 2014; Singh et al., 2015, 2016).

A fair question to ask is: why not use interferometry to cross-correlate the Green’s function at a virtual receiver and at virtual source to get the virtual Green’s function between the virtual source and the receiver? This interferometric method will not retrieve the virtual Green’s function since we do not have a closed surface of sources which is required by seismic interferometry. In Figure 14 (red line), we show the interferometric Green’s function, (cross-correlation of the Green’s functions from the virtual source and receiver to the surface), for the same model (see Figure 7) with the same virtual source $x_j'' = 1.75 \ km$ and virtual receiver $x_i' = 0.25 \ km$ locations in the second 1D example.

Since we have reflectors below the virtual source location $x_j'' = 1.75 \ km$ (see Figure 8) and our physical sources are at the surface, our interferometric Green’s function does not match the modeled or virtual Green’s function (see Figure 14 – white line). This mis-match
Figure 14: Virtual Green’s function with virtual source $x_j''$ at depth 1.75 m and recording at the virtual receiver $x_i'$ at depth 0.75 km retrieved by the method of this paper (white line) and computed by interferometry (red line). The retrieved virtual Green’s function (white line) is almost identical to the modeled virtual Green’s function.

is caused by ignoring contributions from reflectors below the virtual source (we violated the requirement of the closed surface interferometric integral for physical sources that create the virtual source).

For the simple 2D model, the discontinuities and dip in the velocity and density are different. However, we retrieve the two-way and one-way wavefield of the virtual Green’s function without any knowledge of the density model. Figure 12 shows reflections from the density interface (middle interface in Figure 12), even though no density information was
included in our algorithm. We retrieve these reflections because the density information is embedded in the reflection response recorded at the surface and the Marchenko equations are able to retrieve the density reflections from this response.

CONCLUSION

We can retrieve the Green’s function between two points in the subsurface with single-sided illumination. Generally, interferometry gives inaccurate Green’s functions for illumination from above (single-sided) because we do not have the illumination contributions from below. However, the Marchenko equations can be thought of as the mechanism to obviate the need for illumination from below to retrieve the virtual Green’s function. The removal of the requirement for illumination from below (for interferometry) comes from the use of the focusing function, a solution to the Marchenko equations. The events in the focusing function only depend on the truncated medium and this function is solved using illumination only from above. In this paper, we explore this single-side illumination advantage of the focusing function to avoid the illumination from below to retrieve the virtual Green’s function.

ACKNOWLEDGMENTS

We thank Kees Wapenaar (Delft University), Joost van Der Neut (Delft University), Ivan Vasconcelos (Schlumberger Gould Research), Esteban Diaz (CWP), and Nishant Kamath (CWP) for fruitful discussions. We are grateful to Diane Witters for her help in preparing this manuscript. This work was funded by the sponsor companies of the Consortium Project on Seismic Inverse Methods for Complex Structures and by Shell Research. The 2D
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