Full-waveform inversion in a VTI elastic Earth: a parameterization and crosstalk study

Antoine Guittot1 & Tariq Alkhalifah2
1 Center for Wave Phenomena, Colorado School of Mines, Golden, Colorado, USA
2 King Abdullah University of Science and Technology, Thuwal, Saudi Arabia

ABSTRACT
Five parameters are needed to describe VTI anisotropy in 2-D elastic media. With conventional streamer data and considering that we have a kinematically accurate NMO velocity \((v_{	ext{nmo}})\) and \(\eta\) (or horizontal velocity, \(v_h\)) obtained from tomographic methods, we first show that a parameterization given by \(v_h\), \(\eta\) and \(\epsilon\) allows us to focus the inversion to update \(v_h\) (for traveltime and amplitude fitting) and \(\epsilon\) (for amplitude fitting near zero offset). Having an inaccurate \(\delta\) affects the inversion parameterized by the vertical velocity \(v_v\), \(\delta\), and \(\epsilon\) more than the inversion parameterized by \(v_h\), \(\eta\) and \(\epsilon\). Then, we wonder what are the effects of not updating density and shear-wave velocity on the inversion results with the \(v_h\), \(\eta\), and \(\epsilon\) parameterization. For this analysis, using radiation patterns derived in the elastic case and a modified version of the Marmousi II model, we see that PP-waves are mostly helping to recover \(v_h\) and \(\epsilon\), while PS-waves are mostly helping to recover \(\eta\). For all scattering angles and wave modes, \(\epsilon\) and \(\rho\) are strongly coupled. Keeping \(\rho\) and \(v_s\) unchanged during the inversion is a valid strategy if their background models are good enough. For small \(v_s\) and \(\rho\) errors (\(<20\%\)), the \(v_s\) perturbation will map into \(\eta\) while \(\rho\) will map into \(\epsilon\). For large errors (\(\approx100\%\)), \(v_s\) adversely affects the recovery of \(\eta\) and might leak into \(v_h\) while \(\rho\) will preferentially leak into \(\epsilon\), then \(\eta\) and, in smaller proportions, into \(v_h\).

Key words: FWI, elastic, VTI, density, crosstalk, parameterization

1 INTRODUCTION

The scattering potentials of perturbations in the anisotropic parameters reveal the data dependency on the parameters used to describe the anisotropic model (Gholami et al., 2013; Alkhalifah and Plessix, 2014). They also expose our ability to invert for these parameters given the seismic acquisition set-up used in the experiment. The scattering potentials of anisotropic model parameter perturbations are based on the linearized approximation of the wave equation with respect to these parameters given by the first term of the Born series. Since this term constitutes the gradient for full waveform inversion (FWI), it also reveals important information on the parameter tradeoff and their resolvability in FWI (Alkhalifah and Plessix, 2014).

Since the FWI process is highly nonlinear, the story can only be complete when such scattering potential inferences are supported by an FWI implementation. Alkhalifah (2015) studied the short and long wavelength influences of perturbations in the parameters for parameterizations promoted by Alkhalifah and Plessix (2014) for acoustic VTI media. He concluded that a parameterization given by the horizontal velocity \(v_h\), the anellipticity parameter \(\eta\), and the parameter that relates the horizontal-to-the vertical velocity, \(\epsilon\), was optimal for FWI using conventional surface seismic PP-wave data. In this case, the long wavelength information of \(v_{\text{nmo}}\) and \(\eta\) (or \(v_h\)) are assumed to be included in the initial model and we, thus, need to invert only for \(v_h\) and \(\epsilon\). The role of \(\epsilon\) in this case is to provide the perturbations necessary to fit the amplitudes of reflections at short offsets to accommodate the limitations of the acoustic model in properly fitting elastic amplitudes.

In this paper we study the parameterization effects in the elastic case with a marine geometry in mind. Focusing on marine data simplifies the analysis because S-waves are coming from mode conversions only and become second-order events. Also, the inversion of marine data is simpler than the inversion of land data, thus giving us a chance to test our findings on field datasets more easily. Therefore our goal is to bring more insights on the influence of all parameters in the Elastic Full Waveform Inversion (EFWI) of field data, where the Earth is assumed to be closer to an elastic than acoustic medium.

First, we start our paper by comparing the radiation patterns of an optimal \(v_h\), \(\eta\) and \(\epsilon\) parameterization with those
of the more conventional \(v_v, \delta, \epsilon\) parameterization. Then, using a modified Marmousi II synthetic dataset, we analyze both parameterizations when a kinematically accurate NMO velocity and \(\eta\) (or horizontal velocity, \(v_h\)) obtained, for example, from tomographic methods are available. Having an inaccurate \(\delta\) (equal to zero in these first tests) caused the inversion parameterized by \(v_v, \delta, \epsilon\) to yield worse results than the inversion parameterized by \(v_h, \eta\) and \(\epsilon\). Nevertheless, both inversion results are slightly effected by (1) missing long wavelength depth information and (2) the elastic nature of the modeled data. The degradation, however, is far more severe using the conventional \(v_v, \delta, \epsilon\) parameterization.

Finally, in an effort to better understand the influence of all VTI elastic parameters, we also focus on the role played by density (\(\rho\)) and S-wave velocity (\(v_s\)) in EFWI in the proposed \(v_h, \eta\) and \(\epsilon\) parameterization. For this work, we put ourselves in a field dataset mindset and want to understand the consequences of not inverting for density and \(v_s\) in EFWI, focusing only on the retrieval of \(v_h\), \(\eta\) and \(\epsilon\). To this end, we first present the missing radiation patterns for PS-waves for a horizontal layer for all five parameters (\(v_h, v_s, \rho, \eta\), and \(\epsilon\)). Then we illustrate our derivations with inversions of the modified Marmousi II synthetic model. We learn that for small \(\rho\) and \(v_s\) perturbations, density maps preferentially in the \(\epsilon\) field, while S-wave velocity maps in the \(\eta\) field, in agreement with the asymptotic (Born approximated) analysis that yields the derivation of the radiation patterns.

2 COMPARING \(V_v, \delta, \epsilon\) AND \(V_{hi}, \eta, \epsilon\) PARAMETERIZATIONS

In this section, we compare two parameterizations of VTI elastic FWI by first presenting and analyzing radiation patterns and then illustrating our findings on a modified Marmousi II model. This analysis is for marine data and focuses on the inversion of PP-waves only.

2.1 Scattering potentials in VTI elastic media

For acoustic VTI media, Alkhalifah and Plessix (2014) derived such patterns for different anisotropic parameter combinations that they deem to be the most practical. Later, Alkhalifah (2015) made the argument for one of these combinations, \(v_h, \eta\), and \(\epsilon\), for FWI of conventionally acquired surface seismic P-P-wave data. Considering the asymptotic Green’s function, \(G(x, k, \omega)\) expressed in the frequency, \(\omega\), domain, for a plane wave described by the wavenumber vector, \(k\), for either the source or receiver wavefields approaching location \(x\), we can write the single-scattered wavefield (Alkhalifah and Plessix, 2014)

\[
u_s(k_s, k, \omega) = -\omega^2 s(\omega) \int d\mathbf{x} \frac{G(k_s, \mathbf{x}, \omega)G(k, \mathbf{x}, \omega)}{v_0(\mathbf{x})\rho(\mathbf{x})} \cdot \mathbf{a}(\mathbf{x}) \cdot \mathbf{r}(\mathbf{x}) \tag{1}
\]

with \(s\) is the source function, \(\rho\) is the density, \(v_0\) is the background isotropic velocity. The vector \(\mathbf{r}(\mathbf{x})\) includes the perturbations of the individual parameters:

\[
\mathbf{r}(\mathbf{x}) = \begin{pmatrix} r_{v_h}(\mathbf{x}) \\ r_{v_s}(\mathbf{x}) \\ r_{\eta}(\mathbf{x}) \\ r_{\epsilon}(\mathbf{x}) \\ r_{\rho}(\mathbf{x}) \end{pmatrix} \tag{2}
\]

while the coefficients of \(\mathbf{a}(\mathbf{x})\) define the radiation patterns of each parameter for the given parameterization. In this case we...
Figure 2. True (top), initial (middle), and inverted (bottom) models for $v_v$ (left), $\delta$ (middle), and $\epsilon$ (right).

have $a(x) = a_{v_h}^{v_h}(x)$ where

$$a_{v_h}^{v_h}(x) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

(3)

with $\sigma$ is the Poisson’s ratio, $\theta_i$ and $\theta_r$ are the incident and reflection angles, respectively. The vector $r$ includes the perturbations of the individual parameters, $v_h$, $v_s$, $\eta$, $\epsilon$, and $\rho$ from top to bottom. Thus, the coefficients of $a$ define the radiation patterns of each parameter for the given parameterization (Aki and Richards, 1980).

In Figures 1(a) and 1(b), we show the reflection $PP$-wave radiation patterns for perturbations in the elastic VTI parameters for two different parameterizations. The radiation pattern for $v_s$ holds regardless of the parameterization, and the radiation patterns for the rest of the VTI parameters are the same as in the acoustic case. The radiation pattern for a perturbation in $v_h$ has a behavior similar to that of $\delta$ or $\eta$. There is, thus, an unfortunate tradeoff between perturbations in the shear wave velocity and that in either $\eta$ or $\delta$ in each of the parameterization. However, for conventional offset surface seismic data, the scattering influence of $v_s$ (like $\eta$ or $\delta$) on surface $PP$-wave data is small, and thus, can be neglected (Alkhalifah, 1998).

The amplitude disparity will hopefully be absorbed by another parameter, specifically $\epsilon$ in the suggested parameterization.

Figures 1(a) and 1(b) help to better understand our choice of parameterization. In the $v_v$, $\delta$, $\epsilon$ parameterization, $\epsilon$ can only be recovered from the long offsets, large scattering angles, which can be missing from conventional dataset. In addition, density effects will be absorbed by the velocity due the crosstalks between these two parameters at small angles. In contrary, with the $v_h$, $\eta$, $\epsilon$ parameterization, $\epsilon$ can absorb amplitude effects keeping $v_h$ relatively unaffected (as we will show in our examples). Indeed, the radiation pattern for $\epsilon$ shows that small angle scatterings will influence its recovery the most. These small angles are where the amplitude information, from $\rho$ perturbations and other effects, prevails. In other words if we don’t invert for $\rho$, then $\epsilon$ can be used to absorb the reflectivity. There is, however, more to this story when recorded shear waves are involved in the analysis as we will show in the second section of this paper.

2.2 Illustration on an elastic VTI Marmousi II model

The elastic Marmousi II model was developed to provide a challenging dataset to the multi-wave modes enthusiast. The VTI parameters used in the elastic modeling are shown in the top row of Figure 2. The shear wave and density models (not shown here) follow the same structure as the $PP$-wave velocity model. In all the examples, the modeling engine is the same as the one used in the inversion (the so called inversion crime), as the purpose of this section is to focus on the tradeoff only.
Figure 3. True (top), initial (middle), and inverted (bottom) models for \( v_h \) (left), \( \eta \) (middle), and \( \epsilon \) (right).

For the same purpose, we invert eight frequency bands: 0-1 Hz, 0-2 Hz, 0-3 Hz, 0-4 Hz, 0-5 Hz, 0-7 Hz, 0-9 Hz, and 0-11 Hz.

Our synthetic dataset mimics a marine acquisition survey with 67 shots every 225 m and a maximum offset of 5 km. Since we can usually obtain smooth \( v_{nmo} \) and \( \eta \) from surface seismic PP-wave data using, for example, tomographic methods, the starting model is constructed by smoothing the exact \( v_{nmo} \) and \( v_h \) models with a window length of 1.5 km. Such smoothing actually results in even a smoother \( \eta \) (Figure 3 second row middle), which we tend to expect from tomographic inversion methods (compared to velocity). The \( \delta \) model, as usually done without well information, is set to zero (Figure 2 second row middle). In this case, we expect considerable depth error in the inverted parameters. Our objective here is to test the tradeoff and convergence for the various parameterizations, thus we use the true \( \delta \) to map the inverted results to their expected depth. The mapping process given by \( z' = z \sqrt{1 + 2\delta} \) is an approximate correction as lateral variation in \( \delta \) influences data recorded on the surface. Finally, \( v_s \) and density are not updated in the inversion and are equal to a constant value (average value of exact models) in the whole sedimentary section below the water bottom.

2.2.1 Standard \( v_h, \delta \) and \( \epsilon \) parameterization

It is widely used in industry (Vigh et al., 2014; Baumstein, 2014). Figure 1(a) shows the radiation pattern for this parameterization (Gholami et al., 2013). For conventional offset-to-depth ratios (< 2), the scattering wavelengths of \( \delta \) will have little influence on the data. Despite that \( \delta \) has a small imprint for this 5 km offset data, we will invert for it as well. After 15 iterations of EFWI per frequency scale starting from 1 Hz to 11 Hz using an l-BGFS approximation of the Hessian, we end up with the inverted models shown in Figure 2 (bottom row) after applying a depth correction to the inverted models. The inverted vertical velocity shows generally some features of the true model structure, but with higher velocities in some places (Figures 4(a) and 4(b)). The \( \delta \) model, as expected looks erroneous, with limited information added to the initial \( \delta \) model. Finally, the \( \epsilon \) model also, despite the data sensitivity to it with such parameterization, looks erroneous, especially up shallow.

2.2.2 Optimal \( v_h, \eta \) and \( \epsilon \) parameterization

Here, the radiation patterns (Figure 1(b)) resemble that of the previous parameterization with a change in the role that \( \epsilon \) plays. Now \( \epsilon \) helps in fitting the reflectivity whereas before in Figure 1(a), \( \epsilon \) would get mostly updated from the diving waves/long offset data. The parameter \( \eta \), like \( \delta \), has a minor role to play in FWI. After a similar frequency continuation l-BFGS EFWI with the same number of iterations, we end up with the models shown in Figure 3 (bottom row). The horizontal velocity looks more similar to \( v_h \), but with more accurate values (Figures 4(c) and 4(d)). More importantly, \( \epsilon \) now captures the reflectivity, as it absorbed the amplitude mismatch of the elastic assumption, which caused over estimation of velocity in the case of the vertical velocity parameterization.
2.2.3 Obtaining $v_h$ from $v_p$ and $\epsilon$

We advocate a parameterization based on $v_h$ mostly. A question then arises: how does the inverted $v_h$ in our optimal parameterization compares to a $v_h$ computed from $v_p$ and $\epsilon$ in the standard parameterization? Figure 5 brings some answers. In Figure 5a, we see the result of deriving $v_h$ from $v_p$ and $\epsilon$ in Figure 2. In Figure 5b we see the result of our optimal parameterization (similar to Figure 3). We obtain a higher resolution in the top part of Figure 5a due to $v_p$. As we go deeper ($Z > 2.5$ km.), however, $v_h$ in Figure 5a is underestimated because we don’t recover $\epsilon$ very well in the standard parameterization. The direct inversion of $v_h$ in Figure 5b is relatively immune to this defect an yields more accurate results. Therefore, computing $v_h$ from $v_p$ and $\epsilon$ doesn’t provide the same accuracy that a direct inversion of $v_h$ brings.

2.3 Discussion on the effect of parameterization

We numerically tested EFWI with a VTI model parameterized by $v_h$, $\eta$, and $\epsilon$. As the radiation patterns for this parameterization suggest, the shear wave velocity and $\eta$ have minor influence on the inversion of seismic $PP$-wave data with a reasonable offset range (up to 5 km in the test). The density effect is absorbed by $\epsilon$ as they share almost the same scattering behavior. Thus, using an initial velocity model given by an accurate background NMO velocity and $\eta$ ($\delta$ is set to zero), the $v_h$ parameterization, despite the inaccurate $\delta$ model, provides a reasonable velocity, better than that given by the conventional parameterization.

Now, for the optimal $v_h$, $\eta$, and $\epsilon$ parameterization we study the effect of two important parameters (density $\rho$ and $v_s$) when not updated for by the inversion. This strategy is often used in marine data because density effects can be hard to separate from velocity effects (especially at short offsets) and converted waves can be considered as second order events.

3 ANALYZING THE EFFECTS OF $\rho$ AND $V_S$ ON VTI ELASTIC FWI OF MARINE DATA

In this section, we first present radiation patterns for all five parameters ($v_h$, $v_s$, $\rho$, $\eta$ and $\epsilon$) for PS-waves. Then we show how neglecting $\rho$ and $v_s$ affects the inversion using the modified Marmousi II model.

3.1 PS-waves scattering potentials in a VTI elastic media parameterized with $v_h$, $v_s$, $\rho$, $\eta$ and $\epsilon$

For FWI of conventionally acquired surface seismic $PP$-wave data, in an acoustic VTI media, we make the argument for the
combination $v_\rho$, $\eta$, and $\epsilon$. Now we present the PS-waves scattering potentials for this optimal parameterization; a missing piece of the puzzle to better understand the effects of $\rho$ and $v_s$ on the inversion. For PS-waves scattering, we still have Equation 1 with now $a(x) = a_{v_P}^{v_S}(x)$ where

$$a_{v_P}^{v_S} = \begin{pmatrix} 0 \\ \frac{(1-2\sigma)}{1-\sigma} \sin(\theta_r - \theta_i) \cos(\theta_r - \theta_i) \\ \cos(\theta_r) \sin(\theta_r) \cos 2\theta_i \\ -\cos(\theta_r) \sin(\theta_r) \sin(\theta_i - \theta_r) \\ \sin(\theta_r - \theta_i) \left(1 - \frac{(1-2\sigma)}{1-\sigma} \cos(\theta_i - \theta_r)\right) \end{pmatrix}.$$  \hspace{1cm} (4)

For $\sigma = 0.25$ (between carbonates and sandstone), Figure 6(a) shows the reflection PS-wave radiation patterns for Equation 4. We first notice that the scattering amplitudes are weaker than for PP-waves, as expected, and that the patterns are more complex. Second, we notice that $v_\rho$ has no influence on PS-waves scattering. The density has the strongest amplitude, but for very large angles. For short angles we see the dominance of $v_\rho$ and $\eta$ scatterings, with $\eta$ being the weakest. $\epsilon$ scatters at medium-to-large angles. Therefore in our scheme, if $v_s$ is not inverted for and if conventional streamer data are used, we should see a mapping of $v_\rho$ perturbations in $\eta$ for small scattering angles and in $\epsilon$ for large angles.

To complete this analysis, Figure 6(b) shows the reflection PS-wave radiation patterns for the conventional $v_\rho$, $\delta$, $\epsilon$ parameterization. The first thing to notice is that now, $v_\rho$ has influence on the PS-wave scattering. While PP-waves scattering at short angles doesn’t influence $\epsilon$ in Figure 1(a), PS-waves scattering does: $\epsilon$ might be weakly updated by PS-waves. However, there is a strong crosstalk between $\delta$ and $\epsilon$ that might create leakage issues.

Finally, we see that for our parameterization and conventional streamer data, the small wavenumbers of $\eta$ can only be recovered with PS-waves at small scattering angles. We also see that there is strong crosstalk between $\epsilon$ and $\rho$ for both PP- and PS-waves, which validates the strategy to invert for one only thus gathering both informations in one field. The analysis of radiation patterns for PS-waves reinforces the proposed parameterization by showing that converted waves don’t affect $v_\rho$ at all (i.e. Figure 6(a)), while affecting $v_s$ (i.e. Figure 6(b)). Now we illustrate our findings concerning the influence of $v_s$ and $\rho$ on elastic VTI inversion with a modified Marmousi II model.

### 4 ILLUSTRATION ON AN ELASTIC VTI MARMOUSI II MODEL

In this section, we test how $\rho$ and $v_s$ affect a VTI anisotropic EFWI when they are not inverted for. For this, we model a streamer survey with an explosive source and recorded as a pressure component only. $v_\rho$, $\eta$, and $\epsilon$ are shown in the top row of Figure 7. To better understand how $\rho$ affects the inversion, we build a simple density model made of two flat interfaces. The first layer is the water layer ($z=500$ m.) and has a constant density of $1 \text{ kg/m}^3$. The second layer at $z=1.5$ km has a density of $2 \text{ kg/m}^3$. The third layer has a density of $2.4 \text{ kg/m}^3$ (i.e., $20\%$ increase). To better understand how $v_s$ affects the inversion, we also build a simple velocity model made of two flat interfaces as well. The first layer is the water layer ($z=500$ m.) and has a constant velocity of $0 \text{ km/s}$. The second layer at $z=2$ km. (500 m. below the second density layer) has a velocity of $1 \text{ km/s}$ and the third layer has a velocity of $1.2 \text{ km/s}$ (i.e., $20\%$ increase as well).

With these parameters, we model and invert 67 shots every 225 meters with a maximum offset of 5 km. The inversion is conducted in the time domain with five frequency bands (0-3 Hz, 0-5 Hz, 0-7 Hz, 0-9 Hz, 0-11 Hz) and 15 iterations per band, using a l-BFGS solver. The starting models for our first results are shown in the second row of Figure 7. The initial models for the density and velocity are assumed to be correct for the water layer ($\rho = 1 \text{ kg/m}^3$ and $v_s = 0 \text{ km/s}$) and constant in the sediments ($\rho = 2 \text{ kg/m}^3$ and $v_s = 1 \text{ km/s}$). The inversion updates $v_\rho$, $\eta$ and $\epsilon$ only.

EFWI results for this setup are shown in the bottom row of Figure 7. In Figure 7a, $v_\rho$ is well recovered and we don’t see any effect of the density layer (top black arrow) or $v_s$ layer (bottom black arrow). For $\eta$ in Figure 7b, we recover some details of the model due to the converted PS-waves. We can also see the footprint of the $v_s$ layer contrast as shown by the bottom black arrow, especially between 8 km. < X < 10 km. There is a hint of the density layer as well. For $\epsilon$, the model is well recovered and the footprint of the density layer marked by
4.1 Discussion on the effects of $\rho$ and $v_s$

For VTI anisotropic, elastic FWI, we study the effect of $\rho$ and $v_s$ on the retrieval of $v_h$, $\eta$ and $\epsilon$. This study is carrying on with streamer data in mind because $\rho$ and $v_s$ are often ignored and not inverted for during the inversion of such datasets. From an inversion point-of-view and using a modified Marmousi II model, we saw that generally speaking, $v_s$ perturbations would map to $\eta$, while $\rho$ perturbations would map to $\epsilon$. We also see that $v_s$ might not be updated as long as its background values are accurate: for high errors in $v_s$, $\eta$ will be strongly affected (and $v_h$ in a much smaller way). In addition, $\rho$ will map to $\epsilon$ first for all density contrasts and $v_h$ for the highest density contrasts only. Overall, with EFWI applied to conventional streamer data, inverting for $v_h$, $\eta$ and $\epsilon$ while keeping $v_s$ and $\rho$ unchanged seems to be a reasonable approach, more so when the background models for $v_s$ and $\rho$ are accurate enough. Finally, our work focused on the mitigation of crosstalks by careful parameterization of the inversion only. It is now well-known that better multi-parameter inversion methods taking into account the inverse Hessian can help resolving some of these issues as well. One promising technique worth mentioning is the truncated-Newton method which seems to help alleviating leakage effects between parameter spaces.

5 CONCLUSION

The optimal FWI parameterization for VTI media proposed in the acoustic approximation holds in the elastic case for conventional streamer data. Inversion examples assuming accurate background NMO velocity and $\eta$ fields prove that the $v_h$ parameterization yields better results than the $v_v$ one. In addition with the $v_h$ parameterization, not updating $\rho$ and $v_s$ is a valid strategy if their background model is accurate enough. In this case, $\rho$ maps into $\epsilon$ (from mostly PP-waves scattering) and $v_s$ into $\eta$ (from mostly small angle PS-waves scattering).
Figure 7. Top row: exact models. Middle row: starting models. Bottom row: inverted models for (a) $v_h$, (b) $\eta$, and (c) $\epsilon$. The $v_s$ contrast maps preferentially into the inverted $\eta$ model (bottom black arrow) while the $\rho$ contrast (top black arrow) map preferentially into the inverted $\epsilon$ model. $\rho$ and $v_s$ contrasts are equal to 20 %.

Figure 8. Inverted models for (a) $v_h$, (b) $\eta$, and (c) $\epsilon$. $\rho$ and $v_s$ contrasts are now equal to 100 %.

Figure 9. Inverted models for (a) $v_p$, (b) $\delta$, and (c) $\epsilon$. $\rho$ and $v_s$ contrasts are equal to 100 % and map in all three parameter spaces.
REFERENCES


