Seismic shear waves as Foucault pendulum

Roel Snieder\textsuperscript{1,2}, Christoph Sens-Schönfelder\textsuperscript{2}, Elmer Ruigrok\textsuperscript{3,4}, and Katsuhiko Shiomi\textsuperscript{5}

\textsuperscript{(1)} Center for Wave Phenomena, Colorado School of Mines, Golden, USA
\textsuperscript{(2)} GFZ German Research Centre for Geosciences, Potsdam, Germany
\textsuperscript{(3)} Dept. of Earth Sciences, Utrecht University, Utrecht, Netherlands
\textsuperscript{(4)} R&D Seismology and Acoustics, Royal Netherlands Meteorological Institute (KNMI), De Bilt, Netherlands
\textsuperscript{(5)} NIED National Research Institute for Earth Science and Disaster Prevention, Tsukuba, Japan

\textbf{ABSTRACT}

Earth’s rotation causes splitting of normal modes. Wave fronts and rays are, however, not affected by Earth’s rotation as we show theoretically and with observations made with USArray. We derive that the Coriolis force causes a small transverse component for \emph{P}-waves and a small longitudinal component for \emph{S}-waves. More importantly, Earth’s rotation leads to a slow rotation of the transverse polarization of \emph{S}-waves; during the propagation of \emph{S}-waves the particle motion behaves just like a Foucault pendulum. The polarization plane of shear waves counteracts Earth’s rotation and rotates clockwise in the northern hemisphere. The rotation rate is independent of the wave frequency and is purely geometric, like the Berry phase. Using the polarization of \emph{ScS} and \emph{ScS}\textsuperscript{2} waves we show that the Foucault-like rotation of the \emph{S}-wave polarization can be observed. This can affect the determination of source mechanisms and the interpretation of observed \emph{SKS} splitting.

\textbf{Key words:} body waves, Earth’s rotation, Foucault pendulum

1 INTRODUCTION

Physical systems are affected by the rotation of the system. The imprint of rotation is described for classical mechanics (Goldstein, 1980), fluid mechanics (Pedlosky, 1992), electromagnetics (Osmanov and Machabeli, 2002), and quantum mechanics (Xu and Tsai, 1990; Talaghi, 1991). The imprint of rotation on elastic waves in anisotropic media is described by Schoenberg and Censor (1973), and it is known that Earth’s rotation affects Earth’s normal modes (Backus and Gilbert, 1961) and surface waves (Tromp, 1994). The imprint of the Coriolis force on \emph{PKP} travel times is discussed by Maus (2000), but was found to be negligible. In this work we elucidate the imprint of Earth’s rotation on propagating shear waves.

We first discuss the imprint of rotation on the direction of seismic waves (section 2), and show in section 3 the effect of Earth’s rotation of propagating body waves. In section 4 we show observations of the change in polarization of shear waves by comparing the polarization of \emph{ScS} and \emph{ScS2} waves that propagate under Japan. Details of the derivations and of the data analysis are shown in appendices.

2 EARTH’S ROTATION AND THE PROPAGATION OF RAYS

To introduce the imprint of Earth’s rotation on seismic waves we show the wavefield of the direct \emph{P}-wave recorded with USArray (http://www.usarray.org) after a deep earthquake in the Sea of Okhotsk in figure 1a along with the great circle (green line) that connects the event with the center of the used stations. Beamforming of the direct \emph{P}-wave (fig. 1b) shows that these early arriving waves propagate along the great circle. Figure 1c shows the beamforming of the waves recorded between 8 and 9 hours after the event; for this time window the waves that traveled multiple times through the Earth still arrive along the great circle. In 8 hours, Earth rotates over 120°, a deflection of rays over this angle would be detectable as a rotation of the maxima in figure 1c away from the great circle.
Figure 1c suggests that seismic rays are not deflected by the Coriolis force. To explain this, we investigate the imprint of Earth’s rotation on seismic rays. We consider rays whose direction is denoted by the unit vector \( \hat{n} \). The equation of kinematic ray tracing is given by expression (4.44) of Aki and Richards (2002): 
\[
\frac{dS_u}{dt} = -S_u \nabla c ,
\]
with \( S_u = 1/c \).

We next consider a rotating Earth and analyze the equation of kinematic ray tracing in a coordinate system that does not rotate. In that fixed coordinate system, there is no Coriolis force, but the medium rotates with a velocity (Snieder and van Wijk, 2015)
\[
V = \Omega \times r ,
\]
(2)
where \( \Omega \) is Earth’s rotation vector. According to expressions (8-1.5) and (8-1.10) of Pierce (1981), the equation of kinematic ray tracing in moving media is given by
\[
\frac{dS}{dt} = -S \nabla c - S \times (\nabla \times V) - (S \cdot \nabla) V ,
\]
(3)
where \( S \) now is the slowness perpendicular to the wavefront that includes the movement of the medium. For the rigid rotation (2), \( S \times (\nabla \times V) + (S \cdot \nabla) V = -\Omega \times S \), hence the equation of kinematic ray tracing is given by
\[
\frac{dS}{dt} = -S \nabla c + \Omega \times S ,
\]
(4)
The slowness vectors in expressions (1) and (4) are different because \( S \) includes the movement of the rotating Earth, while \( S_u \) does not. But the essential difference in these expressions is the term \( \Omega \times S \); this term describes the co-rotation of the ray direction with Earth’s rotation (Goldstein, 1980). This co-rotation causes the rays to rotate with the Earth, which explains why after 9 hours energy still propagates along the great circle (fig. 1c).

### 3 EARTH’S ROTATION AND THE POLARIZATION OF BODY WAYS

But we know that Earth’s rotation affects Earth’s normal modes (Backus and Gilbert, 1961) and surface waves (Tromp, 1994). This raises the question: what is the exact imprint of Earth’s rotation on seismic body wave propagation? An account of body waves in general rotating elastic media is given by Schoenberg and Censor (1973). Here we simplify the analysis for slowly rotating isotropic media. The fundamental mode of the Earth has a period of about 1 hour, hence \( \Omega/\omega < 0.04 \), with \( \omega \) being the angular frequency of the wave motion. The following analysis is valid to first order in \( \Omega/\omega \). We use a coordinate system with the \( z \)-axis aligned with the direction of propagation, and where the rotation vector \( \Omega \) lies in the \(-y,z\)-plane (figure 2).

In appendix A we solve the Christoffel equation in the presence of the Coriolis force using a time-dependence \( e^{-i \omega t} \). To first order in \( \Omega/\omega \) the \( P \)-velocity is not changed: 
\[
c_P = \frac{\sqrt{2(\lambda + 2\mu)}}{\rho + O(\Omega/\omega)^2} ,
\]
with \( \lambda \) and \( \mu \) the Lamé parameters and \( \rho \) the density. The \( P \)-wave has small transverse component:
\[
q_P = \hat{z} + \frac{2i}{\omega} \frac{\lambda + 2\mu}{\lambda + \mu} \hat{z} \times \Omega ,
\]
(5)
and the \( S \)-wave has a small longitudinal component
\[
q_S = \frac{2i\Omega \sin \theta}{\omega} \frac{\mu}{\lambda + \mu} \hat{z} ,
\]
(6)
where \( \theta \) is the angle between the direction of propagation \( \hat{z} \) and the rotation vector \( \Omega \). Both changes in
the polarization are 90° out of phase with the usual polarization of \(P\) and \(S\) waves, which means that the polarization is slightly elliptical with ellipticity \(\propto \Omega/\omega\). This anomalous polarization is caused by the sideways action of the Coriolis force, which is opposed by restoring elastic forces, hence the dependence on the Lamé parameters.

The most significant imprint of Earth’s rotation is on the transverse polarization of \(S\)-waves, in the following we focus on this transverse polarization. We show in appendix A2 that shear waves with time-dependence \(e^{-i\omega t}\) have two circular polarizations with opposite sense of rotation and different propagation velocities

\[
\mathbf{q}_{S+} = \mathbf{i} - \mathbf{j} \quad \text{with} \quad c_{S} = c_{S}^{(0)} \left( 1 + \frac{\Omega}{\omega} \cos \theta \right),
\]

\[
\mathbf{q}_{S-} = \mathbf{i} + \mathbf{j} \quad \text{with} \quad c_{S} = c_{S}^{(0)} \left( 1 - \frac{\Omega}{\omega} \cos \theta \right), \tag{7}
\]

with \(c_{S}^{(0)} = \sqrt{\mu/\rho}\) the shear wave velocity in an unrotating system.

The shift in the shear velocity in expression (A21) is similar to the frequency shift \(\delta \omega\) of Earth’s normal modes caused by the Coriolis force derived by Backus and Gilbert (1961) who show for toroidal modes that

\[
\frac{\delta \omega_{n}}{\omega_{n}} = \frac{\Omega}{\omega_{n}} \frac{m}{l(l+1)}, \tag{8}
\]

where \(\omega_{n}\) is the normal mode frequency for mode \(n\), while \(l\) and \(m\) are the angular order and degree, respectively. For spheroidal modes a similar expression holds, but the right hand side contains a dimensionless constant that depends on the modal structure. The factor \(m/l(l+1)\) in equation (8) plays the same role as \(\cos \theta\) in expression (A21), because \(m\) is the \(z\)-component of the angular momentum of the spherical harmonics while \(l(l+1)\) is the total angular momentum (Merzbacher, 1970).

The two shear waves with circular polarizations can be superposed to form a linear polarization. A perturbation \(\delta c_{S}\) in velocity corresponds to a perturbation \(\delta k/k = -\delta c_{S}/c_{S}\) in wavenumber, hence with expression (A21)

\[
\delta k = -\frac{\Omega \cos \theta}{\omega} k. \tag{9}
\]

The superposition of the two circularly polarized shear waves is, using expressions (A21) and (9) given by

\[
\mathbf{u}(z,t) = (\mathbf{x} - i\mathbf{y}) \exp i \left( k \left( 1 - \frac{\Omega \cos \theta}{\omega} \right) z - \omega t \right) \\
+ (\mathbf{x} + i\mathbf{y}) \exp i \left( k \left( 1 + \frac{\Omega \cos \theta}{\omega} \right) z - \omega t \right). \tag{10}
\]

Collecting terms multiplying \(\mathbf{x}\) and \(\mathbf{y}\), the expression above can be written as

\[
\mathbf{u}(z,t) = \mathbf{q}_{S}(z)e^{i(kz - \omega t)}, \tag{11}
\]

with

\[
\hat{\mathbf{q}}_{S}(z) = \mathbf{x} \cos \left( \frac{k\Omega \cos \theta}{\omega} z \right) - \mathbf{y} \sin \left( \frac{k\Omega \cos \theta}{\omega} z \right). \tag{12}
\]

This is a linearly polarized wave where the direction of the polarization vector is given by

\[
\varphi = \arctan \left( \frac{q_{y}}{q_{x}} \right) = -\frac{k\Omega \cos \theta}{\omega} z. \tag{13}
\]

The time derivative of the location of the wavefronts is given by

\[
\dot{z} = c_{S} = \omega/k. \tag{14}
\]

Using this in the time-derivative of expression (13) implies that the rate of rotation of the polarization of the \(S\)-waves in the transverse plane is given by

\[
\dot{\varphi} = -\Omega \cos \theta. \tag{15}
\]

The minus sign means that the polarization of the \(S\)-wave rotates in the opposite direction as Earth’s rotation. The projection of the rotation vector along the direction of propagation is \(\Omega \cos \theta\), which is the rotation rate of Foucault’s pendulum (Pérez and Pujol, 2015). As shown by equation (15) this rotation is exactly compensated by the rotation of the polarization vector in the plane perpendicular to the direction of wave propagation. The rate of change of the \(S\)-wave polarization vector follows from expressions (12) and (14) and is given by

\[
\dot{\mathbf{q}}_{S} = -\Omega \cos \theta \mathbf{z} \times \mathbf{q}_{S}. \tag{16}
\]

This amounts to a rotation in the \((x, y)\)-plane with a rotation vector \(-\Omega \cos \theta \mathbf{z}\). This means that shear waves behave exactly like a Foucault pendulum (Pérez and Pujol, 2015) as the change of the polarization plane caused by Earth’s rotation is the same as for Foucault’s pendulum. An elliptically polarized shear wave can be written as the superposition of two linearly polarized shear waves that are 90° out of phase. Each of the linear shear wave polarizations rotates with the same rotation rate (15), and hence the elliptical polarization rotates with

\[
\mathbf{q}_{S}(z) = \mathbf{x} \cos \left( \frac{k\Omega \cos \theta}{\omega} z \right) - \mathbf{y} \sin \left( \frac{k\Omega \cos \theta}{\omega} z \right). \tag{12}
\]
Figure 3. (a) Geometry of the earthquakes (yellow and dark green circles) and stations (squares), with ray paths for ScS (solid lines) and ScS2 (dashed lines) between each earthquake and the furthest station within the array. The mantle is indicated by the green area and the outer core by the red area. The vertical scale is compressed and the rays propagate in the near-vertical direction. The distance between the ScS2 bounce points at the surface is less than 100 km. (b) Vertical cross section in north-south direction. (c) The change in S-wave polarization between ScS2 and ScS waves for the west event (yellow) and the east event (dark green) and their average \( \mu(x) \), light green) as a function of epicentral distance from each event, along with the theoretical value (dashed line). The grey-shaded area denotes the region bounded by \( \mu(x) \pm \sigma \). The determination of the standard deviation \( \sigma \) is discussed in appendix C3.

the same rate. Expressions (15) and (16) therefore are applicable to elliptically polarized shear waves as well.

The circular polarizations in expression (A21) have a direct analog in the Bravais pendulum (Babović and Mekić, 2011). Just as the Foucault pendulum, the Bravais pendulum consists of a mass that is suspended by a long thin wire. In Foucault’s pendulum, the mass oscillates in a plane, while in the Bravais pendulum the mass moves in a circular orbit, either in the clockwise or in the counterclockwise direction. When this orbit moves against Earth’s rotation it takes less time to move over a full circle than when it moves with Earth’s rotation. The difference in the orbit times for the two circular motions of the mass can be used to measure Earth’s rotation.

The rotation rate of the S-wave polarization is independent of the wave frequency and is thus a purely geometric effect resulting from parallel transport along a 3D path, as it is for Foucault’s pendulum (Pérez and Pujol, 2015). It is similar to geometric phases like the Berry phase (Berry, 1984) observed in optics and quantum physics. A straight segment of a seismic ray in the rotating system corresponds, due to Earth’s rotation, with a segment of a helix in a non-rotating system. The resulting geometric phase is the same as if the helix-like shape of the ray was enforced by the structure of a non-rotating medium e.g. in a helical spring (Boulanger et al., 2012).

4 OBSERVATIONS OF THE CHANGE IN POLARIZATION OF S-WAVES

Detecting the change in polarization for S-waves due to Earth’s rotation is challenging because during propagation of the main S-wave phases (about 30 minutes) Earth rotates only over about 7°. We study the change in polarization between ScS and ScS2 waves that have bounced once or twice off the core-mantle boundary. For steeply propagating ScS waves, the change in polarization due to Earth’s rotation for each bounce at colatitude \( \theta = 55° \) is \( \Delta \varphi_{\text{rot}} = -\Omega t \cos \theta = -2.26° \) for a travel time \( t = 940 \text{ s} \). We use tiltmeter records of Hi-net, a seismic network in Japan (figure 3ab) for two earthquakes. The western event (WE) is the Mw=6.2 Tanegashima earthquake (November 21, 2005, depth = 150 km) and the eastern event (EE) is the Mw=7.2 Miyagi earthquake (August 16, 2005, depth = 40 km).

We use earthquakes on both sides of the array for the following reason. The change in polarization \( \varphi \) is caused by a contribution \( \varphi_{\text{rot}} \) due to Earth’s rotation and a contribution \( \varphi_{\text{struc}} \) due to ray bending by laterally varying Earth structure. We show in appendix B that for a horizontal velocity gradient in the upper mantle of
1%/200 km, the resulting change in the polarization direction is about 2°, which is comparable to the change in polarization caused by Earth’s rotation. It follows from time-reversal invariance that the structure-induced rotation of polarization for rays propagating in opposite directions is opposite. This also follows from expression (4.1.5) of Cervený (2001). For Earth’s rotation, the change in polarization is in the same clockwise direction on the northern hemisphere regardless of the direction of propagation. Another way to state this is that the imprint of velocity structure is invariant for time reversal, while that of rotation is not (unless one changes the rotation vector as well). Hence for waves propagating in opposite directions with polarization change $\varphi_\pm$, respectively, $\varphi_\pm = \varphi_{\text{rot}} \pm \varphi_{\text{struc}}$, hence

$$\varphi_{\text{rot}} = (\varphi_+ + \varphi_-)/2.$$  

Averaging the change in polarization for waves propagating in opposite directions thus removes the imprint of lateral velocity variations.

The data processing involves frequency filtering (periods 50-100 s), correcting the SV motion for the reflection coefficients at Earth’s surface and the core mantle boundary so that the $SH$ waves and corrected SV waves are both completely reflected, alignment of the arriving waves, and $f, k$-filtering (appendix C). The resulting change in polarization between the ScS2 and ScS arrivals is shown in figure 3c for the eastern event (dark green line) and the western event (yellow line) as a function of epicentral distance. The polarization in the data is measured clockwise from the great circle direction at each station. These polarization differences are due to a combination of lateral velocity variations and Earth’s rotation. For short distances along the receiver line ($x < 700 \text{ km}$) the rays sample different parts of the mantle, and the change in polarization is masked by the imprint of earth structure. As the distance approaches 1200 km, the rays sample the same region (fig. 3a). The imprint of horizontal variations in Earth structure on polarization is opposite, and the mean of the polarization differences for the two events ($\mu(x)$, light green curve in fig. 3c) is for the larger distances not equal to zero but is close to the change in polarization caused by earth rotation (2.26°). This estimated change in polarization is 3.8 times as large as the standard deviation (0.59°).

5 CONCLUSION

The observed change in $S$-wave polarization between $ScS2$ waves and $ScS$ waves agrees with the change predicted by expression (15). This is an experimental confirmation that $S$-waves in the Earth behave as a Foucault pendulum. Alternatively, one can view our observation of the change in $S$-wave polarization caused by Earth’s rotation as a seismic manifestation of the Berry phase (Pérez and Pujol, 2015; Berry, 1984).

For typical observations of body waves the rotation effect is small because Earth does not rotate much over the propagation time of $S$-waves in the Earth (2.5° in 10 minutes). The change in polarization due to Earth’s rotation should be considered in high precision investigations that rely on polarization, such as the determination of source mechanisms from $S$-wave polarizations. In elastic anisotropic media shear wave splitting may occur, this leads to an elliptical polarization of the $S$-waves, and the orientation of the polarization with the highest velocity provides information on azimuthal anisotropy (Vinnik et al., 1989; Silver and Chan, 1991). Since an elliptically polarized shear wave also rotates with the rotation rate (15), $SKS$ splitting measurements are affected by Earth’s rotation as well. Our analysis provides a simple formula to estimate, and potentially correct for, the Foucault-like rotation of the polarization direction. Our work makes it possible to measure the change in polarization $\varphi_{\text{struc}}$ due to Earth structure. Measurements of this quantity could be used to constrain the horizontal velocity gradients in the Earth. Using cylindrical resonators of materials with low attenuation, one can, in principle, build rotational sensors that measure the rate of change of $S$-wave polarizations or toroidal modes to measure the projection of the rotation vector along the axis of the cylinder.

Acknowledgments. We thank two anonymous reviewers, Frederik Tilmann, and Oleg Godin for their comments, and Kiwamu Nishida for assistance in the data preparation. Roel Snieder was supported by the Alexander von Humboldt Foundation. The data used in this study can be obtained from NIED Japan by sending a request to hinet-admin@bosai.go.jp.

REFERENCES


Boulanger, J., N. Le Bihan, S. Catheline, and V. Rosetto, 2012, Observations of a non-adiabatic geo-


Pedlosky, J., 1992, Geophysical fluid dynamics, 2nd ed.: Springer.


**APPENDIX A: PLANE ELASTIC WAVES IN A ROTATING SYSTEM**

We consider the special case of a homogeneous isotropic elastic medium and assume that that the rotation rate is small, in the sense that $\Omega/\omega \ll 1$, where $\omega$ is the angular frequency of the waves. All results are accurate to first order in $\Omega/\omega$. The equation of motion in an homogeneous elastic medium follows from expression (4.1) of Aki and Richards (2002)

$$\rho \ddot{\mathbf{u}} = \left( \lambda + \mu \right) \nabla (\nabla \cdot \mathbf{u}) + \mu \nabla^2 \mathbf{u} - 2\rho \Omega \times \mathbf{u},$$

where $\rho$ is the mass density, $\lambda$ and $\mu$ the Lamé parameters, and the overdot denotes a time-derivative. We added the last term, which accounts for the Coriolis force. Note that we have not included the centrifugal force $-\rho \Omega \times (\Omega \times \mathbf{r})$ because this force gives a contribution $O(\Omega/\omega)^2$, which we ignore. We seek solutions of the form

$$\mathbf{u} = \mathbf{q} e^{i(k_n r - \omega t)},$$

where the unit vector $\hat{n}$ gives the direction of propagation and $\mathbf{q}$ the polarization. Inserting this in expression (A1) gives the Christoffel equation in a rotating system

$$\mathbf{q} = \frac{\lambda + \mu}{\rho c^2} \hat{n} (\mathbf{q} \cdot \hat{n}) + \frac{\mu}{\rho c^2} \mathbf{q} - \frac{2\Omega}{\omega} \times \mathbf{q},$$

where $c = \omega/k$ is the wave velocity.

We define a coordinate system to describe the polarization as shown in figure 2. We choose the $z$-axis in the direction of wave propagation ($\hat{z} = \hat{n}$), and define the unit vector $\hat{x}$ in the direction of $\hat{z} \times \hat{n}$. The last unit vector is defined by $\hat{y} = \hat{z} \times \hat{x}$, so that the system $\hat{x}, \hat{y}, \hat{z}$ is right-handed. The angle between the rotation vector and the direction of wave propagation is denoted by $\theta$. The vectors $\hat{x}$ and $\hat{y}$ are given by

$$\hat{x} = (\hat{z} \times \hat{n})/\sin \theta \quad \text{and} \quad \hat{y} = (\cos \theta \hat{z} - \hat{n})/\sin \theta.$$

We write the polarization vector as a superposition of the basis vectors:

$$\mathbf{q} = q_x \mathbf{\hat{x}} + q_y \mathbf{\hat{y}} + q_z \mathbf{\hat{z}}.$$

When using this expansion in the Christoffel equation (A3) one needs the cross product of the rotation vector with the basis vectors. If follows from expression (A4) that

$$\Omega \times \hat{x} = \Omega \sin \theta \hat{z} + \Omega \cos \theta \hat{y},$$

$$\Omega \times \hat{y} = -\Omega \cos \theta \hat{x},$$

$$\Omega \times \hat{z} = -\Omega \sin \theta \hat{\mathbf{x}}.$$
Inserting the expansion (A5) into the Christoffel equation (A3), using expressions (A6), and collecting the coefficients multiplying \( \mathbf{x}, \mathbf{y}, \) and \( \mathbf{z} \) gives

\[
\begin{align*}
\left( \frac{\mu}{\rho c^2} - 1 \right) q_x + \frac{2i\Omega}{\omega} (\sin \theta \, q_z + \cos \theta \, q_y) &= 0, \\
\left( \frac{\mu}{\rho c^2} - 1 \right) q_y - \frac{2i\Omega}{\omega} \cos \theta \, q_x &= 0, \\
\left( \frac{\lambda + 2\mu}{\rho c^2} - 1 \right) q_z - \frac{2i\Omega}{\omega} \sin \theta \, q_x &= 0.
\end{align*}
\]

(A7)


A1 The polarization of P-waves

Since the imprint of the rotation is assumed to be small \( (\Omega/\omega \ll 1) \) the P-waves have a polarization that is close to longitudinal. This means that \( q_z = 1 \) and that \( q_x \) and \( q_y \) are small. Since the polarization vector is a unit vector, it is to first order only perturbed in the transverse direction, therefore \( q_x \) is not perturbed. The velocity is close to the P-wave velocity in an unrotating medium, hence

\[
c_P = \sqrt{\frac{\lambda + 2\mu}{\rho} + \frac{\delta c_P}{c_P}},
\]

with \( \delta c_P \ll c_P \). Using a first order Taylor expansion

\[
\frac{1}{c_P} = \frac{\rho}{\lambda + 2\mu} \left( 1 - \frac{2\delta c_P}{c_P} \right),
\]

(A9)

Inserting this in expression (A7) gives

\[
\begin{align*}
\frac{\lambda + \mu}{\lambda + 2\mu} q_x - \frac{2i\Omega}{\omega} \cos \theta \, q_y &= \frac{2i\Omega}{\omega} \sin \theta, \\
\frac{2i\Omega}{\omega} \cos \theta \, q_x + \frac{\lambda + \mu}{\lambda + 2\mu} q_y &= 0, \\
\frac{2i\Omega}{\omega} \sin \theta \, q_x &= -2\frac{\delta c_P}{c_P}.
\end{align*}
\]

(A10)

Since \( q_x \) and \( q_y \) are small, we ignore the products \( (\Omega/\omega) q_x \) and \( (\Omega/\omega) q_y \) in the first two lines, which gives

\[
q_x = \frac{2i\Omega}{\omega} \frac{\lambda + 2\mu}{\lambda + \mu} \sin \theta, \quad q_y = 0.
\]

(A11)

With \( q_z = 1 \) this gives the polarization vector for the P-waves:

\[
\hat{q}_P = \hat{z} + \frac{2i\Omega \sin \theta \, \lambda + 2\mu}{\omega \, \lambda + \mu} \hat{x}.
\]

(A12)

Using the geometry of figure 2 this can also be written as

\[
\hat{q}_P = \hat{z} + \frac{2i \lambda + 2\mu}{\omega \, \lambda + \mu} \hat{z} \times \Omega.
\]

(A13)

The velocity follows by inserting \( q_z \) from expression (A11) into the first line of equation (A10)

\[
\frac{\delta c_P}{c_P} = \frac{2 \lambda + 2\mu}{\lambda + \mu} \left( \frac{\Omega \sin \theta}{\omega} \right)^2,
\]

which means that

\[
c_P = \sqrt{\frac{\lambda + 2\mu}{\rho} + O(\Omega/\omega)^2}.
\]

(A15)

The last expression simply states that the wave velocity of P-waves is to first order in \( \Omega/\omega \) not affected by Earth’s rotation. According to equation (A12) the polarization is not purely in the direction of propagation; the P-wave has a small transverse component. In acoustic media the polarization of P-waves is also affected by Earth’s rotation and setting \( \mu = 0 \) in expression (A13) gives a P-wave polarization \( \hat{q}_P = \hat{z} + (2i/\omega) \hat{z} \times \Omega \).

A2 The circular polarization of S-waves

For the S-waves the longitudinal polarization is small, so we use that \( q_x \) is small. The shear velocity is given by

\[
c_S = \sqrt{\frac{\mu}{\rho} + \delta c_S}.
\]

(A16)

Using a first order Taylor expansion in the perturbation \( \delta c_S \)

\[
\frac{1}{c_S^2} = \frac{\rho}{\mu} \left( 1 - 2\frac{\delta c_S}{c_S} \right).
\]

(A17)

Inserting this in expression (A7) and, ignoring cross terms \( (\delta c_S/c_S) q_x \) and \( (\Omega/\omega) q_x \), gives

\[
\begin{align*}
\frac{\delta c_S}{c_S} q_x &= \frac{\Omega}{\omega} \cos \theta \, q_y, \\
\frac{\delta c_S}{c_S} q_y &= -\frac{i\Omega}{\omega} \cos \theta \, q_x, \\
\frac{\lambda + \mu}{\mu} q_x &= \frac{2i\Omega}{\omega} \sin \theta \, q_x.
\end{align*}
\]

(A18)

Inserting the equation of the middle line into the first expression gives \( (\delta c_S/c_S)^2 = \left( (\Omega/\omega) \cos \theta \right)^2 \), or

\[
\frac{\delta c_S}{c_S} = \frac{\Omega}{\omega} \cos \theta.
\]

(A19)

For the + sign, equation (A18) predicts that \( q_y = -iq_x \), so the normalized polarization vector in the transverse plane in given by \( \hat{q}_S = \hat{x} - iy \). For the − sign in expression (A19), equation (A18) states that \( q_y = +iq_x \), hence the polarization in the transverse plane is given by \( \hat{q}_S = \hat{x} + iy \).

Both transverse polarizations are circular. The first line of expression (A18) states the the S-waves have a small longitudinal component that is for the used value \( q_z = 1 \) given by the last line of expression (A18)

\[
q_z = \frac{2i \Omega \sin \theta}{\omega} \frac{\mu}{\lambda + \mu}.
\]

(A20)

For the S-waves there are thus two solutions that are
both predominantly circularly polarized, the polarization and propagation velocity of the two shear wave solutions is given by
\[ q_{s+} = (\mathbf{x} - i\mathbf{y}) + q_s \hat{z} \quad \text{with} \quad c_s = \sqrt{\frac{\mu}{\rho}} + \delta c_s , \]
\[ q_{s-} = (\mathbf{x} + i\mathbf{y}) + q_s \hat{z} \quad \text{with} \quad c_s = \sqrt{\frac{\mu}{\rho}} - \delta c_s , \]
where the velocity shift \( \delta c_s \) is given by
\[ \frac{\delta c_s}{c_s^{(0)}} = \frac{\Omega}{\omega} \cos \theta , \] (A22)
with \( c_s^{(0)} = \sqrt{\mu/\rho} \). The terms \( q_s \hat{z} \) in equation (A21) denote that the S-waves have a slight elliptical polarization in the longitudinal direction that is akin to the slight elliptical polarization of the P-wave.

**APPENDIX B: ESTIMATION OF RAY BENDING**

To estimate the change in polarization of an ScS2 wave due to lateral velocity variations we use that the shear waves are polarized perpendicular to the rays. The means that when the rays are bent, the shear wave polarizations change accordingly, and we use the ray bending as a proxy for the change in shear wave polarization. We estimate this ray bending using ray perturbation theory. In doing so we use a crude model where the ScS2 ray is straight and propagates through a homogeneous velocity model that is perturbed with a weak velocity perturbation. According to equation (27) of Snieder and Sambridge (1992) the ray perturbation \( \mathbf{r}_1 \) is in this case given by
\[ \frac{d^2 \mathbf{r}_1}{ds^2} = \nabla U , \] (B1)
where \( s \) is the arc length and \( U \) is the relative slowness perturbation. For the purpose of this study we assume that the lateral ray bending is caused by horizontal velocity gradients in the upper mantle only. Taking a straight reference ray of length \( L = 12,000 \) km, we assume in the estimate a constant relative slowness gradient \( \nabla U \) for arc length \( L/2 - D < s < L/2 + D \), where \( D = 600 \) km is used for the depth of the upper mantle. This model only accounts for the ray bending associated with the propagation through the upper mantle near the free surface bounce point, but since we consider the change in polarization between ScS and ScS2 waves, we only need to consider the ray bending of ScS2 caused by the propagation in the upper mantle near the bounce point.

For this model the solution of equation (B1) is, for a ray with fixed endpoints \( (\mathbf{r}_1(s = 0) = \mathbf{r}_1(s = L) = 0) \), for a point beyond the slowness perturbation \( (s > L/2 + D) \) given by
\[ \mathbf{r}_1(s) = \frac{1}{L} \int_{L/2-D}^{L/2+D} (L - s') \nabla U ds' , \] (B2)
where we used the Green's function (17.43) of Snieder and van Wijk (2015). The ray deflection at the receiver follows by taking the derivative with respect to \( s \):
\[ \delta \varphi = \frac{d \mathbf{r}_1}{ds} (L) = \frac{1}{L} \int_{L/2-D}^{L/2+D} s' |\nabla U| ds' , \] (B3)
Using that the slowness gradient is assumed to be constant in the upper mantle
\[ \delta \varphi = D |\nabla U| , \] (B4)
Seismic shear waves as Foucault pendulum

Figure A2. Preprocessed earthquake responses for a time window encompassing ScS and ScS2. The data is bandpass filtered between 0.01 and 0.02 Hz. (a) and (b) show the radial (R) and transverse (T) component for the westernmost event (WE, see Fig. A1(b)). (c) and (d) show the same components for the easternmost event (EE). The red lines denote the ak135 traveltimes (Kennett et al., 1995) of, from early to late times, ScS, sScS, ScS2 and sScS2.

Note that this quantity does not depend on the ray length $L$. For $D = 600$ km and $|\nabla U| = 1000/200$ km, the ray deflection is given by $\delta \phi = 0.03 \approx 2^\circ$. Note that this ray deflection is of the same order of magnitude as the change in polarization between ScS2 and ScS due to Earth’s rotation.

APPENDIX C: DATA PROCESSING OF SCS AND SCS2 WAVES

C1 Used data

We use ScSn waves that have bounced $n$ times between the Core Mantle Boundary (CMB) and the Earth’s surface. ScS2 is a wave that reflects two times at the CMB before it was measured at the Earth’s surface. For an epicentral distance of 6 degrees, one trip back and forth to the CMB takes about 940 seconds. Hence, by detecting the polarization of ScS and of ScS2, phases with the same, but unknown initial polarization, we can detect the polarization alteration due to 940 seconds of propagation in a rotating Earth.

To make sure that the differences in polarization between ScS2 and ScS are not caused by differences in source radiation, we select stations nearby an earthquake (reducing the angles of incidence to a maximum of 3.5\(^\circ\)) and in a line that also intersects the epicenter (reducing the azimuth variation at the largest used epicentral distance to 2\(^\circ\)).

Japan both has large earthquakes and a dense network of seismic stations (Obara et al., 2005). We use tiltmeter recordings of the Hi-net, which is operated by the National Research Institute for Earth Science and Disaster Prevention (NIED). The tiltmeters can be used as seismometers, with a high sensitivity for seismic waves with periods from tens to at least hundreds of seconds (Tonegawa et al., 2006). The tiltmeters detect the tilt in two perpendicular horizontal directions. By use of the gravitational acceleration, the tilt is translated to a horizontal acceleration. After integrating once or twice, either the particle velocity or displacement is found.

Figure A1(a) shows 656 Hi-net tiltmeter stations that were active in 2005. The main tectonic units are identifiable with their imprint on bathymetry and topography. Figure A1(b) shows the locations of the used tiltmeters (green triangles) that are located near a line between two epicenters. At both ends of the line, there is a major (magnitude larger than 6) earthquake illuminating the structure below the line from opposite sides. The westernmost event (WE) is the Mw=6.2 Tane-gashima (Japan) earthquake, which occurred November 21, 2005, on a depth of 150 km. The easternmost event (EE) is the Mw=7.2 Miyagi earthquake (Japan), which occurred August 16, 2005, on a depth of 40 km. The line of stations enables array processing to mitigate interference of other phases than ScSn (see next section).

C2 Processing flow

As a preprocessing, we apply rotation, bandpass filtering, time windowing, and we remove erroneous traces. The two horizontal components are rotated to the transverse and radial component, where the radial component is defined to point away from the source. In the same step, a correction is applied for the tiltmeters not being perfectly oriented to the North and East (Shiomi, 2013). We bandpass filter the earthquake responses be-
Figure A3. Further processing to extract the polarization, exemplified with the ScS time window of the WE: (a) bandpass-filtered, time-windowed and amplitude corrected response, (b) after spatial interpolation applied, (c) after wavenumber bandpass filtering applied and (d) after aligning the radial and transverse response. (e) the waves that have been removed by wavenumber filtering. (f) estimated polarization of ScS after the different processing steps.
between 50 and 80 seconds. Filtering out the measurement at periods smaller than 50 seconds removes much of the complicated scattering in the upper mantle and near the CMB. We time-window the responses to a duration lasting from just before the onset of $ScS$ to after the recording of $ScS2$. As a last preprocessing step we remove the few traces with poor signal to noise ratio, probably caused by instrumental issues or local noise. Fig. A2 shows the resulting responses for the WE and EE, along with expected traveltimes of $ScS$ and $ScS2$, and their depth phases $sScS$ and $sScS2$. The filtered data are dominated by reflections of the CMB. Yet, there are also other phases, some of which may interfere with $ScSn$. These additional phases may affect the estimated polarization of $ScSn$.

We select from the bandpass filtered data (Fig. A2) a time window around $ScS$ and a time window around $ScS2$. The time windows are chosen from minus 60 to plus 120 seconds, with respect to the raytraced phase arrival time for the 1D Earth model ak135 (Kennett et al., 1995). Having two events and two phases results in data in 4 time windows. We exemplify the further processing for the $ScS$ time window of the WE. The processing for the other time windows is identical.

We apply an amplitude correction factor to the radial component. This factor corrects for the amplitude loss of the radial component that is not encountered on the transverse component. This correction does not include geometrical spreading, as this is the same for both components. The correction does include reflectivity losses for $SV$ waves at the free surface and the CMB assuming the Earth is 1D (ak135 model). As a result, the $SH$ waves and the amplitude-corrected $SV$ waves are both completely reflected at the free surface and the CMB. For the largest used epicentral distance (12°) and a two-fold reflection at the CMB, the correction factor for the $SV$ waves is about 7% of the total amplitude.

Fig. A3 shows the further steps in the data processing. Fig. A3(a) is the bandpass filtered $ScS$ recording of the WE, time windowed between -60 and 120 seconds with respect to the ak135 arrival time. To this recording the amplitude correction has already been applied. A long wavetrain starts with $ScS$ and then merges into $sScS$ for later times. The radial component (black traces) has the same polarity as the transverse component, the polarization is only somewhat altered for the larger distances. This last processing step is more important for $ScS2$, for which the components shows larger timing mismatches than for $ScS$.

We spatially interpolate the data using splines to a regularly sampled distribution of epicentral distances with a spacing of 0.1°, yielding Fig. A3(b). From the interpolated data, phases with steep move-outs are removed through wavenumber filtering. Fig. A3(c) shows the data after wavenumber filtering, whereas Fig. A3(e) shows the data removed with wavenumber filtering. We take care choosing the filter settings such that primarily steeply-dipping phases are removed and $ScS$ is not affected. The same filter settings are used for both earthquakes and both phases.

Small timing mismatches occur between the radial and transverse components, either through interference with scattering from nearly flat interfaces (which are not removed by the wavenumber filter) or by anisotropy. These timing mismatches are estimated by crosscorrelating the radial with the transverse component for a time window between -10 and 50 seconds. The found delay times are subsequently used to shift one of the components, yielding Fig. A3(d). The shifts applied are much smaller than the dominant period of $ScS$. Hence, visually, Fig. A3(d) is almost identical to Fig. A3(c).

Finally, the polarization angle is estimated with respect to the radial component (Fig. A4), using the data variance tensor (Aster et al., 1990). The polarization in the horizontal plane is measured clockwise from the radial direction and lies between 0 and 180 degrees. We use a 30 seconds time window which is centered 20 seconds after the ak135 raytraced arrival time of $ScS$. Fig. A3(f) shows the estimated polarizations after the different processing steps:

(i) Red dots: After merely bandpass-filtering, the polarization shows a large scatter from station to station, primarily caused by interference by other phases and local variations in structure. Besides, a small polarization scatter is caused by azimuthal variations from station to station, as the station are not perfectly inline.

(ii) Orange dots: After additionally applying amplitude correction, the polarization angle is slightly reduced. The amplitude correction, and hence the polarization angle reduction, increases with distance.

(iii) Green dots: After additionally applying wavenumber filtering, the polarization becomes much smoother as function of distance. Local perturbations, either caused by structure, by interference, or small azimuth variations, are largely removed by the wavenumber filter.

(iv) Purple dots: After additionally applying time-alignment of both components, the polarization is only somewhat altered for the larger distances. This last processing step is more important for $ScS2$, for which the components shows larger timing mismatches than for $ScS$.

Using identical processing as shown in Fig. A3, the data was prepared and the polarization was extracted for the other three time windows ($ScS2$ of the WE and $ScS$ and $ScS2$ of the EE). The polarization estimated after all the processing (like the purple dots in Fig. A3(f)) are used for estimating the Earth’s rotation. Fig. A4 shows the estimated polarizations as function of epicentral distance.
C3 Error propagation

In the main text, rotation-induced polarization is estimated as the difference of ScS and ScS2 polarization, averaged over the WE and EE. In this section, we estimate the error of measuring the polarization of ScS along the receiver line \( \phi_{\text{ScS}}(x) \) and evaluate how this error propagates into the estimation of rotation-induced polarization. The polarization of ScS along the receiver line is a function of:

- \( \phi_{\text{src}} \): the source polarization, which includes both the moment tensor and the structural perturbation near the source,
- \( \phi_{\text{struc}} \): the structural polarization perturbation (Section B) below the receiver array, and
- \( \phi_{\text{rot}} \): the rotation-induced polarization perturbation (see main text).

The take-off angles at the source vary little over the array, for the different ScS source-receiver paths. Also, the ScS travel times show little variation over the array. Thus, we assume \( \phi_{\text{src}} \) and \( \phi_{\text{rot}} \) to be constant. Hence, when removing the mean from \( \phi_{\text{ScS}}(x) \) a perturbation remains due to receiver structure and to additive noise in waveforms: \( \phi_{\text{struc}}(x) - \bar{\phi}_{\text{struc}} + \phi_{\text{noise}}(x) \), where \( \bar{\phi}_{\text{struc}} \) denotes the mean value over \( x \) and \( \phi_{\text{noise}} \) is a spurious term due to remaining noise. The de-meaned ScS polarization functions are shown in Fig. A5. For the WE, the ScS propagation is from west to east along the receiver line, as indicated with the yellow arrow. For the EE, the propagation direction is opposite. When following the directions of the ray as indicated by the arrows, it can be seen that where the polarization for the WE goes up, it goes down for EE, and vice versa. This observation confirms that the structural polarization perturbations are opposite for oppositely traveling waves.

We use the difference of the functions in Fig. A5 to estimate the error of measuring \( \phi_{\text{ScS}} \). This difference would be zero if the configuration were perfect, the equipment were ideal, the processing perfectly cancelled all the noise and introduced no errors by itself. The mean and standard deviation of the difference of the functions in Fig. A5 are 0° and 0.83°, respectively. Assuming the errors on recording \( \phi_{\text{ScS}}(x) \) from the WE and EE are independent, the standard deviation of the error of \( \phi_{\text{ScS}}(x) \) reads \( \sigma = 0.83°/\sqrt{2} = 0.59° \).

In the main text, we compute the S-wave polarization change by taking the difference between \( \phi_{\text{ScS}}(x) \) and \( \phi_{\text{ScS2}}(x) \). Assuming the errors on measuring \( \phi_{\text{ScS}} \) and \( \phi_{\text{ScS2}} \) to be identical and independent, the stan-
Seismic shear waves as Foucault pendulum

Figure A5. Similarity of ScS polarization along the receiver line (fig. A1b) between the West Event (yellow line) and the East Event (green line). The receiver distance is with respect to the westernmost receiver in the line. The arrows denote the direction of wave propagation along the line.

Standard deviation $\sigma$ of the error of detecting this difference equals $0.83^\circ$. In the main text, the rotation-induced polarization change (light-green line in figure 3c) is estimated by averaging $\varphi_{\text{ScS}}(x) - \varphi_{\text{ScS}}(x)$ over both events. This results in a mean value $\mu(x)$ that is estimated with an error $\sigma = 0.83^\circ / \sqrt{2} = 0.59^\circ$. 