Reverse-time-migration of multiply scattered seismic waves
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SUMMARY

Multiply scattered waves, including internal multiples, are rarely considered as signal when imaging with seismic waves. Reverse-time-migration techniques accurately handle the propagation of such complex scattered waves, but most of these techniques paradoxically do not fully utilize the illumination and energy of multiply scattered waves because these methods rely on the single-scattering assumption. We use a scattering-based approach for reverse-time-migration to overcome the limitations of linear imaging. Our method consists of adapting the conventional backpropagation techniques in order to more accurately reconstruct scattered wavefields and of using a nonlinear adaptive imaging condition in order to map the information contained in these newly extrapolated wavefields into an image of the subsurface. We illustrate these two concepts with a synthetic example of sub-salt imaging.

INTRODUCTION

Complex subsurface structures, such as sub-basalt and sub-salt structures, generate strong scattering which makes them challenging to image with conventional techniques. For such geologic environments, multiply scattered waves contain useful information about the subsurface but are traditionally removed, e.g., with multiple suppression (Foster and Mosher, 1992; ten Kroode, 2002) or surface related multiple elimination (Verschuur et al., 1992; Dragonset et al., 2010). Alternatively, multiply scattered waves are just ignored, e.g., with migration of primaries (Claerbout, 1985), least-square migration (Nemeth et al., 1999) or linearized inversion (Symes, 2008), when linearizing the relation between model and data. The motivations for using multiply scattered waves and their nonlinear relation to the model are preserving amplitudes, providing extra illumination, accounting for energy conservation, and increasing redundancy and sensitivity to model changes.

Advances have been made in using multiples to perform imaging (Brown and Guitton, 2005; Jiang et al., 2007; Malcolm et al., 2009; Verschuur and Berkhout, 2011). These methods apply redatuming techniques (Schuster et al., 2004; Malcolm and de Hoop, 2005; Berkhout and Verschuur, 2006) for extrapolating seismic data and reconstructing scattered wavefields with kinematically correct multiples, but they do not modify the imaging condition to account for the nonlinearity of the resulting wavefields with respect to the model. Like these methods, ours consists in taking into account multiply scattered waves in the extrapolation procedure. We take advantage of computationally affordable reverse-time migration (RTM) engines (Baysal et al., 1983; McMechan, 1989; Farmer et al., 2006) in order to handle complex scattering wave phenomena and adapt the backpropagation step to include sources of scattering in the migration velocity model which results in improving the reconstruction of scattered waves (Vasconcelos et al., 2010). Unlike the previous methods, ours also consists of using a nonlinear imaging condition (Fleury and Vasconcelos, 2010) which allows us to image both primaries and multiples. Our method fully exploits multiple-scattering interactions which improves illumination and amplitude preservation (C. Fleury and I. Vasconcelos, 2011, Geophysics, submitted).

First, we present our RTM method and show how it utilizes the illumination of multiply scattered waves. Then, we apply this method to a synthetic example of sub-salt imaging.

IMAGING WITH ILLUMINATION OF SCATTERED WAVES

Nonlinear imaging condition

The division of earth properties into two subdomains of short and long wavelengths justifies the use of a smooth model \( m_0 \) perturbed by a rough model \( \Delta m \) in seismic migration. We define our model as squared slowness \( (m = 1/c^2) \). The rough model \( \Delta m \) acts as a scattering source responsible for the scattered wavefield \( w_s \) superimposed on the reference wavefield \( w_0 \). An image can be defined as a map of local scattered wavefields at the time of incidence (Claerbout, 1971). We use this notion to define our imaging condition. Rickett and Sava (2002) and Sava and Fomel (2006) discuss the use of extended images which also broadens the scope of such a scattering-based imaging principle (Vasconcelos et al., 2010).

Traditionally, the model \( \Delta m \) is viewed as a small perturbation of the reference model \( m_0 \). This justifies a first-order Born approximation for imaging (Claerbout, 1985; Tarantola, 1986; Esmersoy and Oristaglio, 1988) which gives a linear relation between model and data but neglects multiples, refractions, and other higher-order scattering phenomena. The single-scattering image \( i_0 \) is defined as

\[
i_0 = \sum_{\text{sources}} w_0 \otimes w_s,
\]

where \( \otimes \) denotes a zero-time crosscorrelation.

To go beyond the Born approximation, Fleury and Vasconcelos (2010) add the image \( i_s \) of the autocorrelation of wavefield \( w_s \),

\[
i_s = \frac{1}{2} \sum_{\text{sources}} w_s \otimes w_s,
\]

to the image \( i_0 \) in order to define the nonlinear image \( i \),

\[
i = i_0 + i_s,
\]

that is, an image that maps multiply scattered waves and takes the nonlinear relation between model and data into account. The image \( i_s \) corrects the image \( i_0 \) for the interaction between scattered waves and therefore provides increased illumination with multiples. The image \( i_s \) also maps the energy of multiples into the image, and as a result, the nonlinear imaging condition (3) conserves scattering energy and improves amplitude.
Nonlinear scattering-based imaging of multiples

The limited source aperture makes the illumination of the subsurface uneven. The finite receiver aperture limits the ability to extrapolate the scattered wavefield \( w_s \) with exact amplitudes. As a result, the amplitudes of the images \( i_0 \) and \( i_1 \) are not consistent throughout the entire imaging domain. The addition of these two images needs to be adaptive in practice in order to combine the coherent structures present in both images. We are currently developing an adaptive method for summing the images \( i_0 \) and \( i_1 \) while maximizing focus and coherency of the image \( i \).

Wavefield extrapolation

The reference wavefield \( w_0 \) is obtained by propagating the estimated source signature \( s \) in the migration model \( m_0 \),

\[
H_0 \cdot w_0 = s, \tag{4}
\]

where \( H_0 = \Delta + \omega^2 m_0 \) is the Helmholtz operator. The scattered wavefield \( w_s \) is conventionally obtained in RTM by backpropagating the recorded data \( d_i \) in the same model \( m_0 \),

\[
H_0^\dagger w_s = d_s, \tag{5}
\]

where \( H_0^\dagger (= H_0) \) is formally the adjoint Helmholtz operator. In our method, we extrapolate the wavefield \( w_s \) by using

\[
[H_0^\dagger - P^\dagger] \cdot w_s = P^\dagger \cdot w_0 + d_s, \tag{6}
\]

where \( P = -\omega^2 \Delta m \) is the perturbation operator responsible for scattering. More precisely, we extrapolate the wavefield \( w_s \) with an estimate of equation (6),

\[
[H_0^\dagger - P^\dagger_{est}] \cdot w_s = P^\dagger_{est} \cdot w_0 + d_s, \tag{7}
\]

because the model \( \Delta m \) and perturbation \( P \) are ultimately both unknowns of the imaging problem. The backpropagation of the data \( d_i \), in a perturbed model estimate \( m_{est} (= m_0 + \Delta m_{est}) \) with secondary scattering sources (term \( P_{est} \cdot w_0 \) distributed in the entire imaging volume, more accurately reconstructs scattered waves as part of the wavefield \( w_s \).

To find the perturbation estimate \( P_{est} \), we first consider the fully nonlinear relation between the wavefield \( w_s \) (or, equivalently, the data \( d_i = w_s \) at receivers) and the model perturbation \( \Delta m \). The Born expansion is

\[
w_s = w_0 \cdot P \cdot w_0 + w_0 \cdot P \cdot w_0 \cdot P \cdot w_0 + ... \tag{8}
\]

Additionally, we expand \( P \) as a series in order of \( d_i \),

\[
P = P_1 + P_2 + P_3 + ... \tag{9}
\]

Using the inverse scattering series (Weglein et al., 2003), we can iteratively construct the perturbation estimate \( P_{est} \) by solving a sequence of inverse problems. In practice, we limit ourselves to the first term of the inverse scattering series, i.e., \( P_{est} = P_1 \) and solve the linear inverse problem for \( \Delta m_{est} \):

\[
d_i = w_0 \cdot P_{est} \cdot w_0 = -\omega^2 w_0 \cdot \Delta m_{est} \cdot w_0. \tag{10}
\]

One would solve the same linearized equation (10) if one made the Born approximation. Here, however, our approximation concerns the model estimate and not the data. To find the solution \( \Delta m_{est} \), we implement the method of Symes (2008) that consists of optimal scaling of the migrated image \( i_0 \).

The model \( \Delta m_{est} \) contains high spatial frequency features and introduces scattering (e.g., reflectors and diffractors) in the backpropagation. Applying the imaging condition (3) with a simple crosscorrelation of the wavefields therefore introduces backpropagation artifacts in the images \( i_0 \) and \( i_1 \). Understanding and filtering out these artifacts is part of the imaging process (Youn and Zhou, 2001; Guittion et al., 2007). To reduce them, we modify our imaging condition by only crosscorrelating the up and down components of the wavefields (Liu et al., 2011).

**SUB-SALT IMAGING WITH INTERNAL MULTIPLES**

One application of our method involves imaging complex substructures with the illumination of internal multiples. We illustrate its benefits with a synthetic example of sub-salt imaging (Figure 1).

First, we use the conventional RTM backpropagation of equation (5) to extrapolate the scattered wavefield \( w_s \). Figure 2 shows both the conventional RTM image \( i_0 \) and the image \( i_1 \) of
Nonlinear scattering-based imaging of multiples

Figure 2: Images by using equation (5) for extrapolating $w_s$.

the autocorrelation of the wavefield $w_s$. The image $i_0$ (Figure 2(a)) reconstructs a relatively accurate single-scattering image of the subsurface. The image $i_s$ (Figure 2(b)) reconstructs top-salt reflectors by only using the interaction between scattered waves. We do not observe sub-salt reflectors because the extrapolation of the scattered wavefield $w_s$ is only accurate for internal multiples between the water bottom and the salt body. Improving the wavefield extrapolation is necessary to fully exploit the benefit of the nonlinear imaging condition (3).

As a proof of concept, we repeat the imaging procedure while extrapolating the wavefield $w_s$ using equation (6). We use the exact perturbation operator $P$ to generate sources of scattering in the backpropagation of the data $d_s$. This allows for more accurately reconstructing multiples with their correct kinematics both above and below the salt body. Figure 3 shows the corresponding images $i_0$, $i_s$, and $i$. Figure 4 presents the sub-salt portion of these images (indicated by the green box) which is clipped to emphasize their features. Interestingly, both the images $i_0$ (Figures 3(a) and 4(a)) and $i_s$ (Figures 3(b) and 4(b)) contain coherent information about the subsurface above and below the salt body. We combine this complementary information into the nonlinear image $i$ (Figures 3(c) and 4(d)). We apply a constant weight to the images $i_0$ and $i_s$ before their summation. The image $i_s$ maps the energy of interacting multiples. This results in improving the preservation of scattering amplitudes in the image $i$. The wider angular range of illumination with multiply scattered waves is expected to increase the spatial focus of the wavefields. Figure 4 shows a better resolution of faults and scatterers (highlighted in green) in the nonlinear image $i$. As mentioned earlier, further work is needed to best adaptively exploit the complementarity of the images $i_0$ and $i_s$.

Last, we perform a more realistic extrapolation of the scattered wavefield $w_s$ by using the first-order estimate $P_{est}$ of the perturbation operator. Scaling the conventional image 2(a) gives the model $\Delta m_{est}$. Adding the reference model $m_0$ produces the

Figure 3: Images by using equation (6) for extrapolating $w_s$. 

Figure 4: Sub-salt portion of the images in Figure 3.
Nonlinear scattering-based imaging of multiples

model \( m_{est} \) (Figure 5) which is used for backpropagating the data with equation (7). Figure 6 presents the resulting images \( i_0 \) and \( i_s \). Figure 7 shows the sub-salt portion of these images (indicated by the green box) after clipping. Once again, the images \( i_0 \) (Figures 6(a) and 7(a)) and \( i_s \) (Figures 6(b) and 7(b)) contain coherent and complementary information about the subsurface. This example shows that in addition to primaries, internal multiples can produce an image of the subsurface. More precisely, the crosscorrelation of primaries with multiples and multiples with multiples, present in the scattered wavefields, can produce an image of the subsurface. The adaptive addition of the images \( i_0 \) and \( i_s \) is the next step to perform, which we will optimize for future applications.

CONCLUSION

By adapting the RTM imaging procedure, our method includes the nonlinear scattering interactions of multiples. We more accurately reconstruct internal multiples and other high-order scattering phenomena by modifying the backpropagation. We map the energy of these multiply scattered waves into the nonlinear image by using a nonlinear imaging condition. Multiples provide additional illumination of the subsurface and improve the preservation of amplitudes. This particularly benefits imaging of complex subsurface structures as seen in our sub-salt imaging experiment. Our method forms two images of the subsurface, one that maps the interaction between reference and scattered wavefields and the other that maps the interaction between scattered wavefields. Both images must be adaptively added in order to create one final nonlinear image. This last step, subject to ongoing research, is crucial for achieving practical value for our nonlinear imaging procedure.

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Nonlinear scattering-based imaging of multiples

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