Introduction

Reverse-time migration (RTM) is now a standard imaging technique in the industry. The method provides high-quality images for oil and gas exploration and is of special interest for complex subsurface geologies. Yet, conventional RTM migration does not resolve the problem of imaging waves multiply scattered by complex subsurface structures. This limitation comes in part from the underlying single-scattering assumption of most migration techniques (Fleury and Vasconcelos, 2012). To utilize the energy, illumination and sensitivity contained in multiply scattered data, we propose a nonlinear RTM migration algorithm (Fleury and Snieder, 2011). In this paper, we improve this technique and propose a new strategy for using multiply scattered waves in interpretation and migration velocity analysis.

Theory of nonlinear reverse-time migration

The division of Earth properties into subdomains of short and long wavelengths justifies the use of smooth model $m_0$ perturbed by rough model $\Delta m$ in seismic migration ($m = (\rho, c)$ in the acoustic assumption). Model $\Delta m$ acts as a scattering source for scattered wavefield $w_s$ superimposed on reference wavefield $w_0$. The relation between wavefield $w_s$ (equal to seismic data $d_s$ at receivers) and perturbation $P$ of the Helmholtz operator $H_0$ (function of models $m_0$ and $\Delta m$) is nonlinear:

$$w_s = w_0 \cdot P \cdot w_0 + w_0 \cdot P \cdot w_0 \cdot P \cdot w_0 + ...$$

(1)

To preserve this nonlinear relation between wavefield and perturbation, we modify the conventional RTM extrapolation technique. We introduce both source- and receiver-extrapolated scattered wavefields ($w_{s,sou}$ and $w_{s,rec}$, respectively) and assume to know an estimate of source signature $s$. The modified right-hand side equations (5-7) replace the conventional left-hand side equations (2-4).

$$H_0 \cdot w_0 = s$$

(2)

$$H_0 \cdot w_0 = s$$

(5)

$$H_0^* \cdot w_{s,rec} = d_s$$

(3)

$$(H_0^* - P^*) \cdot w_{s,rec} = d_s + P^* \cdot w_0$$

(6)

$$w_{s,sou} = 0$$

(4)

$$(H_0^* - P^*) \cdot w_{s,sou} = P^* \cdot w_0$$

(7)

Symbol $^\dagger$ denotes the adjoint operation. Operator $P$ being an unknown of the imaging problem, we use an estimate referred to as $P_{est}$ and obtained from a conventional RTM image by either picking reflectors or directly scaling the image in order to build a scattering model ($\Delta m = (\Delta \rho, \Delta c)$). Also, $P_{est}$ can be derived when integrating our nonlinear RTM method with full waveform inversion technology.

With the newly extrapolated reference and scattered wavefields, we define image $i$ as

$$i = \sum_{souces} w_0 \ast w_{s,rec} + \frac{1}{2} \sum_{sources} w_{s,sou} \ast w_{s,rec},$$

(8)

where $\ast$ denotes zero-time crosscorrelation or deconvolution. The imaging condition maps into the subsurface the interaction of reference-extrapolated wavefields $w_0$ and $w_{s,sou}$ with receiver-extrapolated wavefield $w_{s,rec}$. This imaging condition is nonlinear with respect to the perturbation (Fleury and Vasconcelos, 2012) while preserving a linear relation between image and data. We can either choose a crosscorrelation or deconvolution imaging condition. The former relates to an energy-based representation of the subsurface while the latter is reflectivity-based. We use a deconvolution imaging condition following the recommendations of Schleicher et al. (2008). Image $i_0$ maps the interaction of reference and scattered wavefields and is similar to conventional imaging conditions. Image $i_s$ maps the interaction of only scattered wavefields which corresponds to higher orders of scattering. It has no linear contribution to the image and is therefore negligible under the linear single-scattering assumption. To highlight structural contrasts in the images, we modify $i_0$ and $i_s$ to only consider the interactions of up- and down-going wavefields, which we denote with superscripts $(u)$ and $(d)$, respectively. This results in four sub-images $i^{(u)}_0$, $i^{(d)}_0$, $i^{(u)}_s$, and $i^{(d)}_s$ in equations 9 and 10 which are key for our analysis.
Increasing illumination and sensitivity of reverse-time migration with internal multiples

Strategies for nonlinear reverse-time migration

Multiply scattered waves are waves that have been reflected, diffracted, and more generally scattered more than once. They transport energy that contains information about the subsurface but that is not traditionally mapped into conventional RTM images. Multiply scattered waves both illuminate the subsurface with better coverage than singly scattered waves and are generally more sensitive to the Earth model. Sub-images \( i_{(u,d)}^{(a)} \) and \( i_{(u,d)}^{(a)} \) emphasize single scattering events while sub-images \( i_{(u,d)}^{(a)} \) and \( i_{(u,d)}^{(a)} \) emphasize multiple scattering events. Interestingly, all of these sub-images are representations of the same subsurface structure and must consequently share common structural features. A comparative study of these sub-images therefore provides valuable information about the subsurface.

\[
\begin{align*}
\mathbf{i}_0 &= \sum_{\text{sources}} \mathbf{w}_0^{(d)} \ast \mathbf{w}_{s,rec}^{(u)} + \sum_{\text{sources}} \mathbf{w}_0^{(u)} \ast \mathbf{w}_{s,rec}^{(d)} \quad (9) \\
\mathbf{i}_k &= \sum_{\text{sources}} \mathbf{w}_s^{(d)} \ast \mathbf{w}_{s,rec}^{(u)} + \sum_{\text{sources}} \mathbf{w}_s^{(u)} \ast \mathbf{w}_{s,rec}^{(d)} \quad (10)
\end{align*}
\]

Figure 1 Modeling data for experiments (a) one-scatterer velocity model, (b) lense and two-scatterer velocity model, and (c) low-anomaly and one-scatterer velocity model. Density model (d) is the same for all experiments. The red dots and green line indicate the fix-spread source/receiver geometry.

The three synthetic experiments in Figure 1 illustrate this concept. We migrate these data with the same reference model corresponding to background velocity gradient \( c_0 \) in Figure 1(a) and smooth density \( \rho_0 \) in Figure 1(d). Despite small velocity anomalies, conventional RTM images provide a clear image of the density reflector in the three cases. Interpreting this reflector and introducing it into a density contrast model leads to perturbation operator \( \mathbf{P}_{est} \), used in the modified extrapolation procedure. Sub-images \( i_0^{(a,d)} \) and \( i_0^{(u,d)} \) do not contribute to the image of the scatterers because wavefields \( w_0^{(u)} \) and \( w_{s,rec}^{(d)} \) do not illuminate the scatterer. We ignore them in the analysis of the results. Figure 2 shows that with the correct velocity model (experiment (a)), sub-images \( i_0^{(a,d)} \) and \( i_0^{(u,d)} \) focus scattered energy at the correct location (blue dot). In sub-image \( i_0^{(a,u)} \), reference waves illuminate the scatterer from above. In sub-image \( i_0^{(u,d)} \), scattered waves illuminate the scatterer from below. Adding the two contributions improve the spatial resolution of the scatterer. In experiment (b) (Figure 3), the high-velocity lens causes the two scatterers to be focused at the wrong locations in depth in sub-image \( i_0^{(a,u)} \). Sub-image \( i_0^{(a,d)} \) utilizes different illumination and maps the scatterers at their correct locations (blue dots). Sub-image \( i_0^{(a,d)} \) does not
show sensitivity to the velocity lens because the lens does not principally affect waves multiply scattered by the object "reflector-scatterer-reflector" in that order. Experiment (c) shows similar conclusions in Figure 4. Sub-image $i_0^{(d,u)}$ maps the scatterer at the correct location (blue dot) while sub-image $i_s^{(u,d)}$ shows defocus of the multiply scattered energy. Scattered wavefields $w_{s,sou}$ and $w_{s,rec}$ that reflect at the density contrast before or after interacting with the scatterer are sensitive to the low-velocity anomaly. Wavefields $w_0$ and $w_{s,rec}$ that directly interact at the scatterer are not. Experiment (a) illustrates how to utilize illumination and energy of multiply scattered waves to provide extra information about the subsurface. Experiments (b) and (c) demonstrate how to utilize sensitivity of multiply scattered waves to cross-validate the accuracy of a given migration velocity model.

**Figure 2** Point scatterer sub-images for experiment (a): adding sub-images (a) and (b) provides extra-illumination with multiply scattered energy and results in increased spatial resolution in sub-image (c).

**Figure 3** Point scatterer sub-images for experiment (b): contrary to image (a), image (b) focuses the scatterers at their correct locations and shows no sensitivity to the velocity lens.

**Figure 4** Point scatterer sub-images for experiment (c): contrary to image (a), image (b) is sensitive to the low-velocity anomaly and shows defocus of the scatterer.

A practical application for our nonlinear RTM method is target-oriented sub-salt imaging. Sub-salt imaging is challenging because of the structural complexity of salt bodies and because of the general lack of illumination below salt. We consider the Sigsbee 2A data (Paffenholz et al., 2002) and migrate with the velocity models in Figure 5. After conventional RTM, we select a poorly illuminated area (green box in Figure 5) and apply our nonlinear RTM algorithm. Our goal is to improve the image quality and resolve ambiguities in this targeted poorly illuminated sub-salt area. Figure 6 shows some of the resulting images. Despite different wavelets (no Laplacian filtering), sub-image $i_0^{(d,u)}$ exhibits similar structure and amplitude to the conventional image. The illumination is relatively poor apart from the left
Figure 5 Migrating Sigsbee data: (a) reference velocity model and (b) velocity model associated to operator $P_{est}$. Scaling a conventional RTM image provides the contrast $\Delta c_{est}$ used to define the estimate of operator $P$. The red and purple lines indicate the constant-offset source/receiver geometry.

Figure 6 Sigsbee images in poorly illuminated sub-salt area: (a) conventional RTM image after Laplacian filtering compared to (b) sub-image $i_{b}^{(d,a)}$ and (c) sub-image $i_{s}^{(u,d)}$.

side of the image (between 10 and 11 km offset). Sub-image $i_{s}^{(u,d)}$ maps the interaction of multiply scattered waves. This results in better sub-salt illumination. The coherent structures in sub-image $i_{s}^{(u,d)}$ are similar to the ones present in sub-image $i_{b}^{(d,a)}$. Migration velocity model $c_0$ (Figure 5(a)) is therefore quite accurate. Sub-image $i_{s}^{(u,d)}$ also shows better coherence and reveals additional features in the image. For example, sediment layering is more visible especially across the two faults. The nonlinear RTM method helps for interpreting the sub-salt image.

Conclusions

Our strategy for nonlinear RTM imaging outlines the potentials of utilizing energy, illumination, and sensitivity of multiply scattered waves in seismic imaging. The comparative study of nonlinear sub-images shows as a tool for both interpretation and possibly model sensitivity analysis.

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References