Joint migration velocity analysis of PP- and PS-waves for VTI media

Pengfei Cai* and Ilya Tsvankin, Center for Wave Phenomena, Colorado School of Mines

SUMMARY
Combining PP-waves with mode-converted PS-waves in migration velocity analysis (MVA) can help build more accurate VTI (transversely isotropic with a vertical symmetry axis) velocity models. To take advantage of efficient MVA algorithms designed for pure modes, here we generate pure SS-reflections from PP and PS data using the PP+PS=SS method. Then the residual moveout in both PP and SS common-image gathers is minimized during iterative velocity updates. The model is divided into square cells, with the P- and S-wave vertical velocities (\(V_{p0}\) and \(V_{s0}\)) and the anisotropy parameters \(\varepsilon\) and \(\delta\) defined at each grid point. The objective function also includes the differences between the migrated depths of the same reflectors on the PP and SS sections. The replacement of PS-waves with pure SS reflections in MVA allows us to avoid problems caused by the moveout asymmetry and other undesirable features of mode conversions. Synthetic examples confirm that 2D MVA of PP- and PS-waves can resolve all four relevant parameters of VTI media if reflectors with at least two distinct dips are available. After the velocity model has been reconstructed, accurate depth images can be obtained by migrating the recorded PP and PS data.

INTRODUCTION
To resolve the velocity \(V_{p0}\) and anisotropy parameters \(\varepsilon\) and \(\delta\) required for P-wave depth imaging, it is necessary to combine P-wave traveltimes with additional information. Tsankin and Thomsen (1995) demonstrate that long-spread (nonhyperbolic) P- and SV-wave moveouts are sufficient for estimating the parameters \(V_{p0}, V_{s0}, \varepsilon\), and \(\delta\). However, it is more practical to supplement P-waves with converted PS (PSV) data. Several authors discuss joint tomographic inversion of PP- and PS-waves (Stopin and Ehinger, 2001; Audebert et al., 1999; Broto et al., 2003; Foss et al., 2005). However, velocity analysis of mode conversions is hampered by the asymmetry of PS moveout. Therefore, MVA for PS-waves (Du et al., 2012; Foss et al., 2005; Audebert et al., 1999) has to account for the “diodic” nature of PS reflections (Thomsen, 1999). For example, common-image gathers (CIGs) of PS-waves can be computed separately for positive and negative offsets (Foss et al., 2005). To replace mode conversions in velocity analysis with pure SS reflections, Grechka and Tsvankin (2002) suggest the so-called PP+PS=SS method. By combining PP and PS events that share P-legs, that method generates SS reflection data with the correct kinematics. Grechka et al. (2002b) perform inversion of multicomponent (PP and PS) data using stacking-velocity tomography, which operates with NMO velocities for 2D lines and NMO ellipses for wide-azimuth 3D surveys (Grechka et al., 2002a). They compute SS traveltimes with the PP+PS=SS method and then apply stacking-velocity tomography to the PP and SS moveouts.

Figure 1: Matching the horizontal slownesses on common-receiver PP and PS gathers at locations \(x_1\) and \(x_2\) helps find the source-receiver coordinates \(x_3\) and \(x_4\) of the pure SS ray \(x_3Rx_4\). This constructed SS ray has the same reflection point \(R\) as the PP ray \(x_1Rx_2\) and PS rays \(x_1Rx_3\) and \(x_2Rx_4\) (Grechka and Tsvankin, 2002).

To make use of efficient MVA techniques developed for pure modes (Sarkar and Tsvankin, 2004; Wang and Tsvankin, 2011), here we apply the PP+PS=SS method to construct pure SS-wave reflections from PP and PS data. The MVA is performed by minimizing residual moveout of reflection events in both PP- and SS-wave CIGs. PP and SS images of the same reflector do not match in depth if the velocity model is incorrect. Therefore, in addition to flattening image gathers, we penalize depth misties between PP and SS sections.

METHODOLOGY
To build a VTI model for depth migration using P-wave reflection moveout, at least one medium parameter (e.g., \(V_{p0}\)) must be known a priori (Sarkar and Tsvankin, 2004). As discussed in Tsvankin and Grechka (2000), combining P-wave traveltimes with the moveout of PS-waves converted at a horizontal and dipping interface can help constrain the velocities \(V_{p0}\) and \(V_{s0}\) and the parameters \(\varepsilon\) and \(\delta\). The most significant problem in PS-wave velocity analysis is the moveout asymmetry with respect to zero offset in common-midpoint (CMP) geometry (Tsvankin and Grechka, 2011). Therefore, MVA designed for pure modes cannot be directly applied to converted waves. Here, we employ the PP+PS=SS method (Grechka and Tsvankin, 2002) to produce pure SS reflection events from PP and PS reflections.

Implementation of the PP+PS=SS method requires event registration, or identification of PP and PS events from the same interfaces. The main idea of the method is to combine PP and PS reflections that share the same P-legs. This is done by matching time slopes (horizontal slownesses) on common-receiver gathers of PP- and PS-waves (Figure 1). Then the traveltime of the constructed SS wave (sometimes called the “pseudo-S”
Figure 2: Critical (maximum) offset of the constructed SS-waves in a horizontal layer (Tsvankin and Grechka, 2011). arrival) is found from:

\[ t_{SS}(x_3, x_4) = t_{PS}(x_1, x_3) + t_{PS}(x_2, x_4) - t_{PP}(x_1, x_2). \]  

(1)

While the generated SS-wave traveltimes are exact, the method cannot produce correct reflection amplitudes. Hence, we convolve the SS traveltimes with a Ricker wavelet to generate “pseudo” SS reflection data to be used for MVA. The maximum reflection angle of the shear wave generated by mode conversion in a horizontal isotropic layer with the P- and S-wave velocities \( V_P \) and \( V_S \) is \( \theta_{crit} = \sin^{-1}(V_S/V_P) \), and the half-offset \( h_S \) cannot exceed the critical value,

\[ h_{S}^{crit} = D \tan \left[ \sin^{-1} \left( \frac{V_S}{V_P} \right) \right]. \]  

(2)

D is the layer’s thickness. For example, for a typical velocity ratio \( V_S/V_P = 1/2 \), the maximum half-offset is less than 0.6D. Therefore, it is necessary to include long-offset PP and PS data to generate SS-waves suitable for robust velocity analysis. The offset range for the computed SS-waves is more narrow than that for the acquired PP and PS data but may be sufficient for MVA if the survey includes offsets reaching two target depths.

To perform velocity analysis of PP- and SS-waves, we extend the MVA algorithm of Wang and Tsvankin (2011) to multi-component data. The model is divided into source cells, and the parameters \( V_{P0}, V_{S0}, \epsilon, \) and \( \delta \) are defined at each grid point. We apply prestack Kirchhoff depth migration to both PP and SS data, typically starting with an initial isotropic velocity model. The moveouts of migrated PP and SS events in common-image gathers serve as input to the joint MVA. To constrain the anellipticity coefficient \( \eta \) (and, therefore, \( \epsilon \)), the moveout in PP-wave CIGs is described by the nonhyperbolic equation (Sarkar and Tsvankin, 2004):

\[ z^2(h) = z^2(0) + Ah^2 + B \frac{h^4}{h^2 + z^2(0)}, \]  

(3)

where \( z \) is the migrated depth as a function of the half-offset \( h \), and the coefficients A and B are found by a 2D semblance scan. There is no need to apply equation 3 to SS-wave CIGs because the offset-to-depth ratio for the constructed SS events seldom exceeds 1-1.2. For the joint MVA, we not only minimize the residual moveout in PP and SS CIGs, but also perform codephasing, which involves tying PP and SS images of the same reflectors. The objective function includes a term that penalizes the mismatch in depth of migrated PP and SS images using a selection of key reflection points. Those points are chosen on the basis of coherency and focusing (Foss et al., 2005).

To carry out velocity update, it is necessary to compute traveltime derivatives with respect to the model parameters (Wang and Tsvankin, 2011). The exact P- and SV-wave velocities in VTI media can be expressed as (Tsvankin, 2005):

\[ \frac{V^2_{P}}{V_{P0}^2} = 1 + \epsilon \sin^2 \theta - \frac{f}{2} \pm f \sqrt{\left(1 + \frac{2\epsilon \sin^2 \theta}{f}\right)^2 - \frac{2(\epsilon - \delta) \sin^2 2\theta}{f}}, \]  

(4)

where \( f = 1 - V_{S0}^2/V_{P0}^2 \) and \( \theta \) is the phase angle with the symmetry axis. The plus in front of the radical corresponds to P-waves and minus to SV-waves. For purposes of MVA, it is convenient to replace \( \epsilon \) and \( \delta \) with the P-wave horizontal \( (V_{horP}) \) and NMO \( (V_{nmoP}) \) velocities given by \( V_{horP} = V_{P0} \sqrt{T + 2\epsilon} \) and \( V_{nmoP} = V_{P0} \sqrt{T + 2\delta} \). Therefore, we compute the traveltime derivatives with respect to \( V_{horP} \) and \( V_{nmoP} \) instead of \( \epsilon \) and \( \delta \). The MVA algorithm updates \( V_{P0}, V_{S0}, V_{horP}, \) and \( V_{nmoP} \) and then converts \( V_{horP} \) and \( V_{nmoP} \) into \( \epsilon \) and \( \delta \).

Following Sarkar and Tsvankin (2004), the variance \( \text{Var} \) of the migrated depths can be written as

\[ \text{Var} = \sum_{j=1}^{U} \sum_{k=1}^{M} \left[ z(x_j, h_k) - \hat{z}(x_j) \right]^2. \]  

(5)

where \( U \) is the number of image gathers used in the velocity update, \( M \) is the number of offsets in the image gathers, \( x_j \) is the midpoint, \( h_k \) is the half-offset, and the average migrated depth at \( x_j \) is \( \hat{z}(x_j) = \frac{1}{M} \sum_{k=1}^{M} z(x_j, h_k) \). By minimizing the variances \( \text{Var}_{P} \) (for P-waves) and \( \text{Var}_{S} \) (for S-waves), we flatten CIGs for both modes. The difference between the migrated depths of PP- and SS-waves from the same reflector can be estimated as:

\[ \text{Var}_{P-S} = \sum_{j=1}^{U} \left[ z_P(x_j) - \hat{z}_P(x_j) \right]^2. \]  

(6)

Minimizing the variance \( \text{Var}_{P-S} \) makes it possible to tie the PP and SS sections in depth.

The objective function is defined as follows:

\[ F(\Delta \lambda) = \mu_1 ||A_P \Delta \lambda + b_P||^2 + \mu_2 ||A_S \Delta \lambda + b_S||^2 + \mu_3 ||D \Delta \lambda + y||^2 + \xi ||L \Delta \lambda||^2, \]  

(7)

where \( A_P \) and \( A_S \) depend on the derivatives of the PP and SS migrated depths with respect to the medium parameters, the vectors \( b_P \) and \( b_S \) contain elements that characterize the residual moveout in PP- and SS-wave CIGs, the matrix \( D \) describes the differences between the derivatives of the PP and SS migrated depths with respect to the medium parameters, and the vector \( y \) contains the differences between the migrated depths.
Figure 3: VTI model with dipping interfaces. The parameters of the top isotropic layer are $V_P = 2000$ m/s and $V_S = 1000$ m/s; for the second layer, $V_{P0} = 3000$ m/s, $V_{S0} = 1500$ m/s, $\varepsilon = 0.2$ and $\delta = 0.1$. The maximum dip of both reflectors is 27$^\circ$.

SYNTHETIC TEST

We use anisotropic ray-tracing package ANRAY to compute PP- and PS-wave reflection traveltimes. PP and SS images are generated with Kirchhoff prestack depth migration (Seismic Unix program ‘sukdmig2d’). To create traveltime tables for migration of PP- and SS-waves, we perform ray tracing using SU code ‘rayt2dan’.

The algorithm is tested on a model (Figure 3) that includes a reflector (e.g., a fault plane) dipping at an angle approaching 30$^\circ$ near the surface. To avoid instability in ray tracing, we smooth the corner of the dipping interface using bicubic spline interpolation. The synthetic data include PP and PS reflections from the top and bottom of the VTI layer (Figure 4). The maximum offset-to-depth ratio for the bottom reflector is close to two, which is sufficient for applying the PP+PS=SS method. Indeed, the maximum offset for recorded PP data is 4 km and the maximum offset for constructed SS data is 2 km. The whole PP data set is used along with the SS-waves to estimate the residual moveout in CIGs. For codephasing, however, we only use conventional-spread PP data with offsets not exceeding those for SS-waves. Tsvankin and Grechka (2000, 2011) demonstrate that the traveltimes of the PP- and PS-waves reflected from a horizontal and a moderately dipping interface are sufficient to constrain the parameters $V_{P0}$, $V_{S0}$, $\varepsilon$, and $\delta$.

Here we invert only for the parameters of the middle layer. We assume that layer to be weakly heterogeneous and apply strong regularization (the operator $L$ in equation 7). The ini-
A set of CIGs of PP- and SS-waves from both the horizontal and dipping interfaces (for locations from 1 to 4 km) serve as the input to the joint MVA. Since we use gridded tomography, the derivatives of migrated depths with respect to the model parameters are calculated at the vertices of relatively fine grids. Therefore, we follow Wang and Tsvankin (2011) in employing a mapping matrix to convert the model updates into the parameter values at each grid point. After 11 iterations, the image gathers are flat (Figures 7(a) and 7(b)) and the reflectors on the PP and SS sections are correctly positioned (Figures 8(a) and 8(b)). The inversion produces accurate estimates of the interval VTI parameters: $V_P = 3031$ m/s, $V_S = 1515$ m/s, $\epsilon = 0.20$, and $\delta = 0.08$. These results confirm the feasibility of building the VTI depth model using 2D PP and PS reflection data if both horizontal and dipping events are available. We will also discuss application of the algorithm to more complicated VTI models with pronounced lateral heterogeneity.

**CONCLUSIONS**

In the presence of moderate dips, converted PS data may provide essential information for estimating VTI velocity models in depth. Here, we presented an efficient algorithm for joint velocity analysis of PP- and PS-waves from heterogeneous VTI media. After constructing pure SS reflections with the PP+PS=SS method, we update the velocity model by flattening PP- and SS-wave image gathers and tying the PP and SS migrated images in depth. It is essential to acquire long-spread (with the maximum offset-to-depth ratio reaching two) PP and PS data, which ensures that the offset range of the computed SS-waves is sufficient for robust velocity analysis. To better constrain the parameters $\eta$ and $\epsilon$, the residual moveout in PP-wave image gathers is evaluated using nonhyperbolic semblance scan.

Synthetic testing for layered VTI media confirmed that if both horizontal and moderately dipping PP and PS events are available, the joint MVA converges toward the correct depth model. Therefore, PS-waves can play an important role in velocity model-building for prestack depth migration. Multicomponent data also provide accurate estimates of the shear-wave vertical velocity, which can be used in lithology prediction and reservoir characterization.

**ACKNOWLEDGMENTS**

We are grateful to James Gaiser (Geokinetics) for helpful discussions and to the members of the A(nisotropy)-Team of the Center for Wave Phenomena (CWP) for valuable technical assistance. Support for this work was provided by the Consortium Project on Seismic Inverse Methods for Complex Structures at CWP.
REFERENCES


Foss, S., B. Ursin, and M. V. de Hoop, 2005, Depth-consistent reflection tomography using PP and PS seismic data: Geophysics, 70, U51–U65.


Thomsen, L., 1999, Converted-wave reflection seismology over inhomogeneous, anisotropic media: Geophysics, 64, 678–690.


