Estimation of time-lapse velocity change using repeating earthquakes with different locations and focal mechanisms

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SUMMARY

Codas of repeating earthquakes carry information about the time-varying properties of the subsurface or reservoir. Some of the changes within a reservoir change the seismic velocity and thereby the seismic signals that travel through the reservoir. In characterizing these velocity changes, seismic signals are often influenced by changes in other properties of the reservoir such as fluid migration or the properties of the seismic sources of the signals. We investigate the impact of the perturbations in seismic source properties on time-lapse velocity estimation. We suggest a criterion for selecting seismic events that can be used in velocity analysis. This criterion depends on the dominant frequency of the signals, the center-time of the used time window in a signal, and the estimated relative velocity change. The criterion provides a consistent framework for monitoring changes in subsurface velocities using microseismic events.

INTRODUCTION

Monitoring temporal changes within the Earth’s subsurface is a topic of interest in many areas of geophysics. These changes can result from an earthquake and its associated change in stress (Cheng et al., 2010), fluid injection or hydraulic fracturing (Davis et al., 2003), and oil and gas production (Zoback and Zinke, 2002). Some of the subsurface perturbations induced by these processes include temporal and spatial velocity changes, stress perturbations, changes in anisotropic properties of the subsurface, and fluid migration. Many of these changes span over a broad period of time and might even influence tectonic processes, such as induced seismicity (Zoback and Harjes, 1997). A seismic velocity perturbation of the subsurface leads to progressive time shifts across the recorded seismic signals. Various methods and data have been used to resolve the velocity perturbations. These methods include seismic coda wave interferometry (Snieder et al., 2002), doublet analysis of repeating microseismic and earthquake codas (Poupinet et al., 1984), time-lapse tomography (Vesnaver et al., 2003), and ambient seismic noise analysis (Sens-Schönfelder and Wegler, 2006; Cheng et al., 2010; Meier et al., 2010). Earthquake codas have higher sensitivity to the changes in the subsurface because multiple scattering allows these signals to sample the area of interest multiple times. However, doublet analysis of the earthquake (microseismic) codas requires repeating events. Failure to satisfy this requirement can compromise the accuracy of the estimated velocity changes.

Fluid-triggered microseismic events often are repeatable, but then events occur at slightly different positions with somewhat different source mechanisms (Sasaki and Kaieda, 2002; Gondano et al., 2012). Imprints of the source perturbation and the velocity change on the seismic waveforms can be subtle, thereby retaining the similarity between seismic signals. Therefore, we will need to ask, how do the source location, source mechanism, and subsurface perturbations affect the estimated velocity changes?

In this study, we investigate the impact of changes in source properties on the estimation of relative velocity changes. Knowledge of the impact of these perturbations on the estimated velocity change allows for a consistent framework for selecting pairs of earthquakes or microearthquakes used in velocity change analysis. This results in a more robust estimation of velocity change.

MATHEMATICAL CONSIDERATION

In this section, we use the time-shifted cross-correlation (Snieder, 2002, 2006) to develop an expression for the average value of the time perturbation of scattered waves that are excited by sources with varying source properties. This perturbation is due to changes in the velocity of the subsurface and to changes in the source properties. Figure 1 is a schematic figure showing the general setup of the problem we are investigating. Two sources ($S_1$ and $S_2$) represent a microseismic doublet (repeating microseismic events). These events occur at different locations and can have different rupture patterns. We assume that these events can be described by a double couple. We investigate the ability of using the signals of these sources for time-lapse monitoring of velocity changes, assuming that these sources occur at different times. We express the signals of the two sources as unperturbed and perturbed signals.

The unperturbed seismic signal $U(t)$ is given as

$$U(t) = A \sum_T U^{(T)}(t)$$

(1)

and the perturbed seismic signal $\hat{U}(t)$ is given as

$$\hat{U}(t) = \hat{A} \sum_T (1 + r^{(T)}) U^{(T)}(t - t_p^T),$$

(2)

where $A$ and $\hat{A}$ are the amplitudes of the unperturbed and perturbed source signals, respectively. These amplitudes represent the strengths of the sources. The recorded waves are a superposition of wave propagation along all travel paths as denoted by the summation over paths $T$. The change in the source focal mechanism only affects the amplitude of the wave traveling along each trajectory $T$ because the excitation of waves by a double couple is real (Aki and Richards, 2002). The change in the signal amplitudes - due to changes in the source mechanism angles - is defined by $r^{(T)}$ for path $T$, and $t_p^T$ is the
time shift on the unperturbed signal due to the medium perturbation for path $T$. The time-shifted cross-correlation of the two signals is given as

$$C(t) = \int_{t'-t}^{t'+t} U(t')\tilde{U}(t'+t)\,dt',$$  

(3)

where $t$ is the centertime of the employed time window and $2t_c$ is the window length. The normalized time-shifted cross-correlation has a maximum at a time lag equal to average time perturbation $(t_0 = \langle t_p \rangle)$ of all waves that arrive in the used time window:

$$\frac{1}{C(0)} \frac{dC(t_0)}{dt_0} \bigg|_{(t_0 - \langle t_p \rangle)} = 0.$$ 

(4)

Equation (4) allows for the extraction of the average traveltime perturbation from the cross-correlation. In this study, the average of a quantity $F$ is a normalized intensity weighted sum of the quantity (Snieder, 2006):

$$\langle F \rangle = \frac{\sum_i A_T^2 F_T}{\sum_i A_T^2}.$$ 

(5)

where $A_T^2 = \int UT^2(t')^2\,dt'$ is the intensity of the wave that has propagated along path $T$. The average value of the time perturbation and its variances are given by

$$\langle t_p \rangle = -\langle \frac{\delta V \rangle}{V_0} \rangle t_c$$  

(6)

and

$$\sigma^2 = \langle (t_p^2) \rangle - \langle t_p \rangle^2 \approx \frac{D^2 \langle V_0^2 \rangle}{3V_0^2}.$$  

(7)

In the above equations, $\langle \delta V \rangle / V_0$ is the average relative velocity change, $D$ is the shift in the source location, $t_c$ is the centertime of the processed time window and $V_0$ is the unperturbed velocity. Equation 6 suggests that the average time shift in the multiple scattered signals results only from the velocity changes within the subsurface. The variance of the time shifts depends, however, on the perturbations of the source location.

**NUMERICAL VALIDATION**

Figure 2: The experiment geometry of the numerical simulation. The receivers (squares) are surrounding the point scatterers (black dots). The source is positioned in the origin (cross). All perturbations of the source location is done from this position. The stations marked (NW, NE, E, SE, and SW) are used in the presentation of results.

**Description of numerical experiments**

We test the equations in section 2 with numerical simulation using Foldy’s multiply scattering theory (Foldy, 1945) described by (Groenenboom and Snieder, 1995). The theory models multiple scattering of waves by isotropic point scatterers. We conduct our numerical experiments using a circular 2D geometry (Figure 2) with point scatterers surrounded by 96 receiver stations. We uniformly assign the imaginary component of the scattering amplitude $|ImA| = 4$ to all the scatterers. In 2D, this is the maximum scattering strength that is consistent with the optical theorem that allows for conservation of energy (Groenenboom and Snieder, 1995). We assume an acoustic propagating wavefield which is a sum of the direct wavefield $U_0(r)$ and the scattered wavefield from all the scatterers:

$$U(r) = U_0(r) + \sum_{i=1}^{n} G^{(i)}(r, r_i)A_i U(r_i),$$ 

(8)

where the scattered wavefield is described by the second term. The Green’s function at $r$ due to scatterer $i$ at position $r_i$ is $G^{(i)}(r, r_i)$ while $U(r_i)$ is the incoming wavefield at scatterer $i$. The scattering amplitude of each scatterer is given by $A_i$. The direct wave is modulated by the far-field P-wave radiation pattern $F^P(\lambda, \delta, \phi)$ (Aki and Richards, 2002):

$$U_0(r) = F^P(\lambda, \delta, \phi)G^{(i)}(r, r_i),$$ 

(9)

with $r_i$ the source location. where $\lambda$, $\delta$, and $\phi$ are the source parameters (rake, dip and strike, respectively). Sources are located at the center of the scattering area. The source spectrum...
has a dominant frequency $f_d$ of approximately 30 Hz and a frequency range of 10-50 Hz. We assume a reference velocity $V_0 = 3500 \text{ m/s}$. Because the model we are using is an isotropic multiple scattering model, the transport mean free path is the same as the scattering mean free path: $l' = l$ which is approximately 12.2 km. There is no intrinsic attenuation in the numerical model. We generate an unperturbed multiple scattered signals with a reference model defined by the following parameter values: the source angles $\phi = 0^\circ$, $\lambda = 0^\circ$, $\delta = 90^\circ$; change in medium velocity $\Delta V = 0 \text{ m/s}$; and shift in the source location $D = 0 \text{ m}$. In order to understand the effect of the perturbation of these parameters on velocity change estimation, we generate signals from the perturbed version of the model.

**Data processing**

To estimate velocity change imprints on the synthetic signals due to the perturbation of the source location or its radiation properties, we use the stretching algorithm of (Hadziioannou et al., 2009). In the stretching algorithm, we multiply the time of the perturbed signal with a stretching factor $(1 - \varepsilon)$ and interpolate the perturbed signal at this stretched time. We then stretch the perturbed signal at a regular interval of $\varepsilon$ values. To resolve the value of $\varepsilon$, we use an $L_2$ objective function rather than the cross-correlation algorithm as suggested by (Hadziioannou et al., 2009). For events of equal magnitude ($A = A$), the objective function is

$$R(\varepsilon) = ||\hat{U}(t(1 - \varepsilon)) - U(t)||_2,$$

where $||\ldots||_2$ is the $L_2$ norm. The minimum of the objective function depends on the amplitude changes between the two signals and on the travel time perturbations due to velocity changes and shifts in the source location.

**Effect of perturbation of source properties on the estimated velocity change**

To understand the effect of the changes in the source properties on the estimation of the relative velocity changes, we conduct our numerical experiment over a range of parameter changes. We perturb the source location and the orientation of the source parameters. The perturbation of the source radiation parameters is characterized by the weighted root mean square change in source parameters $\langle r \rangle$ (Robinson et al., 2007):

$$\langle r \rangle = -\frac{1}{2}(4\Delta \phi^2 + 4\Delta \lambda^2 + 4\Delta \delta^2),$$

where $\Delta \phi$ is the change in strike, $\Delta \lambda$ is the change in rake, and $\Delta \delta$ is the change in dip. These changes represent the angle differences between the two sources (doublet).

Figure 3 shows that the estimated relative velocity changes $\langle \delta V/V_0 \rangle$ are near the true value ($\delta V = 0$) for models with perturbations of either the source location or the source radiation parameters. The velocity change estimated from individual stations varies around zero, but with a shift in the source location of $D = 0.143\lambda_d$, with $\lambda_d$ the dominant wavelength and source angle perturbations as large as $\Delta \phi = 20^\circ$, $\Delta \lambda = 20^\circ$, and $\Delta \delta = 20^\circ$ ($\langle r \rangle = -0.366$), the magnitude of these variations is smaller than 1/20th of the typical velocity changes inferred from seismic signals (Figure 3). These results agree with equation (6) which predicts that the average value of time shifts in the perturbed signal results only from changes in the medium velocity and is not affected by changes in source properties.

**Limiting regimes of the estimations**

To investigate the extent of the perturbation in the source location and source radiation perturbations that can be allowed in the estimation of relative velocity changes, we generate synthetic signals with models having a 0.1% relative velocity change and various perturbations of the source parameters. Comparing the phase changes due to shift in source location to those due to velocity changes, the shift in the source location has to satisfy

$$\frac{D}{\lambda_d} < \sqrt{2} \left( \frac{\delta V}{V_0} \right) f_d t_e,$$

for an accurate estimation of the velocity changes. Figure 4 shows the estimated relative velocity changes from signals generated from sources at different locations. Using the model parameters $\langle \delta V/V_0 \rangle = 0.001$, $t = 10 \text{ s}$, and $f_d \approx 25 \text{ Hz}$, the constraint on the source location shift for Figure 4 is $D/\lambda_d < 0.35$. Figure 4 shows that for $D/\lambda_d \geq 0.3$, the estimated velocity change deviates significantly from the real velocity change; this is in agreement with equation (12). The criterion in equation (12) imposes a constraint on the spacing requirements for the source locations of the doublets used for time-lapse velocity change monitoring with microearthquakes. Alternatively, equation (12) gives the magnitude of a velocity change that is resolvable with a given shift in the source location. According to equation (12), the allowable source separation increases with the centertime $t_e$ of the employed time window. This is

![Figure 3: Estimated relative velocity change due to perturbation in A. the source location (divided by the dominant wavelength; inset in the top right) and B. the source mechanism measured by $r$ (equation 11; inset in the top right).](image)
due to the fact that the imprint of the velocity change is more pronounced as the waves have propagated over a greater distance through the perturbed medium.

We also investigate the effect of the source radiation properties on the estimated velocity change of the medium of interest. Figure 5 shows the estimated velocity changes from a model with 0.1% velocity change using sources with perturbed radiation angles (measured by $\langle r \rangle$). In Figure 5A, the estimated velocity change at the individual stations progressively deviates from the true velocity change of 0.1% with increasing change in the orientations of the source angles. This deviation is due to the decorrelation between the perturbed and the unperturbed signals as shown in Figure 5B, which shows the maximum normalized cross-correlation of the codas within the processed time window. With an increasing change in the orientation of the sources, the maximum cross-correlation value of the waves excited by the doublets decreases. However, for source angle perturbations as large as $\Delta \phi = 28^\circ$, $\Delta \lambda = 28^\circ$, and $\Delta \delta = 28^\circ$ corresponding to $|\langle r \rangle| = 0.72$ (Figure 5A), the maximum deviation from the 0.1% model velocity change is approximately 0.01%. This is a small change compared to velocity changes resolved from seismic signals in practice. The maximum cross-correlation (Figure 5B) can be used as a diagnostic of the accuracy of the estimated velocity change. In this example, a maximum cross-correlation of 0.7 indicates an error of about 10% in the estimated velocity change.

CONCLUSION

In this study, we investigate the influence of perturbation in source properties (location and radiation) on the estimation of velocity changes. These velocity changes are extracted from multiply scattered signals (codas) of repeating events. We show that we can resolve accurate values of relative velocity changes if the shift in the source location satisfies the constraint (equation 12). This constraint depends on the dominant frequency of the signal, the estimated relative velocity change, and the center-time of the employed time window. This places a restriction on the relative event locations that can be used to estimate the relative velocity change of the subsurface. Using doublets that do not satisfy the constraint result to an inaccurate estimate of the velocity change.

A significant change in the source mechanism of double couple sources can introduce a bias in the estimation of relative velocity change. This bias is due to the decorrelation of the perturbed and unperturbed signals which lowers the accuracy of the estimated velocity change. However, this bias is negligible for the typical velocity changes resolved from seismic signals in practice. This result permits the use of sources of different orientations for the estimation of velocity changes, provided that the maximum cross-correlation of the source signals is greater than 0.7 as shown in Figure 5B. For a consistent estimate of the velocity change, using multiple stations is useful to ascertain the accuracy of the estimated velocity change in an isotropic subsurface.

ACKNOWLEDGMENTS

We are grateful for the financial support of the Department of Energy (DOE) through grant DE-EE0002758.
REFERENCES


