Converted-waves imaging condition for elastic reverse-time migration
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SUMMARY

Polarity changes in converted-wave images constructed by elastic reverse-time migration cause destructive interference after stacking over the experiments of a seismic survey. We derive a simple imaging condition for converted waves designed to correct the image polarity and reveal the conversion strength from one wave mode to another. Our imaging condition exploits pure P- and S-modes obtained by Helmholtz decomposition. Instead of correlating components of the vector S-mode with the scalar P-mode, we exploit the entire S wavefield at once to produce a unique image. We generate PS and SP images using geometrical relationships between the propagation directions for the P and S wavefields, the reflector orientation, and the S-mode polarization direction. Compared to alternative methods for correcting PS and SP images polarity reversal, our imaging condition is simple and robust and does not add significantly to the cost of elastic reverse-time migration.

INTRODUCTION

Seismic acquisition advances, as well as ongoing improvements in computational capability, have made imaging using multi-component elastic waves increasingly feasible. Elastic migration with multi-component seismic data can provide additional subsurface structural information compared to conventional acoustic migration using single-component data.

In elastic media, source and receiver wavefields are constructed using elastic (vector) wave equations. Multi-component wavefields allow for a variety of imaging conditions (Yan and Sava, 2008; Denli and Huang, 2008; Artman et al., 2009; Wu et al., 2010), for example, crosscorrelation of components of the displacement vectors or crosscorrelation of pure wave modes isolated in source and receiver wavefields (Yan and Sava, 2008). This later imaging condition combining incident and reflected wavefield polarization directions to obtain PS and SP images without polarity reversal. The method is simple and robust and operates on separated wave modes, as discussed by Yan and Sava (2008). The method is also cheap to apply since it does not require complex operations, like angle or directional decomposition.

We first compute the incident angles at each image point, using different methods based, for example, on ray tracing (Balch and Erdemir, 1994), Poynting vectors (Sun et al., 2006; Du et al., 2012) or angle gathers (Yan and Sava, 2008; Rosales et al., 2008; Yan and Xie, 2012), and then reverse the polarity using this estimated angle.

In this paper, we propose an alternative 3D imaging condition for elastic reverse-time migration. Our new imaging condition exploits geometric relationships between incident and reflected wave directions, the reflector orientation and wavefield polarization directions to obtain PS and SP images without polarity reversal. The method is simple and robust and operates on separated wave modes, as discussed by Yan and Sava (2008). The method is also cheap to apply since it does not require complex operations, like angle or directional decomposition.

THEORY

Reconstructed source and receiver elastic wavefields can be separated into P- and S-modes prior to imaging (Dellinger and Etgen, 1990; Yan and Sava, 2008). In isotropic media, this separation can be performed using Helmholtz decomposition (Aki and Richards, 2002), which describes the compressional P and transverse S components of the wavefield using the divergence and curl of the displacement vector field \( \mathbf{u} \):

\[
P (x, t) = \nabla \cdot \mathbf{u} (x, t) ,
\]

\[
S (x, t) = \nabla \times \mathbf{u} (x, t) .
\]

PS and SP images can then be obtained by crosscorrelating the P wavefield with each component of the S wavefield (Yan and Sava, 2008). Images produced in this way have three independent components at every location in space, i.e., PS and SP images are vector images. We seek to avoid vector PS and SP images by combining all components of the S wavefield with the P wavefield into an image representing the energy conversion strength from one wave-mode to another at an interface.

To formulate the imaging condition, we define the following quantities, shown in Figure 1:

- Vectors \( \mathbf{D}_P \) and \( \mathbf{D}_S \) indicating the propagation directions of the P- and S-modes, respectively;
- Vector \( \mathbf{n} \) which is the local normal to the interface represented by plane \( \mathcal{I} \);
- Vector \( \mathbf{S} \) which indicates the polarization of the S-mode.

Vectors \( \mathbf{D}_P \) and \( \mathbf{D}_S \) indicating the propagation directions of the P- and S-modes, respectively;
According to Snell’s law, vectors $D_P$, $D_S$ and $n$ belong to the reflection plane $R$. The polarization vector $S$ is orthogonal to the vector $D_S$. In the following, we assume that we know the normal vector $n$, e.g., from a prior image obtained, for example, by imaging pure PP reflections.

The polarization vector $S$ represents waves reflected from incident P or S waves; therefore, the receiver wavefield polarization vector $S$ is not orthogonal, in general, to the plane $R$, and can be decomposed into vectors $S_\perp$ and $S_\parallel$ such that

$$S = S_\perp + S_\parallel.$$  \(3\)

Vectors $S_\perp$ and $S_\parallel$ are orthogonal and parallel to plane $R$, respectively. The S waves reflected from incident pure P waves are confined to the vector field $S_\parallel$, so in the analysis of PS reflections, we can simply assume that the S-mode is orthogonal to the reflection plane $R$. A similar discussion is applicable to SP reflections.

As indicated earlier, the signs of PS and SP reflection coefficients change across normal incidence in every plane, which causes the polarity of the reflected waves to change at normal incidence. Since normal incidence depends on the acquisition geometry, as well as the geologic structure, this polarity reversal can occur at different positions in space, thus making it difficult to stack images for an entire multicomponent survey.

In order to address this challenge, we propose the following imaging conditions for PS and SP images:

$$I^{PS}(x) = \sum_e \sum_t (\nabla P(x,t) \times n(x)) \cdot S(x,t), \quad (4)$$

$$I^{SP}(x) = \sum_e \sum_t ((\nabla \times S(x,t)) \cdot n(x)) P(x,t). \quad (5)$$

Here, $P(x,t)$ and $S(x,t)$ represent the scalar P- and vector S-modes after Helmholtz decomposition, and $I^{PS}(x)$ and $I^{SP}(x)$ are the PS and SP images, respectively.

The physical interpretation of our new imaging condition, equations 4 and 5, is as follows:

**PS imaging** (equation 4): The vector $\nabla P$ characterizes the propagation direction of the P-mode and can be calculated directly on the separated P wavefield. The cross product with the normal vector $n$ constructs a vector orthogonal to the reflection plane $R$; as indicated earlier, this direction is parallel with the polarization vector of the S-mode, $S_\parallel$. Therefore, the dot product of $(\nabla P \times n)$ and $S$ is just a projection of the vector $S$ wavefield on a vector which depends on the incidence direction of the P wavefield. For a P-mode incident in opposite direction, the vector $\nabla P \times n$ reverses direction, thus compensating for the opposite polarization of the S-mode. Consequently, the PS imaging condition has the same sign regardless of incidence direction, and therefore PS images can be stacked without canceling each other at various positions in space.

**SP imaging** (equation 5): The vector $S$ characterizes the polarization direction of the S-mode, and for the situation considered here it is orthogonal to the reflection plane $R$. Therefore, vector $\nabla \times S$ is contained in the reflection plane. The dot product with the normal vector $n$ produces a scalar field characterizing the magnitude of the S-mode, but signed according to its relation with respect to the normal $n$. This scalar quantity can be simply correlated with the scalar reflected P wavefield, thus leading to a single image without sign change as a function of the incidence direction. Thus, SP images produced in this fashion can also be stacked without canceling each other at various positions in space.

In 2D, the $S$ has only one non-zero component, $S_y$, since the vector $S$ is orthogonal to the local reflection plane $R$. Therefore, the PS and SP imaging conditions can be

![Figure 1: Cartoon describing wave propagation in 3D media. The blue and red lines denote the incident P and the reflected S waves. Vector $n$ indicate the normal to the interface $I$, and vectors $D_P$ and $D_S$ indicate the propagation directions of the incident P-mode and of the reflected S-mode, respectively. $D_P$, $D_S$ and $n$ are all contained in the reflection plane $R$.](image1)

![Figure 2: Cartoon describing wave propagation in the local reflection plane $R$. The blue and red lines denote the incident P and the reflected S waves. The wavepaths in the local region around each reflection point can be approximated by straight lines. The black arrow denote the local normal vector to the locally-planar interface.](image2)
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simplified as:

\[ I^{PS} = - \sum_e \sum_t \left( \frac{\partial P}{\partial x} n_z - \frac{\partial P}{\partial z} n_x \right) S_y, \]  
(6)

\[ I^{SP} = \sum_e \sum_t \left( \frac{\partial S_y}{\partial x} n_z - \frac{\partial S_y}{\partial z} n_x \right) P, \]  
(7)

where \( I = I(x, z) \), \( P = P(x, z, t) \), \( S_y = S_y(x, z, t) \), \( n_z = n_z(x, z) \) and \( n_x = n_x(x, z) \). For a horizontal reflector, i.e. \( n = \{0, 0, 1\} \), we can write:

\[ I^{PS} = - \sum_e \sum_t \frac{\partial P}{\partial x} S_y, \]  
(8)

\[ I^{SP} = \sum_e \sum_t \frac{\partial S_y}{\partial x} P, \]  
(9)

which simply indicates that in our new imaging condition we correlate the P or S wavefields with the \( x \) derivative of the S or P wavefields, respectively. These are, of course, just special cases of the general case from equations 4 and 5.

We illustrate our new imaging condition with the synthetic model shown in Figure 3(a), which contains one horizontal reflector embedded in constant velocity. Figures 3(b) and 3(c) show the source P-mode and receiver S-mode for the single source indicated in Figure 3(a), respectively. As expected, the S-mode changes polarity as a function of the propagation direction. Figure 3(d) is the PS image obtained using the conventional imaging condition, i.e. the crosscorrelation of the source P wavefield (Figure 3(b)) with the receiver S wavefield (Figure 3(c)), and inherits the polarity change from the S-mode. In contrast, our new imaging condition leads to the image in Figure 3(e) without polarity reversal. This correction allows us to stack multiple elastic images constructed for different seismic experiments.

EXAMPLE

We also illustrate our method using a modified Marmousi model created by the Institut Français du Pétrole (IFP), Figure 5(a). It contains several major faults and semi-parallel dipping layers. We use 60 explosive sources evenly distributed along the surface, and multicomponent 576 receivers located at \( z = 0.2 \) km. The source function is a Ricker wavelet with peak frequency of 35 Hz.

The source S wavefield is weak, due to the minor energy conversion at the top of the model, therefore the SP image is also weak and we do not discuss it further. Using the conventional and the new imaging condition, we obtain the PS images from a single shot shown in Figures 5(b) and 5(c), respectively. We observe polarity change in Figure 5(b), but not in Figure 5(c). If the polarity changes as a function of source position occur at different locations in subsurface, then stacking leads to destructive interference between images obtained for different shots.

Figure 3: (a) A 2D synthetic model with one horizontal reflector in constant velocity; the dot and horizontal line indicate the location of the source and receivers, respectively. Snapshots of (b) the P source and (c) the S receiver wavefields. PS images using (d) the conventional and (e) our new imaging conditions.
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Figures 5(b) and 5(c) are PS stacked images obtained with the conventional and the new imaging condition, respectively. Compared to the conventional, our new imaging condition corrects the polarity reversal and leads to images better representing the reflection strength as a function of position in space. The PS common image gathers at $x = 2$ km, shown in Figures 4(a) and 4(b), confirm that there is no polarity change as a function of shot position.

![Figure 4](image1.png)

**Figure 4:** PS common image gather at $x = 2$ km obtained from (a) the conventional imaging condition and (b) our new imaging condition.

**CONCLUSIONS**

We derive a new 3D imaging condition for PS and SP images constructed by elastic reverse-time migration. As for more conventional methods, P- and S-modes are obtained using Helmholtz decomposition. However, our imaging condition does not correlate various components of the vector S wavefield with the scalar P wavefield; instead, our method uses geometrical relationships between the wavefields, their propagation direction, the reflector orientation and polarization directions to construct a single image characterizing the PS or SP reflectivity. Our method is simple and robust and leads to accurate images without the need to decompose wavefields into directional components, or to construct costlier images in the angle domain.

**ACKNOWLEDGMENTS**

This work was supported by Consortium Project on Seismic Inverse Methods for Complex Structures. The reproducible numeric examples in this paper use the Madagascar open-source software package (Fomel et al., 2013) freely available from http://www.ahay.org. The authors thank Tariq Alkhalifah for helpful discussions and valuable suggestions.

![Figure 5](image2.png)

**Figure 5:** (a) Marmousi model. PS single-shot images obtained using (a) the conventional and (b) our new imaging conditions. PS stack images obtained using (c) the conventional and (d) the new imaging condition.
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