Time-lapse imaging of localized weak changes with multiply scattered waves

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SUMMARY

Multiply scattered seismic waves, due to their long path length in a limited volume, can increase the detection of weak time-lapse changes within a medium. Such weak changes are usually not resolved with singly scattered waves. Previous use of multiply scattered waves for time-lapse monitoring assume some level of homogeneity about the scattering media. This homogeneity is usually characterized either by a constant mean free path or diffusion coefficient which likely breaks down in complicated medium. We provide a novel method to compute the sensitivity kernel for time-lapse monitoring using a known model of the scattering medium. We also demonstrate the capability of resolving a localized time-lapse velocity change within a 3-layer scattering model using multiply scattered waves. The layers within the model have different scattering properties. We localize the weak velocity change but the resolution of the imaged change degrades with increasing coda traveltime. Also we use multiply scattered waves to resolve weak changes in a laboratory experiment with a 3D concrete block due to stress loadings to the surface of the block.

INTRODUCTION

Multiply scattered seismic waves, due to their long path length in a limited volume, provide information about the subsurface that can be used to increase illumination, especially within a poorly illuminated subsurface (Gaburro et al., 2007), and increase the detection of weak time-lapse changes within the subsurface (Poupinet et al., 1984). This information in the multiply scattered waves can be used to improve the detectability of weak changes especially in cases where the weak changes are not resolvable with singly scattered waves. However, the complexity in the travel paths of the multiply scattered waves, which depends on the scattering properties of the medium of interest, makes it challenging to accurately image and localize the weak changes in the medium. Diffusive models for the scattered intensity have been used successfully in imaging algorithms that use multiply scattered waves such as in medical imaging (Yodh and Chance, 1995) or imaging of missing scatterers (Rossetto et al., 2011). However, the validity of using the diffusion model in explaining multiple scattering intensity depends on the strength of the scattering process and is only valid at large lapse times. We demonstrate an alternative approach to computing the sensitivity kernel that can be used to image weak changes using multiply scattering waves. We numerically simulate the scattered intensity needed for the computation of the kernel using finite difference modeling. We also demonstrate the use of our numerical kernel to image localized weak change both in numerical and in laboratory experiments.

THEORY

Pacheco and Snieder (2005), use the intensity of multiply scattered waves to develop a sensitivity kernel $K(s, x_0, r, t)$ which relates the mean travel time changes $\langle \tau(t) \rangle$ to a localized relative velocity change within a medium $\delta v/v(x_0)$:

$$\langle \tau(t) \rangle = - \int_V K(s, x_0, r, t) \frac{\delta v}{v} (x_0) dV(x_0),$$

where $V$ is the scattering volume, and $s$ and $r$ are the source and the receiver locations, respectively. Rossetto et al. (2011) relate the decorrelation $D(s, r, t)$ between time-lapse multiply scattered waves from a medium to the change in the scattering cross-section of the medium $\delta \sigma(x_0)$.

$$D(s, r, t) = \int_V \frac{\delta \sigma(x_0)}{2} K(s, x_0, r, t) dV(x_0).$$

In both expressions, the sensitivity kernel $K(s, x_0, r, t)$, which depends on the source and receiver locations, the scattering property of the medium, and the lapse time of the scattered wave, is given by

$$K(s, x_0, r, t) = \int [P(s, x_0, t') P(x_0, r, t - t')] dt',$$

where $P$ is the normalized intensity of the multiply scattered waves. The normalized intensity is usually modeled using analytical intensities such as the diffusion intensity and the radiative transfer intensity (Paasschens, 1997). In most scattering models, however, we might need to compute the sensitivity kernel $K(s, x_0, r, t)$ numerically. In these scattering models, we might not have an exact analytical formulation for either the intensity of the scattered waves or the corresponding sensitivity kernel. Using equation 3 and an a-priori scattering model, we can numerically compute the sensitivity kernel by simulating the intensity of the wavefield generated within the scattering model. The characteristics of the a-priori model, such as the scattering mean free path and the average velocity, can be estimated from either using both the ballistic and coda waves in the recorded data or using additional information such as velocity model obtained from other geophysical methods.

COMPUTATION OF THE SENSITIVITY KERNEL

We suggest a novel method to compute the sensitivity kernel (equation 3) by using the source and receiver intensities generated from finite-difference calculation of excited wavefields at the source and receiver locations. The respective intensities are the square of the envelope of the generated wavefields normalized by the spatial integral of the intensity (Sato et al.,
To compute the sensitivity kernel $K(s, x_0, r, t)$, we convolve the source and the receiver intensity fields and normalize with the denominator in equation 3. The wavefields shown here are generated with a random scattering model (Sato et al., 2012) and a 15 Hz Ricker wavelet. The scattering model consists of a realization of a random velocity model defined by the von Karman parameters and constant density. The velocity model consists of a background velocity and a random velocity fluctuations defined by the von Karman model (figure 1: Top). The random velocity fluctuations are defined by these von Karman parameters: correlation length, 0.01 km along the x- and z- axis, fluctuation strength, 0.1%, and exponent, 0.5.

Figure 1 shows the kernel snapshots at $t = 0.95$ s, 2.00 s and 4.00 s for the source (S) and receiver (R). Time $t = 0.95$ s corresponds to the kernel of the first arrival wave. With increasing time, the area covered by the sensitivity kernel progressively increases. The spatial broadening of the kernel with time increases the detectability of any change in the scattering property of the model, especially changes lying away from the S-R line. The shape of the kernel with increasing time depends on the source and receiver locations and the properties of the scattering medium. At time $t > 0.95$ s, the kernel takes an elliptical shape with the major axes along the S-R line and the minor axes perpendicular to the S-R line. The edge of the kernel known as the kernel front is dominated by contributions from single scattering. This kernel has higher sensitivity both at the edge of kernel and at the source and receiver locations. The elevated sensitivity at the source and receiver results from dominant contributions to the multiply scattered waves recorded by a particular S-R pair from the near-source and near-receiver scatterings. Figure 2 compares the numerical kernel with the diffusion and radiative transfer kernels. The radiative transfer kernel more accurately predicts the numerical solution of the sensitivity kernel than the diffusion kernel. This is expected, because the diffusion model is only accurate in a strongly scattering medium and at large lapse times ($vt/|s - r| >> 1$).

INVERTING FOR WEAK CHANGES

Numerical experiment

We explore the use of our numerical kernel to resolve a localized velocity change using the velocity (scattering) and the time-lapse velocity change model in Figure 3. The velocity model is a 3 layer model, with each layer having scattering properties of different statistical characteristics. The statistical characteristics of the top and the bottom scattering layers are homogeneous and structurally isotropic, while the middle layer is heterogeneous characterizing a highly fractured reservoir. The time-lapse change is a 0.5 % rectangular velocity change embedded within the middle layer shown in Figure 3 (bottom). To monitor and resolve this localized change, we use two vertical receiver arrays representing two boreholes. These boreholes record scattered waves generated by the 9 sources that are located along a horizontal line. This source-receiver setup depicts time-lapse monitoring with repeating microseismic events whose scattered waves (cudas) are recorded by the 2 boreholes. We assume acoustic wave propagation hence we are not accounting for the effects of source radiation and elastic seismic wave modes.

To localize the weak velocity change, we minimize the objective function $\phi$:

$$\min \phi = ||\tilde{d} - KS^{-1}\tilde{m}||^2.$$  \hspace{1cm} (4)

This minimization problem corresponds to solving the following weighted least-squares problem:

$$K^T\tilde{d} = (K^TK)S^{-1}\tilde{m}_S = (K^TK)S^{-1}\tilde{m}_S,$$  \hspace{1cm} (5)

where $K$ is the discretized numerical solution of the sensitivity
Figure 2: Comparison between numerical sensitivity kernel (black line) and diffusion- (red line) and radiative transfer- (blue line) based kernel along the source- (at 2km) receiver- (at 5km) line.

kernel, \( \tilde{d} \) is the estimated traveltime change \( \delta t \), and \( \tilde{m} = S^{-1} \tilde{m}_S \) is the fractional velocity change. The sensitivity weighting matrix \( S \) is a diagonal matrix with the elements \( S_{ij} = \delta_{ij} w_j \) that are weighting functions \( w_j \) given by Li and Oldenburg (2000):

\[
w_j = \left( \sum_{k=1}^{N} K_{kj}^2 \right)^{\beta/2}, \quad j = 1, \ldots, M; \beta = 0 - 2, \tag{6}
\]

where \( N \) is the number of data and \( M \) is the number of the discretized model space. We apply \( S \) to suppress elevated sensitivity at the source and receiver locations (Kanu and Snieder, 2014). The sensitivity weighting matrix acts as a preconditioner for the inversion problem. To solve equation 5, we use linear conjugate-gradient method. All the imaged velocity changes in this section are obtained after 5 iterations.

To estimate the traveltime changes, we use 0.6 s time windows with each window overlapping 0.1 s with the previous time window. Using this time window which is 10 times the dominate period helps stabilize the estimated traveltime changes (Snieder et al., 2002). To get more data points for the inversion, we interpolate estimated traveltime changes from 0.5 s to 0.02 s. Figure 4 shows the inverted time-lapse velocity changes using estimated traveltime changes across different coda time windows (the rectangular boxes in Figure 4 top). Each of the coda time window starts 0.3 s before the travelt ime of the first arrival event. Figures 4A, 4B, 4C, and 4D show inverted velocity changes using estimated traveltime changes extending to 0.55 s, 0.80 s, 1.05 s, and 1.30 s, respectively after the traveltime of the first arrival. The inverted velocity images correctly recovers the location of the velocity change. However, with these time windows, the resolution of the inverted velocity change varies with different coda time windows. The image of the first time window (Figure 4A) shows the best spatial resolution of the velocity change within the middle layer but has the weakest amplitude of the resolved velocity change. The weak amplitude of Figure 4A velocity change results from the low sensitivity of the early scattered waves to the velocity change. With longer time windows the inverted velocity changes become closer to the true magnitude of the velocity change (\( \Delta v/v = 0.5 \% \)) and the resolved change lie within the middle layer of the velocity model. However, the inverted velocity change become progressively smeared horizontally within the middle layer. This smearing of the inverted velocity change is due to the spatial broadening of the sensitivity kernel with traveltime and to the strong scattering in the reservoir that might bias the localization of the velocity change. The sensitivity kernel for the velocity model (Figure 3) is given in Kanu and Snieder (2014).

**Laboratory experiment in a 3D concrete block**

Here, we invert for weak changes within a heterogeneous 3D concrete block with an average P-wave velocity of 4 km/s. The weak change in the block is due to a stress loading which is applied at location X on the surface of the concrete block (Figure 5). We use the decorrelations estimated from time-lapse scattered waves arriving between 0.28 ms and 0.58 ms after the first arrival. To invert the change within the block due to the stress loading, we solve equation 5. In this inversion, we use the radiative transfer sensitivity kernel. We estimate the average mean free path by fitting the intensities of the coda waves using radiative transfer intensity which depends on the mean free path. We estimate the mean free path to be approximately 1.52 m. Figure 5 shows the inverted solution of the weak changes within the 3D block at \( z = 0 \) m for stress loading of 5-10 kN and 5-15 kN. The resolved change is centered on the location of the stress loading.
CONCLUSIONS

We suggest a novel approach to compute the sensitivity kernel that can be used to resolve weak changes within a medium, that is changes irresolvable from singly scattered waves. This approach requires simulation of the scattered intensity in the medium of interest rather than approximating the scattering intensity with analytical models such as diffusion and radiative transfer models. Likewise we demonstrate the resolution of the changes using numerical and laboratory experiments. The numerical experiment images localized velocity change within a model that mimics a subsurface containing a highly fractured reservoir. In laboratory experiment, we resolve a change within a 3D concrete block due to a stress loading at the surface of the block using the decorrelation of the time-lapse scattered waves.

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Figure 4: Inverted fractional velocity change using various coda time windows. Top inset: a typical recorded coda signal with time windows use to invert the velocity changes. Inverted velocity change in A: with black time window, B: with red time window, C: with blue time window, and D. with green time window.

Figure 5: Resolved change due to stress loading from 5 kN to 10 kN (left) and from 5 kN to 15 kN (right) at X.
REFERENCES


