Least-squares migration in the presence of velocity errors
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SUMMARY

Seismic migration requires an accurate background velocity model that correctly predicts the kinematics of wave propagation in the true subsurface. Least-squares migration, which seeks the inverse rather than the adjoint of a forward modeling operator, is especially sensitive to errors in this background model, which can result in traveltime differences between predicted and observed data that lead to incoherent and defocused migration images. We propose an alternative misfit function for use in least-squares migration that measures amplitude differences between predicted and observed data, i.e., differences after correcting for nonzero traveltime shifts between predicted and observed data. We demonstrate on synthetic and field data that, when the background velocity model is incorrect, the use of this misfit function results in better focused migration images. Results suggest that our method best enhances image focusing when differences between predicted and observed data can be explained by traveltime shifts.

INTRODUCTION

Seismic migration can be described as the adjoint of a linearized forward modeling operator applied to observed data (Claerbout, 1992). Migration produces a reflectivity image, an image of a perturbation to the background velocity model (Cohen and Bleistein, 1979), that approximates the true reflectivity insofar as the adjoint of the forward operator approximates the pseudoinverse. Typically, the adjoint is a poor approximation, and the accuracy of the computed reflectivity image can be significantly improved by using the pseudoinverse of the forward operator rather than the adjoint. The use of the pseudoinverse of the forward operator in migration is known as least-squares migration (Nemeth et al., 1999; Østmo and Plessix, 2002; Plessix and Mulder, 2004; Kähl and Sacchi, 2003; Dau, 2012).

The quality and accuracy of migration images depends greatly on the accuracy of the background velocity model, and errors in this background model can lead to an incoherent, defocused image. Ideally, the background velocity model should correctly predict the traveltimes of observed data, and should be sufficiently smooth so as not to generate reflected waves. These requirements derive from the conditions under which the Born approximation is valid (Symes, 2009), and under these conditions, migration can accurately image subsurface structures. However, when these conditions are violated, migration images are degraded and become defocused and incoherent. One reason for this degradation is that migration inverts for the perturbation to the background velocity model that controls only the amplitudes of predicted data; if the background model contains errors, then the predicted data will contain errors in both traveltime and amplitude compared to the observed data, and both these types of errors — instead of only the amplitude errors — will contribute to the migration image.

We propose a simple modification of the conventional least-squares misfit function used in iterative least-squares migration. Rather than minimize the difference between predicted and observed data, we propose to minimize their difference after correcting for nonzero traveltime shifts. Assuming estimated traveltime shifts between predicted and observed data are accurate, this misfit function quantifies mostly amplitude differences. We demonstrate that the use of this amplitude misfit function in least-squares migration results in more coherent and better focused images when the background velocity model used for migration differs from the true background velocity model.

METHODS

In this section, we first briefly review linearized waveform inversion before presenting our method for inversion of amplitude differences.

Linearized waveform inversion

Wave propagation in the subsurface is described approximately by the constant-density acoustic wave equation,

$$\frac{\partial^2 u}{\partial t^2} - \Delta u = f,$$

where $u_0$ is the wavefield, $\sigma_0$ is the squared background slowness, and $f$ is the source function. Perturbing $\sigma_0$ by a scattering potential $m$ and linearizing about $m$ yields

$$\frac{\partial^2 u}{\partial t^2} - \Delta u = -m \frac{\partial^2 u_0}{\partial t^2},$$

where $u$ is the scattered or perturbation wavefield. Often $m$ is referred to as the reflectivity model or simply the reflectivity.

Let $u_s$ denote the discretized solution of equation 2 for a source function at position $s$. The wavefield $u_s$ is linear in the reflectivity $m$:

$$u_s = L_s m,$$

where $L_s$ is a linear prediction operator describing the evolution of the scattered wavefield in equation 2. The predicted data $p_{s,r}$ are a subset of the wavefield $u_s$:

$$p_{s,r} = S_s u_s,$$

where $S_s$ is a sampling operator that extracts the wavefield at receiver position $r$.

To solve equation 4 for the reflectivity model $m$, we minimize, in a least-squares sense, the difference between predicted data $p_{s,r}$ and observed data $d_{s,r}$:

$$\min_m J(m) = \frac{1}{2} \sum_{s,r} \| S_s u_s - d_{s,r} \|^2.$$
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To minimize equation 5, we can pursue the negative of the gradient direction

$$\frac{\partial J}{\partial m} = \sum_{s,r} L_s^T S_r^T (S_r u_s - d_{s,r}) .$$

(6)

The adjoint of the prediction operator $L_s$ is a migration operator (Claerbout, 1992), and so we obtain the well-known result (Lailly, 1983; Tarantola, 1984) that the gradient of the least-squares misfit function can be computed by a migration of the residuals.

Inversion of amplitude errors

To formulate an inversion of amplitude errors, we modify the observed data to include a time-shift operator:

$$b_{s,r} = T_s r d_{s,r} .$$

(7)

where $T_{s,r}$ is a linear operator, e.g., a sinc interpolation operator, that shifts the observed data $d_{s,r}$ by the traveltime shifts $\tau_{s,r}$ estimated using dynamic warping (Hale, 2013). Note that $T_{s,r}$ depends implicitly on the model $m$, because the traveltime shifts $\tau_{s,r}$ are computed using the predicted data $p_{s,r}$, which depend on the model.

The shifted observed data $b_{s,r}$ can be viewed as a secondary dataset obtained by processing the observed data. The purpose of this processing is essentially to remove from the observed data any components that are due to an inconsistent model of wave propagation in the true subsurface. Migration using an incorrect background model is equivalent to migration using forward modeling that is inconsistent with the observed data, and so to properly migrate these data, we must first remove those components that cannot be explained by our forward modeling. Those components are the traveltimes.

Thus, we seek to minimize the difference between predicted data $p_{s,r}$ and time-shifted observed data $b_{s,r}$:

$$\min_m J_A(m) = \frac{1}{2} \sum_{s,r} \| S_r u_s - T_s r d_{s,r} \|^2 .$$

(8)

Note that if the estimated traveltime shifts $\tau_{s,r}$ are accurate, then equation 8 measures only amplitude errors between predicted and observed data. If the traveltime shifts are zero, then equation 8 reduces to equation 5. To minimize the misfit function in equation 8, we require its gradient with respect to model
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parameters:
\[
\frac{\partial J}{\partial m} = \sum_{s,r} L_s^T S_r^T (S_r u_s - T_{s,r} d_{s,r}) .
\]  
(9)

Although \( A_{s,r} \) depends on the estimated traveltime shifts \( \tau_{s,r} \), we need not consider this dependence because the shifts obtained with dynamic warping are such that \( \partial A_{s,r} / \partial \tau_{s,r} \) is mostly zero.

RESULTS

We compare conventional least-squares migration (LSM) with the proposed method of least-squares migration of amplitude errors (LSMA) on a 2D synthetic dataset, and on a 2D field dataset.

Synthetic data example

The true background slowness used for modeling and migration is shown in Figure 1a, and is computed by smoothing the true Marmousi model (Lailly and Versteeg, 1990). The true reflectivity is then computed as the difference between the true slowness squared and the true background slowness (Figure 1a) squared. Using the true background slowness and true reflectivity, we simulate observed data by solving equations 1 and 2 for a Ricker source function with peak frequency 10 Hz. To facilitate comparison of LSM and LSMA, all migration images for these synthetic data are computed using 20 nonlinear conjugate gradient iterations (Nocedal and Wright, 2000). Hence, as the cost of dynamic warping (Hale, 2013) is small compared to the cost of modeling and migration, the LSM and LSMA images computed for these synthetic data come at comparable costs.

Figure 1 demonstrates conventional LSM using the true background slowness for migration. The reflectivity image shown in Figure 1b is obtained after 20 nonlinear conjugate gradient iterations of LSM using the true background slowness with 153 shots and 767 receivers evenly spaced along the surface. This computed reflectivity matches well the true reflectivity, because the background slowness model used for migration was exactly the true background slowness.

Figure 2 illustrates the effect of erroneous background slowness model on the reflectivity images obtained using LSM and LSMA. Figure 2a shows the difference between the true background slowness model (Figure 1a) and the erroneous background slowness model that we use for migration. The slowness error shown in Figure 2a was computed by smoothing a random slowness model. Figure 2b shows the reflectivity image computed using 20 iterations of LSM with the erroneous background slowness model with errors shown in Figure 2a. Compared to the reflectivity image (Figure 1b) computed using the true background slowness, the image in Figure 2b is degraded, and shows uneven illumination and defocused reflectors, especially at greater depths where traveltime errors resulting from the erroneous background slowness are more severe.

Figure 2c shows the reflectivity image computed with 20 iterations of LSMA. Compared to the conventional LSM image (Figure 2b), the LSMA images show improved illumination of deeper portions of the model, and better focused and more continuous reflectors throughout. Note, however, that the positions of features in the LSMA image are shifted compared to their positions in the image computed using the true background slowness (Figure 1b). This mispositioning is expected, however, since LSMA images are computed using erroneous background slowness models.

Figure 4: The (a) LSM image and (b) LSMA image computed for the laterally invariant slowness model shown in Figure 3a, and the (c) LSM image computed for the optimized slowness model shown in Figure 3b.
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Figure 5: For the shot located at distance 1.85 km, the (a) observed data, (b) predicted data computed using the laterally invariant slowness model shown in Figure 3a and the LSMA image shown in Figure 4b, and (c) traveltime shifts between (a) and (b).

Field data example

Next we test our method for amplitude-only migration on a subset of a field dataset provided by Eni E&P. The entire 2D dataset contains 3661 shots with a shot spacing of 12.5 m, and was recorded using a streamer with 99 receivers with a receiver spacing of 12.5 m and maximum offset of 1.225 km. The subset of the data that we migrate consists of 431 shots with shot spacing of 25 m. The data have been regularized, and multiples have been attenuated. We estimate a zero-phase wavelet from the amplitude spectrum computed from a subset of the recorded data (Claerbout, 1992), and we apply a bandpass filter to both the estimated wavelet and the recorded data to remove frequency content below 10 Hz and above 40 Hz prior to migration.

We compare LSM and LSMA for two slowness models. The first slowness model, shown in Figure 3a, is laterally invariant (except near the sea floor), while the second, shown in Figure 3b, is an optimized slowness model. The LSM and LSMA images computed for the laterally invariant slowness model (Figure 3a) are shown in Figures 4a and 4b, respectively. Comparing these images, we observe that reflectors in the LSMA image are more continuous and better focused than corresponding reflectors in the LSM image. Moreover, image features in the LSMA image (Figure 4b) are similar to features seen in the LSM image (Figure 4c) computed for the optimized slowness model (Figure 3b), despite the use of a much simpler slowness model for LSMA. Differences between the migration images shown in Figures 4a and 4b are most apparent in the areas enclosed by yellow boxes, in which the slowness differences (Figure 3c) between the models used for migration are relatively large. Elsewhere, where slowness errors are smaller, differences between the migration images are less significant, as one would expect.

Because we compute LSMA images by minimizing the difference between predicted and shifted observed data (equation 8), the predicted data in general will not have the same traveltimes as the original observed data. An example of these travelt ime differences for data corresponding to the shot located at distance 1.85 km is shown in Figure 5. Figure 5a shows the observed data, Figure 5b shows the predicted data computed using the laterally invariant slowness model (Figure 3a) and the LSMA image (Figure 4b), and Figure 5c shows the traveltime shifts between the data shown in Figures 5a and 5b. The maximum frequency content of the data is 40 Hz, which corresponds to a period of 25 ms. Thus we observe from Figure 5c that the remaining traveltime shifts between predicted and observed data exceed one half period. This confirms that LSMA yields an image that explains the dynamics, but not the kinematics, of the observed data.

CONCLUSION

We have presented a method for least-squares migration that minimizes an amplitude misfit function defined with differences between predicted data and shifted observed data, with traveltime shifts between predicted and observed data estimated using dynamic warping. The use of this amplitude misfit function results in a more coherent and better focused migration image when the background slowness model used for migration contains errors. These LSMA images contain image features with amplitudes that match those of the true reflectivity, but with positions that are shifted relative to the positions of corresponding features in the true reflectivity. LSMA images thus are perhaps better suited for interpretation of geologic structures, but in order to correctly position interpreted structures, we would need to first correctly position LSMA image features. One way to correct for the mispositioning of image features is to first align features with measurements of subsurface properties obtained from well logs, and then interpolate alignment shifts between well-log locations to generate shifts for an entire image.

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