Nonhyperbolic moveout analysis using dynamic programming

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Known parameter field

Data created by Hemang Shah and provided courtesy of BP Exploration Operation Company Limited ("BP")
Data created by Hemang Shah and provided courtesy of BP Exploration Operation Company Limited ("BP")
After nonhyperbolic moveout
Estimated vs known

Parameters:

- $V_{nmo}$ (km/s)
- % Difference
- $\bar{\eta}_{eff}$
- Difference

Graphs depicting time (s) against various parameters.
Hyperbolic NMO
CMP gather
After hyperbolic moveout
Hyperbolic moveout equation

\[ t^2(x, t_0) = t_0^2 + \frac{x^2}{V_{nmo}(t_0)} \]

\[ V_{nmo}(t_0) \quad \text{effective NMO velocity} \]
CMP gather

Offset (km)

Time (s)
Peak semblance

$V_{nmo} \text{ (km/s)}$

Time (s)

Semblance

0.0
0.1
0.2
0.3
0.4
0.5

1.5  2  2.5
Peak and known $V_{nmo}$
Semblance $s[i, j[i]]$

Sample index $j$

Sample index $i$
Given a semblance spectrum $s[i, j[i]]$,
Given a semblance spectrum $s[i, j[i]]$, find a sequence of sampled $V_{nmo}$ velocities $j[i]$. 
Given a semblance spectrum $s[i, j[i]]$,

find a sequence of sampled $V_{nmo}$ velocities $j[i]$ to maximize

$$\sum_{i} s[i, j[i]]$$
Given a semblance spectrum $s[i, j[i]]$, find a sequence of sampled $V_{nmo}$ velocities $j[i]$ to maximize

$$
\sum_i s[i, j[i]]
$$

such that

$$
r_{vl} \leq j[i] - j[i - 1] \leq r_{vu}
$$
<table>
<thead>
<tr>
<th>Maximizing semblance</th>
<th>Dynamic warping</th>
</tr>
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<tr>
<td>semblance spectrum</td>
<td>alignment errors</td>
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<tr>
<td>maximize semblance</td>
<td>minimize alignment errors</td>
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<td>NMO velocities</td>
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<td>constrain derivative of NMO velocities</td>
<td>constrain derivative of time shifts</td>
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</table>
Estimated and known $V_{nmo}$

$V_{nmo}$ (km/s)

Time (s)

Semblance
Estimated vs known
Nonhyperbolic NMO
After nonhyperbolic moveout
After nonhyperbolic moveout
After nonhyperbolic moveout
After hyperbolic moveout
Nonhyperbolic moveout equation

\[ t^2(x, t_0) = t_0^2 + \frac{x^2}{V_{nmo}^2} - \frac{2\bar{\eta}_{\text{eff}} x^4}{V_{nmo}^2[t_0^2 V_{nmo}^2 + (1 + 2\bar{\eta}_{\text{eff}})x^2]} \]

\[ V_{nmo}(t_0) \quad \text{effective NMO velocity} \]

\[ \bar{\eta}_{\text{eff}}(t_0) \quad \text{effective anellipticity} \]

Alkhalifah and Tsvankin (1995)
Semblance \( s[i, j[i], k[i]] \)

- **time** \( \rightarrow i \)
- \( V_{nmo} \) \( \rightarrow j \)
- \( \bar{\eta}_{\text{eff}} \) \( \rightarrow k \)
Given a semblance spectrum \( s[i, j[i], k[i]] \),

- time \( \rightarrow i \)
- \( V_{nmo} \) \( \rightarrow j \)
- \( \bar{\eta}_{\text{eff}} \) \( \rightarrow k \)
Given a semblance spectrum $s[i, j[i], k[i]]$, find a sequence of sampled $V_{nmo}$ velocities $j[i]$ and a sequence of sampled $\bar{\eta}_\text{eff}$ anellipticities $k[i]$.
Given a semblance spectrum \( s[i, j[i], k[i]] \), find a sequence of sampled \( V_{nmo} \) velocities \( j[i] \) and a sequence of sampled \( \bar{\eta}_{\text{eff}} \) anellipticities \( k[i] \) to maximize \( \sum_i s[i, j[i], k[i]] \).
Given a semblance spectrum $s[i, j[i], k[i]]$, find a sequence of sampled \( V_{nmo} \) velocities $j[i]$ and a sequence of sampled $\bar{\eta}_{eff}$ anellipticities $k[i]$ to maximize

$$\sum_i s[i, j[i], k[i]]$$

such that

$$r_{vl} \leq j[i] - j[i - 1] \leq r_{vu}$$

and

$$r_{\eta l} \leq k[i] - k[i - 1] \leq r_{\eta u}$$
\[
\begin{align*}
\rho_{vl} & \quad \rightarrow \quad \left( \frac{dV_{nmo}}{dt_0} \right)_{\text{min}} \\
\rho_{vv} & \quad \rightarrow \quad \left( \frac{dV_{nmo}}{dt_0} \right)_{\text{max}} \\
\rho_{\eta l} & \quad \rightarrow \quad \left( \frac{d\bar{\eta}_{\text{eff}}}{dt_0} \right)_{\text{min}} \\
\rho_{\eta v} & \quad \rightarrow \quad \left( \frac{d\bar{\eta}_{\text{eff}}}{dt_0} \right)_{\text{max}}
\end{align*}
\]
\[ r_{vl} \rightarrow \left( \frac{dV_{nmo}}{dt_0} \right)_{\text{min}} \]
\[ r_{\eta l} \rightarrow \left( \frac{d\bar{\eta}_{\text{eff}}}{dt_0} \right)_{\text{min}} \]
\[ r_{vu} \rightarrow \left( \frac{dV_{nmo}}{dt_0} \right)_{\text{max}} \]
\[ r_{\eta u} \rightarrow \left( \frac{d\bar{\eta}_{\text{eff}}}{dt_0} \right)_{\text{max}} \]

\[
\frac{dV_{nmo}}{dt_0} = f[\nu_{nmo}(t_0)]
\]

\[
\frac{d\bar{\eta}_{\text{eff}}}{dt_0} = f[\eta(t_0), \nu_{nmo}(t_0)]
\]
Bounds on interval parameters

$v_{nmo}$ (km/s)

0 1 2 3 4 5 6 7 8

Time (s)

lower upper

0 0.2 0.4

Time (s)

lower upper
Bounds on derivatives

\[ \frac{dV_{nmo}}{dt_0} \]

\[ \frac{d\bar{\eta}_{\text{eff}}}{dt_0} \]
Bounds on derivatives

\[ \frac{dV_{nmo}}{dt_0} \quad \frac{d\tilde{\eta}_{eff}}{dt_0} \]
Estimated vs known
$t_0 = 4.0 \text{ s}$

$\bar{\eta}_{\text{eff}}$

$V_{nmo} (\text{km/s})$

Semblance
$t_0 = 5.0 \text{ s}$

- Known
- Estimated
- Peak
Known
Estimated
Peak

\( t_0 = 6.0 \, \text{s} \)

\( \bar{\eta}_{\text{eff}} \)

\( V_{\text{nmo}} \) (km/s)

Semblance