Moving faults while unfaulting 3D seismic images
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SUMMARY

Unfaulting seismic images to correlate seismic reflectors across faults is helpful in seismic interpretation and is useful for seismic horizon extraction. Methods for unfaulting typically assume that fault geometries need not change during unfaulting. However, for seismic images containing multiple faults and, especially, intersecting faults, this assumption often results in unnecessary distortions in unfaulted images. We developed two methods to compute vector shifts that simultaneously move fault blocks and the faults themselves to obtain an unfaulted image with minimal distortions. We test both methods on a synthetic seismic image containing normal, reverse, and intersecting faults, and we also apply one of the methods to a real 3D seismic image complicated by intersecting faults.

INTRODUCTION

Automatic unfaulting of a seismic image often includes two steps. The first step is to estimate fault surfaces and fault slip vectors from the seismic image. The second step is to extend estimated slip vectors away from samples on faults to all samples in the image, and then simultaneously move fault blocks and even faults to obtain an unfaulted image.

For the first step, several methods (Aurnhammer and Tonnies, 2005; Liang et al., 2010; Hale, 2013) have been proposed to estimate fault slip vectors by correlating seismic reflectors on opposite sides of faults. Fault slip estimated in this way is often dip slip, which is a vector in the fault dip direction, representing displacement of the hanging wall side of a fault surface relative to the footwall side. In a seismic image, fault strike slip is typically less apparent than dip slip and is, therefore, more difficult to estimate by correlating seismic reflectors.

To simplify the second step, some authors (Wei et al., 2005; Wei, 2009; Luo and Hale, 2013) assume that fault geometries need not change when unfaulting a seismic image. This assumption makes the unfaulting processing easier, but it might result in unnecessary distortions when unfaulting images with multiple faults and, especially, intersecting faults. For example, in Figure 1, significant distortions are produced in the unfaulted image (Figure 1c) by fixing fault positions. Clearly, the faults, and especially fault A, must also be moved to obtain the unfaulted image with less distortion shown in Figure 1d.

In this paper, we first use the methods described by Wu and Hale (2015a) to automatically compute fault surfaces and dip slip vectors. We then introduce two methods to compute unfaulting vector shifts by solving equations derived from the slips. These computed shifts simultaneously move footwalls, hanging walls, and faults themselves to undo faulting in a seismic image with minimal distortion, as shown in Figure 1d.

METHODS

Prior to unfaulting a 3D seismic image, we first use the method described by Wu and Hale (2015a) to automatically compute fault surfaces (Figure 1a) and fault dip slips. The vertical component of dip slip is fault throw, which is represented by color in (a) and (b). The seismic image is significantly distorted when unfaulted (c) by moving only fault blocks while fixing fault positions. Faults (especially fault A) must also be moved to obtain an unfaulted image (d) with minimal distortions.

Mappings between input and unfaulted spaces
To compute an unfaulted image \( h(w) \) from an input 3D seismic image \( f(x) \), we must know mappings \( x(w) \) and \( w(x) \) that convert coordinates between \( x \equiv (x_1, x_2, x_3) \) (in the input space) and \( w \equiv (w_1, w_2, w_3) \) (in the unfaulted space). We express the mappings \( x(w) \) and \( w(x) \) in terms of shift vector fields \( r(w) \) and \( s(x) \) defined in the unfaulted space and input space, re-
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![Figure 2: A fault slip vector $\mathbf{t}(x_a)$, estimated at each footwall sample adjacent to a fault, tells us how to correlate the image sample at $x_b$ in the footwall to the corresponding sample $x_0$ in the hanging wall.](image)

respectively:

$$x(w) = w + r(w) \quad \text{(1)}$$

and

$$w(x) = x - s(x). \quad \text{(2)}$$

If applied directly to a uniformly sampled input image $f(x)$, the mapping $w(x)$ yields an irregularly sampled unfaulted image $h(w(x)) = f(x)$. Therefore, we use the inverse mapping $x(w)$ and 3D sinc interpolation of $f(x)$ to compute a uniformly sampled image $h(w) = f(x(w))$. However, it can be difficult to directly compute the shift vector field $r(w)$ defined in the unfaulted space, because fault locations and slip vectors are computed in the input space. Therefore, we first solve for $s(x)$ in the input space, and then convert $s(x)$ to $r(w)$ in the unfaulted space, which is then used to compute the mapping $x(w) = w + r(w)$ and the unfaulted image $h(w)$.

Assuming that the mapping between the input and unfaulted spaces is reversible, equations 1 and 2 imply the shift vector field $r(w)$ can be computed from $s(x)$ by $r(w(x)) = s(x)$. We solve this equation for $r(w)$ using an iterative method beginning with an initial shift vector field $r_0(w) = s(w)$:

$$r_0(w) = s(w), \quad x_0(w) = w + r_0(w)$$

$$r_i(w) = s(x_{i-1}(w)), \quad x_i(w) = w + r_i(w) \quad \text{(3)}$$

In this way, we update the shift vector field $r_i(w)$ until the updates are insignificant in the $m$-th iteration to obtain the shift vector field $r(w) \approx r_m(w)$ in the unfaulted space. This iterative process is fast, as only a nearest neighbor interpolation method is needed when computing $r_i(w) = s(w + r_{i-1}(w))$.

In practice, we find that $m = 20$ iterations are sufficient.

Vector shifts in input space

As discussed by Rice (1983), faults can be considered as surfaces of slip (displacement) discontinuity in surroundings with continuous slip. Therefore, to undo faulting apparent in a seismic image, we must compute unfaulting shifts that are continuous in fault blocks and discontinuous at faults. Accordingly, we define equations of unfaulting differently for image samples alongside faults and for those elsewhere within fault blocks.

Figure 2 shows an example of a slip vector $\mathbf{t}(x_a)$ estimated at a sample $x_a$ adjacent to a fault from its footwall. This slip vector indicates that the image sample at $x_a$ in the footwall and the corresponding sample at $x_0 = x_a + \mathbf{t}(x_a)$ in the hanging wall must be located at the same position in the unfaulted space. We then have $w(x_a) = w(x_0)$, which can be rewritten using equation 2 as $s(x_a) = s(x_0) = \mathbf{t}(x_a)$. Because both the shifts $s$ and slips $t$ are vectors, this equation represents three equations, one for each component:

$$s_k(x_0) - s_k(x_a) = t_k(x_a), \quad \text{(4)}$$

where $k = 1, 2, 3$ represent the components of vectors in the crossline, inline and vertical directions, respectively.

Equation 4 applies only to those samples alongside faults. For other samples away from faults, we expect unfaulting shifts to vary slowly and continuously. Thus, derivatives of each component of the vector shift $s(x)$ should be nearly zero:

$$\omega(x) \nabla s_k(x) \approx 0, \quad \text{(5)}$$

where $\omega(x)$ is a weighting function that is zero at image samples adjacent to faults, and is one elsewhere. Therefore, equation 5 is used for all image samples except those adjacent to faults. Having defined unfaulting equation 4 for image samples alongside faults, and smoothing equation 5 for samples elsewhere, we propose two methods to simultaneously solve these equations for $s(x)$ in two different ways.

Method 1

In practice, automatically estimated slip vectors might be inaccurate for some samples on faults. In such a situation, we want to rewrite equation 4 as an approximation and weight the equation using a measure $c(x)$ of the quality of the estimated slip vectors at faults:

$$c(x_0)(s_k(x_0) - s_k(x_a)) \approx c(x_a)t_k(x_a). \quad \text{(6)}$$

For the examples in this paper, the measure $c(x)$ is fault likehood (Wu and Hale, 2015a), which we compute for every image sample location $x$ where the slip vector $\mathbf{t}(x)$ is also estimated.

To compute unfaulting shifts for all samples in an image, we solve equations 5 and 6 simultaneously:

$$\omega(x) \nabla s_k(x) \approx 0,$$

$$\beta c(x_a)(s_k(x_0) - s_k(x_a)) \approx \beta c(x_a)t_k(x_a), \quad \text{(7)}$$

where we have introduced the parameter $\beta$ to balance the two equations. For all examples in this paper, we use $\beta = N/L$, where $L$ is the number of samples on faults and $N$ is the number of all samples in a seismic image. These equations can be represented in matrix-vector form as

$$\begin{bmatrix} W^G C^M & S \end{bmatrix} \begin{bmatrix} s \\ 0 \end{bmatrix} \approx \begin{bmatrix} 0 \end{bmatrix}. \quad \text{(8)}$$

In total, we have $3N + L$ equations for only $N$ unknowns. Therefore, we compute a least-squares solution of equation 8 by solving $G^T W^T WGs + M^C CMs = M^C Ct$, where the first term corresponds to the smoothing equation 5. In practice, fault dip slips typically vary mainly in dip directions, which are often more consistent in directions normal to seismic reflectors than in directions parallel to those reflectors. Therefore, instead of using isotropic smoothing indicated in $G^T W^T WGs$,
we should smooth less for unfaulting shifts in directions normal to reflectors than in directions parallel to reflectors:
\[
G^\top W^\top DWGs + M^\top C^\top CMs = M^\top C^\top Ct. \tag{9}
\]

The matrix \(D\), constructed from structure tensors (Fehmers and Höcker, 2003), is used to implement the anisotropic smoothing for unfaulting shifts. We solve equation 9 for each component of shifts \(s\) using a conjugate gradient (CG) method.

**Method II**

For method I, we assumed that slip vectors are estimated using an automatic method for most samples on faults, and might be inaccurate for some samples. However, in an interactive interpretation system, one might manually pick pairs of points, for example \(x_a\) and \(x_b\) in Figure 2, alongside a fault, and then simply compute corresponding slip vectors \(t(x_a) = x_b - x_a\).

In this case, we expect the unfaulting equation 4 with interpreted slip vectors to be strictly satisfied for manually picked pairs of points alongside a fault. At the same time, however, we still expect shifts to vary smoothly within fault blocks, for all image samples located away from faults. Therefore, for method II, instead of solving equation 9, we compute the unfaulting shifts by solving
\[
G^\top W^\top DWGs = 0 \text{ subject to } Ms = t. \tag{10}
\]

Similar to Wu and Hale (2015b), we use a preconditioned CG method to solve this linear system with hard constraints.

For example, we use slip vectors, estimated on fault surfaces shown in Figure 1a, for both the methods to compute vertical, inline, and crossline components of unfaulting shifts. Figures 3a and 3b show the vertical components of shifts computed using method I and method II, respectively. We observe that the shifts are discontinuous at faults and continuous elsewhere, as expected for both methods.

**Vector shifts in the unfaulted space**

The shifts \(s(x)\) computed by the two methods above are all in the input space. To obtain an unfaulted image, we must use \(r(w)\) that can be computed from \(s(x)\) using the iterative method in equation 3. Figures 4a and 4b show vertical components of vector shifts \(r(w)\) obtained in this way from the vector shifts \(s(x)\) computed in the input space using method I and method II, respectively. After converting to the unfaulted space, the discontinuities of shifts in Figures 4a and 4b are displaced relative to those in Figures 3a and 3b.

Using the converted vector shifts \(r(w)\), we obtain the corresponding unfaulting vector \(x(w) = w + r(w)\) and then compute the unfaulted images shown in Figures 5a (method I) and 5b (method II). In both unfaulted images, seismic reflectors are more continuous than those in the input seismic image. We also observe that the faults are shifted in the unfaulted space relative to the input space. For example, fault A is dislocated in the original seismic image (Figure 1b) by fault B, but is relocated in both unfaulted images, as shown in Figures 5a and 5b.

**APPLICATION**

The synthetic example shown in Figure 5 demonstrates that both methods work well in unfaulting normal, reverse, and intersecting faults. As an additional test, method I was further applied to a real seismic image (Figure 6a) complicated by intersecting faults. From the 3D seismic image, we first used the methods described by Wu and Hale (2015a) to compute the intersecting fault surfaces (Figure 6a) and dip slip vectors, with which we then computed unfaulting vector shifts \(r(w)\) in unfaulted space using method I. The vertical components of the shifts are displayed in Figure 6b.

We then compute the unfaulting mapping \(x(w) = w + r(w)\) and the unfaulted image shown in Figure 6c. For the fault with large slips highlighted by the red arrow in Figure 6a, footwall and hanging wall sides are moved significantly to align the reflectors across the fault, as shown in Figure 6c. From an unfaulted image with seismic reflectors that are continuous across
Figure 6: Fault surfaces and slip vectors (a) are first estimated from a 3D seismic image, and then are used to compute unfaulting shift vectors (b) to obtain an unfaulted image (c), which is finally flattened (d) by using an unfolding method. Only vertical components of slip vectors and unfaulting shift vectors are shown in (a), and (b), respectively.

faults, seismic horizon interpretation is more straightforward.
Here we used the method described by Luo and Hale (2013) to compute vector shifts that undo the folding in the unfaulted image to obtain the flattened image shown in Figure 6d. As discussed by Luo and Hale (2013), using the unfolding and unfaulting vector shifts, we can extract any number of seismic horizons from the input seismic image. Figure 7 shows two extracted seismic horizons colored by depth. Our unfaulting processing facilitates the extraction of such complicated horizon surfaces by aligning seismic reflectors across faults.

CONCLUSION
We have described two methods to efficiently compute vector shifts that simultaneously move fault blocks and faults themselves to undo faulting in 3D seismic images. We suggest using method I when fault slips are estimated automatically for most samples, because the estimated slips might be inaccurate for some samples and this method computes a least-squares solution of the unfaulting equations constructed from the slips. Method II is preferable if fault slips are manually interpreted for only a limited number of samples at faults, because this method considers the interpreted slips as hard constraints when computing unfaulting shifts.

ACKNOWLEDGMENTS
This research is supported by the sponsors of the Consortium Project on Seismic Inverse Methods for Complex Structures. The real 3D seismic image was graciously provided by Kees Rutten and Bob Howard, via TNO (Netherlands Organisation for Applied Scientific Research).

Figure 7: Two horizon surfaces (colored by depth) are extracted using unfaulting and unfolding shift vectors.
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