Automatic and simultaneous correlation of multiple well logs

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Well locations

Courtesy of Rocky Mountain Oilfield Test Center
Well locations

Courtesy of Rocky Mountain Oilfield Test Center
Density logs
After alignment
Aligned with static shifts
Aligned with static shifts
Aligned with static shifts
Aligned with static shifts
Aligned with static shifts
After alignment
After alignment
Log correlation

Depth (km)

Log index

Density (g/cc)
Two logs

Density (g/cc)

2  2.5  3

Depth (km)

0  0.5  1  1.5

log 1

log 2

Density (g/cc)

2  2.5  3

Two logs
Two logs

Density (g/cc)

2.5  3

Depth (km)

1.35
1.4
1.45
1.5
1.55
1.6
1.65
1.7

log 1

log 2
Two logs

Density (g/cc)

2.5  3

Depth (km)

1.35
1.4
1.45
1.5
1.55
1.6
1.65
1.7

\[ f[i, I] \]
\[ I = 1 \]

\[ f[j, J] \]
\[ J = 2 \]
Alignment error

\[ e_{IJ}[i, j] \equiv |f[i, I] - f[j, J]|^{\frac{1}{8}} \]
Alignment error

\[ e_{IJ}[i, j] \equiv |f[i, I] - f[j, J]|^{\frac{1}{8}} \]
Alignment error

\[ e_{IJ}[i, j] \equiv |f[i, I] - f[j, J]|^{\frac{1}{8}} \]
Alignment error

$e_{IJ}[i, j] \equiv |f[i, I] - f[j, J]|^{\frac{1}{8}}$
Alignment error

\[ e_{IJ}[i, j] \equiv \left| f[i, I] - f[j, J] \right|^{\frac{1}{8}} \]
Alignment error

\[ e_{I,J}[i,j] \equiv |f[i,I] - f[j,J]|^2 \]
Alignment error

\[ e_{IJ}[i, j] \equiv |f[i, I] - f[j, J]|^{\frac{1}{8}} \]
$ij$-coordinate system
$ij$-coordinate system
$ij$-coordinate system
$ij$-coordinate system
$k\ell$-coordinate system
$kl$-coordinate system
$k\ell$-coordinate system
Alignment error $e_{IJ}[k, l]$
Alignment error $e_{IJ}[k, l]$
Alignment error $e_{IJ}[k, l]$
Alignment error $e_{IJ}[k, l]$
Pairwise correlation

\[ I = 1 \]

\[ f[i, I] \]

\[ I = 1 \]

\[ f[j, J] \]

\[ J = 2 \]
Well locations

Well locations shown on a crossline and inline map.
Density logs
Density logs

Log index

Depth (km)

Density (g/cc)
Density logs
$13! = 6$ billion possible orderings
13x12/2 = 78 possible log pairs
13x12/2 = 78 possible log pairs
Pairwise correlation

\[ f(z, 1) \]

\[ f(z, 2) \]
Pairwise correlation

\[ f(z, 1) \]

\[ f(z, 2) \]
Pairwise correlation

\[ s(z_i, 1) \]

\[ s(z_j, 2) \]

\[ f(z, 1) \]

\[ f(z, 2) \]
Relative geologic time

\[ \tau(z_i, 1) = z_i + s(z_i, 1) \quad \tau(z_j, 2) = z_j + s(z_j, 2) \]
Relative geologic time

\[ \tau(z_i, 1) = z_i + s(z_i, 1) \quad \tau(z_j, 2) = z_j + s(z_j, 2) \]

\[ \tau(z_i, 1) = \tau(z_j, 2) \]

\[ z_i + s(z_i, 1) = z_j + s(z_j, 2) \]
Relative geologic time

\[ \tau(z_i, 1) = z_i + s(z_i, 1) \quad \tau(z_j, 2) = z_j + s(z_j, 2) \]

\[ \tau(z_i, 1) = \tau(z_j, 2) \]

\[ z_i + s(z_i, 1) = z_j + s(z_j, 2) \]

\[ s(z_i, 1) - s(z_j, 2) = z_j - z_i \]
\[ s(z_i, I) - s(z_j, J) = z_j - z_j \]
\[ s(z_i, I) - s(z_j, J) = z_j - z_j \]

\[ \frac{L(L - 1)}{2} \times N_z = 150,000 \text{ equations} \]

\[ L \times N_z = 25,000 \text{ unknown shifts} \]

\[ N_z : \text{number of depths} \quad L : \text{number of logs} \]
\[ s(z_i, I) - s(z_j, J) = z_j - z_j \]

\[ D_s s \approx d_z \]
\[ s(z_i, I) - s(z_j, J) = z_j - z_j \]

\[ D_s s \approx d_z \]

\[ D_s^T D_s s = D_s^T d_z \]
After alignment
After alignment
After alignment
Squeezed
\[ \tau(z, l) = z + s(z, l) \]
\[ \tau(z, l) = z + s(z, l) \]

\[ z(\tau, l) = \tau - r(\tau, l) \]
\[ \tau(z, l) = z + s(z, l) \]
\[ z(\tau, l) = \tau - r(\tau, l) \]
\[ s(z, l) = r(\tau(z, l), l) \]
\[ r(\tau, l) = s(z(\tau, l), l) \]
\[ s(z_i, I) - s(z_j, J) = z_j - z_j \]
\[ s(z_i, I) - s(z_j, J) = z_j - z_j \]

\[ r(\tau(z_i, I), I) - r(\tau(z_j, J), J) = z_j - z_i \]
\[ r(\tau(z_i, I), I) - r(\tau(z_j, J), J) = z_j - z_i \]
\[ r(\tau(z_i, I), I) - r(\tau(z_j, J), J) = z_j - z_i \]

\[ D_r r \approx d_z \]

\[ D_r^T D_r r = D_r^T d_z \]
Density logs
After alignment
After alignment
After alignment
After alignment
13x12/2 + 11x10/2 + 5x4/2 = 142 possible log pairs
\[ r(\tau(z_i, I), I) - r(\tau(z_j, J), J) = z_j - z_i \]
\[ r(\tau(z_i, I), I) - r(\tau(z_j, J), J) = z_j - z_i \]

\[ \sum_{m=1}^{N_m} \frac{L_m(L_m - 1)}{2} = 270,000 \text{ equations} \]

\[ W \times N_z = 25,000 \text{ unknown shifts} \]

\( N_z \): number of depths  \hspace{1cm} \( W \): number of wells  
\( N_m \): number of log types  \hspace{1cm} \( L_m \): number of logs of type \( m \)
Summary

1. pairwise correlation
   not L$_2$-norm
   rotated coordinate system
2. consistency
   least-squares solution
   relative geologic time shifts
<table>
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<tr>
<th>Era</th>
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<th>Depth</th>
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<td>Permian</td>
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<td>Carlisle Shale</td>
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Formation properties identified by Beyer & Clutsom, 1978

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<th>Porosity (%)</th>
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Formation properties identified in 13 Teapot Dome wells

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</table>
\[ \tau(z, l) = z + s(z, l) \]
\[ \tau(z, l) = z + s(z, l) - \bar{s}(z) \]
\[ \tau(z_i, I) = z_i + s(z_i, I) - \bar{s}(z_i) \]
\[ \tau(z_j, J) = z_j + s(z_j, J) - \bar{s}(z_j) \]
\[ \bar{s}(z_i) \neq \bar{s}(z_j) \]
\[ \tau(z_i, I) \neq \tau(z_j, J) \]
$$\tau(z, l) = z + s(z, l)$$
$$\tau(z, l) = z + r(\tau(z, l), l)$$
$$\tau(z, l) = z + r(\tau(z, l), l) - \bar{r}(\tau(z, l))$$
$$\tau(z_i, I) = z_i + r(\tau(z_i, I), I) - \bar{r}(\tau(z_i, I))$$
$$\tau(z_j, J) = z_j + r(\tau(z_j, J), J) - \bar{r}(\tau(z_j, J))$$
$$\tau(z_i, I) = \tau(z_j, J)$$
$$\bar{r}(\tau(z_i, I)) = \bar{r}(\tau(z_j, J))$$
\[ r(\tau, l) = c(l) + q(\tau, l) \]

\[ c(I) - c(J) = z_j - z_i \]

\[ \frac{L(L-1)}{2} \times N_z = 150,000 \text{ equations} \]

\[ L = 13 \text{ unknown shifts} \]

\[ N_z : \text{number of depths} \quad L : \text{number of logs} \]
\[ r(\tau, l) = c(l) + q(\tau, l) \]

\[ c(I) - c(J) = z_j - z_i \]

\[ D_c c = d_c \]
\[ r(\tau(z_i, I), I) - r(\tau(z_j, J), J) = z_j - z_i \]

\[ (c(I) + q(z_i, I)) - (c(J) + q(z_j, J)) = z_j - z_i \]

\[ q(z_i, I) - q(z_j, J) = z_j - z_i + c(J) - c(I) \]

\[ D_r q \approx d_u \]

\[ D_r^T D_r q = D_r^T d_u \]