Moving faults while unfaulting 3D seismic images

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1: Center for Wave Phenomena
2: BP America Inc
Unfaulting and unfolding
Step one: fault positions and slips
Step two: unfaulting
Step three: unfolding

(Simon Luo and Dave Hale, 2013)
Step two: unfaulting
Input seismic image
Four faults
Fault surfaces and slips
Fault positions and slips
Input seismic image
Fix faults while unfaulting

(Simon Luo and Dave Hale, 2013)
Move faults while unfaulting
Input and unfaulted spaces

input image: $f(x)$

unfaulted image: $h(w)$

$h(w)$ = $w + r(w)$

$x(w)$ = $s(x)$

foot wall

hanging wall
Vector shifts $s(x) \& r(w)$

input image: $f(x)$

unfaulted image: $h(w)$

$h(w) = f(x)$

$x(w) = w + r(w)$

$w(x) = x - s(x)$

hanging wall

foot wall
w(x) or x(w)?

irregularly sampled \( h(w(x)) = f(x) \)

regularly sampled \( h(w) = f(x(w)) \)

\[
\begin{align*}
w(x) &= x - s(x) \\
x(w) &= w + r(w)
\end{align*}
\]
Vector shifts $s(x) \& r(w)$

input image: $f(x)$

unfaulted image: $h(w)$

$h(w) = f(x)$

$h(w) = f(x - s(x))$

$h(w) = f(x + r(w))$

$h(w) = f(x)$

$h(w) = f(x - s(x))$

$h(w) = f(x + r(w))$
Vector shifts $s(x)$ & $r(w)$

\[
\begin{align*}
w(x) &= x - s(x) \\
x(w) &= w + r(w)
\end{align*}
\]

\[
\begin{align*}
s(x) &= r(w(x)) \\
r(w) &= s(x(w))
\end{align*}
\]
Compute $r(w)$ from $s(x)$

\[ r(w) = s(x(w)) \]

\[ x(w) = w + r(w) \]
Compute shifts in the input space
Fault slip vector $\mathbf{t}(\mathbf{x}_f)$

$\mathbf{x}_f$: adjacent to faults from foot wall

$\mathbf{x}_h$: adjacent to faults from hanging wall

$x_h = x_f + t(x_f)$
Unfaulting equation

\[ w(x_f) = w(x_h) \]

same coordinates in unfaulted space \( w \)

\[ x_h = x_f + t(x_f) \]
Unfaulting equation

\[ w(x_f) = w(x_h) \]

\[ x_f - s(x_f) = x_h - s(x_h) \]

\[ w(x) = x - s(x) \]

\[ x_h = x_f + t(x_f) \]
Unfaulting equation

\[ w(x_f) = w(x_h) \]
\[ x_f - s(x_f) = x_h - s(x_h) \]
\[ s(x_h) - s(x_f) = t(x_f) \]
\[ x_h = x_f + t(x_f) \]
Unfaulting equation

\[ s(x_h) - s(x_f) = t(x_f) \]

\[ sk(x_h) - sk(x_f) = tk(x_f) \]

\[ k = 1, 2, 3 \]

inline, crossline and vertical components
Vertical component

\[ s_3(x_h) - s_3(x_f) = t_3(x_f) \]

\( x_f \): adjacent to faults from foot wall

\( x_h \): adjacent to faults from hanging wall
$t_3(\mathbf{x}_f)$ might be inaccurate

$$s_3(\mathbf{x}_h) - s_3(\mathbf{x}_f) \approx t_3(\mathbf{x}_f)$$

$\mathbf{x}_f$: adjacent to faults from foot wall

$\mathbf{x}_h$: adjacent to faults from hanging wall
Weighted unfaulting equation

\[ c(\mathbf{x}_f) \left[ s_3(\mathbf{x}_h) - s_3(\mathbf{x}_f) \right] \approx c(\mathbf{x}_f) t_3(\mathbf{x}_f) \]

- \( c(\mathbf{x}_f) \): quality of \( t_3(\mathbf{x}_f) \)
- \( \mathbf{x}_f \): adjacent to faults from foot wall
- \( \mathbf{x}_h \): adjacent to faults from hanging wall
Smooth equation

$$\nabla s_3(x_b) \approx 0$$

$$x_b : \text{samples away from faults}$$

shifts vary slowly for samples away from faults
Equations for all samples

\[ \nabla s_3(x_b) \approx 0 \]

\[ c(x_f)[s_3(x_h) - s_3(x_f)] \approx c(x_f)t_3(x_f) \]

- \( x_b \): samples away from faults
- \( x_f \): samples adjacent to faults from foot wall
- \( x_h \): samples adjacent to faults from hanging wall
Matrix-vector form

\[
\begin{bmatrix}
WG \\
CM
\end{bmatrix} \begin{bmatrix} \mathbf{s} \end{bmatrix} \approx \begin{bmatrix} \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{Ct} \end{bmatrix}
\]

\(N\) : number of all samples

\(L\) : number of samples on faults

\(3N + L\) equations for \(N\) unknowns
Least-squares solution

$$(\mathbf{WG})^\top \mathbf{WGs} + (\mathbf{CM})^\top \mathbf{CMs} = (\mathbf{CM})^\top \mathbf{Ct}$$
Convert shifts into unfaulted space
Compute $r(w)$ from $s(x)$

\[ r(w) = s(x(w)) \]

\[ x(w) = w + r(w) \]
Iterative method

initial: \( r_0(w) = s(w) \)

\[
\begin{align*}
    r(w) &= s(x(w)) \\
    x(w) &= w + r(w)
\end{align*}
\]
Iterative method

initial: \[ r_0(w) = s(w) \]

1st iteration: \[ r_1(w) = s(x_0(w)) \]
Iterative method

initial: \[ r_0(w) = s(w) \]

1st iteration: \[ x_0(w) = w + r_0(w) \]
\[ r_1(w) = s(x_0(w)) \]

\( \ddots \)

\( i \)-th iteration: \[ x_{i-1}(w) = w + r_i(w) \]
\[ r_i(w) = s(x_{i-1}(w)) \]

\( \ddots \)

\[ r(w) = s(x(w)) \quad x(w) = w + r(w) \]
Iterative method

initial: \[ r_0(w) = s(w) \]

1st iteration: \[ x_0(w) = w + r_0(w) \]
\[ r_1(w) = s(x_0(w)) \] \[ \text{nearest interpolation} \]

\[ \vdots \]

\[ \vdots \]

\[ N\text{-th iteration: } r(w) \approx r_N(w) = s(w + r_{N-1}(w)) \]

\[ r(w) = s(x(w)) \]
\[ x(w) = w + r(w) \]
Fault positions and slips
Vertical shifts (input space)
Vertical shifts (unfaulted space)
Inline shifts (input space)
Inline shifts (unfaulted space)
Crossline shifts (input space)
Crossline shifts (unfaulted space)
Input seismic image
Unfaulted seismic image
Input 3D real seismic image
Fault surfaces and slips
Fault positions and slips
Vertical unfaulting shifts
Seismic image
Unfaulted image
Horizon 1
Horizon 2
Horizon 3
Horizon 4
Unfaulting and unfolding
Backup slides
Matrix-vector form

\[
\begin{bmatrix}
WG \\
CM
\end{bmatrix} s \approx \begin{bmatrix}
0 \\
C_t
\end{bmatrix}
\]

\(N\) : number of all samples

\(L\) : number of samples on faults

\(3N + L\) equations for \(N\) unknowns
\[
\begin{bmatrix}
WG \\
\beta CM
\end{bmatrix}
\sim
\begin{bmatrix}
0 \\
\beta Ct
\end{bmatrix}
\]

\[
\beta = \frac{N}{L}, \quad N > L
\]
Least-squares solution

\[ (WG)^\top WG_s + \beta^2(CM)^\top CM_s = \beta^2(CM)^\top Ct \]

\( (WG)^\top WG_s \): smoothing term

\( \beta^2(CM)^\top CM_s \): unfaulting term
Anisotropic smoothing

\[(WG)^\topDWGs + \beta(CM)^\topCMs = \beta(CM)^\topCt\]

\[D(x) : \text{structure tensor computed from a seismic image}\]
Linear system with hard constraints

\[(WG)^\top WGs = 0 \text{ subject to } Ms = t\]
Two methods

method one:

\[(WG)^\top WG_s + \beta^2 (CM)^\top CM_s = \beta^2 (CM)^\top Ct\]

method two:

\[(WG)^\top WG_s = 0 \text{ subject to } Ms = t\]
Vertical shifts (input space)

method two
Vertical shifts (unfaulted space)

method two
Seismic image
method two
method one

Unfaulted image