Waveform inversion for microseismic source parameters: Synthetic and field data applications

Oscar Jarillo Michel* and Ilya Tsvankin

Center for Wave Phenomena
Colorado School of Mines
Objectives

- test with limited aperture
- test with noise
- application to field data
Wave equation

\[ \rho \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial}{\partial x_j} \left( c_{ijkl} \frac{\partial u_k}{\partial x_l} \right) = -M_{ij} \frac{\partial [\delta(x - x^s)]}{\partial x_j} S(t) \]

\[ M = \begin{pmatrix} M_{11} & M_{13} \\ M_{13} & M_{33} \end{pmatrix} \]
2D fault geometry
Moment-tensor elements

\[ M_{11} = -\frac{\Sigma D}{2} (c_{13} - c_{11}) \sin 2\theta \]

\[ M_{13} = \Sigma D c_{55} \cos 2\theta \]

\[ M_{33} = -\frac{\Sigma D}{2} (c_{33} - c_{13}) \sin 2\theta \]
Model parameters

- source coordinates: $x_1^s$ and $x_3^s$
- origin time: $t_0$
- moment-tensor elements: $M_{11}$, $M_{33}$, and $M_{13}$
Input data

- in-plane polarized waves (P and SV)

- horizontal and vertical displacements:

  \[ u_1(x^s, x^{fn}, t), \quad u_3(x^s, x^{fn}, t) \]
Objective function

\[ F(m) = \frac{1}{2} \| d_{\text{pre}}(m) - d_{\text{obs}} \|^2 \]

Adjoint-state method

\[
\frac{\partial F}{\partial x_i^s} = \int_0^T \frac{\partial [M : \epsilon^\dagger(x^{ts}, t)]}{\partial x_i} \bigg|_{x^{ts}} S(T - t) \, dt
\]
Dislocation source in VTI medium

\[ \epsilon = 0.4, \delta = 0, \sigma = 0.94 \]
Adjoint wavefield: $u^\dagger$
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Adjoint wavefield: $u^\dagger$
Homogeneous VTI model

$\epsilon = 0.4$, $\delta = 0$, $\sigma = 0.94$
Limited aperture

\[ \epsilon = 0.4, \ \hat{\delta} = 0, \ \sigma = 0.94 \]
Inversion for $x^s$ and $M$ ($t_0$ fixed)
Inversion for $x^s$ and M (full aperture)
Estimated $\chi_1^S$
Estimated $x_3^S$
Estimated $M_{11}$
Estimated $M_{33}$
Estimated $M_{13}$

Iteration number

$M_{13}$ (GN.m)
Homogeneous VTI model

$\epsilon = 0.4, \delta = 0, \sigma = 0.94$
Vertical displacement with noise
Inversion for $x^s$ and M ($t_0$ fixed)
Inversion for $x^S$ and M (without noise)
Estimated $\chi_1^S$
Estimated $M_{11}$
Estimated $M_{13}$

![Graph showing the estimated $M_{13}$ with iteration number on the x-axis and $M_{13}$ (GN.m) on the y-axis. The graph displays a trend where the values fluctuate around a horizontal line at $M_{13}$ value of 14.](image-url)
Case study

- Bakken field
- five-layer VTI model
- two near-vertical monitor wells
- direct P- and SV-waves
### VTI velocity model

<table>
<thead>
<tr>
<th>Layer</th>
<th>( V_{P0} ) (km/s)</th>
<th>( V_{S0} ) (km/s)</th>
<th>( \epsilon )</th>
<th>( \delta )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>4.56</td>
<td>2.72</td>
<td>0.10</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>UB</td>
<td>3.16</td>
<td>2.01</td>
<td>0.37</td>
<td>-0.01</td>
<td>0.33</td>
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<tr>
<td>MB</td>
<td>4.63</td>
<td>2.83</td>
<td>0.01</td>
<td>0.17</td>
<td>-0.12</td>
</tr>
<tr>
<td>LB</td>
<td>2.81</td>
<td>1.97</td>
<td>0.27</td>
<td>0.19</td>
<td>0.35</td>
</tr>
<tr>
<td>TF</td>
<td>4.17</td>
<td>2.38</td>
<td>0.09</td>
<td>0.16</td>
<td>0.13</td>
</tr>
</tbody>
</table>

(Grechka and Yaskovich, 2014)
Geometry of experiment

- Source
- Receivers
Vertical displacement

![Graph showing vertical displacement over time](image-url)
Field data

- entire coda cannot be simulated
- wavelet unknown
- no initial estimates of $M$
- high frequencies (400-500 Hz)
Field data

- entire coda cannot be simulated
- wavelet unknown
- no initial estimates of $M$
- high frequencies (400-500 Hz)
Vertical displacement
Windowed vertical displacement

\[ \text{x}_3 (\text{m}) \]

\[ t (\text{s}) \]

P

SV
Field data

- entire coda cannot be simulated
- wavelet unknown
- no initial estimates of $M$
- high frequencies (400-500 Hz)
Windowed vertical displacement

![Graph showing windowed vertical displacement with time (t) on the x-axis and displacement (x3) on the y-axis. The graph has horizontal lines indicating displacement at different depths and vertical lines showing time intervals.]
Extracted wavelet
Field data

- entire coda cannot be simulated
- wavelet unknown
- no initial estimates of $M$
- high frequencies (400-500 Hz)
Synthetic data \((M_{11} = M_{33} = 0, M_{13} = -7 \cdot 10^{10} \text{ N} \cdot \text{m})\)
Inversion for $x^s$ and $M$ ($t_0$ fixed)
Synthetic data \( (M_{11} = M_{33} = 0, M_{13} = -7 \cdot 10^{10} \, \text{N} \cdot \text{m}) \)
Data with inverted model
Windowed vertical displacement

![Graph showing windowed vertical displacement with time (t) on the x-axis and displacement (x3) on the y-axis, with P and SV waves indicated.](image-url)
Estimated $x_1^S$
Estimated $x_3^s$
Summary

- elastic WI for microseismic data
- source location, origin time, and moment tensor
- aperture can be limited
- stable in presence of moderate noise
- preliminary field-data results
Future work

- multiscale approach
- anisotropic velocity analysis
- 3D inversion
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