Expanded abstracts submitted by CWP to the

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Vienna, Austria
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16 - 21 October 2016

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An optimal parameterization for full waveform inversion in anisotropic media

Tariq Alkhalifah (King Abdullah University of Science and Technology) and Antoine Guitton (Center for Wave Phenomena)

Summary

Full waveform inversion (FWI) in transversely isotropic media usually requires abundant a priori information, like well data and smoothness assumptions, to make the FWI converge to a plausible solution. The proper model parameterization in transversely isotropic (TI) media with a vertical symmetry axis (VTI) can alleviate some of these limitations. Considering the limitations of our field data acquisition to constrain the long wavelength vertical velocity (the depth information) in VTI media, we test a parameterization using the horizontal velocity, $\eta$ and $\varepsilon$ that is relatively immune from this specific lack of long wavelength description of the background model. We test our claim with a VTI Marmousi II data set modeled under the elastic assumption. The initial velocity is extracted from a smoothed version of the true normal moveout and horizontal velocity models to represent what we expect from the long wavelength analysis of our data (traveltimes). Having an inaccurate $\delta$ (equal to zero in this test) caused the inversion parameterized by the vertical velocity, $\delta$, and $\varepsilon$ to yield worse results than the inversion parameterized by the horizontal velocity, $\eta$ and $\varepsilon$. 
The vector seismic wavefield approaching location \( x \), we can write the single-scattered wavefield (Alkhalifah and Plessix, 2014)

\[
u_s(k_s, k_r, \omega) = -\omega^2 s(\omega) \int d^3x \frac{G(k_s, x, \omega)G(k_r, x, \omega)}{v_0^2(x)p(x)} a(x) \cdot r(x)
\]

with \( s \) is the source function, \( p \) is the density, \( v_0 \) is the background isotropic velocity, and

\[
r = \begin{pmatrix} r_{v_h} \\ r_{\eta} \\ r_{\epsilon} \\ r_{v_i} \end{pmatrix}, \quad a = \begin{pmatrix} 2 \\ -n_z^2 n_{xx} - n_x^2 n_{zz} \\ -n_x^2 n_{zz} + n_z^2 n_{xx} \\ 4n_z^2 n_{xx}^2 + 4n_x^2 n_{zz}^2 \end{pmatrix}
\]

The role of \( \epsilon \) in this case is \( \epsilon = \epsilon_0 \) in this case is.

The scattering potential in elastic media

For acoustic VTI media, Alkhalifah and Plessix (2014) derived such patterns for different anisotropic parameter combinations that do seem to be the most practical. Later, Alkhalifah (2015) made the argument for one of these combinations, \( v_{\eta h}, \eta, \xi \), for FWI of conventionally acquired surface seismic P-wave data. Considering the asymptotic Greens function, \( G(x, k, \omega) \) expressed in the frequency, \( \omega \), domain, for a plane wave described by the wavenumber vector, \( k \), for either the source or receiver wavefields approaching location \( x \), we can write the single-scattered wavefield (Alkhalifah and Plessix, 2014)

\[
u_s(k_s, k_r, \omega) = -\omega^2 s(\omega) \int d^3x \frac{G(k_s, x, \omega)G(k_r, x, \omega)}{v_0^2(x)p(x)} a(x) \cdot r(x)
\]

The vector \( r \) includes the perturbations of the individual parameters, \( v_{\eta h}, \eta, \epsilon, \) and \( v_i \), from top to bottom. Thus, the coefficients of \( a \) are the parameters for each parameter for the given parameterization (Aki and Richards, 1980). The components of the source, \( \{n_x, n_z\} \), and the receiver, \( \{n_x, n_z\} \), plane wave unit vectors for reflection from a horizontal reflector are given by \( \{\sin(\theta_i), \cos(\theta_i)\} \) and \( \{-\sin(\theta_j), \cos(\theta_j)\} \), respectively, where \( \theta_i \) is the source incident angle (for horizontal reflectors, it is half the scattering angle).

In Figures 1a and 1b, we show the reflection P-wave radiation patterns for perturbations in the anisotropy VTI parameters for two different parameterizations. The radiation pattern for a perturbation in \( v_{\eta h} \) has a behavior similar to that of \( \delta \) or \( \eta \). However, for conventional offset surface seismic data, the scattering influence of \( v_i \) (like \( \eta \) or \( \delta \)) on surface P-wave data is small, and thus, can be neglected. Since the long wavelength components of \( v_i \) have little influence on P-wave propagation (Alkhalifah, 1998), and for simplicity, we can ignore shear waves all together for P-wave inversion. The amplitude disparity will
Figure 1 The reflection radiation patterns from a horizontal reflector for the two sets of parameters describing an elastic VTI model. The polar component describes the opening angle between the incidence and reflected wave path, while the radial component expresses the relative amplitude of scattering.

hopefully be absorbed by another parameter, specifically $\epsilon$ in the suggested parameterization. There is, however, more to this story when recorded shear waves are involved in the analysis.

Figure 2 True (top), initial (middle), and inverted (bottom) models for $v_r$ (left), $\delta$ (middle), and $\epsilon$ (right).

An elastic VTI Marmousi II model

The VTI parameters used in the elastic modeling are shown in the top row of Figure 2. The shear wave and density models (not shown here) follow the same structure as the $P$-wave velocity model. In all the examples, the modeling engine is the same as that used in the inversion (the so called inversion crime), as the purpose of this study is to focus on the tradeoff. For the same purpose, our analysis we utilize frequencies below 1 Hz in the inversion. We will show real data examples of the inversion with the various parameterization in the presentation. Our synthetic dataset mimics a marine acquisition survey with 67 shots every 200 m. and a maximum offset of 5 km. Since we can usually obtain smooth $v_{nmo}$ and $\eta$ from surface seismic $P$-wave data using for example tomographic methods, the starting model is constructed by smoothing the exact $v_{nmo}$ and $v_h$ models with a window length of 1.5 km. Such smoothing actually results in even a smoother $\eta$ (Figure 3 second row middle), which we tend to expect from tomographic inversion methods (compared to velocity). The $\delta$ model, as usually done without well information, is set to zero (Figure 2 second row middle). In this case, we expect a considerable
depth error in the inverted parameters. Our objective here is to test the tradeoff and convergence for the various parameterizations, thus we use the true $\delta$ to map the inverted results to their expected depth. The mapping process given by $z' = z\sqrt{1 + 2\delta}$ is an approximate correction as lateral variation in $\delta$ influences data recorded on the surface (Alkhalifah et al., 2001). Finally, $v_s$ and density are not updated in the inversion and are equal to a constant value (average value of exact models) in the whole sedimentary section below the water bottom.

**Standard $v_s$, $\delta$ and $\epsilon$ parameterization:** It is widely used in industry (Vigh et al., 2014; Baumstein, 2014). Figure 1a shows the radiation pattern for this parameterization (Gholami et al., 2013). For conventional offset-to-depth ratios ($< 2$), the scattering wavelengths of $\delta$ will have little influence on the data. Despite that $\delta$ has a small imprint for this 5 km offset data, we will invert for it as well. After 15 iterations of EFWI per frequency scale starting from 1 Hz to 11 Hz using an LBGFS approximation of the Hessian, we end up with the inverted models shown in Figure 2 bottom row after applying a depth correction to the inverted models. The inverted vertical velocity shows generally some features of the true model structure, but with higher velocities in some places (Figures 4a and 4b). The $\delta$ model, as expected looks erroneous, with limited information added to the initial $\delta$ model. Finally, the $\epsilon$ model also, despite the data sensitivity to it with such parameterization, looks erroneous, especially up shallow.

**Optimal $v_h$, $\eta$ and $\epsilon$ parameterization:** Here, the radiation patterns (Figure 1b) resemble that of the previous parameterization with a change in the role that $\epsilon$ plays. Now $\epsilon$ helps in fitting the reflectivity whereas before in Figure 1a, $\epsilon$ would get mostly updated from the diving waves/long offset data. The parameter $\eta$, like $\delta$, has a minor role to play in FWI. After a similar frequency continuation LBFGS EFWI with the same number of iterations, we end up with the models shown in Figure 3 bottom row. The horizontal velocity looks more similar to $v_h$, but with more accurate values (Figures 4c and 4d). More importantly, $\epsilon$ now captures the reflectivity, as it absorbed the amplitude mismatch of the elastic assumption, which caused over estimation of velocity in the case of the vertical velocity parameterization.

**Discussions**

The radiation patterns studied in previous papers for acoustic VTI media considered a background isotropic model. Since, we were looking at local perturbations (smaller than the dominant wavelength), we assumed that the scattering behavior would generally hold for an anisotropic background. Thus, many conclusions were drawn from the radiation patterns to extract proper inversion strategies. This
study was conducted to test these assertions and hopefully justify the suggested parameterization even for anisotropic background. Since the convergence of EFWI in the performed complicated tests required updates based on anisotropic backgrounds, we can safely conclude that the suggested parameterization has many of the promoted benefits. In another test, we ran EFWI with our optimal parameterization inverting for \( v_h \) and \( \varepsilon \) only: the results came out generally similar to what is shown in Figure 3 bottom row, at the reduced cost of not bothering with updating \( \eta \).

**Figure 4** Vertical profiles at 8 km (a and c) and 12 km (b and d) laterally of the \( v_v \) model (a and b) from Figure 2 (left) and the \( v_h \) model (c and d) from Figure 3 (left).

**Conclusions**

We numerically tested EFWI with a VTI model parametrized by \( v_h \), \( \eta \), and \( \varepsilon \). As the radiation patterns for this parameterization suggested, the shear wave velocity and \( \eta \) have minor influence on the inversion of seismic P-wave data with a reasonable offset range (up to 5 km in the test). The density effect is absorbed by \( \varepsilon \) as they share almost the same scattering behavior. Thus, using an initial velocity model given by an accurate background NMO velocity and \( \eta \) (\( \delta \) is set to zero), we compared EFWI results using the \( v_h \) parameterization, with the commonly used Thomsen’s parameter representation of the model. The \( v_v \) parameterization results, despite the inaccurate \( \delta \) model, provided a reasonable velocity, better than that given by the conventional parameterization.

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Full-waveform inversion in an anisotropic elastic Earth - Can we isolate the role of density and shear wave velocity?

Antoine Guitton (Center for Wave Phenomena) and Tariq Alkhalifah (King Abdullah University of Science and Technology)

Summary

Five parameters are needed to describe VTI anisotropy in 2-D elastic media. With conventional streamer data, an optimal parameterization for full-waveform inversion (FWI) consists in using the horizontal velocity $v_h$, the anellipicity parameter $\eta$, and the parameter that relates the horizontal-to-vertical velocity, $\varepsilon$. This parameterization, derived with a pseudo-acoustic formulation of the wave equation, minimizes crosstalks. We extend this analysis to the elastic case and wonder what are the effects of density and shear-wave velocity on the inversion results if ignored. Using radiation patterns derived in the elastic case and a modified version of the Marmousi II model, we see that PP-waves are mostly helping to recover $v_h$ and $\varepsilon$, while PS-waves are mostly helping to recover $\eta$. For all scattering angles and wave modes, $\varepsilon$ and $\rho$ are strongly coupled. Keeping $\rho$ and $v_s$ unchanged during the inversion is a valid strategy if their background models are good enough. For small $v_s$ and $\rho$ errors, the $v_s$ perturbation will map into $\eta$ while $\rho$ will map into $\varepsilon$. For large errors, $v_s$ adversely affects the recovery of $\eta$ and might leak into $v_h$ while $\rho$ will preferentially leak into $\varepsilon$, then $\eta$ and, in smaller proportions, into $v_h$. 
Introduction

In Alkhalifah and Guitton (2016), we analyze the scattering potentials of perturbations of VTI parameters to (1) unravel the crosstalks between parameter classes and (2) propose an optimal parameterization using the horizontal velocity \( v_h \), the anellipcity parameter \( \eta \), and the parameter that relates the horizontal-to-vertical velocity, \( \varepsilon \). This analysis, contrary to the work of Gholami et al. (2013), is based on the elastic wave equation. As such, it brings more insights on the influence of all parameters in the Elastic Full Waveform Inversion (EFWI) of field data, where the Earth is assumed to be closer to an elastic than acoustic medium. In an effort to better understand the influence of all VTI elastic parameters, now we switch our focus on the role played by density (\( \rho \)) and S-wave velocity (\( v_s \)) in EFWI for marine data acquisitions only in the proposed \( v_h, \eta \) and \( \varepsilon \) parameterization. Focusing on marine data simplifies the analysis because S-waves are coming from mode conversions only, and thus, are second-order events in our data. Also, the inversion of marine data is simpler than the inversion of land data, thus giving us a chance to test our findings on field datasets more easily. For this work, we set ourselves in a field dataset mindset and want to understand the consequences of not inverting for density and \( v_s \) in EFWI, focusing only on the retrieval of \( v_h, \eta \) and \( \varepsilon \). To this end, we first present radiation patterns for PP and PS-waves for a horizontal layer for all five parameters \((v_h, v_s, \rho, \eta \text{ and } \varepsilon)\). Then we illustrate our derivations with inversions of a modified Marmousi II synthetic model. We learn that for small \( \rho \) and \( v_s \) perturbations, density maps preferentially in the \( \varepsilon \) field, while S-wave velocity maps in the \( \eta \) field, in agreement with the asymptotic (Born approximated) analysis that yields the derivation of the radiation patterns.

Scattering potentials in a VTI elastic media parameterized with \( v_h, v_s, \rho, \eta \text{ and } \varepsilon \)

For FWI of conventionally acquired surface seismic P-wave data, in an acoustic VTI media, Alkhalifah (2015) made the argument for the combination \( v_h, \eta, \text{ and } \varepsilon \). Considering the asymptotic Greens function, \( G(x, k, \omega) \) expressed in the frequency, \( \omega \), domain, for a plane wave described by the wavenumber vector, \( k \), for either the source or receiver wavefields approaching location \( x \), we write the single-scattered wavefield (Alkhalifah and Plessix, 2014)

\[
\begin{align*}
\hat{u}^\text{PP,PS}(k_x, k_z, \omega) &= -\omega^2 s(\omega) \int dx \frac{G(k_x, x, \omega)G(k_z, x, \omega)}{v_0^2(x)\rho(x)} a_{\text{PP,PS}}(x) \cdot r(x) \\
\end{align*}
\]

with \( s \) is the source function, \( \rho \) is the density, \( v_0 \) is the background isotropic velocity, and \( r^t = (r_{v_h}, r_{v_s}, r_{\rho}, r_{\eta}) \) with (\( \sigma \) is the Poisson’s ratio, \( \theta_i \) and \( \theta_r \) are the incident and reflection angles, respectively)

\[
\begin{align*}
\mathbf{a}_\text{PP} &= \begin{pmatrix}
2 \\
-2(\sin 2\theta_i + \sin 2\theta_r) \\
2\cos(\theta_i)\cos(\theta_r)
\end{pmatrix}, \\
\mathbf{a}_\text{PS} &= \begin{pmatrix}
0 \\
\frac{(1-2\sigma)}{1-\sigma} \sin(\theta_i - \theta_r) + \cos(\theta_i - \theta_r) \\
\frac{(1-2\sigma)}{1-\sigma} \sin(\theta_i - \theta_r)
\end{pmatrix}.
\end{align*}
\]

The vector \( r \) includes the perturbations of the individual parameters, \( v_h, v_s, \eta, \varepsilon \), and \( \rho \) from top to bottom. Thus, the coefficients of \( \mathbf{a}_\text{PP} \) and \( \mathbf{a}_\text{PS} \) define the radiation patterns of each parameter for the given parameterization (Aki and Richards, 1980).

For \( \sigma = 0.25 \) (between carbonates and sandstones), Figure 1a shows the reflection PP-wave radiation patterns while Figure 1b shows the reflection PS-wave radiation patterns. For PP-waves we see the close similarity between \( \rho \) and \( \varepsilon \): they both absorb the amplitude information contained in the short-offsets data. Therefore, in our proposal of inverting for \( v_h, \eta \) and \( \varepsilon \), only, \( \rho \) will contain both density and \( \varepsilon \) information. On the other hand, \( v_s \) and \( \eta \) have a weak influence on small scattering angles and only appear for large angles: for streamer data, \( v_s \) and \( \eta \) have a small imprint and can’t be easily inverted for. Now, for PS-waves, we first notice that the scattering amplitudes are weaker than for PP-waves, as expected, and that the patterns are more complex. Second, we notice that \( v_h \) has no influence on PS-waves scattering. The density has the strongest amplitude, but for very large angles. For short angles

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Figure 1 Radiation patterns for (a) PP-waves and (b) PS-waves for opening angles $\theta_i + \theta_r$ from $0^\circ$ to $180^\circ$. Although not shown, the patterns are symmetric around the vertical axis for the remaining angles. Note that the amplitude of the patterns in (b) are multiplied by 1.5 compared to (a) for better display.

we see the dominance of $v_s$ and $\eta$ scatterings, with $\eta$ being the weakest. $\epsilon$ scatters at medium-to-large angles. Therefore in our scheme, if $v_s$ is not inverted for and if conventional streamer data are used, we should see a mapping of $v_s$ perturbations in $\eta$ for small scattering angles and in $\epsilon$ for large angles.

Finally, we see that for our parameterization and conventional streamer data, $\eta$ is basically unaffected by the PP-waves, in agreement with Alkhalifah (2015), and its small wavenumbers can only be recovered with PS-waves at small scattering angles. We also see that there is strong crosstalk between $\epsilon$ and $\rho$ for both PP- and PS-waves, which validates the strategy to invert for one only thus gathering both informations in one field. Now we illustrate our findings with a modified VTI Marmousi II model.

An elastic VTI Marmousi II model

In this section, we test how $\rho$ and $v_s$ affect a VTI anisotropic EFWI when they are not inverted for. For this, we model a streamer survey with an explosive source and recorded as a pressure component only. $v_h$, $\eta$ and $\epsilon$ are shown in the top row of Figure 2. To better understand how $\rho$ affects the inversion, we build a simple density model made of two flat interfaces. The first layer is the water layer ($z=500$ m.) and has a constant density of $1\, \text{kg/m}^3$. The second layer at $z=1.5$ km with a density of $2\, \text{kg/m}^3$ and the third layer has a density of $2.4\, \text{kg/m}^3$ (i.e., 20% increase). To better understand how $v_s$ affects the inversion, we also build a simple velocity model made of two flat interfaces as well. The first layer is the water layer ($z=500$ m.) and has a constant velocity of $0\, \text{km/s}$ and the second layer at $z=2$ km. (500 m. below the second density layer) has a velocity of $1\, \text{km/s}$ and the third layer has a velocity of $1.2\, \text{km/s}$ (i.e., 20% increase as well).

With these parameters, we model and invert 67 shots every 225 meters with a maximum offset of 5 km. The inversion is conducted in the time domain with five frequency bands (0-3 Hz, 0-5 Hz, 0-7 Hz, 0-9 Hz, 0-11 Hz) and 15 iterations per band, using a l-BFGS solver. The starting models for our first results are shown in the second row of Figure 2. The initial models for the density and velocity are assumed to be correct for the water layer ($\rho=1\, \text{kg/m}^3$ and $v_s=0\, \text{km/s}$) and constant in the sediments ($\rho=2\, \text{kg/m}^3$ and $v_s=1\, \text{km/s}$). The inversion updates $v_h$, $\eta$ and $\epsilon$ only.

EFWI results for this setup are shown in the bottom row of Figure 2. In Figure 2a, $v_h$ is well recovered and we don’t see any effect of the density layer (top black arrow) or $v_s$ layer (bottom black arrow). For $\eta$ in Figure 2b, we recover some details of the model due to the converted PS-waves. We can also see the footprint of the $v_s$ layer contrast as shown by the bottom black arrow, especially between X=8-10Km. There is a hint of the density layer as well. For $\epsilon$, the model is well recovered and the footprint of the density layer marked by the top black arrow is pronounced. Contrary to the $\eta$ inversion result, we don’t see any noticeable leakage of the $v_s$ layer into $\epsilon$. 
Figure 2 Top row: exact models. Middle row: starting models. Bottom row: inverted models for (a) $v_h$, (b) $\eta$, and (c) $\varepsilon$. The $v_s$ contrast maps preferentially into the inverted $\eta$ model (bottom black arrow) while the $\rho$ contrast map preferentially into the inverted $\varepsilon$ model. $\rho$ and $v_s$ contrasts are equal to 20%.

Next, we decided to invert a new dataset where the initial $v_h$, $\eta$ and $\varepsilon$ models are still as in Figure 2 but where $\rho$ and $v_s$ have stronger contrasts between the second and third layers. Now, with flat layers still at the same position ($\rho$ layer at $z=1.5$ km. and $v_s$ layer at $z=2$ km.), the bottom density layer has a value of now $4 \text{ kg/m}^3$ and the bottom $v_s$ layer has a value of $2 \text{ km/s}$, for a 100% increase. The results of EFWI are shown in Figure 3. The inversion of $v_h$ in Figure 3a is accurate, but not as good as the one we saw in Figure 2 for weaker $\rho/v_s$ contrasts. Now, we can see the density footprint a little bit better and a hint of the $v_s$ layer as well. For $\eta$, the dominant effect comes from the $v_s$ perturbation: it has a strong effect below $z=2$ km where the structural information is quite affected. This is expected since $\eta$ is mostly updated from PS-waves and the initial $v_s$ is twice as low as the true $v_s$ below 2 km. For $\eta$, we see a larger footprint of $\rho$ compared to Figure 2 as well. For $\varepsilon$ in Figure 3, the density layers is quite visible throughout with only a hint of the $v_s$ layer. The structural information is not affected as much as it is with $\eta$ in Figure 3b. Finally and most importantly, the inverted horizontal velocity did not suffer from such leakage, indicating that such a parametrization can help obtain a clean high resolution model of the velocity. In contrast, a similar test shown in Figure 4 using the often utilized Thomsen’s parametrization to describe the VTI model yields leakage of $\rho$ and $v_s$ into the inverted vertical velocity, $\delta$ and $\varepsilon$.

Figure 3 Inverted models for (a) $v_h$, (b) $\eta$ and (c), $\varepsilon$. $\rho$ and $v_s$ contrasts are now equal to 100%.
Discussion - Conclusions

For VTI anisotropic, elastic FWI, we study the effect of $\rho$ and $v_s$ on the retrieval of $v_h$, $\eta$ and $\varepsilon$. This study is carrying on with streamer data in mind because $\rho$ and $v_s$ are often ignored and not inverted for during the inversion of such datasets. The $v_h$, $\eta$ and $\varepsilon$ parameterization was justified by a careful analysis of radiation patterns using a pseudo-acoustic representation of the wave equation. In this study, we move away from the acoustic world to the more realistic elastic world and analyze radiation patterns that take into account all five parameter classes necessary to describe a 2-D VTI elastic anisotropic medium ($v_h$, $v_s$, $\eta$, $\varepsilon$ and $\rho$). First, conventionally recorded marine PP-waves are first-order (Born approximated) sensitive to mainly $v_h$ perturbations, as well as $\varepsilon$ near zero offset, while PS-waves are sensitive to $\eta$ perturbations at moderate offsets. Second, unlike in the pseudo-acoustic approximation, the small wavenumbers of $\eta$ can be updated with streamer data if (1) converted-waves are present and (2) the $v_s$ model is accurate. Third, $\varepsilon$ and $\rho$ have strong crosstalks for both PP- and PS-waves at any offset. Choosing $\varepsilon$ only (or eventually $\rho$) makes sense to absorb the information for both. From an inversion point-of-view, these findings are corroborated on a synthetic, modified Marmousi II model: we saw that generally speaking, $v_s$ perturbations would map to $\eta$, while $\rho$ perturbations would map to $\varepsilon$. We also see that $v_s$ might not be updated as long as its background values are accurate: for high errors in $v_s$, $\eta$ will be strongly affected (and $v_h$ in a much smaller way). In addition, $\rho$ will map to $\varepsilon$ first for all contrasts and $v_h$ for the highest contrasts only. Overall, with EFWI applied to conventional streamer data, inverting for $v_h$, $\eta$ and $\varepsilon$ while keeping $v_s$ and $\rho$ unchanged seems to be a reasonable approach, more so when the background models for $v_s$ and $\rho$ are accurate enough. We would like to mention that our work focuses on the mitigation of crosstalks from a parameterization view only. It is now well-known that better multi-parameter inversion methods taking into account the inverse Hessian can help resolving some of these issues as well.

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References

Anisotropy signature in P-wave extended images for VTI media

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Summary

Extended images obtained from reverse-time migration (RTM) contain information about the accuracy of the velocity field and subsurface illumination at different incidence angles. Here, we evaluate the influence of errors in the anisotropy parameters on the residual moveout (RMO) in P-wave extended images obtained with RTM for VTI (transversely isotropic with a vertical symmetry axis) media. Assuming the actual spatial distribution of the zero-dip normal-moveout velocity, we analyze extended images computed with distorted fields of the parameters $\eta$ and $\delta$. Differential semblance optimization (DSO) and stack-power criteria are employed to study the sensitivity of focusing to the anisotropy parameters. The results show that the signature of $\eta$ is dip-dependent, whereas errors in $\delta$ cause defocusing in extended images only if that parameter varies laterally. We also obtain and analyze the gradients of the DSO objective function with respect to the anisotropy parameters. The results of this work provide the foundation for anisotropic wavefield tomography operating with extended images.
Introduction

The extended imaging condition retains information about the wavefield directionality and angle-dependent reflector illumination by preserving the spatial and/or temporal correlation lags in the output. For example, one can obtain space-lag (Rickett and Sava, 2002) or time-lag (Sava and Fomel, 2006) extended common-image gathers (CIG), which are computed at fixed horizontal coordinates.

Wavefield tomography based on minimizing the residual energy at nonzero lags in the extended domain has recently attracted considerable attention in the literature (Yang and Sava, 2011; Li et al., 2014). The defocusing of energy can be quantified using differential semblance optimization (DSO) (Symes and Carazzone, 1991) and/or a measure of stack power (Chavent and Jacewitz, 1995). The gradients of the corresponding objective function can be efficiently found by applying the adjoint-state method (ASM) (Plessix, 2006).

Velocity analysis in the extended image domain has significant potential for anisotropic velocity model building in structurally complex areas. P-wave kinematics in VTI media is controlled by the vertical velocity $V_{P0}$ and Thomsen parameters $\epsilon$ and $\delta$. An alternative parameter set includes the normal-moveout velocity for a horizontal interface ($V_{NMO} = V_{P0}\sqrt{1 + 2\delta}$), the anellipticity parameter $\eta = (\epsilon - \delta)/(1 + 2\delta)$, and $\delta$. Sava and Alkhalifah (2012) study the $\eta$-signature in extended image domain for TI media and conclude that $\eta$-errors cause consistent “V”-shape defocusing for horizontal reflectors regardless of the complexity of the $V_{NMO}$-field.

Here, we evaluate the defocusing in space-lag CIGs caused by errors in $\eta$ and $\delta$ for VTI models with curved interfaces and laterally varying $\delta$-fields. We also obtain the gradient of the DSO objective function with respect to the parameter $\eta$ and present the corresponding sensitivity kernels.

Methodology

Alkhalifah and Tsvankin (1995) demonstrate that P-wave reflection moveout and time-domain processing for a laterally homogeneous VTI medium above the target horizon are governed by $V_{NMO}$ and $\eta$. In the case of a horizontal VTI layer, $\eta$ controls the nonhyperbolic (long-offset) portion of the P-wave moveout (Tsvankin, 2012). For a dipping reflector beneath VTI media, the P-wave NMO velocity depends on both $V_{NMO}$ and $\eta$; if $\eta > 0$ (typical case), $V_{NMO}$ increases much faster with dip compared to elliptical ($\epsilon = \delta$) or purely isotropic models.

P-wave moveout and time-domain processing still depend on just $V_{NMO}$ and $\eta$ even when these parameters vary laterally above the target horizon, but $\delta$ changes only with depth (Alkhalifah et al., 2001). However, if $\delta$ is laterally variable, P-wave traveltimes become sensitive to all three relevant parameters - $V_{NMO}$, $\eta$, and $\delta$ (or $V_{P0}$, $\epsilon$, and $\delta$) (Alkhalifah and Tsvankin, 1995; Tsvankin, 2012).

Parameter estimation in VTI media is often accomplished by applying ray-based reflection tomography. Ray theory, however, may break down for complicated structures and should be replaced with wave-equation-based methods. Inexpensive and kinematically accurate reconstruction of P-wavefields in TI models can be achieved by solving a system of equations adjoint to equations 1. The 2D version of the formulation proposed by Fowler et al. (2010) can be written as:

\[
\frac{\partial^2 p}{\partial t^2} = V_{hor}^2 \frac{\partial^2 p}{\partial x^2} + V_{P0}^2 \frac{\partial^2 q}{\partial z^2},
\]

\[
\frac{\partial^2 q}{\partial t^2} = V_{NMO}^2 \frac{\partial^2 p}{\partial x^2} + V_{P0}^2 \frac{\partial^2 q}{\partial z^2},
\]

where $V_{hor} = V_{NMO}\sqrt{1 + 2\eta} = V_{P0}\sqrt{1 + 2\epsilon}$ is the P-wave horizontal velocity. Both the $p$- and $q$-components contain a wavefield with accurate P-wave kinematics and a shear-wave artifact caused by eliminating $V_{SO}$. One way to remove the S-wave artifact, used here, is to place sources and receivers in a purely isotropic or elliptical ($\epsilon = \delta$, $\eta = 0$) medium (Duveneck and Bakker, 2011).

The derivatives of the objective function with respect to the model parameters can be efficiently computed with the adjoint-state method. The adjoint wavefields needed to obtain the gradient of the DSO objective function $J_{DSO}$ are found by solving a system of equations adjoint to equations 1. The amplitude and spatial distribution of the adjoint sources are defined by the residual energy in the extended.
images. The $\eta$-gradient is given by:

$$
\frac{\partial J_{DSO}}{\partial \eta} = \int \int 2 \frac{\partial \lambda_s}{\partial x} V_{nmo}^2 \frac{\partial u_s}{\partial x} ds dt + \int \int 2 \frac{\partial \lambda_r}{\partial x} V_{nmo}^2 \frac{\partial u_r}{\partial x} ds dt,
$$

where $u_s$ and $u_r$ are the source- and receiver-side forward wavefields, and $\lambda_s$ and $\lambda_r$ are the source- and receiver-side adjoint wavefields, respectively. One can notice that, for the chosen parameterization, the $\eta$-gradient involves the lateral derivatives and depends on the background value of $V_{nmo}$, which indicates the well-known trade-off between $V_{nmo}$ and $\eta$ in the horizontal direction.

**Signature of $\eta$ and $\delta$ in extended images**

Here, we analyze how the anisotropy parameters $\eta$ and $\delta$ influence the residual moveout in RTM extended images for a modified segment of the BP 2007 TTI model with an anticline structure (Figure 1). The model, which includes a tilted symmetry axis, is simplified as follows:

- The symmetry-axis tilt is removed to make the model VTI.
- The original $V_{nmo}$-field is smoothed, and only the two strongest reflectors are retained to avoid reflections from multiple interfaces.
- The parameter $\eta$ is taken to be constant ($\eta = 0.15$) throughout the model.

The spatially varying $\delta$-field in the original BP model is left unchanged, and density is assumed to be constant. Sources and receivers are located at the surface, and the near-surface layer is taken to be isotropic to suppress the shear-wave artifact. We obtain RTM space-lag CIGs for $\eta$-values ranging from 0 to 0.3 with a 0.05 increment. The signature of $\eta$ in space-lag CIGs computed with the actual $\delta$-field for the dipping interface segments deviates from the “V”-shape and resembles the residual caused by an inaccurate velocity model for isotropic media (Figure 2d, f). This is explained by the fact that for dipping reflectors $\eta$ changes the NMO velocity and, therefore, conventional-spread moveout. Repeating the test with the erroneous $\delta = 0$ shows that for subhorizontal reflector segments the signature of $\eta$ maintains the “V”-shape even if $\delta$ is incorrect. As expected, the residual moveout (RMO) due to the $\eta$-errors in space-lag CIGs for dipping reflector segments does not have the “V”-shape.

Figure 3 demonstrates that if $\delta$ is distorted, the extrema of the DSO and stack-power objective functions
Figure 2 Space-lag CIGs computed with (a, d) $\eta = 0$, (b, e) $\eta = 0.15$ (actual value), and (c, f) $\eta = 0.3$ at: (a, b, c) $x = 2.1$ km and (d, e, f) $x = 3.5$ km.

Figure 3 Influence of $\eta$ on the DSO (solid) and stack-power (dashed) objective functions calculated from space-lag extended images. The images are obtained with (a) the actual $\delta$-field and (b) $\delta = 0$.

are shifted toward lower $\eta$-values (close to 0.1). Indeed, the laterally varying $\delta$ influences the focusing in extended images, and, therefore, the shape of the DSO and stack-power objective functions computed as a function of $\eta$. However, because in this model the lateral variation in $\delta$ is relatively mild, that parameter does not change the shape of RMO caused by errors in $\eta$ for both subhorizontal and dipping interface segments.

Gradient of DSO objective function

Here, we discuss the $\eta$-gradient for a model that includes a horizontal interface beneath a homogeneous VTI layer. First, we compute the gradient for the trial $\eta = 0$ using a single source-receiver pair and a single image point (Figure 4). These gradients are often referred to as sensitivity kernels, which describe the spatial distribution of the parameter update for given acquisition geometry. Figure 5 shows the $\eta$-gradients computed using sources and receivers uniformly distributed at the surface. The gradient changes sign depending on the sign of the error in $\eta$. One can notice that even for the actual $\eta$-field, the gradient does not go to zero (Figure 5b) because imaging with the actual velocity model still produces residual energy at nonzero lags due to the aperture limitations. We are currently implementing the gradients with respect to the VTI parameters in a DSO-based inversion algorithm.

Conclusions

We presented a study of the anisotropy signature in RTM extended images for VTI models with curved interfaces and laterally varying $V_{\text{nmo}}$- and $\delta$-fields. The residual moveout due to errors in $\eta$ maintains
Figure 5 Gradients for the homogeneous VTI model from Figure 4 computed with different values of $\eta$: (a) $\eta = 0$, (b) $\eta = 0.15$ (actual value), and (c) $\eta = 0.3$.

a linear shape for subhorizontal interfaces, regardless of the overburden complexity. For a dipping reflector, $\eta$-errors cause stronger defocusing, and the shape of the residual moveout is similar to that caused by velocity distortions for isotropic media. The different signature of $\eta$ for dipping interfaces is explained by the influence of $\eta$ on NMO velocity, which becomes pronounced for dips reaching 25-30°.

The DSO and stack-power objective functions demonstrate that the energy focusing in extended images is sensitive to the lateral variation of $\delta$. For a simplified segment of the BP TI model, accurate estimation of $\eta$ with either objective function requires including $\delta$ in the inversion. However, since the lateral variation in $\delta$ for the BP model is relatively mild, setting $\delta = 0$ does not noticeably change the shape of RMO caused by $\eta$-errors. Also, we obtained and analyzed the gradient of the DSO objective function with respect to the coefficient $\eta$.

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References


Feasibility of high-resolution fracture characterization using waveform inversion

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Summary

Wide-azimuth surface seismic data contain valuable information about the fracture properties, such as crack density, fluid content and fracture direction. Although this information can be obtained from standard techniques such as tomographic velocity analysis or amplitude variation with offset and azimuth, full-waveform inversion (FWI) has the potential of higher-resolution fracture characterization. Building a wavefield-based inversion framework for the fracture parameters requires identifying parameter combinations that minimize trade-offs and ensure high-resolution of the results. Using the Born approximation, which is the central ingredient of the FWI updating process, we derive the radiation patterns of different parameters of the effective orthorhombic medium formed by a vertical fracture set embedded in a VTI (transversely isotropic with a vertical symmetry axis) background. The radiation patterns describe the angular influence of each parameter, in the considered parametrization, on seismic data, and thus give insights into the resolution of the fracture properties. We show that the high-resolution recovery of crack density depends on the fracture infill, while the fracture direction might not be well-resolved because of the trade-off between different parameters.
Introduction

Rocks with naturally aligned fractures contain substantial oil and gas reserves. The ability to characterize fracture properties, such as crack density, fluid content and fracture direction is crucial in reservoir management and optimizing well locations.

According to the linear slip-theory, effective models of fractured rocks can be described by the compliance matrix of the host rock and the excess compliance matrix of the fractures (Schoenberg, 1980). The excess fracture compliance is conveniently expressed in terms of the dimensionless parameters called fracture weaknesses (Hsu and Schoenberg, 1993). A proper characterization of the fracture properties depends on our ability to efficiently estimate the fracture parameters. For instance, Bakulin et al. (2000) derive linearized relationships between the fracture weaknesses and the anisotropy parameters that can be constrained from surface seismic data. They also suggest different approaches to estimate the fracture parameters. Most of those methods are based on velocity analysis (Tsvankin, 1997) which can be accomplished, for example, through reflection tomography. These approaches are generally reliable, but produce only smooth models with relatively low spatial resolution. On the other hand, amplitude-variation-with-offset (AVO) analysis (Rüger and Tsvankin, 1997) may provide high-resolution information, but requires generally low relief structures and careful amplitude handling. Full-waveform inversion (FWI) has the potential of higher-resolution fracture characterization, albeit it has been shown that the resolution of the inverted models varies with parametrization (Alkhalifah and Plessix, 2014). This is because model parameters may have significantly different influence on wave propagation and amplitude. Therefore, in an attempt to build wavefield-based inversion for the fracture properties, it is essential to identify which fracture parameters can be reliably estimated and to devise a proper inversion strategy. Analyzing the scattering potential of these parameters (radiation patterns) is helpful in elucidating the dependence of seismic data on different combinations of parameters as well as in predicting the resolution of each parameter.

In this study, we investigate the feasibility of high-resolution fracture characterization using FWI. We derive the radiation patterns for different parameters with the goal of constraining the weaknesses and orientation of a set of aligned fractures. For simplicity, we assume a formation with a single vertical fracture set embedded in a transversely isotropic background medium with a vertical symmetry axis (VTI). Our results show both the potential and limitations of FWI in constraining fracture parameters.

Fracture parameters of a vertical fracture set in a VTI background

The effective model resulting from a single vertical fracture set in a VTI background is orthorhombic with one of its symmetry plane coinciding with the fracture plane. In this case, fractures can be described by three weaknesses denoted by $\Delta_V$, $\Delta_H$ and $\Delta_N$. The normal weakness $\Delta_N$ contains information about the fluid content and hydraulic connectivity between fractures and pores, while the tangential weaknesses $\Delta_V$ and $\Delta_H$ depend on the crack density (Schoenberg and Douma, 1988).

A general elastic orthorhombic model can be described by nine anisotropy parameters (Tsvankin, 1997); for the model used here only eight of them are independent. However, if the ratio $g = V_s^2/V_p^2$ of the squared vertical S- and P-wave velocities of the background medium is known (for example, from well logs), just three combinations of the anisotropy parameters can constrain the fracture weaknesses. Assuming the fracture set to be orthogonal to the $\chi_1$-axis, Bakulin et al. (2000) develop the following weak-anisotropy approximations for the medium parameters:

$$
\varepsilon^{(2)} - \varepsilon_b \approx -2g(1-g)\Delta_N, \\
\delta^{(2)} - \delta_b \approx -2g((1-2g)\Delta_N + \Delta_V), \\
\eta^{(2)} - \eta_b \approx 2g(\Delta_V - g\Delta_N), \\
\gamma^{(2)} - \gamma_b \approx -\Delta_H/2, \\
\delta^{(3)} \approx 2g(\Delta_N - \Delta_H).
$$

Here, $\varepsilon_b$, $\delta_b$ and $\gamma_b$ are the Thomsen anisotropy parameters of the background medium, $\varepsilon^{(2)}$, $\delta^{(2)}$, $\gamma^{(2)}$ and $\delta^{(3)}$ are Tsvankin’s (1997) parameters of the effective orthorhombic medium, and $\eta_b$ and $\eta^{(2)}$ are the anellipticity parameters defined as $\eta_b = \frac{\varepsilon_b - \delta_b}{1 + 2\delta_b}$ and $\eta^{(2)} = \frac{\varepsilon^{(2)} - \delta^{(2)}}{1 + 2\delta^{(2)}}$.

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Radiation patterns and choice of parameters

Since equations (1) - (5) are weak-anisotropy approximations (the exact equations are very complicated), it is more convenient to derive the radiation patterns in terms of anisotropy parameters rather than fracture weaknesses. The radiation patterns of the anisotropy parameters provide insights into the resolution of the fracture weaknesses.

In principle, only three relations from (1) - (5) allow us to determine the fracture weaknesses (assuming that an estimate of the ratio g is available). Hence, here we derive the radiation patterns for an acoustic orthorhombic model described by six parameters (e.g., in Tsvankin’s (1997) notation: it is the vertical velocity \( V_p \), and the anisotropy coefficients \( \epsilon^{(1,2)} \) and \( \delta^{(1,2,3)} \)) and the azimuth \( \phi_F \) of the fracture plane. In this case, we choose three anisotropy parameters describing the VTI background medium and three parameters that measure the deviation from that background. This representation in terms of "deviation" parameters is convenient because we are focused on the dependence of the data on fracture weaknesses. Based on the parametrization suggested for VTI media (Alkhalifah, 2015), we describe the "deviation" parameters in terms of the background medium, and three parameters that measure the deviation from that background. We derive the radiation patterns for the orthorhombic parameters based on the Born approximation and investigate different parametrizations of our effective orthorhombic model, which include (in addition to the background parameters), different combinations of three "deviation" parameters defined as follows (Masmoudi and Alkhalifah, 2016):

\[
\begin{align*}
\epsilon_d &= \frac{\epsilon^{(2)} - \epsilon_b}{1 + 2\epsilon_b}, \\
\delta_d &= \frac{\delta^{(2)} - \delta_b}{1 + 2\delta_b}, \\
\eta_d &= \frac{\eta^{(2)} - \eta_b}{1 + 2\eta_b} = \frac{\epsilon_d - \delta_d}{1 + 2\epsilon_d}.
\end{align*}
\]

Note that the linearized parameters \( \epsilon_d, \delta_d, \) and \( \eta_d \) are proportional to the fracture weaknesses (see equations (1) - (3)). The coefficient \( \delta^{(3)} \) and the fracture azimuth angle \( \phi_F \) are also included in the parameter set.

We derive the radiation patterns for the orthorhombic parameters based on the Born approximation and the acoustic wave equation (Alkhalifah, 2003). In this approach, we define the perturbation vector \( \mathbf{r}_p \) as

\[
\mathbf{p} = \mathbf{p}_0 (1 + \mathbf{r}_p),
\]

where \( \mathbf{p} \) is a seven-row vector corresponding to the orthorhombic parameters, \( \mathbf{p}_0 \) includes the parameters of the background medium, and \( \mathbf{r}_p \) is the vector containing the perturbation for the individual parameters. By substituting the perturbations (7) into the wave equation and using the Born approximation (for a small perturbation \( \mathbf{r}_p \)), we end up with the scattered wavefield \( P_s \):

\[
P_s = \omega^6 \int G(\mathbf{k}_s, \mathbf{x}, \omega) G(\mathbf{k}_r, \mathbf{x}, \omega) \mathbf{a}_p(\mathbf{x}) \cdot \mathbf{r}_p(\mathbf{x}) \, d\mathbf{x}.
\]

Here \( \omega \) is the angular frequency, and \( \mathbf{k}_s \) and \( \mathbf{k}_r \) are the wavenumber vectors for the source and receiver wavefields, respectively (see Figure 1). The vector \( \mathbf{a}_p \) defines the radiation patterns for the perturbation parameters \( \mathbf{r}_p \). This procedure assumes a locally (around the scatterer) smooth background, which is also the assumption for the plane-wave approximation. If the orthorhombic model is parametrized by \( v_b, \epsilon_b, \eta_b, \epsilon_d, \eta_d, \delta^{(3)} \) and \( \phi_F \), the vectors \( \mathbf{a}_p \) and \( \mathbf{r}_p \) take the following form:

\[
\mathbf{a}_p = \begin{pmatrix}
2 & 2 \cos^2 \theta \sin^2 \theta \\
-2 \cos^2 \theta \\
2 \cos^2 \theta \sin^3 \theta \sin^2 \phi \\
2 \sin^2 \theta \sin^2 \phi (1 - \sin^2 \theta \cos^2 \phi) \\
2 \sin^4 \theta \cos^2 \phi \sin^2 \phi \\
-\beta \sin^2 \theta \sin \phi \cos \phi
\end{pmatrix},
\quad \mathbf{r}_p = \begin{pmatrix}
v_b \\
\eta_b \\
\epsilon_b \\
\eta_d \\
\epsilon_d \\
\delta^{(1)} \\
\phi_F
\end{pmatrix},
\]

where \( \mathbf{a}_p \) includes (from top to bottom) the perturbations of the parameters \( v_b, \eta_b, \epsilon_b, \eta_d, \epsilon_d, \delta^{(3)} \) and \( \phi_F \). The coefficient \( \beta \) is proportional to the degree of ellipticity of the background model (specifically to \( \delta_d \)); the scattering angle \( \theta \) and the source-to-receiver-azimuth angle \( \phi \) are shown in Figure 1. We use equations (9) and (6) to obtain the radiation patterns of different combination of the anisotropy parameters.
Figure 1 Schematic plot of the source and receiver rays as they connect at a model point and the corresponding angles. The gray surface indicates a horizontal reflector.

Analysis of the radiation patterns

By considering a horizontal reflector, we plot the radiation patterns of three different parametrizations (Figure 2). These parametrizations are \((v_h, \varepsilon_b, \eta_b, \varepsilon_d, \delta^3, \phi_F)\), \((v_h, \zeta_b, \varepsilon_b, \delta_d, \delta^3, \phi_F)\) and \((v_h, \zeta_b, \eta_d, \varepsilon_d, \delta_d, \delta^3, \phi_F)\), denoted \(P_1\), \(P_2\) and \(P_3\), respectively. The first three plots in Figure 2 show the radiation patterns of the background parameters \((v_h, \varepsilon_b, \eta_b)\) which are the same for all three parameterizations. The scattering potential of these parameters is independent from the source-to-receiver azimuth \(\phi\). This behaviour is convenient for waveform inversion that starts with updating the parameters having the most influence on the data. In our acoustic representation, \(v_h\) should be well resolved, \(\varepsilon_b\) can be used to improve the amplitude fit at small scattering angles, while \(\eta_b\) might not be recovered because of its weak scattering potential (Alkhalifah, 2015). Therefore, we still need to rely on velocity analysis (e.g. tomographic inversion) to obtain at least a smooth \(\eta_b\)-field.

Once the background parameters have been estimated, we can update the deviation parameters needed to characterize the fracture properties. Figure 2 shows that the radiation patterns of \(\delta^3\) and \(\phi_F\) are the same for the different parametrizations. Also, the scattering potential is associated with large scattering angles and reaches maximum for the source-to-receiver-azimuth equal to 45°. Therefore, there is a trade-off between these two parameters, which cannot be mitigated at other angles. On the other hand, we notice that \(\varepsilon_d\) (in parametrizations \(P_1\) and \(P_2\)) is always associated with large scattering angles and the source-to-receiver-azimuth equal to 0° (corresponding to the plane perpendicular to the fractures). In this case, \(\varepsilon_d\) should be well resolved since \(\eta_d\) (in \(P_1\)) or \(\delta_d\) (in \(P_2\)) are scattering in different angular ranges. If we rely on parametrization \(P_1\) or \(P_2\), we expect that both \(\eta_d\) and \(\delta_d\) to be poorly resolved (similarly to \(\eta_b\)). Finally, in parametrization \(P_3\), \(\eta_d\) and \(\delta_d\) are combined, so we might not resolve \(\eta_d\) and \(\delta_d\) either because of the trade-off between them.

In general, the parameter \(\varepsilon_d\), which is sensitive to the fracture infill, should be better resolved than \(\delta^3\), \(\eta_d\) or \(\delta_d\) which give access to the tangential weaknesses. For dry cracks, \(\varepsilon_d\) provides a direct measure of the crack density, but if the cracks are filled with fluid, \(\varepsilon_d\) vanishes. In this case, we can estimate the crack density from the tangential weaknesses. On the other hand, the scattering potential of the fracture azimuth depends on the ellipticity of the model. In the studied combination of parameters, we might not be able to resolve \(\delta^3\) and \(\phi_F\) using FWI, and have to rely on conventional methods such as velocity analysis.

Conclusions

For purposes of high-resolution fracture characterization, we have analyzed the scattering patterns of different combination of anisotropy parameters constraining the fracture properties. Our results suggest that the high-resolution recovery of crack density depends on the fracture infill, while the fracture direction might not be constrained because of the trade-off between different parameters.

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Anisotropic elastic wavefield imaging using the energy norm

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Summary

Based on the energy conservation principle, we derive a scalar imaging condition for anisotropic elastic wavefield migration. Compared to conventional imaging conditions that simply correlate displacement components or potentials from source and receiver wavefields, the proposed imaging condition does not suffer from polarity reversal, which might degrade the image quality after stacking over shots. Our imaging condition also accounts for the directionality of the wavefields in space and time, leading to attenuation of backscattering artifacts, which commonly appear in elastic reverse-time migration images with strong model contrasts. In addition, our new imaging condition does not require wave-mode decomposition, which demands significant additional cost for anisotropic wavefields. This new imaging condition relies on knowledge of the anisotropic model parameters used during migration, and is applicable for any kind of anisotropy. We show the quality of the energy image compared to its conventional counterparts by numerical experiments that simulate complex geological settings.
Introduction

A typical assumption in seismic wavefield imaging is that the subsurface is acoustic and isotropic. However, larger offsets and the increasing number of acquired azimuths require that we account for the influence of anisotropy; at the same time, the search for subsurface information such as fracture distribution encourages multicomponent elastic imaging. Although this technique has progressed greatly in recent years, a robust and effective method for elastic anisotropic imaging remains an elusive goal.

Wavefield imaging is implemented in two steps: wavefield extrapolation in the subsurface, using recorded data and an Earth model, and the application of an imaging condition to extract the Earth’s reflectivity from the extrapolated wavefields (Claerbout, 1971). If a two-way elastic wave equation is used in the wavefield extrapolation, followed by the zero-lag cross-correlation of the wavefields as the imaging condition, the procedure is called elastic reverse time migration (ERTM) (Chang and McMechan, 1987; Hokstad et al., 1998). Correlating the displacement fields for each component of the source and receiver wavefields is the most common imaging condition for elastic wavefields, generating images with a mixture of wave modes. Wave-mode decomposition for anisotropic wavefields is possible, but requires additional computational cost and robust techniques (Yan and Sava, 2009, 2011; Cheng and Fomel, 2013). In addition, polarity reversal occurs if one correlates different components of the displacement fields, and damages image quality after stacking over experiments (Balch and Erdemir, 1994). Similarly to wave-mode decomposition, polarity reversal corrections can be implemented with techniques that require additional cost for imaging (Duan and Sava, 2014). Moreover, backscattering artifacts occur if the migration velocity contains sharp contrasts, thus contaminating the image with low-wavenumber content (Yan and Xie, 2012). Ideally, these artifacts should be attenuated during imaging, without the use of post-imaging artificial filters such as the Laplacian operator (Zhang and Sun, 2009).

By defining an imaging condition that yields an attribute of the Earth’s reflectivity into a single image without wave-mode decomposition, polarity reversal and backscattering artifacts, we seek to facilitate interpretation and provide a concise description of the subsurface structures. Our proposed imaging condition is formulated using the energy conservation principle, and extends previous work on isotropic elastic imaging (Rocha et al., 2015b).

Theory

For a general anisotropic medium, we can write the wave equation:

\[
\rho \ddot{U} = \nabla \cdot \left[ \xi \nabla U \right],
\]

(1)

where \(U(x,t)\) is the elastic wavefield, \(\xi(x)\) is the fourth-rank stiffness tensor, and \(\rho (x)\) is the medium density. The double dot indicates second derivative in time. Analogously to the acoustic and isotropic elastic cases discussed in Rocha et al. (2015a,b), we develop the energy function

\[
E(t) = \frac{1}{2} \int_{\Omega} \rho ||\dot{U}||^2 + \left( \xi \nabla U \right) : \nabla U \, dx,
\]

(2)

where \(\Omega\) is the physical domain that contains the wavefield, and the symbol \(\cdot\) indicates the element-wise inner product between matrices called Frobenius product. Equation 2 measures the total energy of the wavefield within a domain. Each one of the terms in equation 2 incorporates a particular physical meaning. The first term represents the kinetic energy of the wavefield, and the second term is the strain energy (Slawinski, 2003), which represents the potential energy of the wavefield.

Based on the energy conservation expression in equation 2, we can define a norm, and then, an inner product of two arbitrary elastic wavefields \(U\) and \(V\):

\[
< U, V >_E = \int_0^T \int_{\Omega} \rho \dot{U} \cdot \dot{V} + \left( \xi \nabla U \right) : \nabla V \, dx,
\]

(3)
and an elastic imaging condition:

$$I_E = \sum_{e j} [\rho U \cdot V + (e\nabla U) : \nabla V]. \quad (4)$$

Here, $e$ is the experiment index, which is commonly a shot record. $U(e, x, t)$ and $V(e, x, t)$ are the source and receiver displacement fields, respectively, and $I_E(x)$ is a scalar image obtained from the wavefields. We can also define the following multidimensional vector

$$\Box U = \left\{ \rho^{1/2} U, \xi^{1/2}(\nabla U) \right\}. \quad (5)$$

The vector in equation 5 contain twelve components, of which three components are from the particle velocity, $U$, and nine are from the displacement gradient, $\nabla U$. A similar multidimensional vector can be defined for the receiver wavefield $V$; therefore, we can rewrite equation 4 as the dot product between these vectors:

$$I_E = \sum_{e j} \Box U \cdot \Box V. \quad (6)$$

This expression indicates that the energy elastic image is represented by the projection of the two multidimensional vectors onto one-another. Therefore, an image is formed when these space-time vectors are not orthogonal.

Vectors $\Box U$ and $\Box V$ are related to the polarization and propagation directions of the elastic wavefields $U$ and $V$. We can represent an elastic wavefield by

$$U = u_0 e^{i\omega (p \cdot x - t)} , \quad (7)$$

where $u_0$ is the polarization vector, $p$ is the slowness vector indicating propagation direction, and $\omega$ is the frequency. We assume that $\omega$ is large and that the vectors $u_0$ and $p$ are slowly varying in space and time, which makes their spatial and temporal derivatives small compared to $\omega$. Equation 7 can be rewritten in index form as $U_i = u_0 e^{i\omega (p_i x_j - t)}$, where index $i = \{1, 2, 3\}$ represents the Cartesian components of wavefield $U$. Substituting the plane wave definition in equation 7 as trial solution of the elastic equation in 1, we obtain the Christoffel equation:

$$\rho u_i = u_0 e^{i\omega (p \cdot x - t)} \cdot \Box V, \quad (8)$$

where $i, j, k, l = \{1, 2, 3\}$. We seek an imaging condition that attenuates waves of the source and receiver wavefields propagating along the same path and with the same polarization, i.e., elastic backscattering. Therefore, the backscattering events are characterized by $V_i = U_i = u_0 e^{i\omega (p_i x_j - t)}$. Defining $(\Box V)^\dagger$ with negative sign on the component containing the time derivative:

$$(\Box V)^\dagger = \left\{ -\rho^{1/2} \hat{V}, \xi^{1/2}(\nabla V) \right\}, \quad (9)$$

we can compute the dot product between $\Box U$ and $(\Box V)^\dagger$ for backscattering events as

$$\Box U \cdot (\Box V)^\dagger = \omega^2 \left[ \rho u_i u_l - u_0 u_0 e_{ijkl} p_j p_l \right] e^{2i\omega (p_i x_j - t)} . \quad (10)$$

Using the relation in equation 8, we obtain

$$\Box U \cdot (\Box V)^\dagger = 0 , \quad (11)$$

which means the dot product is zero everywhere except at locations where $p$ and $u_0$ are different for $U$ and $V$, i.e., at reflectors. Therefore, the dot product in equation 11 nullifies the events that propagate on the same path and have the same polarization. Such events include reflection backscattering, diving, direct and head waves from the same wave modes. Therefore, the imaging condition

$$I_E^\dagger = \sum_{e j} \Box U \cdot (\Box V)^\dagger \quad (12)$$

attenuates backscattering artifacts in anisotropic elastic RTM images, while retaining similar properties of the isotropic images discussed in Rocha et al. (2015b).
Example

Figure 1 shows the Marmousi-II (Martin et al., 2002) model parameters used to simulate synthetic data, the migration model parameters and the resulted images. The synthetic data are obtained from a horizontal line of receivers that record the displacement field at every grid point of the water bottom ($z = 0.5$ km), and from 76 pressure sources located near the surface of the water layer ($z = 0.05$ km) with a horizontal spacing of 150 m. This acquisition geometry resembles an ocean bottom seismic survey (OBS), recording multicomponent data at the water bottom. The migration model parameters are slightly smoothed versions of the true model parameters. As expected, all images contain backscattering artifacts, and Figure 1(h) also shows the effects of a non-constructive stack due to polarity reversal artifacts. In contrast, we attenuate all these artifacts in Figure 1(j) using the imaging condition from equation 12.

Conclusions

The energy imaging condition produces a scalar image without the need for wave-mode decomposition and does not suffer from polarity reversal, as is the case for displacement images using different components of the source and receiver wavefields. The proposed imaging condition incorporates propagation and polarization directions in its application, as inferred from the definition of the multidimensional vectors $\square U$ and $\square V$ built using the extrapolated wavefields $U$ and $V$. By properly applying a dot product between these multidimensional vectors, we are able to attenuate the backscattering artifacts in the imaging process, and obtain robust, cheap and high-quality elastic images.

Acknowledgements

We thank the sponsor companies of the Center for Wave Phenomena, whose support made this research possible. The reproducible numeric examples in this paper use the Madagascar open-source software package, freely available at http://www.ahay.org.

References

Figure 1 Migration model parameters used for the Marmousi-II experiment. (a) P- and (b) S-wave velocity along the local symmetry axis ($V_{P0}$ and $V_{S0}$, in km/s), (c) $\epsilon$, (d) $\delta$, (e) local tilt angle $\nu$ (in degrees). (f) Stratigraphic density model used to generate synthetic data (in g/cm$^3$). Images obtained using the correlation between displacement components (g) $U_xV_x$, (h) $U_xV_z$, and (i) $U_zV_z$; (j) energy image using the energy imaging condition in equation 12. The energy image shows strong reflectors undisturbed by either polarity reversal or backscattering events, with no additional post-imaging processing.
OpenSeaSeis: A simple open-source seismic data processing system

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(Note: This abstract was submitted as a 2016 EAGE Meeting Workshop)

Summary

OpenSeaSeis is an open-source seismic data processing system owned and distributed by CWP/Colorado School of Mines since 2013. The package is based on “Seaseis” which was begun in 2006, and publicly released in 2009 by Bjorn Olofsson.

The package consists of an interactive 2D seismic viewer, with some simple processing options, as well as a batch flow processing environment permitting a number of seismic processing operation, some taken from Seismic Unix.
Summary

OpenSeaSeis is an open-source seismic data processing system owned and distributed by CWP/Colorado School of Mines since 2013. The package is based on "Seaseis" which was begun in 2006, and publicly released in 2009 by Bjorn Olofsson. The package consists of an interactive 2D seismic viewer, with some simple processing options, as well as a batch flow processing environment permitting a number of seismic processing operation, some taken from Seismic Unix.

Strengths

1. Data reading/viewing. The viewer permits SEGY, limited SEGd, and SU format data to be read easily and displayed as either image plots or as wiggle traces.
2. Simple processing. Demo processing flows are provided so that simple batch processing sequences may be performed.
3. Run and Manage a large number of production flows. Such as for a 3D survey.
5. Module addition. Programmers (C, C++, Fortran) can copy-paste the module skeleton to allow new modules (such as from Seismic Unix) to be added.
6. Java GUI modification. Programmers can quickly create a new interactive application for visualization/processing/manipulation of data.

Drawbacks

1. Its current limited geophysical module library.
2. Its current lack of parallel programming. Though the modular nature may facilitate adding MPI/parallelism.

Opportunities

Because there has been only a small amount of development, this presentation is an appeal for interested parties to consider taking on the task of improving this package. See (Olofsson, 2012) for more information.
References


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Data denoising and interpolation using synthesis and analysis sparse regularization

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SUMMARY

Missing trace reconstruction is a challenge in seismic processing due to incomplete and irregular acquisition. Noise is a concern, due to the many sources of noise that occur during seismic acquisition. Most of the recent research on denoising and interpolation focuses on transform domain approaches using $L_1$ norm minimization. A specific kind of constraint, called the synthesis approach, is widely used in geophysical problems. The analysis approach, which can be considered as the synthesis’ dual problem, is an alternative for sparsity-constrained inversion. Although less popular than the synthesis solution, the analysis approach is more effective in several problems, such as denoising of natural images. We compare and contrast the analysis and synthesis approaches as sparsity constraints for the missing trace reconstruction and denoising problems. We show that the analysis approach yields more accurate results than the synthesis approach, which makes it a viable approach for sparsity-constrained inversion for noise related geophysical problems.

INTRODUCTION

Seismic acquisition ideally seeks to sample densely and regularly in every spatial direction, with the intent of obtaining signals that adequately represent data observed at the surface. Good sampling has important consequences for many applications, such as reverse-time migration and multiple removal. However, acquisition costs and field obstacles can easily make the acquisition both irregular and sparse. The purpose of missing trace reconstruction is to fill the data gaps or to resample the data as accurately as possible.

Transform domains for missing trace estimation have been proposed by several authors, (Hennenfent and Herrmann, 2008; Naghizadeh and Sacchi, 2010). This approach assumes that the analyzed wavefield is sparse in some domain and involves the minimization of a convex function with an $L_1$ norm, which is known to promote sparsity. This approach is also used to denoise complex data corrupted by incoherent noise (Herrmann et al., 2007; Zhu et al., 2015). Hennenfent and Herrmann (2008) suggest that both problems are very similar when the considered transform domain is Fourier related, as missing trace reconstruction can be regarded as a denoise problem in the Fourier domain.

The transform domain approach is usually related to the compressive sensing problem (Candès et al., 2006). In this setting, one tries to find the sparsest representation that describes the signal to be estimated in a transform domain. This formulation is known as the synthesis approach and it has been widely used for several geophysical problems, such as full-waveform inversion (Li et al., 2012), deblending (Wason et al., 2011) and salt body detection (Ramirez et al., 2016).

The analysis approach provides a different way of solving the $L_1$ minimization problem by estimating the signal in the ambient domain using a set of forward transforms. Although less popular than the synthesis approach, the analysis approach and its geometry has been studied theoretically from the perspective of compressive sensing (Candes et al., 2011) and the cosparse transform model (Nam et al., 2013). In geophysics, the analysis approach has been used sparingly to solve problems such as denoising with data-driven frames (Chen et al., 2016) and multiple removal (Yang and Fomel, 2015).

We compare the analysis and synthesis approaches for missing trace reconstruction and for denoising, and illustrate both techniques with a complex shot gather from the Sigsbee 2A model, for different spatial sampling schemes.

THEORY

The underconstrained inverse problem can be formulated mathematically as

$$\mathbf{y} = \mathbf{Φ}\mathbf{x}, \quad (1)$$

where $\mathbf{Φ}$ is a linear operator that maps a high dimensional vector $\mathbf{x} \in \mathbb{R}^n$ to a low dimensional vector $\mathbf{y} \in \mathbb{R}^m$. A general minimization problem that estimates $\mathbf{x}$ is (Cai et al., 2012)

$$\min_{\mathbf{u}} \| \mathbf{Φ}\Theta^*\mathbf{u} - \mathbf{y}\|_2 + \kappa \|(\mathbf{I} - \Theta\Theta^*)\mathbf{u}\|_2 + \gamma \|\mathbf{u}\|_1, \quad (2)$$

where $\kappa$ and $\gamma$ are regularization parameters. In equation 2, $\mathbf{x}$ is assumed to be sparse with respect to a transform $\Theta$, and the estimated signal in the ambient domain is recovered by $\mathbf{x} = \Theta^*\mathbf{u}$.

Two special cases of equation 2 are of particular interest in the literature. If $\kappa = 0$, then the equation simplifies to

$$\min_{\mathbf{u}} \gamma \|\mathbf{u}\|_1 + \|\mathbf{y} - \mathbf{Φ}\Theta^*\mathbf{u}\|_2. \quad (3)$$

This equation defines the synthesis approach, because the estimated signal is synthesized using $\mathbf{x} = \Theta^*\mathbf{u}$. When $\kappa \rightarrow \infty$, the second term of equation 2 is zero, which gives rise to the analysis approach

$$\min_{\mathbf{x}} \gamma \|\Theta\mathbf{x}\|_1 + \|\mathbf{y} - \mathbf{Φ}\mathbf{x}\|_2. \quad (4)$$
Denoising and interpolation using sparsity-constrained regularization

If $\Theta$ is an isometry, both formulations yield the same solution. However, in the general case, this is not true because $\Theta\Theta^* \neq I$. One of the main differences between the approaches comes from the fact that the solutions are estimated in different domains: while the synthesis approach seeks a sequence $u$ in the transform domain, the analysis approach seeks to find a dataset $x$ in the ambient domain. In particular, the solution of equation 3 is closely related to the soft-threshold operator (Daubechies et al., 2004), which implies that it must be the sparsest $u$ that represents a given $x$ in the transform domain. On the other hand, the solution of equation 4 is dominated by the second term in equation 2 as $\kappa \rightarrow \infty$. Thus, it is related to the orthogonal projector $I - \Theta\Theta^*$, which implies that this formulation seeks the sparsest $u$ that belongs to the null space of this orthogonal projector. Thus, in general, the solution of the synthesis approach is much sparser than that of the analysis approach.

More sparsity can be an advantage for the synthesis approach in terms of descriptive power, because its solution needs fewer elements to describe a signal in the transform domain when compared to that of the analysis approach. However, this can be a drawback when we consider erroneous estimation: every coefficient of the synthesis solution carries more significance than those of the analysis solution. Thus, the misestimation of the magnitude or support of coefficients of the synthesis solution, for example, in the presence of noise, might yield a solution that is very different from the desired result (Elad et al., 2007). This motivates us to study the performance of both approaches when geophysical problems involving noise are considered.

EXAMPLES

We evaluate the performance of equations 3 and 4 when applied to the missing trace reconstruction and denois-
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Figure 2: (a) Shot gather after random undersampling, and (b) corresponding f-k spectrum.

Figure 2: (a) Shot gather after random undersampling, and (b) corresponding f-k spectrum.

ing problems. We specify \( \Phi \) as a restriction matrix that spatially undersamples the data by 50% for the former and as the identity matrix for the latter, while \( \Theta \) is taken as the curvelet transform (Candes et al., 2006). We use a complex shot gather from the Sigsbee 2A model, which features a wide range of amplitudes and conflicting dips. To solve both approaches in similar grounds, we use the NESTA solver (Becker et al., 2011) for all the experiments.

For the denoising problem, we contaminate the considered synthetic shot gather with random (incoherent) Gaussian noise. Figure 1 shows the solution and corresponding difference plot for the synthesis and analysis approaches. We can see that both approaches do well in removing the noise present in the data, although the difference plots also show some damage to the seismic reflections that should stay untouched. This is common in transform domain approaches, even when data-driven transforms are used (Chen et al., 2016). However, the SNR of the estimated solutions indicates that the analysis approach obtains a better solution than the synthesis approach for removing incoherent noise.

For the missing trace reconstruction problem, we evaluate the results for three undersampling schemes: uniform, random and jittered (Hennenfent and Herrmann, 2008), but in the following we illustrate only the random sampling results, for simplicity. Figures 2a and 2b show the randomly undersampled data and the corresponding f-k spectra. Note that, although the f-k spectrum features no coherent aliases, the resulting spectral leakage from the random undersampling assumes a somewhat coherent pattern, i.e., it is not completely incoherent noise as in the prior application. Because we use the curvelet transform as the transform domain, we can consider the interpolation of the missing traces as the denoising of the noisy f-k spectrum. Figure 3 shows the solutions for the synthesis and analysis approaches along with the corresponding f-k spectra and difference plots for the random undersampling scheme. The SNR for the uniform and jittered undersampling experiments are: 13.29 dB and 16.26 dB for the synthesis solutions; 14.83 dB and 17.93 dB for the analysis solutions. The SNR and difference plots confirm that the analysis approach is superior to the synthesis approach for every undersampling scheme.

CONCLUSIONS

The state-of-the-art solutions for the denoising and missing trace reconstruction problems are based on transform domain approaches. In the context of sparsity-promoting inversion, we identify two key techniques: the analysis and the synthesis approaches. We compare these methods on two noise related problems. For the denoising case, our examples show that the analysis approach is superior to the synthesis approach, obtaining a clean image with less damage to the signal and thus attaining a higher SNR than that of the synthesis approach. For the missing trace reconstruction problem, our examples show that the analysis approach consistently finds better solutions than the synthesis counterpart for any of undersampling scheme. Our analysis indicates that the analysis approach is a strong candidate for solving inverse problems in geophysical applications involving noise when a sparsity-promoting constraint is efficient and it should be considered along with the synthesis approach.

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Figure 3: Random undersampling: (a), (b) Synthesis and analysis solutions. SNR of 16.24 dB and 17.87 dB, respectively. (c), (d) Corresponding f-k spectra. (e), (f) Corresponding difference plots scaled by 10.
Denoising and interpolation using sparsity-constrained regularization

REFERENCES

Waveform inversion for attenuation estimation in anisotropic media
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SUMMARY

Robust estimation of attenuation coefficients remains a challenging problem, especially for heterogeneous and anisotropic models. Here, we apply full-waveform inversion (FWI) for attenuation analysis in heterogeneous VTI (transversely isotropic with a vertical symmetry axis) media. A time-domain finite-difference algorithm based on the standard linear solid model simulates nearly constant quality-factor values in a specified frequency band. We employ the adjoint-state method to derive the gradients of the objective function in the Born approximation. Four parameters describing the attenuation coefficients of P- and SV-waves are updated simultaneously with the L-BFGS technique. The inversion algorithm is tested on homogeneous models with a Gaussian anomaly in one of the Thomsen-style attenuation parameters. Accurate knowledge of the velocity parameters and sufficient aperture of the experiment make it possible to resolve the anomaly, with better estimates of its shape and peak magnitude obtained for a lower-frequency source wavelet.

INTRODUCTION

Attenuation coefficients can be utilized to obtain important information about reservoir rocks such as the fluid type, saturation, and mechanical properties of the rock matrix (Johnston et al., 1979; Tiwari and McMechan, 2007). It is also essential to correct for attenuation in seismic imaging and inversion (e.g., Causse et al., 1999).

Formations that exhibit velocity anisotropy are often characterized by even stronger attenuation anisotropy (Zhu et al., 2006; Best et al., 2007; Chichinina et al., 2009). Laboratory experiments have shown that attenuation anisotropy may help estimate the orientation and properties of aligned fractures and the presence of organic laminae inside the rocks (Best et al., 2007; Chichinina et al., 2009; Ekanem et al., 2013).

In general, conventional attenuation-analysis techniques (e.g., the centroid frequency shift and spectral-ratio methods) are limited to structurally simple subsurface models (Hackert and Parra, 2004; Reine et al., 2012). Originally introduced to build high-resolution velocity fields, full-waveform inversion (FWI) can potentially provide more robust Q-estimates for realistic structures. Brossier (2011) performs viscoelastic FWI for isotropic media and observes that while attenuation has little impact on velocity estimation, velocity and density can leave a strong imprint on the inverted attenuation coefficients. Therefore, viscoelastic FWI can be implemented using a hierarchical approach, in which the velocity parameters are recovered prior to attenuation analysis (Prieux et al., 2013). Bai and Yingst (2013) apply multiscale FWI to estimate the attenuation coefficients for a viscoacoustic version of the Marmousi model.

Using the actual velocity field, they obtain a high-resolution isotropic Q-image, which somewhat deteriorates at depth.

By incorporating the generalized standard linear solid model, Bai and Tsvankin (2016) devise an anisotropic time-domain finite-difference algorithm to simulate nearly constant elements of the quality-factor matrix within a specific frequency band. Here, we employ their modeling methodology to perform FWI for attenuative VTI media. Synthetic transmission experiments with Gaussian anomalies illustrate the feasibility of the proposed algorithm.

METHODOLOGY

Forward modeling for viscoelastic VTI media

Wavefields in attenuative media can be simulated in the time domain by superposing several rheological bodies, each providing a relaxation mechanism (Emmerich and Korn, 1987; Carcione, 1993; Bohlken, 2002). Two to three relaxation mechanisms typically are sufficient for a frequency-independent Q-simulation (Emmerich and Korn, 1987; Bohlken, 2002). Zhu et al. (2013) observe that even one mechanism with properly chosen parameters can produce adequate Q-factors for surface seismic surveys. Here, primarily for purposes of computational efficiency, we employ only one relaxation mechanism.

As discussed by Bai and Tsvankin (2016), anisotropic attenuation can be described by the following relaxation function (shown here with only one relaxation mechanism):

$$
Ψ_{ijkl}(t) = C^R_{ijkl} \left( 1 + \tau_{ijkl} e^{-t/τ^R} \right) H(t),
$$

where

$$
C^R_{ijkl} = \Psi_{ijkl}(t \to \infty)
$$

is called the “relaxed stiffness,” which corresponds to the low-frequency limit ($a = 0$), $τ^R$ denotes the stress relaxation time determined by the dominant frequency, the $τ_{ijkl}$-parameters measure the difference between the stress and strain relaxation time and quantify the magnitude of anisotropic attenuation, and $H(t)$ is the Heaviside function. The relaxation function at zero time yields the “unrelaxed stiffness”:

$$
C^U_{ijkl} ≡ Ψ_{ijkl}(t = 0) = C^R_{ijkl} \left( 1 + τ_{ijkl} \right).
$$

The stiffness difference

$$ΔC_{ijkl} = C^U_{ijkl} - C^R_{ijkl}
$$

is proportional to $τ_{ijkl}$ and, therefore, reflects the magnitude of attenuation.

The P- and SV-wave attenuation in VTI media can be described by the Thomsen-style parameters $A_{P0}$, $A_{S0}$, $ε_p$, and $δ_0$ (Zhu and Tsvankin, 2006). $A_{P0} \approx 1/(2Q_{33})$ and $A_{S0} \approx 1/(2Q_{55})$ denote the P- and S-wave attenuation coefficients in the vertical (symmetry-axis) direction. The parameter $ε_p$ quantifies the fractional difference between the horizontal and vertical P-wave attenuation coefficients, and $δ_0$ controls the curvature of
the P-wave attenuation coefficient at the symmetry axis. Combined with the unrelaxed stiffnesses $C_{ijkl}^{II}$ (used as the reference elastic parameters), these attenuation parameters can be converted into the quality-factor elements $Q_{ijkl}$ and then into $\Delta C_{ijkl}$.

The viscoelastic stress-strain relationship can be expressed as

$$\sigma_{ij} = C_{ijkl}^{II} e_{kl} + \Delta C_{ijkl} r_{kl},$$

where $r_{kl}$ are the memory variables, which satisfy the following partial differential equations (Bai and Tsvankin, 2016):

$$\frac{\partial r_{kl}}{\partial t} = -\frac{1}{\tau^2} (r_{kl} + \varepsilon_{kl}).$$

**Viscoelastic full-waveform inversion**

FWI utilizes the entire waveforms of certain arrivals (e.g., diving waves and/or reflections) to iteratively update the model parameters. The degree of data fitting is usually evaluated with the $\ell_2$-norm objective function (e.g., Tarantola, 1988; Tromp et al., 2005):

$$F(m) = \frac{1}{2} \sum_{r=1}^{N} \| u(x_r, t, m) - d(x_r, t) \|^2,$$

where $u(x_r, t, m)$ denotes the data computed for the trial model $m$, $d(x_r, t)$ is the observed data, $r$ is the receiver index, and $t$ is the time. Summation over shots is omitted. Instead of calculating the Fréchet derivatives, which can be prohibitively expensive, the gradient of the objective function is typically computed with the adjoint-state method (Tarantola, 1988; Tromp et al., 2005; Fichtner, 2005). Then just two simulations of wave propagation (one forward and one adjoint) are required to update the model at each iteration.

In viscoelastic media, the adjoint wavefield is “propagated backward in time, with numerically stable negative attenuation” (Tarantola, 1988). Tromp et al. (2005) and Fichtner and van Driel (2014) present the adjoint equations for general anisotropic attenuative media, but implement them only for isotropic attenuation. By applying the Born approximation, the gradients for the viscoelastic parameters $\Delta C_{ijkl}$ can be expressed as the cross-correlation of the memory variables from the forward simulation with the adjoint strain fields (Tarantola, 1988):

$$\frac{\partial F}{\partial \Delta C_{ijkl}} = -\sum_{\text{sources}} \int_0^T \frac{\partial u^s_j}{\partial x_j} r_{kl} dt,$$

where $u^s_j$ denotes the adjoint displacement field.

FWI algorithms for attenuative media are usually parameterized by coefficients inversely proportional to the quality factor $Q$ (e.g., Liao and McMechan, 1995; Kamei and Pratt, 2013). Here, we invert for $A_{P0}$, $A_{S0}$ and two more parameters of similar magnitude ($A_{Pb}$, $A_{Ph}$) dependent on attenuation anisotropy. The P-wave horizontal attenuation coefficient $A_{Ph}$ is given by:

$$A_{Ph} = A_{P0} (1 + \varepsilon_o) \approx \frac{1}{2Q_{11}}.$$

To account for the attenuation-anisotropy coefficient $\delta_o$, we define another attenuation parameter, $A_{Ph}$:

$$A_{Ph} = A_{P0} (1 + \delta_o),$$

which governs the variation of P-wave attenuation near the symmetry axis, and has a form similar to the normal-moveout (NMO) velocity for a horizontal VTI layer. The gradients for the stiffness differences $\Delta C_{ijkl}$ (equation 6) can be converted into those for the attenuation coefficients $A_{P0}$, $A_{S0}$, $A_{Ph}$ and $A_{Ph}$ by applying the chain rule.

To reduce the ambiguity of the inverse problem, we assume the velocity parameters ($C_{ijkl}^{II}$) and density to be known. This prevents cycle-skipping in the inversion because the influence of attenuation-induced dispersion in the seismic frequency band is typically small (Zhu and Tsvankin, 2006; Kurzmann et al., 2013). Hence, the FWI algorithm can operate with relatively high frequencies to increase the sensitivity of the wavefield to attenuation. The L-BFGS method is applied to scale the gradients by an approximate inverse Hessian matrix.

**SYNTHETIC EXAMPLES**

We conduct a series of transmission experiments for Gaussian anomalies in the Thomsen-style attenuation parameters embedded in a homogeneous VTI background. The velocity parameters $V_{P0}$, $V_{S0}$, $\varepsilon$, and $\delta$ and the density are constant and kept at the actual values during the inversion. The reference frequency, which determines the stress relaxation time needed in the viscoelastic wave equation 4, is set equal to the central frequency of the wavelet. Starting from the homogeneous background model, we conduct simultaneous inversion for the attenuation parameters $A_{P0}$, $A_{S0}$, $A_{Ph}$, and $A_{Ph}$. To illustrate the influence of frequency on the inversion results, each test is performed for two wavelets with a different central frequency (100 Hz and 30 Hz).

First, we introduce a Gaussian anomaly in $A_{P0}$ into a homogeneous VTI background and place horizontal source and receiver arrays above and below the anomaly (Figure 1). Because the parameters $\varepsilon_o$ and $\delta_o$ are constant, there are anomalies in $A_{Ph}$ and $A_{Ph}$ as well (equations 7 and 8). Although the value of the objective function rapidly decreases for both wavelets (Figure 2), not all parameters are accurately resolved. The shape and peak magnitude of the anomaly in $A_{P0}$ are reasonably well recovered using the 100-Hz wavelet (Figure 3(a)). Also, the update in $A_{S0}$ is negligible (Figure 3(b)), as expected. However, the parameters $A_{Ph}$ and $A_{Ph}$, which are supposed to vary along with $A_{P0}$, remain almost unchanged (Figure 3(c) and 3(d)). The same experiment with a lower-frequency (30 Hz) wavelet produces different inversion results (Figure 4). The peak magnitude of $A_{P0}$ (Figure 4(a)) is less accurate ($A_{P0} = 0.019$ or $Q_{P0} = 26$), but the $A_{Ph}$- and $A_{Ph}$-fields (Figure 4(c) and 4(d)) have an update proportional to that in $A_{P0}$. Analysis of the objective function shows that the frequency dependence of the inversion results is partly related to the trade-off between $A_{Ph}$ and $A_{Ph}$. The reduction in frequency mitigates the interplay between these parameters and improves their estimates (Figure 4(c) and 4(d)).
Figure 1: Gaussian anomaly in the parameter $A_{P0}$ embedded in a homogeneous VTI medium. The plot shows the fractional difference between $A_{P0}$ and its background value, 0.005 ($Q_{P0} = 100$); at the center of the anomaly, $A_{P0} = 0.025$ ($Q_{P0} = 20$). The other parameters are constant: $A_{S0} = 0.005$, $\varepsilon_{Q} = 0.2$, $\delta_{Q} = 0.4$, $V_{P0} = 4000$ m/s, $V_{S0} = 2000$ m/s, $\varepsilon = 0.15$, $\delta = 0.1$, $\rho = 2.0$ g/m$^3$. The yellow dots denote vertical displacement sources, and the magenta line marks the receivers placed at each grid point.

Next, we introduce a Gaussian anomaly in the shear-wave coefficient $A_{S0}$ using the same background model and source-receiver geometry as in Figure 1, but with horizontal displacement sources. The peak magnitude of the $A_{S0}$-anomaly is 0.025 ($Q_{S0} = 20$), and the background value is $A_{S0} = 0.005$ ($Q_{S0} = 100$). The algorithm produces no updates in the other three attenuation parameters ($A_{P0}$, $A_{Ph}$, and $A_{Pn}$), which indicates the absence of trade-offs. When the 100-Hz wavelet is used (Figure 5(a)), the shape of the anomaly is somewhat distorted and its peak ($A_{S0} = 0.019$) deviates from the actual value ($A_{S0} = 0.025$). Still, the data residuals are substantially reduced after the inversion (Figure 6), and the inverted model in Figure 5(a) provides a close fit to the observed data (Figure 7). As before, a more accurate recovery of the peak magnitude ($A_{S0} = 0.022$) and shape of the anomaly (Figure 5(b)) is achieved with the 30-Hz wavelet.

Finally, the algorithm is tested on a model with a negative Gaussian anomaly in $\varepsilon_{Q}$ (Figure 8). As expected, the most significantly updated parameter is $A_{Ph} = A_{P0} (1 + \varepsilon_{Q})$, but for the 100-Hz wavelet, both the shape and the peak magnitude of the $\varepsilon_{Q}$-anomaly are distorted (Figure 9(a); the peak $\varepsilon_{Q} = -0.57$ instead of $-0.8$). Yet again, the 30-Hz wavelet yields better inversion results (Figure 9(b)) with a closer estimate of the peak value ($\varepsilon_{Q} = -0.72$).
Figure 6: Differences between the observed and simulated data for the model with the $A_{S0}$-anomaly. The source is located at $x = 0.25$ km and excites the 100-Hz wavelet. The residuals for the initial model: (a) z-component; (b) x-component. The residuals for the inverted model after 40 iterations: (c) z-component; (d) x-component.

Figure 7: Horizontal displacement computed with the 100-Hz wavelet for the model with the $A_{S0}$-anomaly. Both the source and receiver are located at $x = 0.25$ km. The solid red line and the dashed blue line mark the data generated with the initial and inverted models, respectively; the dotted black line is the observed data.

CONCLUSIONS

We presented a time-domain FWI methodology for attenuation estimation in transversely isotropic media. The finite difference modeling algorithm simulates a nearly constant $Q$-matrix using one relaxation mechanism. Applying the adjoint state method and the Born approximation, we obtained the gradients for the viscoelastic parameters.

The inversion algorithm was tested on homogeneous VTI models with a Gaussian anomaly in one of the Thomsen-style attenuation parameters. A perturbation in the P-wave vertical attenuation coefficient $A_{P0}$ (with fixed $\varepsilon_Q$ and $\delta_Q$) leads to the corresponding anomalies in the parameters $A_{Ph}$ and $A_{Pn}$. The inversion using a 100-Hz wavelet accurately recovers $A_{P0}$, but because of the trade-off between $A_{P0}$ and $A_{Ph}$, these parameters are barely updated. Reducing the central frequency of the wavelet to 30 Hz improves the inversion results for $A_{P0}$ and $A_{Ph}$, although the peak values of the anomalies are underestimated. The algorithm was more successful in reconstructing the anomalies in $A_{S0}$ and $\varepsilon_Q (A_{Ph})$. In the absence of obvious cross-talk with the other parameters, the peak magnitude and shape of each anomaly were accurately recovered, with better results for the low-frequency (30-Hz) wavelet.

Our initial results generally confirm the feasibility of estimating anisotropic attenuation models with FWI, provided the velocity parameters are known with sufficient accuracy. The sensitivity of the algorithm to velocity errors and application to more complicated models are topics of ongoing research.

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Extended imaging, deconvolution, and two-way wavefields: a comparison
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SUMMARY

Multiple-scattered waves contain information that is commonly disregarded during imaging and tomography. Marchenko wavefields are superior to time-reverse wavefields by handling primaries together with internal and surface-related multiples. Using all types of waves for imaging can greatly improve the illumination and augment the sensitivity of the data to errors in the background velocity model. We compare extended images computed with reverse time and Marchenko wavefields, and investigate the potential of using multidimensional deconvolution for extended images in order to obtain higher image resolution. Our experiments show that the Marchenko wavefields are sensitive to errors in the background model in a way that is similar to the sensitivity of time-reverse wavefields. The main difference between these imaging strategies is the improved angle illumination with the Marchenko wavefields due to the correct use of multiples; this improvement can eliminate the bias of tomography operators towards lower velocities when the data are contaminated with multiples.

INTRODUCTION

The quality of the seismic image depends on the choice of wave propagator, but more importantly on the quality of the background model. The better the velocity, the more focused the seismic image becomes. In order to retrieve the information about kinematics errors, one must extend the image. In the context of two-way operators, the extension is usually performed using extended images (Rickett and Sava, 2002; Sava and Fomel, 2006; Sava and Vasconcelos, 2011a), or the angle domain (Sava and Fomel, 2003; Yoon et al., 2004; Jin et al., 2014; Yoon et al., 2011; Vyas et al., 2011), or the surface offset domain (Giboli et al., 2012). Another important factor that determines the quality of a seismic image is the amount of data illuminating a target in the subsurface.

Most of the current imaging (and modeling) technologies utilize the primaries in the data but disregard the multiples which can better illuminate the targets with conventional acquisition geometries. In order to incorporate the multiples into the seismic images, a conventional approach is to use the primaries as an areal source for the surface-related multiples; this allows one to image the first order surface-related multiples in the same position as the primaries (Guitton, 2002; Grion et al., 2007; Verschuur and Berkhout, 2011; Whitmore et al., 2010; Wong et al., 2015). However, these techniques require prior separation of multiples and primaries. A recent alternative to incorporate the multiples into the imaging process is the Marchenko modeling framework in which the multiples are used together with the primaries in a global prediction procedure that characterizes focusing at an arbitrary location in the subsurface (Broggini et al., 2014; Behura et al., 2014; Wapenaar et al., 2014b; Singh et al., 2015). The focusing solutions are guaranteed to exist regardless of the background velocity model. With focusing functions, the up- and down-going Green’s functions are retrieved through convolutions and correlations between the focusing solutions and the reflection response (the data) including internal multiples (Broggini et al., 2014; Behura et al., 2014; Wapenaar et al., 2014b) and surface-related multiples (Singh et al., 2015).

We show that the Marchenko wavefields contain sensitivity to the background model that is similar to conventional wavefields obtained by time reversal or downward continuation. We assess the sensitivity by using extended images, which is the first step towards a formulation of a tomographic operator with Marchenko wavefields. We also pose the extended imaging process as an inversion problem which can be thought of as an extended deconvolution imaging condition. We illustrate our method with synthetic examples comparing the sensitivity of both approaches to the background model.

MODELING DIFFERENCES

In this section, we review similarities and differences between Marchenko and RTM propagators. While both propagators are two-way, Marchenko separates the total wavefield $u$ into up- and down-going components:

$$u(x, x_s, t) = u_i(x, x_s, t) + u_e(x, x_s, t).$$

This separation is necessary because the method relies on one-way reciprocity theorems (Wapenaar et al., 2014a). Singh et al. (2015) shows how to recover the total wavefield $u$, and its directional components $u_i$ and $u_e$ with a free-surface condition.

An iterative procedure finds the focusing solutions $f_{1+}^i(x_s, x, t)$ and $f_{1-}^i(x_s, x, t)$ such that when they are injected at the surface locations $x_s$ produce a focus at an arbitrary point $x$ in the subsurface. The focusing solutions always exist regardless of the background model. In order to obtain the focusing solutions, the method utilizes the reflectivity response $R(x_s, x_s, t)$ together with an estimate of the direct arrival $T_d(x, x_s, t)$ which goes from the subsurface point $x$ to the surface locations $x_s$. The direct arrival information contains the kinematic information from the background model. The recovered wavefield satisfies the wave equation

$$\rho \nabla \cdot \left( \frac{1}{\rho} \nabla u \right) + \frac{\omega^2}{v^2} u = -j\omega \rho \delta(x - x_s)$$
for a volumetric injection source \(-j\omega \rho \delta(x - x_s)\), density \(\rho\), and velocity \(v\).

Once the focusing functions are obtained through the iterative scheme described in Singh et al. (2015), the individual components are then retrieved through the following equations:

\[
u_s(x, x_r, \omega) = -f_1^s(x_s, x_s, \omega) + \sum_{x_s} f_1^s(x_s, x_s, \omega) R(x_r, x_s, \omega) - rf_1^s(x_s, x_s, \omega) R(x_r, x_s, \omega),
\]

\[
u_r(x, x_r, \omega) = +f_1^r(x_s, x_s, \omega)^* - \sum_{x_s} f_1^r(x_s, x_s, \omega)^* R(x_r, x_s, \omega) - rf_1^r(x_s, x_s, \omega) R(x_r, x_s, \omega).
\]

Note that the inputs for retrieving either wavefield are the up-going component of the Green’s function, whereas the focusing solutions \(f_1^s\) and \(f_1^r\) represent the down-going field, \(\nu_s\), interacting with the up-going field, \(\nu_r\); which in a matrix-vector form it reads

\[
u_r = U_s^\top \nu_r,
\]

where the matrix \(U_s^\top\) is the extended-imaging operator applied to the receiver field \(\nu_r\). Alternatively, one can think about the (extended) imaging process as an inversion problem such that \(R(x, \lambda, \tau)\) satisfies the relation

\[U_s R \approx \nu_r,\]

where \(\lambda\) and \(\tau\) are space- and time-lags, respectively.

The extended-imaging operator is linear and represents the down-going field, \(\nu_s\), interacting with the up-going field, \(\nu_r\); which in a matrix-vector form it reads

\[R = U_s^\top \nu_r,
\]

Figure 1: Free surface experiment with a constant velocity background: (a) variable density model and (b) the recorded data for a shot at \(x_s = (0, 0)\)km. Note how the data contain primaries, internal and surface-related multiples.

**SENSITIVITY TO THE BACKGROUND MODEL**

In the Marchenko context, the wavefield \(\nu_r\) represents the up-going component of the Green’s function, whereas \(\nu_s\) represents the up-going component. Following the imaging principle (Claerbout, 1971) the image \(R(x)\) is constructed when wavefields \(\nu_s\) and \(\nu_r\) coincide in space and time:

\[R(x) = \sum_{x_s, t} u_s(x_s, x, t) u_r(x_s, x, t).
\]

Although helpful for understanding the subsurface, \(R(x)\) does not contain much information about the quality of the background velocity model. Instead, one can use the extended-imaging condition (Rickett and Sava, 2002; Sava and Fomel, 2003; Sava and Vasconcelos, 2011b) to analyze the interaction between source and receiver wavefields at the vicinity of the image location \(x\) and \(t = 0\):

\[R(x, \lambda, \tau) = \sum_{c, t} u_s(c, x - \lambda, t - \tau) u_r(c, x + \lambda, t + \tau),
\]

\[\epsilon R \approx 0.
\]

The composite system can be written as

\[(U_s^\top U_s + \epsilon^2 I) R = U_s^\top \nu_r,
\]

where \(\lambda\) and \(\tau\) are space- and time-lags, respectively.

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Figure 1: Free surface experiment with a constant velocity background: (a) variable density model and (b) the recorded data for a shot at \(x_s = (0, 0)\)km. Note how the data contain primaries, internal and surface-related multiples.
Figure 2: Space-lag gathers obtained from RTM wavefields for (a) low, (b) correct, and (c) fast velocity. Corresponding angle gathers for (d) low, (e) correct, and (f) fast velocity.

Figure 3: Deconvolved space-lag gathers obtained from RTM wavefields for (a) low, (b) correct, and (c) fast velocity. Corresponding deconvolved angle gathers for (d) low, (e) correct, and (f) fast velocity.

and can be solved by the conjugate gradient method. Note that the matrix $U_s^T U_s$ does not need to be formed; instead one needs to compute the action of the matrix to the input reflectivity $R(x, \lambda, \tau)$. The deconvolution process is similar to the method proposed by Valenciano et al. (2009) and is also referred to as multi-dimensional deconvolution (MDD) by van der Neut et al. (2011). The deconvolution approach produces better focusing and better approximates the reflectivity.

**EXAMPLES**

We compare the behavior of RTM and Marchenko images for errors in the background velocity using a synthetic model with constant background velocity and variable density profile, Figure 1a. The pressure data containing primaries and multiples is shown in Figure 1b. Our main objective is to investigate the sensitivity of the Marchenko images to errors in the background model, which is the first step towards the formulation of a tomography problem.

We construct extended images with $\tau = 0$, i.e. space-lag gathers, for slow, correct, and fast background velocities. Figures 2a-2c show the gathers computed using the conventional RTM propagator, where one can observe the strong response of the multiples present in the data below the imaged reflector. Figures 2d-2f show the corresponding common angle gathers for slow, correct, and fast models, respectively. The transformation from space-lag $\lambda$ to angle $\theta$ is described by Sava and Fomel (2006). For this particular model, the reflection coefficient should remain constant as a function of the scattering angle. We recompute the same set of images with the deconvolution framework in equation 13: Figures 3a-3c show the deconvolved space-lag gathers with RTM wavefields, and Figures 3d-3f show the corresponding angle gathers with the RTM wavefields. In these figures, the amplitude variation across angles is more even. The deconvolved space-lag gathers contain more energy at larger lags; this can be beneficial for improved tomographic updates because during tomography, the objective function measures the energy away from $\lambda = 0$ (Shen and Symes, 2008). Hence, with the deconvolved gathers one can better highlight aspects of the gathers useful for tomography.

We repeat the same experiment using the Marchenko wavefields. Figures 4a-4c show the space-lag gathers for low, correct, and fast background velocity, respectively. The main difference between these gathers and those in Figures 2a-2c is the correct handling of the surface-related and internal multiples. Despite the characteristics of the method, where the focusing solutions $f_1^+$ and $f_2^+$ always exist, the gathers present moveout, indicating errors in the background model. Also, the focusing for the correct velocity model, Figure 4b, is improved.
CONCLUSIONS

We compare image gathers obtained with Marchenko and time-reverse propagators. The Marchenko gathers correctly image primaries, internal multiples, and surface-related multiples. In contrast, RTM gathers treat the multiples as primaries; hence, the events are imaged in the wrong location and can bias tomographic inversion towards lower velocities. The Marchenko and RTM gathers show similar sensitivity to errors in the background model. However, the Marchenko gathers possess increased illumination as shown by the angle gathers in our experiments.

We also show the value of posing the extended imaging problem using deconvolution instead of correlation. The focus in the gather is improved, and the energy distribution is more homogeneous across angles. The use of Marchenko wavefields together with extended deconvolution can benefit tomography. The dependence of the Marchenko wavefield with respect to the background velocity is not as explicit as with the RTM wavefields; in Marchenko, the kinematics are embedded in the direct arrival information used as input for the framework.

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Imaging the model through the wave equation
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SUMMARY

Two-way wavefields generated with the Marchenko method are able to reproduce complex wave phenomena that includes primaries, internal multiples and surface-related multiples. The wavefield contains reflections and information from the true model, but propagating with the kinematics of an input background velocity model. We design an inverse problem to find a model that explains the scattering phenomena present in the reconstructed wavefield. The two-way nature of the Marchenko wavefields allows us to use them with the homogeneous wave equation and obtain images that are indicative of subsurface model parameters. Unlike conventional imaging methods, where the image is indicative of the interfaces in the subsurface, our method inverts for the properties within the subsurface. Our tests show that one can invert for images indicative of the true model even when the background velocity model is partially inaccurate. Furthermore, we are able to image properties from the true model that are not part of the inputs used to generate the wavefield.

INTRODUCTION

Seismic imaging methods aim to estimate the Earth’s reflectivity, i.e., a map of the interfaces that scatter the propagating wavefields. However, a seismic image does not only have structural information, but also dynamic properties that contain important information about the subsurface parameters (Etgen et al., 2009). Despite great advances in seismic exploration technology, the underlying imaging mechanisms (the imaging condition) are essentially the same as the one proposed by Claerbout (1971).

The wavefield reconstruction step is where many advances occurred recently. The source and receiver wavefields can be extrapolated with integral formulations as in Kirchhoff migration (Schneider, 1978), one-way equations as in wave equation migration (WEM) (Gazdag, 1978; Gazdag and Sguazzero, 1984), or two-way equations as in reverse time migration (RTM) (Baysal et al., 1983; Whitmore, 1983; McMechan, 1983). The three outlined families have a basic assumption in common: the single-scattering (Born) approximation. Hence, the multiple-scattered waves present in the data are imaged in the wrong position. Therefore, the multiples are usually eliminated pre imaging (Verschuur et al., 1992; Weglein et al., 1997; Guitton, 2005; Lin and Herrmann, 2013) or post imaging (Sava and Guitton, 2005; Wang et al., 2010; Weibull and Arnsen, 2013).

Marchenko imaging retrieves the Green’s function and properly handles primaries and internal multiples given a background velocity model with the appropriate kinematics. The Marchenko modeling framework (Broggini and Snieder, 2012; Behura et al., 2014; Wapenaar et al., 2014; Singh et al., 2015a) solves for focusing functions that are used to retrieve the total Green’s function, which can be decomposed into up- and down-going components. This framework is also applicable to surface-related multiples (Singh et al., 2015a,b). The total Green’s function satisfies the acoustic wave equation with all the scattering present in the data.

In this paper, we use the total Green’s function instead of the individual up- or down-going components (or source and receiver wavefields) to obtain an image of the model parameters (instead of the reflectivity). We use the total Green’s functions together with relations given by the wave equation to obtain images sensitive to other subsurface properties like density or impedance.

Fig. 1: Variable velocity model: the points at the surface denote source locations and the highlighted box indicates the area in which the Marchenko wavefield is retrieved.

IMAGING THE MODEL PARAMETERS

The Marchenko framework outlined in Singh et al. (2015a) allows for the Green’s function retrieval with pressure-normalized fields. The method reconstructs the Green’s function from any point \( x \) inside the medium to the surface. The method requires the reflectivity response \( R(x_s, x_r, t) \) for sources \( x_s \) and receivers \( x_r \) located at \( z = 0 \). It also requires an estimate of the transmission response \( T_d(x, x_r, t) \) from the point of interest \( x \) to the surface. In practice, \( T_d \) is approximated by the first arrivals in the background medium, which can be obtained with either a finite difference approach or an eikonal traveltime data convolved with an appropriate wavelet.

From the data and the transmission response, the Marchenko
Imaging acoustic parameters

Figure 2: Velocity retrieval: (a) is a smooth background model used for computing the transmission response $T_d$, (b) the retrieved velocity model, and (c) a snapshot of the wavefield $u(x, x_s = (0,0), t = 0.6s)$. Note how the retrieved model converges to the background model.

framework retrieves the Green’s function from all the points in the subsurface to the surface. The retrieved wavefield $u(x, x_s, t)$ satisfies the acoustic wave equation.

If the transmission response is accurate enough, the wave equation is satisfied for the true model parameters.

We use the total wavefield $u$ to retrieve model parameters, like density and velocity, present in the wave equation. The retrieved wavefields are used as an input to the wave equation from which we solve for specific model parameters. This idea have been successfully implemented in the medical community for finding elastic parameters of human tissues (Manduca et al., 2001). Here we show an inversion framework for acoustic parameters.

**Constant velocity acoustics**

Consider the constant density homogeneous wave equation

$$s^2 \frac{\partial^2 u}{\partial t^2} - \nabla^2 u = 0,$$

where $s^2(x)$ is the slowness squared and $u(x_s, x, t)$ the Marchenko wavefield generated from the source location $x_s$. Note that the wavefield can be obtained inside the medium away from the source location $x_s$ where the homogeneous equation is satisfied.

In order to solve for the unknown field $s^2(x)$, one can set up a least-squares linear problem:

$$J(s^2) = \left\| s^2 \frac{\partial^2 u}{\partial t^2} - \nabla^2 u \right\|_2^2,$$

which has the following solution:

$$s^2(x) = \frac{\sum_{t, x_s} \frac{\partial^2 u(x_s, x, t)}{\partial t^2} \nabla^2 u(x_s, x, t)}{\sum_{t, x_s} \left(\frac{\partial^2 u(x_s, x, t)}{\partial t^2}\right)^2} - \nabla \cdot \left(\frac{1}{\rho} \nabla u\right) = 0.$$

Observe how this expression closely resembles the least squares solution for the deconvolution imaging condition (Claerbout, 1971; Guitton et al., 2007).

The kinematics embedded in the Marchenko wavefield $u$ are driven by the first arrival information $T_d(x_s, x, t)$ which is computed using a background velocity model. The inversion procedure outlined in equation 2 utilizes the geometric attributes of the wavefield; therefore, the wave equation is satisfied for the same background velocity model used for computing the transmission response $T_d(x_s, x, t)$.

**Variable density acoustics**

For variable density, the Marchenko wavefield $u$ satisfies the wave-equation

$$s^2 \frac{\partial^2 u}{\partial t^2} - \nabla \cdot \left(\frac{1}{\rho} \nabla u\right) = 0.$$

Note how the density and velocity model are decoupled inside the box.

The relation of the wavefield $u(x_s, x, t)$ with the model parameters $s^2(x_s, x, t)$ and density $\rho(x)$ is not as trivial as in the previous case. This problem does not have a direct least squares solution. Considering that the kinematics are controlled by the model used as input to compute $T_d$, we can focus our attention on the density model $\rho$. Using only the homogeneous wave equation (for pressure), one cannot retrieve the magnitude of the density field because it scales all the terms in the equation. However, one should be able to retrieve the relative changes in the density field. As with the constant density case above, we set up a least-squares linear problem:

$$J(\rho) = \left\| \frac{s^2}{\rho} \frac{\partial^2 u}{\partial t^2} - \rho \frac{\nabla u}{\rho} \right\|_2^2.$$

The solution is given by:

$$\rho(x) = \frac{\sum_{t, x_s} \rho(x_s) \frac{\partial^2 u(x_s, x, t)}{\partial t^2} \nabla^2 u(x_s, x, t)}{\sum_{t, x_s} \left(\rho(x_s) \frac{\partial^2 u(x_s, x, t)}{\partial t^2}\right)^2} - \nabla \cdot \left(\frac{1}{\rho} \nabla u\right) = 0.$$
Imaging acoustic parameters

Figure 4: Density retrieval: (a) exact velocity model used as background, (b) inverted density model, and (c) a snapshot of the wavefield \( u(x, x_s, t = 0.0s) \), Note how the velocity layer leaks into the inverted model. However, the density layer and interface are clearly defined in the inverted model.

Density case, we set the following optimization goal:

\[
J_{1/\rho} = \left\| \frac{s^2}{\rho} \nabla u - \nabla \cdot \left( \frac{1}{\rho} \nabla u \right) \right\|_2^2. \tag{5}
\]

Numerically, the problem is implemented as follows:

\[
J(m) = \left\| D_1 m + D G_u m \right\|_2^2 = \left\| L(m) \right\|_2^2, \tag{6}
\]

where \( D_1 = \text{diag} \{ s^2(x) \frac{\partial^2 u(x, x_s, t)}{\partial x^2} \} \), \( G_u \) is a block diagonal matrix that contains the gradient of \( u \), and \( D \) is the divergence matrix.

The gradient with respect to the buoyancy \( m = 1/\rho \) is given by

\[
\nabla_m J = D_1^T L(m) + G_u^T D^T L(m), \tag{7}
\]

where \( ^\top \) denotes adjoint. Once the gradient is defined, the model is updated iteratively using a non-linear solver.

**Pressure and particle velocity relations**

Another option for retrieving an image of the buoyancy is to use the relations between pressure fields and particle velocity. The Marchenko framework described in Singh et al. (2015a, 2016) allows for the reconstruction of pressure \( p \) and the vertical component of the particle velocity \( v_z \) fields. Hence, we can find the model that best matches the acoustodynamics relation:

\[
\frac{\partial V_z}{\partial t} = -\frac{1}{\rho} \frac{\partial u}{\partial z}. \tag{8}
\]

Similarly to as in the first case, we can solve a linear least-squares problem to find the appropriate solution:

\[
J(m) = \left\| \frac{\partial V_z}{\partial t} + m \frac{\partial u}{\partial z} \right\|_2^2, \tag{9}
\]

which has a solution that resembles equation 3:

\[
m = -\frac{\sum_t x_s \frac{\partial V_z}{\partial t}(x, x_s, t) \frac{\partial u}{\partial z}(x, x_s, t)}{\sum_t x_s \left( \frac{\partial u}{\partial z}(x, x_s, t) \right)^2}. \tag{10}
\]

We have presented two options to invert for parameters given the pressure wavefield (first two cases) or the pressure and particle velocity wavefields (last case). These approaches are not the only options; one could try to reparametrize the wave equation in terms of acoustic impedance or other suitable combinations.

**Examples**

We test the imaging procedure described above for two cases: constant and variable density. For each case, we test the sensitivity to the background velocity model and compute acoustic data with a free surface boundary condition. In order to record both pressure and particle velocity data, we use the finite difference implementation of the coupled acoustodynamics system of equations from Thorbecke and Draganov (2011).

We place 500 sources and receivers at \( z = 0m \) in the range \( x = [-1000, +1000]m \) (Figure 1) to generate data. The input transmission data \( T_d(x, x_s, t) \) are computed using an eikonal solver.

Figure 1 shows the velocity model and the highlighted box depicts the area where the Marchenko wavefields are retrieved. The dots in the surface indicate the source locations used for the imaging process (a subset of the sources present in the data).

Figure 2a depicts a smooth version of the true velocity used as the background model. The Marchenko wavefields show propagating waves, with the corresponding reflections, even if the background velocity model is smooth (Figure 2c). One could say that the retrieved wavefield is an expression of the true Green’s function with the kinematics of the background model. Figure 2b is the inverted velocity model using the same procedure as discussed above. Note that the inverted model converges to the background model. This is because the kinematics of the retrieved wavefield are given by the background model, but the reflections present in the wavefield are due to the discontinuities of the original model.

The model parameters for the variable density experiment are shown in Figures 3a-3b. The density and velocity model contain complementary structure to see how well the inverted model can decouple reflections from either parameter. Since the kinematics of the wavefield converge to those of the background model, we focus our attention on the independent parameter, which is density in this case. Figure 4a shows the background model inside the box (the true model), whereas Fig-
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Figure 4b shows the inverted density model. The velocity interfaces leak into the inverted model, but, the density layers and the interface are well-imaged. Note that the density model is not an input for the computation of the wavefield shown in Figure 4c and yet the inverted image clearly delineates the density layer. In this example, we use the relation between pressure and particle velocity fields (equation 10) to compute the density image.

Figure 5a shows a smooth version of the true model used as background velocity field, and Figure 5b depicts the inverted density model. The density layer is clearly imaged despite the smooth background velocity model. The velocity interfaces again leak into the inverted model; note that the background velocity model does not contain any interface and yet the velocity interfaces from the original (true) model are imaged. The wavefield snapshot in Figure 5c is propagated in the smooth background model and contains all the reflections from the true model. Figure 5d is a density inversion from the pressure field (equation 6). By only using the pressure field, there is ambiguity with the magnitude of the density model because many different density models can yield the same wavefield. The model is clearly imaged with this method as the density layers are visible and there is only a hint from the velocity interface leaking into the inverted model. This approach is a valid alternative when one only have the pressure data as input, instead of the pressure and particle velocity data needed for the previous variable density examples.

CONCLUSIONS

We present a framework to obtain images of acoustic model parameters that are different from reflectivity. Our method relies on two-way wavefields computed with Marchenko modeling. The wavefields are input into the wave equation together with the background velocity to find the model that explains, in a least-squares sense, the homogeneous wave equation. We show different options one could use depending on which type of data are available. One can exploit the relations between pressure and particle velocity and invert for the density model or one could use an iterative approach if pressure is the only available data. Our tests show that it is possible to find an image representing the true model even when the background model is smooth. This method is able to image the properties within the subsurface instead of the properties contrasts as it is done with conventional images.

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Figure 5: Density retrieval: (a) smooth velocity model used as background, (b) inverted density model from pressure and particle velocity data, (c) a snapshot of the wavefield $u(x, x_s = (0,0), t = 0.6s)$, and (d) density image from pressure only inversion. Note how the velocity layer leaks into the inverted model. However, the density layer and interface are clearly defined in the inverted models.
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Elastic least-squares reverse time migration
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SUMMARY

Least-squares migration (LSM) can produce images with improved resolution and reduced migration artifacts. We propose a method for elastic least-squares reverse time migration (LSRTM) based on a new perturbation imaging condition that yields scalar images of squared P and S velocity perturbations. These perturbation images are simply related to physical subsurface properties, and in addition, they do not suffer from polarity reversals seen with other more conventional elastic imaging methods. We use 2D examples to demonstrate the proposed LSRTM algorithm using our perturbation imaging condition. Results show that elastic LSRTM increases the image resolution and attenuates artifacts, while providing images where the relative amplitudes of the reflectors can be used for reservoir characterization.

INTRODUCTION

Advances in seismic acquisition and ongoing improvements in computational capability make imaging using elastic waves increasingly feasible (Sun et al., 2006; Yan and Sava, 2008; Denli and Huang, 2008; Artman et al., 2009; Wu et al., 2010; Du et al., 2012; Duan and Sava, 2015; Rocha et al., 2015). Compared to acoustic images, elastic images can provide more information about the subsurface, e.g., fracture distributions and elastic properties. However, elastic migration also suffers from issues that negatively affect the quality of the images. Because it is in general difficult to separate all arrivals in the recorded data by wave mode, some arrivals are migrated using an incorrect velocity model. Such nonphysical modes lead to artifacts (i.e. cross-talk) in the image (Duan et al., 2014).

Least-squares migration (LSM) is an improved imaging algorithm that reduces these migration artifacts and also improves the resolution of migration images. LSM is a linearized wave-form inversion that seeks to find the image that best predicts, in a least-squares sense, the recorded seismic data (Schuster, 1993; Nemeth et al., 1999; Dai et al., 2011). Schuster (1993) proposes LSM for cross-well data while Nemeth et al. (1999) apply this technique to surface data. Their studies show that LSM can significantly improve the spatial resolution of the images, and can also reduce migration artifacts arising from limited aperture, coarse sampling, and acquisition gaps.

LSM can be implemented using a Kirchhoff engine (Nemeth et al., 1999; Dai et al., 2011), one-way wave equation (Kuehl et al., 2002; Kaplan et al., 2010; Huang and Schuster, 2012), or two-way wave equation, i.e., least squares reverse-time migration (LSRTM) (Dai and Schuster, 2013; Dong et al., 2012; Luo and Hale, 2014; Wong et al., 2015). Although computationally expensive, RTM is advantageous for velocity models with complicated geologic structures that result in wavefield multipathing.

A key component for elastic LSRTM is the imaging condition, and many different types of imaging conditions for elastic media have been proposed. For example, Yan and Sava (2008) propose a displacement imaging condition that crosscorrelates each component of source and receiver displacement wavefields. They also propose a potential imaging condition that crosscorrelates P- and S-wave modes in source and receiver wavefields. One issue with this potential imaging condition is that the image components for converted waves change polarity at normal incidence. Stanton and Sacchi (2015) use a LSRTM method based on this imaging condition, including an additional polarity correction in the angle domain. Duan and Sava (2015) propose a scalar imaging condition for converted waves that produces scalar images without polarity reversal; however, this imaging condition requires knowledge of the geologic dip.

In this paper we propose an elastic LSRTM method based on a new perturbation imaging condition, which we derive for squared P and S velocities. Images computed using this new imaging condition can be simply related to physical subsurface properties, and in addition, these images do not suffer from polarity changes and thus can be stacked over experiments without an additional polarity correction. Using the perturbation imaging condition, we demonstrate that we obtain elastic LSRTM images with higher resolution and less migration artifacts than RTM.

THEORY

LSM aims to find the image that best predicts, in a least-squares sense, the recorded seismic data. For elastic migration, we consider a vector image \( \mathbf{m} \) which contains both compressional and shear wave information. Migration is an adjoint operator \( \mathbf{F}^\dagger \) that maps recorded data \( \mathbf{d} \) to an image \( \mathbf{m} \), and corresponding linearized forward process can be expressed as

\[
\mathbf{Fm} = \mathbf{d},
\]

where \( \mathbf{F} \) is the demigration operator.

LSM updates the model iteratively by minimizing the objective function

\[
J(\mathbf{m}) = \sum \frac{1}{2} \| \mathbf{D}(\mathbf{Fm} - \mathbf{d}_e) \|^2,
\]

which evaluates the misfit between observed data \( \mathbf{d}_e(\mathbf{e}, \mathbf{x},t) \) and predicted data \( \mathbf{Fm} \) for each experiment \( e \). Matrix \( \mathbf{D}(\mathbf{e}, \mathbf{x},t) \) denotes a data weighting operator, which can be applied for various purposes. For example, Trad et al. (2015) use matrix \( \mathbf{D} \) to eliminate the impact of high-amplitude noise or missing traces on inversion; Wong et al. (2015) use matrix \( \mathbf{D} \) to down-weight salt reflection energy. In this paper, we use the data weighting term to balance the amplitudes of the recorded data.

Perturbation models are derived using the Born approximation (Hudson and Heritage, 1981; Jaramillo and Bleistein, 1999;
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Ribodetti et al., 2011). We consider the homogeneous elastic isotropic wave-equation:

\[ \mathbf{u}_n - \alpha \nabla (\nabla \cdot \mathbf{u}_n) + \beta \nabla \times (\nabla \times \mathbf{u}_n) = \mathbf{d}_s, \]  

(3)

where \( \mathbf{u}_n (e, x, t) = [u_n(x, t)] \) is the source displacement wavefield, which is a function of experiment \( e \), space \( x \), and time \( t \). Vector \( \mathbf{d}_s (e, x, t) \) is the source function. Parameters \( \alpha (x) = \frac{\lambda + 2\mu}{\rho} \) and \( \beta (x) = \frac{\mu}{\rho} \) are squared P- and S-wave velocities, respectively. \( \lambda \) and \( \mu \) are the Lamé parameters, and \( \rho \) is the density.

The perturbation \( \mathbf{m} = \begin{bmatrix} \alpha \mathbf{I}^\alpha & \beta \mathbf{I}^\beta \end{bmatrix}^T \) to the background model gives the perturbed model \( \begin{bmatrix} \alpha + \mathbf{I}^\alpha & \beta + \mathbf{I}^\beta \end{bmatrix}^T \). The total wavefield \( \mathbf{u}_n + \delta \mathbf{u}_n \) is computed using the same source term \( \mathbf{d}_s \):

\[
\begin{align*}
(\mathbf{u}_n + \delta \mathbf{u}_n) - (\alpha + \mathbf{I}^\alpha) \nabla (\nabla \cdot (\mathbf{u}_n + \delta \mathbf{u}_n)) \\
+ (\beta + \mathbf{I}^\beta) \nabla \times (\nabla \times (\mathbf{u}_n + \delta \mathbf{u}_n)) = \mathbf{d}_s,
\end{align*}
\]

(4)

where \( \delta \mathbf{u}_n \) is the perturbed wavefield:

By ignoring higher order terms \( \mathbf{I}^\alpha \nabla (\nabla \cdot \delta \mathbf{u}_n) \) and \( \mathbf{I}^\beta \nabla \times (\nabla \times \delta \mathbf{u}_n) \), and subtracting equation 3 from equation 4, we obtain a relation for the perturbed wavefield \( \delta \mathbf{u}_n \):

\[
\delta \mathbf{u}_n - \alpha \nabla (\nabla \cdot \delta \mathbf{u}_n) + \beta \nabla \times (\nabla \times \delta \mathbf{u}_n) \\
= [\nabla (\nabla \cdot \mathbf{u}_n) - \nabla \times (\nabla \times \mathbf{u}_n)] \begin{bmatrix} \alpha \mathbf{I}^\alpha \\ \beta \mathbf{I}^\beta \end{bmatrix}.
\]

(5)

The predicted data are extracted from the perturbed wavefield \( \delta \mathbf{u}_n \) at the receiver locations.

We define a matrix \( \mathbf{W} \) that, at each time and space position, is given by

\[ \mathbf{W} = [\nabla (\nabla \cdot \mathbf{u}_n) - \nabla \times (\nabla \times \mathbf{u}_n)]. \]  

(6)

In this matrix, the first and second elements are the decomposed P- and S-modes of the source wavefield \( \mathbf{u}_n \), respectively. The demigration operator in equation 1 thus becomes

\[ \mathbf{F} = \mathbf{K} \mathbf{P}^T \mathbf{W} \]  

(7)

where \( \mathbf{P} \) represents an elastic forward modeling operator that computes the perturbed wavefield \( \delta \mathbf{u}_n \) for a source term \( \mathbf{W} \mathbf{m} \); \( \mathbf{K} \) is an operator that restricts the perturbed wavefield \( \delta \mathbf{u}_n \) to the known receiver positions. Equation 7 maps the image \( \mathbf{m} \) to the data \( \mathbf{d}_s \), and its adjoint operator

\[ \mathbf{F}^T = \mathbf{W}^T \mathbf{P}^T \mathbf{K}^T. \]  

(8)

maps the data \( \mathbf{d}_s \) to the image \( \mathbf{m} \); the operator \( \mathbf{K}^T \) injects the recorded data \( \mathbf{d}_s \) into the wavefield, and the adjoint wave propagation operator \( \mathbf{P}^T \) computes the receiver displacement wavefield \( \mathbf{u}_n = \mathbf{P}^T \mathbf{K}^T \mathbf{d}_s \). Equation 8 describes a perturbation imaging condition for elastic RTM, because the application of the adjoint operator \( \mathbf{F}^T \) to the recorded data \( \mathbf{d}_s \) yields the images

\[ I^{\alpha} = \sum_{e,t} [\nabla (\nabla \cdot \mathbf{u}_n)] \cdot \mathbf{u}_r, \]  

(9)

\[ I^{\beta} = \sum_{e,t} [\nabla \times (\nabla \times \mathbf{u}_n)] \cdot \mathbf{u}_r, \]  

(10)

for the \( \alpha \) and \( \beta \) models, respectively. From equation 8, we see that images \( I^{\alpha} \) and \( I^{\beta} \) are computed by taking the zero-lag crosscorrelation of elements of the matrix \( \mathbf{W} \) with the displacement receiver wavefield \( \mathbf{u}_s \). These perturbation images do not suffer from polarity reversal, which is a common issue for elastic images whose values are related to angle-dependent reflectivities.

Figure 1: (a) Synthetic experiment with \( \alpha \) (left) and \( \beta \) (right) models depicting reflectors at different depths. Inverted \( \alpha \) (left) and \( \beta \) (right) models after (b) 1 and (c) 10 iteration(s) of elastic LSRTM.

EXAMPLES

We use two examples to demonstrate our method for elastic migration. The first example is layered and each of the \( \alpha \) and \( \beta \) models contains one horizontal reflector at different depths, as shown in Figure 1a. We generate 30 two-component shot gatherings using a vertical displacement source with a 30 Hz peak frequency Ricker wavelet. Using the perturbation imaging condition (equation 7), we obtain the images for \( \alpha \) and \( \beta \) shown in Figure 1b. Notice that additional reflectors appear in both \( \alpha \) and \( \beta \) images; these reflectors are generated by unphysical modes in the constructed receiver wavefield. Figures 1c show the LSRTM images after 10 iterations. Compared to the images shown in Figure 1b, the LSRTM images have higher resolution and contain fewer artifacts.

We also demonstrate our method with a modified version of the Marmousi-II model (Martin et al., 2006). This model is fully elastic, and supports not only compressional waves, but also shear waves and converted waves. The model simulates hydrocarbon reservoirs that dramatically decrease the value of \( \alpha \),
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but slightly increase the value of $\beta$. Figures 2a and 2b show the background $\alpha$ and $\beta$ models, respectively; both model contain a homogeneous layer at the top. Figures 3a and 3b show the corresponding true perturbation models for $\alpha$ and $\beta$, respectively, which are inconsistent in reservoir areas, for example, the highlighted box in Figures 3a and 3b. This inconsistency poses a challenge for elastic LSRTM, e.g. if the inversion allows a leakage between model parameters.

We model 40 shots evenly spaced on the surface using a displacement source with a 30 Hz peak frequency Ricker wavelet. The horizontal and vertical components of the source function have the same amplitudes in order to generate strong shear waves. The receiver spread is fixed for all shots and spans from 0 to 3.0 km with a 5 m sampling.

The recorded data are modeled according to equation 8. In order to obtain a uniform update using all arrivals, we use the data weighting term $D$ to balance the arrivals with weak amplitudes. We define the data weighting function based on the inverse of the data envelope. Figures 4a is the horizontal component of the weighted recorded shot gather, demonstrating that the amplitudes of all arrivals are well balanced.

Figures 5a and 5b show the RTM images for $\alpha$ and $\beta$, respectively. We observe that the events for the shallow reflectors in both models have stronger amplitudes compared to those of the deeper reflectors, and strong backscattering is present due to the sharp interfaces in the background model. We apply an illumination compensation based on the source wavefield $u_\alpha$, to the RTM images and LSRTM gradients, in order to balance nonuniform data coverage.

The LSRTM images after 112 iterations, where the conver-
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Figure 5: RTM (a) $\alpha$ and (b) $\beta$ images with illumination compensation based on the source wavefield.

Figure 6: Updated (a) $\alpha$ and (b) $\beta$ images after 112 iterations. Note that the reservoir near coordinates $\{1.9, 0.4\}$ km is correctly recovered in the updated $\alpha$ image, without any leakage in the updated $\beta$ image.

Figure 7: Comparison between inverted images (solid lines) and true perturbation images (dashed lines) for (a) $\alpha$ and (b) $\beta$ models.

CONCLUSIONS

We propose a method for elastic least-squares reverse time migration using a perturbation imaging condition. The images computed using our method represent perturbations of squared P and S velocities. Compared to RTM, LSRTM has higher computational cost, and more work is needed to improve its convergence rate. Nevertheless, elastic LSRTM produces high-resolution images that provide correct relative amplitudes, which makes our algorithm especially suitable for applications such as reservoir characterization.

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3D angle decomposition for elastic reverse time migration
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SUMMARY

We propose 3D angle decomposition methods from elastic reverse time migration using time- and space-lag common image point gathers, time-lag common image gathers, and space-lag common image gathers computed by elastic wavefield migration. We compute time-lag common image gathers at multiple contiguous locations, instead of isolated positions as is commonly done with common image gatherers. Then, we transform the extended images to the angle domain using slant stacks along surfaces that connect neighboring positions, instead of line slant stacks for isolated common image gatherers. We demonstrate our methods using 2D and 3D synthetic examples and show that our techniques provide accurate opening angle and azimuth angles, and that they can handle steeply dipping reflectors and converted wave modes.

INTRODUCTION

Common reflection-angle gathers are useful in migration velocity analysis and amplitude-versus-angle analysis. Among angle decomposition techniques, wavefield-based methods are preferred because they accurately simulate wave propagation in complex geologic structures. Wavefield-based methods generally fall into two categories: pre-migration and post-migration algorithms (Vyas et al., 2011). Pre-migration methods decompose the wavefields prior to the imaging condition, for example by f − k domain decomposition, (Xu et al., 2011), or by Poynting vector methods (Yoon and Marfurt, 2006; Yan et al., 2013). Post-migration methods, transform extended images to angle gatherers after the imaging condition, using space-lag or time-lag common image gatherers (CIG), and mixed time- and space-lags common image point (CIP) gatherers (De Bruin et al., 1990; Sava and Fomel, 2003; Biondi and Symes, 2004; Sava and Vlad, 2011). Compared to pre-migration algorithms, post-migration methods are cheaper and can handle complicated wavefields.

Angle decomposition methods can also be applied to elastic wavefields. In contrast to acoustic wavefields, elastic wavefields contain various wave modes (e.g. P and S waves). Yan and Xie (2012) suggest a pre-migration method for angle decomposition in elastic media, while Yan and Sava (2008) propose post-migration decomposition methods involving wave mode decomposition. Duan and Sava (2015) further develop a scalar imaging condition for converted waves, which generates 3D PS and SP scalar images cheaply and without explicit polarity correction.

In this paper, we present methods for opening and azimuth angle decomposition from elastic RTM images. We develop the extended scalar imaging condition which generalizes the zero-lag scalar imaging condition (Duan and Sava, 2015). Our methods utilize local plane wave decompositions of extended images and exploit the relationships between the time-lag, space-lag, and image shift in space, which are measured in extended images. These relationships lead to opening and azimuth angle decomposition methods using either time-lag common image gather (τCIGs), space-lag common image gatherers (λCIGs), or time- and space-lag common image point (CIP) gatherers. We demonstrate the algorithms with 3D elastic time- and space-lag CIPs and 2D elastic time-lag extended images evaluated at multiple contiguous positions.

ELASTIC SCALAR IMAGING CONDITION

Duan and Sava (2015) propose a scalar imaging condition using geometrical relationships between the propagation directions for the P and S waves and the reflector orientation. The PS and SP images computed using this imaging condition are scalars without polarity reversals. Here, we develop a more general form of extended imaging conditions as a function of space and crosscorrelation lags (space-lag λ and time-lag τ):

\[ I^{P}(x,\tau) = \sum_{e,t} P_e(x, x - \lambda, t - \tau) P_t(x, x + \lambda, t + \tau), \]

\[ I^{PS}(x,\tau) = \sum_{e,t} [\nabla P_e(x, x - \lambda, t - \tau) \times n(x)] \cdot S_t(x, x + \lambda, t + \tau), \]

\[ I^{SP}(x,\tau) = \sum_{e,t} [\nabla \times S_e(x, x - \lambda, t - \tau) \cdot n(x)] \cdot P_t(x, x + \lambda, t + \tau), \]

\[ I^{SS}(x,\tau) = \sum_{e,t} S_e(x, x - \lambda, t - \tau) \cdot S_r(x, x + \lambda, t + \tau). \]

Vector \( n(x) \) is the normal to the reflector plane. Quantities \( S(x,e,t) \) and \( P(x,e,t) \) represent P and S wavefields obtained by wave-mode decomposition, as functions of experiment e, time t, and space x. Subscripts s and r indicate the wavefield origins at source or receivers, respectively. P and S wavefields are obtained from displacement field using Helmholtz decomposition (Dellinger and Etgen, 1990; Yan and Sava, 2008):

\[ P(x,e,t) = \nabla \cdot u(x,e,t), \]

\[ S(x,e,t) = \nabla \times u(x,e,t). \]

MOVEOUT ANALYSIS

The extended imaging condition in equations 1-4 preserves necessary information for decomposing images in angle-dependent components, which can be used for velocity analysis. In 3D, the PP, PS, SP, and SS images generated using this imaging condition are scalars, which can be used directly to construct angle gatherers. We formulate our method under the assumption that the reflector is locally planar, and that the incident and
reflected wavefields are also locally planar. However, the derived algorithms are not limited to cases where the wavefronts are planar, because waves can always be decomposed to planar components, even in complex wavefields characterized by multipathing.

We define the angle domain as the set of half-opening angle \( \theta \) and azimuth \( \phi \). The (planar) source wavefield propagates along the direction indicated by vector \( \mathbf{n}_s \), with speed \( v_s \), and the (planar) receiver wavefield propagates along the direction indicated by vector \( \mathbf{n}_r \), with speed \( v_r \). Quantity \( x_0 \) indicates the intersection of the two wavefronts at time \( t = t_0 \):

\[
\begin{align*}
\mathbf{n}_s \cdot (x - x_0) &= v_s (t - t_0), \\
\mathbf{n}_r \cdot (x - x_0) &= v_r (t - t_0).
\end{align*}
\]

The time-lag \( \tau \) and space-lags \( \lambda \) in equations 7 and 7 represent movement of the wavefronts in time and space, respectively. Then, the intersection of the two wavefronts shifts to a new position \( x \). The geometrical expressions for the extended imaging condition using the planar source and receiver wavefields are

\[
\begin{align*}
\left( \frac{\mathbf{n}_s}{v_s} - \frac{\mathbf{n}_r}{v_r} \right) \cdot (x - x_0) &= \left( \frac{\mathbf{n}_s}{v_s} + \frac{\mathbf{n}_r}{v_r} \right) \cdot \lambda = -2\tau, \\
\left( \frac{\mathbf{n}_s}{v_s} + \frac{\mathbf{n}_r}{v_r} \right) \cdot (x - x_0) &= \left( \frac{\mathbf{n}_s}{v_s} - \frac{\mathbf{n}_r}{v_r} \right) \cdot \lambda = 0,
\end{align*}
\]

where vector \( (x - x_0) \) measures the spatial shift of the image point, corresponding to time-lag \( \tau \) and space-lag \( \lambda \).

We consider three types of extended images, using both space- and time-lags, or using time-lag, or using space-lags. In the following, we derive the equations that link the space and/or time lags with half-opening \( \theta \) and azimuth \( \phi \) angles from equations 9 and 10.

Angle decomposition using CIPs

For CIP gathers, we set the image shift \( x - x_0 = 0 \) in equations 9 and 10, and replace vectors \( \mathbf{n}_s \) and \( \mathbf{n}_r \) with vectors \( \mathbf{q}, \mathbf{n} \) and angle \( \theta \). Quantity \( \mathbf{n} \) is the local normal vector, and \( \mathbf{q} \) is the vector lying at the intersection of the interface and the reflection plane, thus it is a function of the azimuth angle \( \phi \). We obtain the system of equations that describes the relationships between image shift \( (x - x_0) \) and time-lag \( \tau \):

\[
\begin{align*}
\frac{\gamma \sin 2\theta}{\sqrt{\gamma^2 + 1 + 2\gamma \cos 2\theta}} (\mathbf{q} \cdot \lambda) &= v_r \tau, \\
\mathbf{n} \cdot \lambda &= 0.
\end{align*}
\]

where quantity \( \gamma \) is the ratio of \( v_r \) and \( v_s \). For angle decomposition, we loop over all possible values of the reflection angles, \( \theta \) and \( \phi \), and slant-stack the extended image \( I(\lambda, \tau) \) using equations 11 and 12 to produce the angle-domain image \( I(\phi, \theta) \).

Equations 11 and 12 simplify to the acoustic case, with velocity ratio \( \gamma = 1 \) Sava and Vlad (2011):

\[
\begin{align*}
\sin \theta (\mathbf{q} \cdot \lambda) &= v_r \tau, \\
\mathbf{n} \cdot \lambda &= 0.
\end{align*}
\]

We illustrate our angle decomposition methodology using a simple example of a 3D homogeneous model with a horizontal reflector at depth \( z = 0.2 \) km. The acquisition geometry consists of a vertical displacement source at \( \{0.4, 0.4, 0.02\} \) km and a 2D network of receivers at \( z = 0.02 \) km. We compute the PS image (Figure 1a) and an angle gather (Figure 1b) at coordinates \( \{0.2, 0.2, 0.2\} \). The dot overlain on the angle gather image is the analytical solution of the opening and azimuth angles for this specific acquisition geometry.

Angle decomposition using time-lag CIGs

For COGIs, we set the space-lag \( \lambda = 0 \) in equations 9 and 10, and replace vector \( \mathbf{n}_s \) and \( \mathbf{n}_r \) with vector \( \mathbf{q}, \mathbf{n} \) and angle \( \theta \). The system of equations that describes the relationships between image shift \( (x - x_0) \) and time-lag \( \tau \) is:

\[
\begin{align*}
\sqrt{\gamma^2 + 1 + 2\gamma \cos 2\theta} \left[ \mathbf{n} \cdot (x - x_0) \right] &= v_r \tau, \\
\sin 2\theta \sqrt{\gamma^2 + 1 + 2\gamma \cos 2\theta} \left[ \mathbf{q} \cdot (x - x_0) \right] &= v_r \frac{1 - \gamma^2}{\gamma} \tau.
\end{align*}
\]

where equation 15 corresponds to the projection of the image shift \( (x - x_0) \) on the reflector normal \( \mathbf{n} \), and equation 16 corresponds to the projection of the image shift \( (x - x_0) \) on vector \( \mathbf{q} \). For angle decomposition, we loop over all possible values of the reflection angles, \( \theta \) and \( \phi \), and slant stack the extended image \( I(x, \tau) \) using equations 15 and 16 to produce the angle-domain image \( I(\phi, \theta) \). Equations 15 and 16 simplify to
Angle decomposition for elastic RTM

Figure 2: Predicted spatial shift for (a) dipping and (b) horizontal reflectors imaged using PP and PS data, respectively. The large red dots represents the source locations, and the line of smaller green dots represents receiver positions.

the acoustic case, with velocity ratio \( \gamma = 1 \) (Sava and Fomel, 2006):

\[
\begin{align*}
\cos \theta \mathbf{n} \cdot (\mathbf{x} - \mathbf{x}_o) &= v_s \tau, \quad (17) \\
\mathbf{q} \cdot (\mathbf{x} - \mathbf{x}_o) &= 0. \quad (18)
\end{align*}
\]

Note that in equations 17 and 18, time-lag \( \tau \) is not related to vector \( \mathbf{q} \), i.e., the time-lag extended images do not contain azimuthal information for the acoustic case. However, for converted waves (\( \gamma \neq 1 \)), time-lag extended images contain azimuthal information according to equations 15 and 16.

We illustrate the time-lag algorithm with 2D examples. Consider a model with a dipping reflector in the P-velocity. Using equations 17 and 18, we can predict the spatial shift of the reflector as a function of time-lag \( \tau \), Figure 2a. The dashed line is the intersection of two planes (equations 17 and 18), which illustrates the spatial shift for image point at \( x = 0.0 \) km, \( z = 0.0 \) km. The solid line through the image point represents the analytical moveout of the extended image in a vertical CIG. The dashed and solid lines do not coincide with one-another, i.e., the method that applies a slant stack to a vertical CIG at a single \( x \) location does not measure the image shift \( (\mathbf{x} - \mathbf{x}_o) \) accurately. The angle gathers computed using this method and the method using a slant stack along the surface are shown in the middle and right panels of Figure 3a, respectively. The surface slant stack leads to more focused angle gathers, indicating significantly higher accuracy.

Similarly, for converted waves, the image does not only shift vertically, but also horizontally in the time-lag gather even for a horizontal reflector. The predicted spatial shift of the reflector is shown in Figure 2b. The first panel in Figure 3b shows the computed extended image gather at \( x = 0.0 \) km. Note that the shape of the extended image in this panel is curved. The angle gathers computed using a line slant stack for this gather and a surface slant stack are shown in the middle and right panels of Figure 3b, respectively. As for the preceding example, we obtain more focused angle gathers using the surface slant stack.

ANGLE DECOMPOSITION USING SPACE-LAG CIG

For \( \lambda \) CIGs, we set \( \tau = 0 \) in equations 9 and 10, and replace vector \( \mathbf{n} \), \( \mathbf{n}_s \) and \( \mathbf{n}_r \) with vector \( \mathbf{q}, \mathbf{n} \) and angle \( \theta \). The system of equations that describes the relationships between image shift \( (\mathbf{x} - \mathbf{x}_o) \) and space-lag \( \lambda \) is:

\[
\begin{align*}
\gamma^2 + \frac{1 + 2 \cos 2\theta}{2\gamma} \mathbf{n} \cdot (\mathbf{x} - \mathbf{x}_o) + \sin 2\theta \mathbf{q} \cdot \lambda &= \frac{1 - \gamma^2}{2\gamma} \\
\mathbf{n} \cdot \mathbf{x} - \mathbf{x}_o &= 0, \quad (19) \\
\gamma^2 + \frac{1 + 2 \cos 2\theta}{2\gamma} \mathbf{n} \cdot \lambda + \sin 2\theta \mathbf{q} \cdot (\mathbf{x} - \mathbf{x}_o) &= 0. \quad (20)
\end{align*}
\]

For angle decomposition, we loop over all possible values of the reflection angles, \( \theta \) and \( \phi \), and slant stack the extended image \( I(\mathbf{x}, \lambda) \) using equations 19 and 20 to produce the angle-domain image \( I(\phi, \theta) \).

EXAMPLES

Figure 4: 3D salt body based on the SEG/EAGE model, showing the position of the image point at \{0.3, 0.4, 0.3\} km, and the sources on the surface.

For simple geologic models, subsurface illumination from different opening and azimuth angles tends to be uniform. However, in areas with complex geologic structures, such as salt
bodies, severe wavefield distortions lead to irregular subsurface illumination, even if acquisition is uniform. Angle gathers characterize subsalt illumination, which benefits reservoir characterization and acquisition design.

We show an angle gather computed using the proposed elastic angle decomposition methodologies for a modified version of the SEG/EAGE salt model (Aminzadeh et al., 1996). We add a horizontal reflector at \( z = 0.3 \) km, and compute the opening and azimuth angles for an image point on the horizontal reflector at coordinates \( x = 0.3 \) km, \( y = 0.4 \) km. The acquisition geometry contains 91 sources and a 2D network of receivers at \( z = 0.02 \) km. We generate three-component shot gathers using a vertical displacement source with a 100 Hz peak frequency Ricker wavelet. As shown in Figure 4, due to the existence of the salt body, it is difficult to illuminate the considered image point from certain azimuths, as seen in Figures 5a-5b. Both angle gathers show similar illumination patterns. The PP image is mainly illuminated from north-west and south-east, and the PS image is illuminated from west and south-east (0\(^\circ\) indicates due East). Compared to the PP angle gather, the PS angle gather has higher resolution, since the S wavelength is generally smaller than the P wavelength.

CONCLUSIONS

Elastic 3D angle decomposition can be performed on time-lag or space-lag common image gathers, as well as on mixed time-/space-lag common image point gathers. We propose extended imaging conditions to compute scalar PP, PS, SP, and SS images, and algorithms for computing opening and azimuth angle gathers. Our algorithms generate accurate angle gathers if we consider the complete slant-stack surface, instead of the more popular slant stacks along lines defined in common image gathers at fixed surface positions. Our 2D and 3D examples show that the algorithms can handle dipping reflectors and images generated using converted waves.

ACKNOWLEDGMENTS

We thank the sponsors of the Center for Wave Phenomena, whose support made this research possible. The reproducible numeric examples in this paper use the Madagascar open-source software package (Fomel et al., 2013) freely available from http://www.ahay.org. The authors thank Paul Fowler for helpful discussions and valuable suggestions.

Figure 5: Opening and azimuth angle gather computed using (a) PP and (b) PS elastic reflections.
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Anisotropic waveform inversion for microseismic velocity analysis and event location
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SUMMARY

Waveform inversion (WI) is extensively used in reflection seismology and could provide improved velocity models and event locations for microseismic surveys. Here, we develop an elastic WI algorithm for anisotropic media designed to estimate the 2D velocity field along with the source parameters (location, origin time, and moment tensor). The gradient of the objective function is obtained with the adjoint-state method, which requires just two modeling simulations at each iteration. In the current implementation, either the source parameters or the velocity model are fixed to minimize parameter trade-offs. Synthetic examples illustrate the accuracy of the inversion for layered VTI (transversely isotropic with a vertical symmetry axis) media and the sensitivity of the results to perturbations in the initial parameters.

INTRODUCTION

Although most waveform-inversion algorithms are acoustic, WI has been extended to multicomponent data from elastic anisotropic media (Lee et al., 2010; Kamath et al., 2015). These methods are designed for velocity analysis of reflected and diving waves acquired at the surface, but elastic anisotropic WI is also an attractive tool for microseismic data, which are typically recorded with multicomponent geophones. For microseismic studies, velocity models can be built simultaneously with event location using anisotropic traveltime inversion (Gei et al., 2011; Grechka and Yaskевич, 2014). However, WI can potentially improve the resolution of velocity analysis and accuracy of event location because it operates with entire waveforms and could include multiples, scattered waves, etc., in addition to the direct arrivals.

In a previous publication, (Jarillo Michel and Tsvankin, 2014), we employ the adjoint-state method to compute the gradient of the WI objective function with respect to the microseismic source location x0, origin time t0, and moment tensor M. Because it is essential to account for anisotropy in microseismic studies (Grechka and Yaskевич, 2014), the algorithm is implemented for layered VTI media. Jarillo Michel and Tsvankin (2015) iteratively minimize the WI objective function and estimate the parameters x0, t0, and M from 2D multicomponent microseismic data, with the VTI velocity model assumed to be known. A nondimensionalization technique (Kim et al., 2011) is used to update all model parameters simultaneously. Here, we extend the WI methodology of Jarillo Michel and Tsvankin (2015) to velocity analysis of microseismic data and apply it to estimation of the interval parameters of horizontally layered VTI media.

WAVEFORM-INVERSION METHODOLOGY

Our algorithm operates with the elastic wave equation for an arbitrary source in a heterogeneous anisotropic medium:

$$\rho \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial}{\partial x_j} \left( c_{ijkl} \frac{\partial u_k}{\partial x_l} \right) = -M_0 \frac{\partial \delta(x - x_s)}{\partial x_j} S(t),$$

where $$u_i(x,t)$$ is the displacement field, t is time, $$c_{ijkl}$$ is the stiffness tensor ($$i,j,k,l = 1,2,3$$), $$\rho(x)$$ is density, M is the moment tensor of the source, $$x_s$$ is the source location, S(t) is the source time function, and $$\delta(x - x_s)$$ is the spatial delta function; summation over repeated indices is implied. The finite-difference code sfewefd in MADAGASCAR is used to solve equation 1 for 2D heterogeneous VTI media.

The data residuals are measured by the $$\ell_2$$-norm objective function $$F$$ commonly used in WI:

$$F(m) = \frac{1}{2} \sum_{n=1}^{N} ||d_{\text{pre}}(m) - d_{\text{obs}}||^2,$$

where $$d_{\text{obs}}$$ is the observed displacement and $$d_{\text{pre}}(m)$$ is the displacement simulated for a trial model. The wavefield excited by each microseismic event is recorded by N receivers positioned at $$x^n_s$$ ($$n = 1,2,...,N$$); the function $$F$$ also involves summation over all available sources.

Although the examples shown below are for 2D models, in principle this methodology is applicable to 3D data from layered-cake VTI media. Indeed, if the source-receiver azimuth can be estimated from the polarization of the direct P-wave, the in-plane horizontal displacement component employed in our algorithm can be computed by a simple receiver rotation. Then the wavefields from sources at different azimuths can be inverted simultaneously using the proposed 2D WI methodology.

Inversion for medium parameters

The gradient of the objective function $$F$$ with respect to the source parameters is discussed in Jarillo Michel and Tsvankin (2014, 2015). The WI algorithm proposed here is designed to also recover the anisotropic velocity model. The gradient of the objective function $$F$$ with respect to the stiffness coefficients $$c_{ijkl}$$ can be obtained using the adjoint-state method (Liu and Tromp, 2006; Kamath and Tsvankin, 2014):

$$\frac{\partial F}{\partial c_{ijkl}} = -\int_0^T \frac{\partial u_i}{\partial x_j} \frac{\partial \psi_k}{\partial x_l} \, dt,$$

where u and $$\psi$$ are the forward and adjoint displacement fields, respectively. The derivatives of $$F$$ with respect to the chosen model parameters $$m_n$$ can be found from the chain rule:
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\[ \frac{\partial F}{\partial m_n} = \sum_{ijkl} \frac{\partial F}{\partial c_{ijkl}} \frac{\partial c_{ijkl}}{\partial m_n}. \]  

The gradient for the source and velocity parameters can be computed from the same forward and adjoint wavefields generated by two finite-difference simulations. In the current implementation, the source and medium parameters are estimated separately, which helps avoid parameter trade-offs. Next, we plan to perform sequential inversion where the source parameters are obtained first using the initial velocity model. Then the inverted source parameters are employed to update the velocity model, and the process continues in iterative fashion.

The model is updated with the $l-$BFGS method (Byrd et al., 1995), which approximates the inverse of the full Hessian matrix using the gradient of the objective function computed at previous iterations. This method generally produces better convergence than the more common steepest-descent and non-linear conjugate-gradient techniques.

Parameterizations proposed for WI in acoustic VTI media include three combinations of the P-wave vertical ($V_{P0}$), NMO ($V_{NMO} = V_{P0}\sqrt{1 + 2\delta}$), and horizontal ($V_{H0} = V_{P0}\sqrt{1 + 2\epsilon}$) velocities and the coefficients $\epsilon$, $\delta$, and $\eta = (\epsilon - \delta)/(1 + 2\delta)$. Describing P- and SV-waves in elastic VTI media requires an additional parameter such as the S-wave vertical velocity $V_{S0}$ (Kamath and Tsvankin, 2014). We parameterize the medium by $(V_{H0}/V_{hor})^2$, $(V_{S0}/V_{S0})^2$, $(1 + 2\eta)$, and $(1 + 2\epsilon)$; the subscript $i$ stands for the initial value. The sensitivity analysis by Alkhalifah and Plessix (2014) shows that this parameterization is optimal for near-horizontal wave propagation typical for microseismic surveys.

SYNTHETIC EXAMPLES

We apply the algorithm to estimate the interval parameters of the horizontally layered (1D) VTI model from Figure 1. Although the medium is laterally homogeneous, the inversion is carried out for a 2D gridded model with a spacing of 6 m and no smoothness constraints. The initial 1D model for all parameters is obtained by smoothing the actual fields in the vertical direction; in addition, $V_{hor}$ in the middle layer is perturbed by 10% (Figure 2). Given the low frequency of the source wavelet and the short distance between the sources and receivers, the initial model produces no cycle skipping. The source parameters of all 10 microseismic events in the model are assumed to be known. The gradient with respect to the gridded VTI parameters was reduced near the sources by applying an appropriate mask.

Although the largest error is in the initial $V_{hor}$-field, the derivatives $\partial F/\partial m$ for the initial model are nonzero for all model parameters, and are particularly large for $V_{S0}$ because the S-wave amplitudes are higher than those of P-waves. However, the gradient for $V_{S0}$ changes sign with iterations, and the final update in the S-wave vertical velocity is accurate. The search is terminated after about 25 iterations when the objective function becomes sufficiently small (Figure 3). As expected, the largest update is in the horizontal velocity in the middle layer, where we introduced the 10% error. The inverted parameters $V_{hor}$, $V_{S0}$, $\eta$, and $\epsilon$ are close to the actual interval values (Figures 4 and 5). Indeed, $V_{hor}$ should be well-constrained for the predominantly near-horizontal P-wave propagation in this experiment. As discussed by Alkhalifah and Plessix (2014), there are no trade-offs between $V_{hor}$, $\eta$, and $\epsilon$ for relatively large P-wave propagation angles with the symmetry axis. Also, the results benefit from the wide aperture of source-receiver directions.

To study the sensitivity of the inversion results to the accuracy of the initial anisotropy coefficients, we set the initial $\epsilon$ in the middle layer to 0.1 (the actual value is 0.2; Figure 6). As before, the source parameters are assumed to be known. The rate of reduction in the objective function is lower than that in Figure 3 and the parameter $\epsilon$ in the middle layer is not accurately recovered (Figures 7 and 8). There is also a distortion in the inverted horizontal velocity for this layer, which suggests a trade-off between $V_{hor}$ and $\epsilon$. P-wave WI for our parameterization is weakly sensitive to $\epsilon$ for near-horizontal propagation directions (Alkhalifah and Plessix, 2014), and the addition of SV-waves does not help resolve this parameter. Well-posed inversion for $\epsilon$ requires near-vertical raypaths missing in our experiment. Still, we are able to estimate both $V_{hor}$ and $\epsilon$ with a sufficiently accurate initial model for the anisotropy coefficients (Figures 2 and 5).

CONCLUSIONS

We extended the elastic waveform-inversion algorithm presented in our previous publications to velocity model-building from microseismic data. The gradient of the objective function with respect to the source and medium parameters is obtained with the adjoint-state method for 2D gridded anisotropic media. The algorithm is applied to velocity analysis of multicomponent P and SV data for layered VTI models assuming that the source parameters are known. We employed the parameterization that includes $V_{hor}^2$, $V_{S0}^2$, $(1 + 2\eta)$, and $(1 + 2\epsilon)$, which is beneficial for minimizing trade-offs when waves travel predominantly in the horizontal direction. Although the current algorithm is 2D, it requires only a minor modification for 3D microseismic data from layer-cake VTI media.

Synthetic testing for a vertical receiver array showed that WI can accurately recover all four interval VTI parameters from the horizontal and vertical displacement components, if the initial model is sufficiently accurate. The algorithm can tolerate substantial errors (10%) in the initial values of $V_{hor}$ because this parameter is well-resolved for near-horizontal P-wave propagation. In contrast, the initial $\epsilon$-field should be close to the actual model. Ongoing work includes joint WI for the anisotropic velocity model and event locations.

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Figure 1: Sources (black dots) and receivers (magenta line) embedded in a layered VTI medium with the parameters (a) $V_{\text{hor}}$, (b) $V_{\text{S0}}$, (c) $\eta$, and (d) $\epsilon$ (the velocities are in m/s). The source mechanism is described by the moment-tensor elements $M_{11} = 0$, $M_{33} = 0$, and $M_{13} = 1 \cdot 10^{10} \text{N} \cdot \text{m}$ (i.e., it is a dip-slip source with a horizontal fault plane). The central frequency of the source signal is 20 Hz.

Figure 2: Actual (black) and initial (magenta) parameters for the model in Figure 1: (a) $V_{\text{hor}}$, (b) $V_{\text{S0}}$, (c) $\eta$, and (d) $\epsilon$ (the velocities are in m/s). There is a substantial distortion in the initial $V_{\text{hor}}$ in the middle layer.

Figure 3: Change of the normalized objective function $\mathcal{F}(\mathbf{m})$ with iterations for the model in Figure 1.

Figure 4: 2D field of the inverted velocity $V_{\text{hor}}$ (in m/s).
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Figure 5: Actual (black) and inverted (blue) parameters for the model in Figure 1: (a) $V_{\text{hor}}$, (b) $V_{S0}$, (c) $\eta$, and (d) $\epsilon$ (the velocities are in m/s). The inversion is performed using the initial model from Figure 2. The profiles are plotted at $x_1 = 750$ m.

Figure 6: Actual (black) and initial (magenta) parameters for the model in Figure 1: (a) $V_{\text{hor}}$, (b) $V_{S0}$, (c) $\eta$, and (d) $\epsilon$ (the velocities are in m/s). In addition to smoothing the actual values, the initial parameter $\epsilon$ in the middle layer is distorted by 0.1.

Figure 7: 2D field of the inverted parameter $\epsilon$.

Figure 8: Actual (black) and inverted (blue) parameters for the model in Figure 1: (a) $V_{\text{hor}}$, (b) $V_{S0}$, (c) $\eta$, and (d) $\epsilon$ (the velocities are in m/s). The inversion is performed using the initial model from Figure 6 where the parameter $\epsilon$ is distorted. The profiles are plotted at $x_1 = 750$ m.
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Elastic FWI for VTI media: A synthetic parameterization study

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SUMMARY

A major challenge for multiparameter full-waveform inversion (FWI) is the inherent trade-offs (or cross-talk) between model parameters. Here, we perform FWI of multicomponent data generated for a synthetic VTI (transversely isotropic with a vertical symmetry axis) model based on a geologic section of the Valhall field. A horizontal displacement source, which excites intensive shear waves in the conventional offset range, helps provide more accurate updates to the SV-wave vertical velocity. We test three model parameterizations, which exhibit different radiation patterns and, therefore, create different parameter trade-offs. The results show that the choice of parameterization for FWI depends on the availability of long-offset data, the quality of the initial model for the anisotropy coefficients, and the parameter that needs to be resolved with the highest accuracy.

INTRODUCTION

Full-waveform inversion (FWI) can provide a higher resolution model of the subsurface (on the order of wavelength) compared to reflection or traveltime tomography (Pratt, 1999). Application of FWI to VTI media are typically limited to the acoustic approximation (Plessix and Cao, 2011; Gholami et al., 2013). Alkhalifah and Plessix (2014) and Alkhalifah (2015) analyze FWI radiation (sensitivity) patterns in acoustic VTI media obtained for different combinations of the P-wave vertical, NMO (\(V_{nmo}\)), and horizontal (\(V_{hor}\)) velocities and anisotropy coefficients \(\epsilon\), \(\delta\), and \(\eta\). The parameter combination \((V_{hor}, \eta, \delta)\) minimizes the trade-offs for long offsets and provides the most accurate estimate of the horizontal velocity.

Most of the P-wave diving-wave energy is limited to depths less than 0.5 km (Figure 1(b)). The initial models for all parameterizations are computed by smoothing the actual fields from the shallow gas layers (represented by low-velocity horizons) above the reservoir, which is located at a depth of 2.5 km. Hence, we expect to achieve a relatively high spatial resolution for the top 1.5 km of the section. A multiscale preliminary inversion results for this model employing one of these parameterizations (Kamath et al., 2015). The benefits and shortcomings of each parameterization are discussed using both the FWI results and the corresponding radiation patterns.

METHODOLOGY

Inversion methodology

FWI is performed in the time domain by minimizing the least-squares objective function \(F\) defined as the L2-norm of the difference between the observed data and those computed for a trial model. The gradient of the objective function with respect to the stiffness coefficients \(c_{ijkl}\) is obtained using the adjoint-state method (Kamath and Tsankin, 2016):

\[
\frac{\partial F}{\partial c_{ijkl}} = -\int_0^T \frac{\partial u_i}{\partial x_j} \frac{\partial \psi_k}{\partial x_l} \, dt,
\]

where \(T\) is the trace length, and \(\mathbf{u}\) and \(\psi\) are the source and adjoint wavefields, respectively. The gradient with respect to a chosen model parameter \(m_n\) is then found as:

\[
\frac{\partial F}{\partial m_n} = \sum_{ijkl} \frac{\partial F}{\partial c_{ijkl}} \frac{\partial c_{ijkl}}{\partial m_n}
\]

Finite-difference modeling is employed to compute the multicomponent displacement field from the elastic wave equation for VTI media. The model is iteratively updated with the LBFGS-B technique of Byrd et al. (1995).

Synthetic model and data processing

The synthetic model used in this paper is fashioned after the geology of the Valhall Field in the North Sea (Munns, 1985). The sampling is increased from 3.125 m to 20 m in both the vertical and horizontal directions to reduce computational cost. We make the entire model elastic by removing the water column and clip the minimum value of the S-wave vertical velocity \(V_{s0}\) at 700 m/s (Figure 1(b)). The initial models for all parameterizations are computed by smoothing the actual fields of \(V_{p0}, V_{s0}, V_{nmo}\), and \(V_{hor}\) (Figure 2).

We use 109 displacement sources placed with an increment of 80 m, and receivers at every grid point; both sources and receivers are at a depth of 20 m. The source signal is a Ricker wavelet with a peak frequency of 3.5 Hz.

To better understand the wavefield, we shoot a fan of rays from the middle of the survey using a VTI ray-tracer (Figure 3). Most of the P-wave diving-wave energy is limited to depths down to 1.5 km. In addition to diving waves, we record reflections from the shallow gas layers (represented by low-velocity horizons) above the reservoir, which is located at a depth of 2.5 km. Hence, we expect to achieve a relatively high spatial resolution for the top 1.5 km of the survey.
Elastic FWI for 2D VTI media

Figure 1: Parameters (a) $V_{P0}$, (b) $V_{S0}$, (c) $\varepsilon$, and (d) $\delta$ of a synthetic VTI model based on sections from the Valhall field. The velocities have units of km/s.

Figure 2: Initial model for FWI: (a) $V_{P0}$, (b) $V_{S0}$, (c) $V_{nmo}$, and (d) $V_{hor}$.

Figure 3: Fan of rays from a source at x=4.4 km superimposed on the actual $V_{P0}$-field.

Figure 4(a) indicates that the shallow-gas layers are generally well delineated by the updated $V_{P0}$-field. The relatively low amplitude of shear waves produced by a vertical source does not allow the algorithm to properly update $V_{S0}$ (Figure 4(b)). Although the inverted velocity $V_{nmo}$ matches the trend of the actual curve (Figure 4(d)).

The P-wave radiation pattern (Figure 5(a)) indicates that an anomaly in $V_{P0}$ scatters most of the energy close to the vertical axis of symmetry. Hence, the estimated $V_{P0}$-field has relatively high vertical resolution (Wu and Toksöz, 1987). The maximum energy scattered by a $V_{nmo}$-anomaly is only 25% of that for $V_{P0}$ (at incidence angles near 45°, which correspond to opening angles close to 90°). Consequently, the updates in the $V_{nmo}$-field are limited to a depth of 2.4 km. An anomaly in $V_{nmo}$, on the other hand, scatters energy close to the isotropy plane, which results in low-wavenumber updates mostly in the central part of the model. Although the velocity $V_{hor}$ has a lower spatial resolution than $V_{P0}$ and $V_{nmo}$, it is estimated with a higher accuracy than $V_{nmo}$. Based on the S-wave radiation patterns (Figure 5(b)), we expect significant updates in the $V_{S0}$-field from both near- and far-offset shear-wave data. However, a vertical displacement source does not generate intensive shear waves in the recorded offset range, which suppresses updates in $V_{S0}$.

Overall, parameterization I helps resolve the P-wave vertical velocity $V_{P0}$, but the other Thomsen parameters for our synthetic survey are not well-constrained. There is no trade-off between $V_{P0}$ and the other velocities for relatively small opening angles, which ensures accurate $V_{P0}$-updates. Hence, if only conventional-offset data (with the maximum offset-to-depth ratio close to unity) are available, it is advantageous to invert for $V_{P0}$ using parameterization I.

Next, we perform FWI for data generated with an oblique displacement source, whose vertical and horizontal components are equal. Such a source generates more intensive shear waves in the recorded offset range, which results in better updates for

NUMERICAL TESTS

Parameterization I

The first test is performed for a parameterization that includes $V_{P0}$, $V_{S0}$, $V_{nmo}$, and $V_{hor}$ (Kamath and Tsvankin, 2016), with the wavefield generated using a vertical displacement source. Figure 4(a) indicates that the shallow-gas layers are generally well delineated by the updated $V_{P0}$-field. The relatively low amplitude of shear waves produced by a vertical source does not allow the algorithm to properly update $V_{S0}$ (Figure 4(b)). Although the inverted velocity $V_{nmo}$ matches the trend of the actual curve more closely than the initial model (Figure 4(c)), the spatial resolution in $V_{nmo}$ is lower than that for $V_{P0}$, with noticeable deviations at depths of 1.2 km, 1.5 km, 2.2 km, and 2.5 km. The low-frequency trend of the $V_{hor}$-field, especially at depths of 1.5 km, 1.8 km, 2 km, and 2.2 km, is close to the actual curve (Figure 4(d)).

The P-wave radiation pattern (Figure 5(a)) indicates that an anomaly in $V_{P0}$ scatters most of the energy close to the vertical axis of symmetry. Hence, the estimated $V_{P0}$-field has relatively high vertical resolution (Wu and Toksöz, 1987). The maximum energy scattered by a $V_{nmo}$-anomaly is only 25% of that for $V_{P0}$ (at incidence angles near 45°, which correspond to opening angles close to 90°). Consequently, the updates in the $V_{nmo}$-field are limited to a depth of 2.4 km. An anomaly in $V_{nmo}$, on the other hand, scatters energy close to the isotropy plane, which results in low-wavenumber updates mostly in the central part of the model. Although the velocity $V_{hor}$ has a lower spatial resolution than $V_{P0}$ and $V_{nmo}$, it is estimated with a higher accuracy than $V_{nmo}$. Based on the S-wave radiation patterns (Figure 5(b)), we expect significant updates in the $V_{S0}$-field from both near- and far-offset shear-wave data. However, a vertical displacement source does not generate intensive shear waves in the recorded offset range, which suppresses updates in $V_{S0}$.
the $V_{0}$-field, especially in the shallow part of the model. The SV-wave radiation patterns for anomalies in $V_{nmo}$ and $V_{hor}$ are identical; both have trade-offs with $V_{0}$ for intermediate opening angles, which correspond to incidence angles of around $45^\circ$ (Figure 5(b)). Overall, intensive shear waves bring in more information and help resolve the $V_{0}$-field, but also create additional trade-offs for parameterization I. Still, to exploit the shear-wave information, the oblique source is employed in all tests for the other two parameterizations.

**Parameterization II**

Next, we perform the inversion using parameterization II (after Alkhalifah and Plessix, 2014), which includes $V_{nmo}^{2}$, $V_{S0}^{2}$, $(1 + 2\eta)$, and $(1 + 2\delta)$; the squared velocities are normalized by the respective initial values (Figure 2). The P-wave radiation pattern of an anomaly in $V_{nmo}$ is “isotropic” (Figure 6(a)), with energy scattered evenly over the full range of angles. The radiation patterns exhibit trade-offs between $V_{nmo}$ and $\delta$ for small opening angles and between $V_{nmo}$ and $\eta$ for large angles (Figure 6(a)).

The P- and SV-wave radiation patterns for $V_{0}$ are similar to those for parameterization I. The SV-wave radiation patterns (Figure 6(b)) indicate trade-offs between $V_{0}$ and $\eta$ for intermediate opening angles. The updates in the $V_{0}$-field are accurate down to a depth of 2 km (Figure 7(b)), which indicates that the objective function is more sensitive to the velocity $V_{0}$ than the coefficient $\eta$. 

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**Figure 4:** Actual (black), initial (magenta), and inverted (green) velocities for parameterization I: (a) $V_{0}$, (b) $V_{S0}$, (c) $V_{nmo}$, and (d) $V_{hor}$. The data were generated by an array of vertical displacement sources. The profiles here and in the subsequent plots are displayed at location $x = 3.5$ km.

**Figure 5:** (a) P- and (b) SV-wave radiation patterns obtained with parameterization I for reflections from a horizontal interface. The patterns here and in the subsequent plots are computed as functions of the opening angles at the diffractor with the background velocity ratio $V_{P}/V_{S} = 2$.

**Figure 6:** (a) P- and (b) SV-wave radiation patterns obtained with parameterization II.

**Figure 7:** Actual (black), initial (magenta), and inverted (green) parameters for parameterization II: (a) $V_{nmo}$, (b) $V_{0}$, (c) $\eta$, and (d) $\delta$. 

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A purely isotropic P-wave radiation pattern for $V_{\text{mno}}$ combined with a higher sensitivity to this velocity (Plessix and Cao, 2011) makes parameterization II most suitable for estimating this parameter from data recorded for a wide offset range. Indeed, employing parameterization II results in a marked improvement (compared to parameterization I) in the $V_{\text{mno}}$-field. At depths of 1.2 km, 2.2 km, and 2.5 km (Figure 7(a)), the inverted $V_{\text{mno}}$ matches the actual values better than for parameterization I. Because the initial $\eta$- and $\delta$-fields are relatively close to the actual models, the updates in both anisotropy coefficients are insignificant (Figures 7(c) and 7(d)). Therefore, parameterization II should be useful in cases when not only a wide range of offsets is recorded, but also prior estimates of the $\eta$- and $\delta$-fields are sufficiently accurate.

**Parameterization III**

Finally, the inversion is carried out for parameterization III $[V_{\text{hor}}^2, V_{S0}^2, (1+2\eta), \text{and } (1+2\delta)]$, with the squared velocities normalized by their initial values (after Alkhalifah and Plessix, 2014). The P- and SV-wave radiation patterns for an anomaly in $V_{S0}$ are similar for all three parameterizations, and the SV-wave pattern for $\eta$ coincides with that for parameterization II. For parameterization III, the velocity $V_{\text{hor}}$ has a purely isotropic P-wave radiation pattern (Figure 8(a)), which was the case for $V_{\text{mno}}$ in parameterization II. Trade-offs exist between $V_{\text{hor}}$ and $\epsilon$ for small opening angles, and between $V_{\text{hor}}$ and $\eta$ for intermediate opening angles. Because $V_{\text{hor}}$ has no trade-offs with any other parameter for large opening angles, diving waves can be employed to accurately update the long-wavelength $V_{\text{hor}}$-field (Alkhalifah, 2015).

In agreement with the results of Alkhalifah (2015) for acoustic media, for this parameterization $V_{\text{hor}}$ is the best constrained parameter (Figure 9(a)). The updates in the $\eta$- and $\epsilon$-fields, which are relatively close to the actual values to begin with, are insignificant (Figures 9(c) and 9(d), respectively). The main advantage of this parameter combination over parameterization II is that diving waves can be inverted for an accurate low-wavenumber $V_{\text{hor}}$-field. High-resolution estimates of $V_{\text{hor}}$ can be obtained from conventional-offset data if the coefficient $\epsilon$ is well-constrained a priori.

**CONCLUSIONS**

We performed time-domain elastic FWI for a synthetic model based on a geologic section of the Valhall field. Parameterization I ($V_{P0}$, $V_{S0}$, $V_{\text{mno}}$, and $V_{\text{hor}}$) yields a high-resolution $V_{P0}$-field even with conventional-offset data, if the initial model does not produce cycle-skipping. To build an accurate low-wavenumber model of $V_{\text{hor}}$, it is necessary to use diving waves or wide-angle reflections. For parameterization II $[V_{\text{mno}}^2, V_{S0}^2, (1+2\eta), \text{and } (1+2\delta)]$ the velocity $V_{\text{mno}}$ has a purely isotropic radiation pattern, which allows the algorithm to generate adequate low-wavenumber updates in $V_{\text{mno}}$, provided the initial $\eta$-field is sufficiently accurate. The main advantage of parameterization III $[V_{\text{hor}}^2, V_{S0}^2, (1+2\eta), \text{and } (1+2\delta)]$ is that diving waves tightly constrain the $V_{\text{hor}}$-field because there are no trade-offs for long-offset data. A high-resolution model of $V_{\text{hor}}$ can, however, be obtained only with an accurate a priori estimate of the $\epsilon$-field. The insights from these synthetic experiments should help in choosing the most suitable parameterization for different inversion scenarios.

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SUMMARY

Wavefield tomography can handle complex subsurface geology better than ray-based techniques and, ultimately, provide a higher resolution. Here, we implement forward and adjoint wavefield extrapolation for VTI (transversely isotropic with a vertical symmetry axis) media using a pseudospectral operator that employs a separable approximation of the P-wave dispersion relation. This operator is employed to derive the gradients of the differential semblance optimization (DSO) and modified stack-power objective functions. We also obtain the gradient expressions for the data-domain objective function, which can incorporate borehole information necessary for stable VTI velocity analysis. These gradients are compared to the ones obtained with a space-time finite-difference (FD) scheme for a system of coupled wave equations. Whereas the kernels computed with the two wave-equation operators are similar, the pseudospectral method is not hampered by the imprint of the shear-wave artifact. Numerical examples also show that the modified stack-power objective function produces cleaner gradients than the more conventional DSO operator.

INTRODUCTION

In accordance with the semblance principle (Al-Yahya, 1989), image-domain wavefield tomography enforces the consistency of migrated images for different experiments (Sava, 2014). The corresponding objective function usually involves evaluating the energy focusing in the extended domain (Rickett and Sava, 2002; Sava and Fomel, 2006; Sava and Vasconcelos, 2011), which can be done with DSO (Symes and Carazzone, 1991) or image-power estimates (Chavent and Jacewitz, 1995).

Anisotropic wavefield tomography is typically implemented under the pseudoacoustic assumption originally proposed by Alkhalifeh (1998). Acoustic modeling in TI media can be performed with space-time FD schemes applied to coupled second-order equations (Duvaneck et al., 2008; Fletcher et al., 2009; Zhang et al., 2011). Pseudospectral methods provide an alternative way to propagate only P-waves without the shear-wave artifacts (Etgen and Brandsberg-Dahl, 2009; Du et al., 2014). Separable P-mode dispersion-relation approximations for TI media are described in Pestana et al. (2011), Du et al. (2014), and Schleicher and Costa (2015).

Weibull and Ariet (2014) and Li et al. (2016) attempt to estimate the VTI parameters using different image-domain tomographic algorithms. Li et al. (2015) study the defocusing in the extended domain caused by errors in the VTI parameters. Here, we derive and test the wavefield-tomography gradients in the image and data domains using a wave-equation operator based on a separable P-mode approximation for VTI media.

SEPARABLE P-MODE APPROXIMATION

P-wave kinematics in VTI media is controlled by the vertical velocity $V_{P0}$ and Thomsen parameters $\epsilon$ and $\delta$ (Tsvankin, 2012). Alternative parameter combinations for acoustic VTI media also involve the horizontal velocity $[V_{hor} = V_{P0} \sqrt{1 + 2\epsilon}]$, the anellipticity parameter $\eta = (\epsilon - \delta) / (1 + 2\delta)$, and the zero-dip normal-moveout (NMO) velocity $[V_{nmo} = V_{P0} \sqrt{1 + 2\delta}]$.

Pseudospectral methods are designed to evaluate the spatial wavefield derivatives in the wavenumber domain. Anisotropic extrapolation requires approximate dispersion relations with separable wavenumber and model-parameter terms. Employing the first-order Padé expansion, the P-wave separable dispersion relation for VTI media is obtained as (Schleicher and Costa, 2015):

$$\omega^2 = (1 + 2\epsilon)V_{P0}^2 k_x^2 + V_{P0}^2 k_z^2 - 2(\epsilon - \delta)V_{P0}^2 \frac{k_x^2 k_z^2}{k_x^2 + k_z^2}$$

$$\times \left[ 1 - 2\epsilon \frac{k_z^2}{k_x^2 + k_z^2} + 2(\epsilon - \delta) \frac{k_x^2 k_z^2}{(k_x^2 + k_z^2)^2} \right],$$

where $k_x$ and $k_z$ are the vertical and horizontal wavenumbers.

OBJECTIVE FUNCTIONS IN THE IMAGE DOMAIN

Extended images are produced by retaining the correlation lags between the source and receiver wavefields in the output. The general imaging condition can be formulated as follows (Sava and Vasconcelos, 2011):

$$I(x, \lambda, \tau) = \sum_{e,t} W_e(x - \lambda, t - \tau) W_r(x + \lambda, t + \tau),$$

where $I(x, \lambda, \tau)$ is the extended image, $W_e$ and $W_r$ are the source and receiver wavefields, respectively, $\lambda$ is the space lag, and $\tau$ is the time lag. Nonzero-lag energy can be used to update the migration velocity model by applying the DSO operator (Symes and Carazzone, 1991). For the horizontal space-lag extended image $I$, that operator has the form:

$$J_{DSO} = \frac{1}{2} \| \lambda_e I(x, z, \lambda_e) \|^2,$$

where $\lambda_e$ is a penalty operator. Another commonly used (stack-power) objective function measures zero-lag energy:

$$J_{ST} = \frac{1}{2} \| I(x, z, \lambda_e = 0) \|^2.$$

Zhang and Shan (2013) propose a “partial” stack-power objective function that combines the criteria in equations 3 and 4.
without the need to find optimal weights for $J_{DSO}$ and $J_{ST}$:

$$J_{PSF} = \frac{1}{2} \| H(\lambda_a) I(x, z; \lambda_a) \|^2_\ell_2,$$  \hspace{1cm} (5)

where $H$ is a Gaussian operator centered at zero-lag.

**GRADIENT COMPUTATION USING THE ADJOINT-STATE METHOD**

Here, we employ the adjoint-state method (Plessix, 2006) to obtain the gradient expressions. For most TI models, the gradients can be computed with the three leading terms of the separable dispersion relation 1. We truncate equation 1 only for deriving the gradients and not for modeling.

**Data domain**

We define the data-domain model vector as $\hat{m} = \{V_{\text{hor}}, \eta, \epsilon\}$. The gradient of the $\ell_2$-norm data-difference objective function is given by:

$$\frac{\partial J}{\partial \epsilon} = \sum_{e, r} K(\tau) \frac{2V^2_{\text{hor}}}{(1 + 2\epsilon)^2} k^2_x \mathbf{u} \ast \mathbf{a}$$

$$\frac{\partial J}{\partial \eta} = \sum_{e, r} K(\tau) \frac{2V^2_{\text{hor}}}{(1 + 2\eta)^2} k^2_x k^2_y \mathbf{u} \ast \mathbf{a}$$

$$\frac{\partial J}{\partial V_{\text{hor}}} = \sum_{e, r} K(\tau) 2V_{\text{hor}} \left[ k^2_x \mathbf{u} \ast \mathbf{a} + \frac{k^2_y}{1 + 2\epsilon} \mathbf{u} \ast \mathbf{a} - \frac{2\eta k^2_x}{1 + 2\eta} k^2_y \mathbf{u} \ast \mathbf{a} \right],$$  \hspace{1cm} (6)

where $K(\tau)$ is the Dirac delta-function, and $\mathbf{a}$ and $\mathbf{a}$ are the forward and adjoint wavefields, respectively.

**Image domain**

For waveform inversion of reflection data, Alkhalilah and Plessix (2014) suggest to define the model vector as $\mathbf{m} = \{V_{\text{nmo}}, \eta, \epsilon\}$. The adjoint sources for the partial stack-power objective function (2014) suggest to define the model vector as $\hat{m} = \{V_{\text{hor}}, \eta, \epsilon\}$.

The adjoint sources for the partial stack-power objective function (2014) suggest to define the model vector as $\hat{m} = \{V_{\text{hor}}, \eta, \epsilon\}$.

**SYNTHETIC EXAMPLES**

Below, we test the derived gradient expressions on two simple VTI models. The medium parameters are specified on a rectangular grid, and the density is assumed to be constant. The gradients obtained with the pseudospectral operator are compared with the ones for the space-time finite-difference (FD) algorithm proposed by Fletcher et al. (2009); the corresponding adjoint extrapolation is developed by Wang and Sava (2015).

**Model 1**

First, we compute the gradients in the data domain for a model that includes a constant $V_{\text{hor}}$-field and Gaussian anomalies in the parameters $\eta$ (reaching 0.2 at the center) and $\epsilon$ (reaching 0.15) (Figure 1). Only transmitted waves are employed to generate parameter updates. The source function is a Ricker wavelet with a central frequency of 2 Hz. Using the actual $\eta$-field, we compute the gradients for understated and overstatement at peak values of the $\epsilon$-anomaly ($\epsilon = 0$ and 0.3; the background $\epsilon = 0$ is correct). Note that for the peak frequency of the source signal (2 Hz) and the model size, the $\epsilon$-errors cause data residuals that do not exceed half a cycle.

For the chosen parameterization ($V_{\text{hor}}, \eta, \epsilon$), the coefficient $\epsilon$ should be constrained for near-vertical propagation, if $V_{\text{hor}}$ has been estimated from long-offset data (Alkhalilah and Plessix, 2014). We compute the gradients using the vertical (“borehole”) receiver array shown in Figure 1d. In general, P-wave reflection moveout must be supplemented with borehole (Wang and Tsvankin, 2013) or other information to resolve the VTI parameters. The gradients generated by both operators are similar and, as expected, change sign depending on the sign of the $\epsilon$-error (Figure 2). Because the background $\eta$-field is positive, the gradients produced with the FD extrapolator contain a pronounced shear-wave artifact. In the data domain, the gradient for the actual $\epsilon$-field goes to zero. However, the data-difference estimate is questionable for real-data applications because the acoustic approximation does not accurately model P-wave amplitudes.

**Model 2**

Next, we compute the $\eta$-gradient in the image domain using reflection data. The model includes a horizontal interface beneath a homogeneous VTI layer with $V_{\text{nmo}} = 2$ km/s, $\eta = 0.15$, and a thickness of 2 km. The near-surface layer 0.2 km-thick is assumed to be elliptic ($\epsilon = \delta$) to suppress the shear-wave artifact produced by the FD extrapolator. We generate horizontal-space-lag extended images (Figure 3) and obtain the $\eta$-gradients for understated and overstated values of $\eta$. The $\eta$-errors induce residual energy in extended images (Figure 3) that has a linear (“V”-like) shape, which is typical for near-horizontal interfaces (Sava and Alkhalilah, 2012; Li et al., 2015). For both extrapolators, the extended images computed with the understated and even actual $\eta$-fields also contain considerable residual energy that spreads from the image point up to the surface. These kinematic artifacts, caused by having the reflector illuminated only from the surface, may introduce bias in the image-domain objective function and lead to false model updates.
The DSO gradients computed using surface acquisition geometry and the entire extended image are shown in Figure 4. With either extrapolation operator, the gradient of the DSO objective function (equation 3) for the understated $\eta$-field is strongly influenced by the kinematic artifacts in the extended image. The contribution of the artifact is even larger than that of the residual induced by the $\eta$-error because the artifact is located closer to the physical sources and receivers. The partial stack-power objective function (equation 5) significantly reduces the artifact (Figure 5). Nevertheless, robust anisotropic inversion may require additional suppression of kinematic artifacts, which can be achieved by proper accounting for illumination in the imaging or DSO penalty operators (Lameloise et al., 2015; Hou and Symes, 2015; Yang and Sava, 2015).

**CONCLUSIONS**

We implemented forward and adjoint pseudospectral extrapolation operators based on a VTI separable dispersion-relation approximation and derived the corresponding wavefield-tomography gradients. This work is mostly focused on image-domain tomography, which is less susceptible to amplitude distortions produced by acoustic algorithms. However, because estimation of all three relevant VTI parameters (e.g., $V_{P0}$, $\eta$, and $\delta$) is seldom feasible using only P-wave reflection moveout, we also derived data-domain gradients, which can easily incorporate borehole information. The similarity between the gradients obtained with the pseudospectral and FD operators validates our analytic results. For a model where the sources and receivers were placed in a layer with $\eta > 0$, the gradients computed with the pseudospectral algorithm do not contain the imprint of the shear-wave artifact that contaminates the FD results.

The image-domain example reveals illumination-related issues with the DSO objective function applied to cross-correlation extended images. Kinematic artifacts caused by insufficient illumination substantially distort the gradients and should be suppressed prior to updating the model. The partial stack-power objective function helps reduce the false updates caused by these artifacts. Ongoing work involves implementing the imaging and inversion steps of anisotropic image-domain tomography and an extension of the algorithm to tilted TI media.

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Figure 3: Space-lag CIGs for a horizontal VTI layer (model 2) computed at $x = 4$ km using the pseudospectral extrapolator with (a) $\eta = 0$, (b) $\eta = 0.15$ (actual value), and (c) $\eta = 0.3$.

Figure 4: Gradients of the DSO objective function (equation 3) for model 2 computed using (a,b) the pseudospectral extrapolator and (c,d) the FD extrapolator for (a,c) $\eta = 0$ and (b,d) $\eta = 0.3$.

Figure 5: Gradients of the partial stack-power function (equation 5) for model 2 computed using (a,b) the pseudospectral extrapolator and (c,d) the FD extrapolator for (a,c) $\eta = 0$ and (b,d) $\eta = 0.3$. 
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Yang, T., and P. Sava, 2015, Image-domain wavefield tomography with extended common-image-point gathers: Geophysical Prospecting, 1086–1096.
INTRODUCTION

Distributed Acoustic Sensing (DAS) is rapidly gaining popularity in the oil and gas industry especially for Vertical Seismic Profile (VSP) imaging and for reservoir monitoring (Mestayer et al., 2011; Cox et al., 2012; Mateeva et al., 2012, 2013; Daley et al., 2013; Madsen et al., 2013). The advantages of DAS for borehole applications in terms of costs, the deployment mechanism, and spatial resolution make its usage attractive over conventional geophone acquisition (Lumens et al., 2013; Mateeva et al., 2013).

Lumens (2014) indicates that the DAS system is more sensitive in the axial direction compared to the radial direction, thus reducing the DAS measurable data to the axial strain which can be acquired with acceptable signal-to-noise ratio. Published work does not detail mechanisms for extracting multicomponent data from the axial strain measurement using DAS. To acquire multicomponent data, alternative measuring devices such as geophones are deployed, which are costly and do not provide the dense spatial sampling of DAS. In this paper, we investigate possibilities to use different optical fiber configurations to obtain multicomponent information from DAS data.

Our first approach is to use optical fiber as a shape sensing tool. The relationship between bending and the difference in strain between two Fiber Bragg Gratings (FBG) has been discussed by Gander et al. (2000) as a curvature sensor. Flockhart et al. (2003) and Fender et al. (2006) demonstrate positive experimental results for using this theory as a curvature sensor for structural monitoring. However, their discussion is limited to using the optical fiber as a shape monitoring tool. We exploit shape sensing capabilities to obtain data components that cannot be measured using the axial strain along a single fiber.

Our second approach is to use a helical-shaped optical fiber. Using the characteristics of the helix and axial strain measurement of the optical fiber, we can calculate the entire strain tensor at every measuring location under the assumption that the seismic wavelength is much longer than the helix period. We show the theoretical relationships between the measured axial strain in the optical fiber and the full strain tensor in the surrounding area, and demonstrate the applicability of this strategy using a synthetic example.

MULTIPLE PARALLEL OPTICAL FIBERS

The axial strain measurement by the optical fiber is a projection of the strain tensor from the surrounding area as a function of the position of the optical fiber. Using the optical fiber system illustrated in Figure 1a, we can measure the bending of two optical fibers by the relationship between their difference in axial strain and curvature $\kappa$ (Gander et al., 2000)

$$\kappa = \frac{\Delta \varepsilon_{nm}}{r},$$

where $\Delta \varepsilon_{nm}$ is the difference in strain and $r$ is the distance between two optical fibers. At least two optical fibers are needed to resolve bend measurement of one axis. We show the $n$-axis bending calculation for two horizontal optical fibers denoted with the superscript top and bottom

$$\kappa = \frac{\varepsilon_{\text{bottom}} - \varepsilon_{\text{top}}}{2\Delta n}.$$  \hspace{1cm} (2)

We recognize that equation 2 effectively represents the centered finite-difference approximation of the transverse derivative of the axial strain

$$\kappa = \frac{\varepsilon_{nm} (n + \Delta n) - \varepsilon_{nm} (n - \Delta n)}{2\Delta n} \approx \frac{\partial \varepsilon_{nm}}{\partial n}. $$  \hspace{1cm} (3)

![Figure 1: (a) An example of 3D parallel fiber optic configuration with local coordinates and (b) the cross section of a cable with (b) three optical fiber core (circles) with respective curvature vectors (solid line). The resultant vector (dotted line) is the sum of the respective curvature vectors.](image)

We can generalize this idea to 3D by calculating the axial strain gradient between the optical fibers. Moore and Rogge (2012)
show that the 2-axis bend measurement calculation can be obtained with an arbitrary circular arrangement. An example of the arrangement is shown in Figure 1a. The expression needed to calculate local curvature vector for the respective optical fibers is (Moore and Rogge, 2012)

\[ \kappa_i = \frac{\varepsilon_{nnn}}{2\Delta n} \left( \cos(\theta_i) \hat{n} + \sin(\theta_i) \hat{l} \right), \]

where \( i \) is the optical fiber index, \( \theta_i \) is the angle between the positive \( l \)-axis of the respective optical fiber as shown in Figure 1b. Expression equation 4 decomposes the scalar curvature into a vector curvature based on a local coordinate system of the cable as the reference axis. The resultant curvature vector \( \kappa \) represents the overall bending of the optical fiber by summing all the curvature vectors of the respective optical fibers.

\[ \kappa = \sum_{i=1}^{N} \kappa_i. \]

Calculating the angle made by \( \kappa \) with respect to the reference axis gives

\[ \varphi = \tan^{-1} \left( \frac{\kappa_n}{\kappa_l} \right), \]

and its derivative of the angle we can obtain the torsion of the optical fibers as

\[ \tau = \frac{d\varphi}{dm}. \]

The curvature \( \kappa \) allow us to compute the displacement components through the geometrical definition

\[ \kappa = \frac{\partial^2 u_l}{\partial m^2} \hat{l} + \frac{\partial^2 u_n}{\partial m^2} \hat{n}. \]

Solving equation 8 using the calculated curvature yields the transverse displacement components \( (u_l \text{ and } u_n) \). The axial displacement \( u_m \) is the integration of the measured axial strain \( \varepsilon_{mm} \) along the optical fiber. Therefore, we can obtain all the displacement components with respect to the local coordinate system of the optical fiber given that we have at least three parallel optical fibers.

### Plane wave example

We consider planar P and S waves impinging at an angle on the optical fiber to illustrate the measures of axial strain. This example shows compression and elongation along the optical fiber for both wave modes which is observed through the axial strain tensor shown in Figures 2a and 2b. The axial strain for both P and S waves show that they share the same sign although they are different in terms of polarization, therefore both P and S waves share the same gradient. As the curvature measurement is ultimately the gradient of axial strain in the transverse direction, P and S waves have the same curvature to calculate the displacement \( u_n \) in the n-direction as shown in Figures 3c and 3d. The consequence of the similar gradient of axial strain, we are unable to reconstruct the transverse displacement component with the correct sign for S-waves as illustrated by Figure 3f where the S-wave points in the direction of P-waves.

### Discussion

Using the analytical plane waves example, we show the lack of information using the axial strain itself when we calculate the curvature for both P and S waves that are used to reconstruct the displacement components. The axial strain does not contain information to evaluate the polarization direction of different wave modes. This creates an ambiguity in reconstructing the displacement field.
disagreement between the curvature sign and the displacement for S-wave is not a result of an incorrect curvature calculation, but that of insufficient information to provide the reconstruct the displacement with the appropriate sign.

Based on the reconstructed displacement results, multiple parallel optical fibers do not give us enough information to accurately reconstruct the multicomponent DAS data. In the following section, we propose a different optical fiber layout that can provide adequate information, under certain assumptions, for multicomponent DAS data reconstruction.

**HELICAL OPTICAL FIBER**

A helical optical fiber configuration for Distributed Acoustic Sensing (DAS) (Den et al., 2013) is designed to detect broadband acoustic signals, which refers to waves that arrive perpendicular to the direction along the optical fiber. Here, we exploit the helical shape as a tool to measure different projections of the strain field along the optical fiber.

![Figure 4: An example of the helical optical fiber with the associated local coordinate system.](image)

In this configuration, we make the assumption that the wavelength of the wavefield is much greater than the spatial wavelength of the helical optical fiber. This assumption is important so that we can group multiple measurements along the optical fiber to refer to the same strain tensor field.

The relationship between the axial strain measured by the optical fiber and the strain tensor of the surrounding area via the angles can be expressed as (Young and Budynas, 2002)

\[
\varepsilon' = \text{Re} \mathbf{R}^T, \quad (9)
\]

where the symbol \( (i) \) denotes rotated strain tensor and \( \mathbf{R} \) represents the rotation matrix. Rotation of a coordinate system can be written

\[
\begin{bmatrix}
m_1 & l_1 & n_1 \\
m_2 & l_2 & n_2 \\
m_3 & l_3 & n_3
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
r_{11} & r_{12} & r_{13} \\
r_{21} & r_{22} & r_{23} \\
r_{31} & r_{32} & r_{33}
\end{bmatrix}
\begin{bmatrix}
m_1 & l_1 & n_1 \\
m_2 & l_2 & n_2 \\
m_3 & l_3 & n_3
\end{bmatrix}, \quad (10)
\]

where \( \varepsilon' \) denotes axes of the rotated coordinate system and \( \varepsilon \) represents axes of the original coordinate system which is an identity matrix. Therefore, the rotation matrix is represented by the local coordinate system of the optical fiber. Since the primary strain measurement of the optical fiber is due to axial strain only, we can narrow the problem to the rotated axial strain measurement in 3D at a location along the optical fiber as follows

\[
[d] = [R_{11}^2 \, R_{12}^2 \, R_{13}^2 \, 2R_{11}R_{12} \, 2R_{11}R_{13} \, 2R_{12}R_{13} \, G] \begin{bmatrix}
E_{xx} \\
E_{xy} \\
E_{xz} \\
E_{yx} \\
E_{yy} \\
E_{yz} \\
E_{zx} \\
E_{zy} \\
E_{zz}
\end{bmatrix} \cdot m, \quad (11)
\]

where the observed data \( d \) vector represents axial strain measurements along a segment of the optical fiber of length \( h \) which is assumed to be much smaller than the wavelength of the wavefield. The matrix \( G \) represents the expanded rotation matrix from equation 10. The number of rows in the data vector and matrix \( G \) refers to the number of measurements that refers to the same strain tensor of the surrounding area.

The expression in equation 11 can be solved using least-squares inversion where the kernel or forward operator \( (G) \) is known. The data \( (d) \) are the recorded axial strains along a segment of the helical optical fiber (this gives axial strain as a function of angles, given the assumption that the seismic wavelength is much larger than the helix period holds) and the model \( (m) \) is the strain tensor of the surrounding area. Therefore, the model can be expressed as

\[
m = (G^T G)^{-1} G^T d. \quad (12)
\]

The number of axial strain measurements with the associated angles can be varied accordingly with the parameters of the helix such as the radius of the helix, the distance between the windings and the interval at which the measurements are made. In Figure 5f, we show an example of the number of measurable axial strain with the respectively associated angles. The example is designed with a radius and a distance between the windings which are much smaller than the wavelength of the wavefield. Using the same parameters, Figure 5e shows a 3D example with the associated angle and azimuth of the measured locations along the fiber. The 3D plot shows a constant measuring angle due to the characteristic of the helix with a constant wrapping angle thus, this configuration could not provide a range of angle dependent measurement to determine the strain tensor of the surrounding area.

By introducing a varying winding distance along the fiber, we obtain a better coverage for the measured angle dependent axial strain. This alternative configuration in 2D represents a chirp function as shown in Figure 5d. Using the chirping helix measurements, we can solve equation 12 to obtain the strain tensor of the surrounding area.

**Plane wave example**

Assuming that the seismic wavelength is much larger than the helix period, we are able to measure axial strain \( \varepsilon_{\text{axial}}(\theta) \) as a function of angle. The strain tensors of P- and S-waves are different as shown in Figure 7a and 7b. Therefore, the measured axial strain as a function of angles for P- and S-waves are different due to the contribution of all the elements in the strain tensor as expressed in equation 11. An example of a rotated strain field at a specific time for all angles is shown in Figure 6a for P-wave and Figure 6b for S-wave that illustrates the
Multicomponent Distributed Acoustic Sensing

Figure 5: Example of a helical configuration for a constant wrap angle in (a) 3D and (b) 2D together with the (e) angles and azimuth for 3D and (f) angles for 2D. An example of a chirping helical configuration in (c) 3D and (d) 2D together with the (g) angles and azimuth for 3D and (h) angles for 2D.

Figure 6: Axial strain of optical fiber for plane (a) P-wave and (b) S-wave with a wavelength of 100m as a function of angles \( \theta \) at a specific time. The difference in the rotated strain field of P- and S-wave is due to the difference its respective strain tensor contributing in the projection to the axial strain of the optical fiber.

Discussion

Our synthetic example indicates that we can reconstruct the strain tensor field if we have the angle dependent axial strain measurements. As seen in Figure 5f and 5h, the chirping helix can provide a wider range of angles with respect to the winding direction of the traditional helix with a constant wrap angle. In situations where the period of the helix is equivalent to the Distributed Acoustic Sensing (DAS) system measuring distance \( \Delta h \), the traditional helix can only provide a constant angle for the axial strain measurement whereas the chirping helix can provide a range of angles due to the change in the helix coil frequency.

Figure 7: Strain field for plane (a) P-wave and (b) S-wave propagating at 30\(^\circ\). The reconstructed (c) P-wave and (d) S-wave by solving the least-squares inversion problem using the angle dependent axial strain measurements. The vertical axis is time and the horizontal axis is the receivers.

CONCLUSIONS

Multiple parallel optical fibers or a single helical optical fiber have the potential to provide multicomponent DAS data. However, the shape sensing method applied by the parallel optical fibers is unable to accurately reconstruct correct displacements when multiple wave modes are involved. The reliance of this method on the gradient of the axial strain with respect to multiple parallel optical fibers is insufficient due to lack of additional information to relate curvature to the polarization of the incident wavefields.

However, the strain tensor can be reconstructed given enough angle dependent axial strain measurement associated with a fix position along the optical fiber. This is true even for multiple wave modes as the strain tensor of the respective wave mode contributes to the axial strain along the optical fiber. The angle dependent measurement can be obtained using the helical optical fiber under the assumption that the seismic wavelength is much larger than the helix period. Alternative to the optical fiber helix shape allow us to measure a wider range of angle dependent axial strains compared to the traditional helix and can reduce the possibility of repeated angle dependent measurements.

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Full-waveform inversion with reflected waves for 2D VTI media  
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SUMMARY

Full-waveform inversion in anisotropic media using reflected waves suffers from the strong non-linearity of the objective function and trade-offs between model parameters. Estimating long-wavelength model components by fixing parameter perturbations, referred to as reflection-waveform inversion (RWI), can mitigate nonlinearity-related inversion issues. Here, we extend RWI to acoustic VTI (transversely isotropic with a vertical symmetry axis) media. To minimize trade-offs between model parameters, we employ a new hierarchical two-stage algorithm that operates with the P-wave normal-moveout velocity $V_{nmo}$ and anisotropy coefficients $\delta$ and $\eta$. First, $V_{nmo}$ is estimated using a fixed perturbation in $\delta$, and then we invert for $\eta$ by fixing the updated perturbation in $V_{nmo}$. The proposed 2D algorithm is tested on a horizontally layered VTI model.

INTRODUCTION

Full-waveform inversion (FWI) is a powerful technique capable of building high-resolution velocity models, especially for shallow layers (Virieux and Operto, 2009). Conventional FWI heavily relies on diving waves to update the model. However, existing FWI algorithms often fail to provide a satisfactory background model update for deeper horizons, which are illuminated mostly by reflections.

Conventional FWI uses a least-squares objective function, which has multiple local minima, especially when reflected waves dominate the data. An initial model that does not produce cycle-skipping can be obtained by such optimization methods as migration-based traveltome tomography (Plessix et al., 1995), reflection tomography (Wang and Tsvankin, 2013a,b), or migration velocity analysis (e.g., Biondi and Symes, 2004). Xu et al. (2012) develop an algorithm similar to migration-based traveltime tomography called reflection-waveform inversion (RWI), which is designed to invert reflection data for the long-wavelength components of the velocity model. A migration/demigration approach (Zhou et al., 2012) can be employed to update the background model along the source and receiver wavetpath. Wang et al. (2013) implement RWI in the frequency domain and demonstrate that low-frequency data are as essential for RWI as they are for conventional FWI. A spatial- and temporal-correlation-based objective function that can handle phase delays larger than half a period is presented by Chi et al. (2015). Wu and Alkhalifah (2015) propose a new optimization approach that estimates both the background and perturbed model simultaneously.

Here, we extend RWI to anisotropic models and devise a two-stage algorithm to invert reflection data for the parameters of VTI media. We employ parameterization in terms of the P-wave normal-moveout (NMO) velocity $V_{nmo}$, the anellipticity parameter $\eta$ and the Thomsen parameter $\delta$, which yields optimal waveform-inversion results when the data contain high-quality reflections in the conventional offset range (Alkhalifah et al., 2015a).

METHODOLOGY

Two-stage inversion algorithm

The proposed algorithm is designed to estimate just the NMO velocity and parameter $\eta$, because $\delta$ typically is not well-constrained by reflection data. Stable inversion for $\delta$ cannot be performed without additional (e.g., borehole) information (Wang and Tsvankin, 2013a,b). The method operates with P-wave reflection data and includes the following steps:

1. First, we perturb the parameter $\delta$ while keeping $V_{nmo}$ and $\eta$ fixed. This perturbation model is then fixed while the reference NMO velocity is updated. The main goal of this step is to fit the data residuals at the near offsets caused by incorrect initial values of $\delta$ and $V_{nmo}$. 

2. Second, the updated NMO velocity is used to generate the perturbation model (image). At this stage, the algorithm inverts only for $\eta$ to fit the data residuals at the far offsets.

At both stages, we use a correlation-based objective function and compute the gradient from the adjoint-state method. If high-quality reflections are available, parameterization in terms of $V_{nmo}$, $\eta$, and $\delta$ helps reduce the trade-offs and provide adequate resolution (Alkhalifah and Plessix, 2014). The velocity $V_{nmo}$ has a purely isotropic (uniform) radiation pattern, while the perturbation in $\eta$ depends mostly on the horizontal wavenumber. A perturbation in $\delta$ is associated primarily with the vertical wavenumber, so $\delta$ can be used to fit the near-offset amplitude information. Employing $\delta$ helps compensate, to a certain extent, for inadequate amplitude fitting of reflection data in the acoustic approximation. Therefore, we perturb $\delta$ and use the Born approximation of the pseudoacoustic wave equation to generate the scattered wavefield.

Kinematically accurate reconstruction of P-wavefields in TI media is provided by a system of two second-order coupled equations (Fletcher et al., 2009; Fowler et al., 2010). Fowler et al. (2010) suggest to describe pseudoacoustic wave propagation by

$$
\frac{\partial^2 u^p}{\partial t^2} = V_{nmo}^2 \frac{\partial^2 u^p}{\partial x^2} + V_{p0}^2 \frac{\partial^2 u^\delta}{\partial z^2},
$$

$$
\frac{\partial^2 u^\delta}{\partial t^2} = V_{nmo}^2 \frac{\partial^2 u^\delta}{\partial x^2} + V_{p0}^2 \frac{\partial^2 u^\delta}{\partial z^2},
$$

(1)
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where \( V_{\text{hor}} = V_{\text{nmo}} \sqrt{1 + 2\eta} = V_{p0} \sqrt{1 + 2\varepsilon} \) is the P-wave horizontal velocity. Both the p- and q-components of the wavefield contain a P-wave with the correct kinematics and a shear wave “artifact” caused by setting the S-wave vertical velocity to zero. This artifact can be eliminated by placing sources and receivers in a purely isotropic or elliptic (\( \eta = 0 \)) medium (Alkhalifah, 2000; Duveneck et al., 2008).

Representing the parameter \( \delta \) as the sum of the background value (\( \delta_0 \)) and a perturbation (\( \delta_p \)), the wavefields can be expressed as \( u^p = u^{0p} + u^{p1} \) and \( u^q = u^{0q} + u^{q1} \), where \( u^{p1} \) and \( u^{q1} \) are the perturbations and \( u^{0p} \) and \( u^{0q} \) are the wavefields for the background medium.

For a small perturbation in \( \delta \), equation 1 can be expressed in matrix form as:

\[
L \begin{bmatrix} u^{p1} \\ u^{q1} \end{bmatrix} = \begin{bmatrix} 2 \frac{\partial^2 u^{0p}}{\partial z^2} \\ 2 \frac{\partial^2 u^{0q}}{\partial z^2} \end{bmatrix}, \quad (2)
\]

where \( L = \begin{bmatrix} \frac{V_{\text{hor}}^2}{V_{\text{nmo}}} \frac{\partial^2}{\partial z^2} & \frac{V_{\text{hor}}^2}{V_{\text{nmo}}} \frac{\partial^2}{\partial z^2} \\ \frac{V_{\text{hor}}^2}{V_{\text{nmo}}} \frac{\partial^2}{\partial z^2} & \frac{V_{\text{hor}}^2}{V_{\text{nmo}}} \frac{\partial^2}{\partial z^2} \end{bmatrix}. \)

Here, we refer to the term \( 2\delta V_{p0}^2 \) as the “\( \delta \)-image.” The dot product of the \( \delta \)-image with the squared double-derivative of the q-component of the source wavefield produces secondary sources in the model space. The Born-scattered data (i.e., predicted data) are computed by forward modeling the secondary sources using equation 2. Next, we use this \( \delta \)-image as our perturbation model and invert only for \( V_{\text{nmo}} \) to get an updated velocity model; this update is based primarily on short-offset data. In contrast, in existing RWI algorithms the image is obtained from near-offset least-squares migration. This completes the first stage of the inversion.

Perturbation in the parameter \( \eta \) mostly depends on the horizontal wavenumber, so the sensitivity to \( \eta \) increases with the offset-to-depth ratio. Hence, at the second inversion stage, the estimated \( V_{\text{nmo}} \) is used to generate the perturbation model (image) and invert for \( \eta \) by fitting the far-offset data residuals.

Correlation objective function

The data are generated with the constant-density pseudo-acoustic wave equation, which does not properly model reflection amplitudes. In addition, because the actual reflectivity cannot be obtained by cross-correlating the source and receiver wavefields, the amplitude matching of the observed and predicted data using a least-squares objective function may be problematic. The requirement of amplitude matching can be relaxed by using a normalized cross-correlation objective function \( C \) that evaluates the similarity between the observed and Born-modeled data (Choi and Alkhalifah, 2012; Xu et al., 2012a):

\[
C = -d_m \cdot d_{\text{obs}}, \quad (3)
\]

where \( d_m = \frac{d_m}{\|d_m\|} \) is the normalized Born-simulated data and \( d_{\text{obs}} = \frac{d_{\text{obs}}}{\|d_{\text{obs}}\|} \) is the normalized observed data.

Adjoint-state method

The adjoint-state method (Tromp et al., 2005; Plessix, 2006) provides an efficient way of computing the derivatives of the objective function with respect to the model parameters. The components of the adjoint-state method are the objective function, state equations, and adjoint equations. The objective function depends on the model parameters through the state equations.

The adjoint-state method involves four main steps:

(i) Computation of the state variables (forward wavefield) by solving the state equations.

(ii) Computation of the adjoint source functions.

(iii) Computation of the adjoint-state variable (adjoint wavefield) by solving the adjoint equations.

(iv) Computation of the gradient of the objective function.

The problem in hand includes two state equations: the pseudo acoustic wave equation that generates the forward wavefield and the Born approximation of the wave equation for a small perturbation in the parameter \( \delta \). RWI can be posed as the following optimization problem:

\[
\begin{align*}
\text{minimize} & \quad J(m, u^p, u^q, u^{p1}, u^{q1}), \\
\text{subject to} & \quad F(m, u^p, u^q, u^{p1}, u^{q1}) = 0,
\end{align*}
\]

where \( u^p \) and \( u^q \) are the forward-modeled wavefields and \( u^{p1} \) and \( u^{q1} \) are the Born-scattered wavefields. The functions \( F \) and \( F_1 \) are the state equations for the forward-modeled and the Born-simulated data, respectively. The model vector for our problem is defined as \( m = \{V_{\text{nmo}}, \eta, \delta \} \).

Using equation 3 and 4, the Lagrangian can be formulated as:

\[
\Lambda = C(x, t) - \langle Lu_t - f, \lambda_{u_t} \rangle - \langle Lu_1 - I u_1, \lambda_{u_1} \rangle, \quad (5)
\]

where \( C(x, t) = -\frac{d_m}{\|d_m\|} \cdot \frac{d_{\text{obs}}}{\|d_{\text{obs}}\|} \), \( L \) is the wave operator, \( I \) is the identity, \( u_t \) and \( u_1 \) are the state variables, and \( \lambda_{u_t} \) and \( \lambda_{u_1} \) are the adjoint variables. We generate the source wavefield \( u_t \) and the Born-scattered wavefield \( u_1 \) by solving the following equations:

\[
Lu_t - f = 0, \quad (6)
\]

\[
Lu_1 - I u_1 = 0, \quad (7)
\]

Next, the adjoint variables for both the source and receiver sides are obtained from:

\[
L^T \lambda_{u_t} - I \lambda_{u_1} = 0, \quad (8)
\]

\[
L^T \lambda_{u_1} - \frac{1}{\|d_m\|} (d_m (d_m \cdot d_{\text{obs}}) - d_{\text{obs}}) = 0. \quad (9)
\]

Here, the adjoint sources are:

\[
r_d = \frac{1}{\|d_m\|} (d_m (d_m \cdot d_{\text{obs}}) - d_{\text{obs}}). \quad (10)
\]

Equation 10 shows that the Born-simulated data are scaled by the dot product of the observed data and the Born-simulated data.
data. This controls the amplitude matching depending on the similarity between these two data sets. Finally, the gradient for the model parameters $m$ is computed from $\frac{\partial \Lambda}{\partial m} = \frac{\partial J}{\partial m}$.

Then the source-side gradients for $V_{\text{nmo}}$, $\eta$, and $\delta$ take the form:

$$\frac{\partial J}{\partial V_{\text{nmo}}} = \iint 2 u^p_{\text{sx}} V_{\text{nmo}} (1 + 2\eta) u^p_{\text{rx}} dsdt$$

$$+ \iint 2 u^q_{\text{sx}} \frac{V_{\text{nmo}}}{(1 + 2\delta)} u^q_{\text{rx}} dsdt$$

$$+ \iint 2 u^p_{\text{sx}} \frac{V_{\text{nmo}}}{(1 + 2\delta)} u^q_{\text{rx}} dsdt$$

$$+ \iint 2 u^q_{\text{sx}} \frac{V_{\text{nmo}}}{(1 + 2\delta)} u^p_{\text{rx}} dsdt,$$

(11)

$$\frac{\partial J}{\partial \eta} = \iint 2 u^p_{\text{sx}} V^2_{\text{nmo}} u^p_{\text{rx}} dsdt,$$

(12)

$$\frac{\partial J}{\partial \delta} = \iint -2 u^p_{\text{sx}} \frac{V^2_{\text{nmo}}}{(1 + 2\delta)^2} u^q_{\text{rx}} dsdt$$

$$+ \iint -2 u^q_{\text{sx}} \frac{V^2_{\text{nmo}}}{(1 + 2\delta)^2} u^p_{\text{rx}} dsdt,$$

(13)

where the integration is performed over the sources and time, the subscript “s” denotes the source-side state variables and subscript “r” denotes the adjoint variables. The corresponding expressions for the receiver-side gradient are obtained in a similar way.

**Sensitivity kernel**

The sensitivity kernel for a particular model parameter is the response in the model space to the data perturbations for a single source and a single receiver. The sensitivity kernel describes the model areas that can provide updates for a particular receiver location. For our problem, the source-side sensitivity kernel is the cross-correlation of the source wavefield with the adjoint-source wavefield, whereas the receiver-side sensitivity kernel is the cross-correlation of the residual receiver wavefield with the demigrated source wavefield. The data residuals are back-propagated along the “rabbit-ear” wavepath to generate the sensitivity kernel. Model of the parameters $\delta$, $V_{\text{nmo}}$, and $\eta$ are shown in Figure 1 (a), 3 (a), and 4 (a), respectively. The sensitivity kernels for the parameters $V_{\text{nmo}}$, $\eta$, and $\delta$ are shown in Figure 1.

**SYNTHETIC EXAMPLE**

The algorithm is tested on the layered VTI model in Figure 1. The horizontal and vertical grid spacing is 25 m. The data are excited by 16 sources positioned at 50 m intervals on the surface with a maximum source-receiver offset of 7.5 km. The source signal is a Ricker wavelet with a central frequency of 5 Hz. There are no diving waves in this model, and we use only reflections by muting the direct arrivals in the recorded data. The sources and receivers are embedded in a thin isotropic layer to suppress the shear-wave artifact. The modeled wavefield is computed for a smooth background medium free from reflections, and the adjoint sources are injected back into the model to generate the adjoint wavefield. For both the forward and the adjoint wavefield extrapolation, we use a finite difference algorithm developed within the framework of MADA-GASCAR.

The gradients in this example are obtained by perturbing $\delta$, while $V_{\text{nmo}}$ and $\eta$ are kept constant. Note that the gradient illuminates even the third (deepest) layer without applying any illumination compensation (Figure 2). The gradient for $\eta$ (equation 12) depends on the background $V_{\text{nmo}}$ and the first lateral derivatives of the forward and adjoint wavefields. Therefore, there is a trade-off between $V_{\text{nmo}}$ and $\eta$ in the horizontal direction. The gradient for $\delta$ (equation 13) is controlled by the background $V_{\text{nmo}}$, $\delta$ and the first vertical derivatives of the forward and adjoint wavefields, which causes a trade-off between $V_{\text{nmo}}$ and $\delta$ in the vertical direction. The gradient for $V_{\text{nmo}}$ (equation 11) is dependent on the NMO velocity itself, $\eta$, $\delta$ and the first derivatives of the forward and adjoint wavefields in both the horizontal and vertical directions. Because the gradient for $V_{\text{nmo}}$ contains $\delta$, we invert for $V_{\text{nmo}}$ first.

The initial NMO velocity is taken equal to the velocity in the first layer, whereas the initial $\delta = \eta = 0$. As described above, first we fix the $\delta$-image and invert for $V_{\text{nmo}}$. The algorithm was able to recover $V_{\text{nmo}}$ in the second and the third layers, although the velocity in the deepest layer is somewhat overestimated (Figure 3). After the second stage, the inverted $V_{\text{nmo}}$ is used to generate the image and invert only for $\eta$ by fitting the data residuals at the far offsets. The estimated $\eta$ in the first layer is accurate because the offset-to-depth ratio for its bottom is large ($x/z = 7$), whereas the inverted $\eta$-values in the second and third layer are slightly distorted. Even better estimates of $\eta$ could be obtained if the model contained dipping reflectors. Clearly, the hierarchical inversion approach makes it possible to handle the nonlinearity of the objective function and the trade-offs between the model parameters.
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Figure 2: Gradients of the objective function (equation 3) with respect to (b) $V_{\text{nmo}}$, (c) $\eta$, and (d) $\delta$ for a small perturbation in the $\delta$-model from Figure 1(a).

Figure 3: First stage of the inversion: (a) the actual $V_{\text{nmo}}$-field, (b) the initial $V_{\text{nmo}}$, and (c) the inverted $V_{\text{nmo}}$.

Figure 4: Second stage of the inversion: (a) the actual $\eta$-field, (b) the initial $\eta$, and (c) the inverted $\eta$.

CONCLUSIONS

We proposed a hierarchical two-stage RWI (reflection waveform inversion) algorithm to reduce the nonlinearity of the objective function and mitigate the tradeoffs between the model parameters for VTI media. At the first stage of RWI, the $\delta$-image is employed to estimate $V_{\text{nmo}}$ from near-offset data. Then we use the updated NMO velocity to generate the image which includes the far-offset velocity information needed to constrain $\eta$. An example for a layered VTI model confirms that the algorithm can resolve the interval $V_{\text{nmo}}$ and $\eta$ from reflection waveforms. As is the case in moveout analysis, either dipping interfaces or large offset-to-depth ratios are required to estimate $\eta$ with sufficient accuracy.

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Passive wavefield imaging using the energy norm
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SUMMARY
Wavefield imaging offers a robust approach for source evaluation in microseismic monitoring. The coexistence of P- and S-wave modes at the source location after time-reversal leads to an imaging condition from which the source position and radiation pattern can be identified. We propose a new imaging condition that is based on energy conservation and is directly related to the source mechanism. Unlike the correlation between decomposed P- and S-wave fields typically used in passive elastic imaging, our imaging condition compares wave modes present in the displacement field without costly wave-mode decomposition, and produces a strong and focused correlation at the source location. Numerical experiments demonstrate the advantages of the proposed imaging condition (compared to PS correlation), its sensitivity with respect to velocity inaccuracy and sparse acquisition, and its quality and efficacy in estimating the source location.

INTRODUCTION
Passive seismic monitoring uses signals caused by natural or induced seismicity to infer subsurface properties. The main difference from conventional exploration seismology is the absence of a controlled source. Although the seismic source location for passive seismic is not known, one can apply methods similar to those in active seismic acquisition to achieve an image and an Earth model (Duncan and Eisner, 2010; Maxwell et al., 2010; Xuan and Sava, 2010; Behura et al., 2013; Blias and Grechka, 2013; Witten and Shragge, 2015; Bazargani and Snieder, 2016). Microseismic monitoring is frequently used for description of unconventional reservoirs (Warpinski et al., 2012), and its most common application is for hydraulic fracturing. Fluid injection induces microseismic events, which can be observed by monitoring either from the surface or from boreholes. Using the recorded data, one can estimate the microseismic source location, and locate hydraulic fracturing in the subsurface (Maxwell, 2010; Michel and Tsvankin, 2013). Joint estimation of source location and mechanism potentially provides information about faults and fractures orientation (Zhebel and Eisner, 2012; Jeremic et al., 2014).

Wavefield imaging is usually implemented in two steps: (1) backpropagation of the recorded wavefield into the earth; and (2) application of an imaging condition to extract the source location and/or origin time (McMechan, 1982; Gajewski and Tessmer, 2005; Xuan and Sava, 2010; Nakata and Beroza, 2016). For multicomponent data, one typically employs imaging procedures that exploit the different wave modes present in elastic data. The spatial and temporal coexistence of P- and S-wave fields at the source allows for a PS imaging condition implemented in three steps: (1) backpropagation of the multicomponent wavefield; (2) wave-mode decomposition of the wavefield; and (3) the zero-lag crosscorrelation of the decomposed P- and S- wavefields (Artman et al., 2010; Witten and Artman, 2010; Douma and Snieder, 2015).

We propose an imaging condition for passive seismic data similar to the PS imaging condition, but without step (2) wave-mode decomposition. Our imaging condition attenuates the correlation between identical waves in the displacement field and highlights the correlation of different modes at the source. Based on energy conservation of elastic wavefields, we define an imaging condition as the difference between the kinetic and potential terms of a wavefield, which are computed using just the displacement field. This imaging condition is successful in elastic imaging with attenuated artifacts for active experiments, as shown in previous work (Rocha et al., 2016a,b,c). Here, we explain how a similar imaging condition is applicable to passive imaging, and we demonstrate its effectiveness in locating seismic sources using synthetic experiments in realistic settings.

THEORY
The anisotropic elastic wave equation is

$$\rho \ddot{U} = \nabla \cdot \left[ \frac{\epsilon}{P} \nabla U \right],$$

(1)

where $U(x,t)$ is the displacement field as a function of space ($x$) and time ($t$), $\rho(x)$ is the density, and $\epsilon(x)$ is the 4th-order stiffness tensor. The superscript dot indicates time differentiation. For an isotropic and slowly varying medium, equation 1 reduces to

$$\ddot{U} = V_P^2 \nabla \cdot (\nabla \cdot U) - V_S^2 \nabla \times (\nabla \times U),$$

(2)

where $V_P$ and $V_S$ are the P- and S-wave velocities, respectively. Using recorded multicomponent data, the displacement field $U$ can be extrapolated in the subsurface using either equation 1 or 2 depending on the assumptions about the medium anisotropy. Our imaging condition works equally well for both types of wavefields.

Different imaging conditions using the displacement field directly have been proposed to estimate the source location. Steiner et al. (2008) propose to use the absolute value of the particle velocity. One can also implement wave-mode decomposition, i.e. separate the displacement field into P- and S-wave fields. For isotropic media, wave-mode decomposition is cheap and typically implemented by Helmholtz decomposition (Dellinger and Etgen, 1990; Yan and Sava, 2009):

$$P = \nabla \cdot U,$$

(3)

$$S = \nabla \times U,$$

(4)

where

$$\nabla \cdot U = \frac{\epsilon}{P} \nabla U,$$

$$\nabla \times U = \frac{\epsilon}{S} \nabla U,$$

and $\epsilon$ is the same 4th-order stiffness tensor.
Passive imaging using the energy norm

where \( P(x,t) \) is a scalar wavefield containing the compressional wave mode, and \( S(x,t) \) is a vector wavefield containing the transverse wave mode. For anisotropic media, wave-mode decomposition is implemented by solving the Christoffel equation (Dellinger and Etgen, 1990); using techniques with significant additional cost (Yan and Sava, 2009, 2011; Cheng and Fomel, 2013).

Separated wave modes can be imaged using

\[
I_{PP}(x) = \sum_t P(x,t) P(x,t), \quad (5)
\]

\[
I_{SS}(x) = \sum_t S(x,t) \cdot S(x,t). \quad (6)
\]

These imaging conditions consist of an autocorrelation of a given wave mode, which produces low-wavenumber content along the path where the waves propagate. Alternatively, one can use different wave modes to form an image free of low-wavenumber artifacts. For a non-scattering medium, \( P- \) and \( S- \) waves propagate at different speeds and coexist in space and time only at the source (Yan and Sava, 2008; Artman et al., 2010):

\[
I_{PS}(x) = \sum_t P(x,t) S(x,t). \quad (7)
\]

\( I_{PS}(x) \) is a multicomponent image, whose components represent the correlation between \( P(x,t) \) and the corresponding component from \( S(x,t) \). In 3-D, three images can be computed instead of a single one that concisely shows the source location.

In contrast to PS imaging, the energy norm (Rocha et al., 2016a,c) allows one to define an imaging condition for elastic reverse time migration (ERTM) that concisely exhibits the reflectors in subsurface. For passive imaging, in which only one wavefield is extrapolated, we can define an imaging condition as

\[
I_E(x) = \sum_t \rho \ddot{U} - (\varepsilon \nabla U) : \nabla U. \quad (8)
\]

Although the imaging condition in equation 8 involves autocorrelation of the displacement field \( U \), the interaction between the same wave modes is attenuated as shown by Rocha et al. (2016b,c). The first and second terms in equation 8 represent the kinetic and potential wavefield energies, respectively. Equation 8 is related to the Lagrangian operator, which is expressed as the difference between kinetic and potential energy terms:

\[
\mathcal{L}(U,x,t) = \frac{1}{2} \rho \ddot{U} - \frac{1}{2} (\varepsilon \nabla U) : \nabla U. \quad (9)
\]

In the presence of sources, the Lagrangian for small displacement fields is associated with the action from these sources (Ben-Menahem and Singh, 1981):

\[
\int_0^T \mathcal{L}(\delta U, x, t) \, dt = - \int_0^T \nabla \cdot (\mathbf{t} \cdot \delta U) \, dt - \int_0^T \rho \mathbf{F} \cdot \delta U \, dt, \quad (10)
\]

where \( \mathbf{t}(x,t) \) and \( \mathbf{F}(x,t) \) are external stress or body forces, respectively. Equation 10 implies that, at the source, the imaging condition in equation 8 is directly related to the source mechanism.

Moreover, the energy imaging condition for passive imaging correlates \( P- \) and \( S- \) modes using the displacement field directly, without wave-mode decomposition. For isotropic media, the costs of the PS imaging condition (equation 7) and the energy imaging condition (equation 8) are comparable, since in both cases one needs to compute wavefield derivatives; for anisotropic media, computing equation 8 is quite cheaper than decomposing modes during extrapolation.

EXAMPLES

An imaging condition for passive seismic should deliver a strong and focused correlation at the source location if true model parameters are used. Figure 1a shows an ideal 2-D passive experiment where multicomponent receivers surround the source from all possible angles. The source mechanism consists of a stress field generated by a fault displacement oriented at 45° with respect to the horizontal. Note that the energy imaging condition (Figure 1e) results in a stronger and more focused correlation at the source location compared to the PS imaging condition (Figure 1c).

Figure 1b shows a similar experiment geometry but containing a sparse array of receivers only at the surface. The PS (Figure 1d) and energy images (Figure 1f) have similar resolution and quality considering the same acquisition limitations. In addition, Figures 1d and 1f illustrate that one should consider smearing and truncation artifacts when estimating a source location using wavefield imaging methods for passive seismic, since acquired data capture an incomplete wavefield, which prevents the extrapolated waves from collapsing into a focal point.

Figure 2 shows the effect of the source mechanism on PS and energy images, for a geometry similar to Figure 1b but with receivers at every grid point of the surface. For sources describing fault or fracture planes at steep angles (Figures 2c-2f), both imaging conditions show smearing along their source orientation. For a horizontal source (Figures 2a-2b), the energy image exhibits better focusing compared to the PS image. The analysis of radiation patterns for different source orientations is important because it can help us infer the fault slip or fracture direction.

We also illustrate our method with the Marmousi II model (Martin et al., 2002) in order to simulate more realistic passive acquisition in an area subject to hydraulic fracturing (Figure 3). The experiment consists of a microseismic source representing a fracture oriented at 30° (Figure 3a). Data are recorded with an array of multicomponent receivers with 80 m spacing in a borehole, and another array of receivers with 40 m receiver...
Passive imaging using the energy norm

Figure 1: (a) Circular acquisition with equal coverage from all angles and (b) with sparse coverage at the surface only. Multicomponent receivers in red, stress source oriented by $45^\circ$ in the center, and zoom area represented by the box. PS images (c)-(d), and energy images (e)-(f) for the circular and surface acquisition, respectively. Using the circular acquisition, the energy image in (e) has a strong and focused peak at the source location compared to the PS image in (c). Using the sparse surface acquisition, the PS (d) and energy (f) images are comparable in quality and show truncation artifacts compared to the images in (c) and (e).

Figure 2: Radiation patterns for the acquisition with complete surface coverage. PS images for a stress source oriented at (a) $0^\circ$, (c) $30^\circ$, and (e) $60^\circ$. Energy images for a stress source oriented at (b) $0^\circ$, (d) $30^\circ$, and (f) $60^\circ$. Both PS and energy imaging condition have analogous radiation patterns, with exception to $0^\circ$, where the energy image exhibits better focusing.

spacing on the surface. The combination of surface and borehole arrays improves passive seismic results due to the larger effective acquisition aperture (Thornton and Duncan, 2012). The Marmousi II model contains P and S velocities (Figure 3b) that are not proportional to each other, i.e., a spatially variable velocity ratio $(V_P/V_S)$ (Figure 3c). We smooth the velocities to obtain only P and S direct arrivals in the synthetic data and we further smooth the velocities and scale by 2% to simulate inaccurate migration velocity.

Figures 3d-3f show the application of the PS and energy imaging conditions when using the correct velocity. In Figure 3d, the energy correlation is strong at the source location compared to the surrounding artifacts, which is not the case for the corresponding PS image. Comparing the zoomed images (Figures 3e and 3f), note that the energy image is more focused at the source location than the PS image. In Figure 3f, the smearing in the source correlation compared to the radiation patterns in Figure 2d and 2f potentially infers that the source mechanism is due to an oblique fracture, which matches the simulated stress source oriented at $30^\circ$. Using the incorrect velocity (Figures 3g-3i), the artifacts increase and both images are more unfocused (Figures 3h and 3i), although the energy image retains partially its shape from the accurate velocity case. The Marmousi II experiment demonstrates that the energy imaging condition involves an indirect correlation of P and S waves that produces a stronger correlation at the source location compared to the direct PS correlation.
CONCLUSIONS

For passive wavefield imaging with multicomponent data, the energy imaging condition offers an elegant solution to locate seismic sources for an arbitrary Earth model. In contrast, the PS imaging condition requires a costly decomposition of the wavefields in Earth models that incorporate anisotropy. Based on the energy conservation for extrapolated wavefields, our imaging condition represents the temporal integral of the Lagrangian operator (which is the difference between kinetic and potential terms from the wavefield) and produces an image that is directly related to the source mechanism. We demonstrate for simple models that the energy and PS imaging conditions are comparable in terms of image quality and characterization of radiation patterns. For more realistic settings, we show that the energy imaging condition handles imaging artifacts and source focusing better than its conventional PS counterpart. Future work involves exploring the cost and further benefits of the energy imaging condition for anisotropic media, and developing a velocity inversion procedure using the unfocused energy on extended image gathers.

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Strategies for imaging with Marchenko-retrieved Green’s functions
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SUMMARY

Recent papers show that imaging with the retrieved Green’s function constructed by the Marchenko equations, called Marchenko imaging, reduces the artifacts from internal and free-surface multiples compared to standard imaging techniques. Even though the artifacts are reduced, they can still be present in the image, depending on the imaging condition used. We show that when imaging with the up- and down-going Green’s functions, the Multidimensional Deconvolution (MDD) imaging condition yields better images compared to correlation. Better in this case means improved resolution, fewer artifacts and a closer match with the true reflection coefficient of the model. The MDD imaging condition only uses the primaries to construct the image, while the multiples are implicitly subtracted in the imaging step. Consequently, combining the first arrival of the down-going Green’s function with the complete up-going Green’s function produces superior (or at least equivalent) images than using the one-way Green’s functions. We also show that standard imaging algorithms which use the redatumed reflection response, constructed from the retrieved Green’s functions, produce images with reduced artifacts from multiples compared to standard imaging with surface reflection data. All imaging methods that rely on the Marchenko equations require the same inputs as standard imaging techniques: the reflection response at the surface and a smooth version of the velocity.

INTRODUCTION

The Marchenko equations can be used to retrieve the up- and down-going Green’s function between an arbitrary virtual receiver in the subsurface and a source on the surface. However these equations do not prescribe how to use the Green’s function in imaging. The purpose of this abstract is to explain and compare different strategies for imaging with these up- and down-going Green’s functions. Imaging with these Green’s functions is called Marchenko imaging. Standard imaging techniques assume single scattering, and therefore misposition multiple-reflection events in the image. However, imaging with the Marchenko-retrieved Green’s functions significantly reduces (if not eliminates) the artifacts associated with multiple reflections (Broggini et al. (2014); Slob et al. (2014); Wapenaar et al. (2014); Singh et al. (2015) and Singh et al. (2016)). There are many types of imaging conditions that can be used to image the subsurface with the Marchenko Green’s functions; they are, however, mostly restricted to imaging with Green’s functions that include only primaries and internal multiples and they have never been systematically compared for imaging artifacts.

In this paper, we investigate different imaging conditions for the Green’s functions that include primaries, internal multiples, and also free-surface multiples. In addition to correlation and deconvolution imaging algorithms using the up- and down-going Green’s function $G^\pm$, we also image the subsurface with the first arrival of the down-going Green’s function $G_f^-$ and the up-going Green’s function $G^-$. Bakulin and Calvert (2006) and Mehta et al. (2007) show that muting the wavefield recording at the virtual source location, so that it is limited to its first arrival, improves the virtual source method. This muting suppresses spurious events in the virtual source gather. We use the same concept of muting the first arrival of $G^+$ to further reduce the imaging artifacts from multiples in the Marchenko image. For simplicity, we separate our investigation on Marchenko imaging strategies into 1D and 2D. The conclusions of our 1D analysis is applicable to the 2D scenario. In our 2D investigation we compare imaging with a standard imaging technique, reverse time migration, and Marchenko imaging.

1D STRATEGIES FOR IMAGING

In this section we restrict our imaging and theory to 1D. The ideas, conclusions, and analysis are, however applicable, for the most part, to multidimensions. The imaging conditions analyzed in this section are 1) correlation and 2) deconvolution. We also compare the application of these imaging conditions to either the up-going Green’s function $G^+$ and down-going Green’s function $G^+$ or to the up-going Green’s function $G^-$ and first arrival of the down-going Green’s function $G_f^-$.

For our 1D models, we denote the depth as $z_i$, where the subscript $i = 0, 1, 2, \ldots$ corresponds to the depth in 1D; for instance $z_0$ is the acquisition surface. Superscript $(\pm)$ refers to down-going waves and $(\pm)$ to up-going waves at the depth $z_i$. Any variable with a subscript 0 that is not the coordinate field $z$, (e.g., $R_0$) indicates that no free surface is present.

We use a two layer 1D model that has variable density (2 g/cm² and 4.5 g/cm³ in the first and second layers at a depth of 1.5 km and 2.2 km, respectively) and constant velocity (3 km/s), to image the subsurface using the Marchenko retrieved Green’s functions. Although we are using a constant velocity model, the Marchenko equations and the corresponding imaging is not limited to a constant velocity. We compute the image at 5 m intervals in the 1D model. Each image point corresponds to a virtual receiver location of the Green’s function. We compute $G^+$ and $G^-$ at each virtual receiver location in the subsurface for sources on the surface. The governing equation for imaging with up- and down-going wavefields in 1D is

$$G^-(z_i, z_0, t) = \int_{-\infty}^{\infty} G^+(z_i, z_0, t - t') R_0(z_i, z_i, t')dt', \tag{1}$$

where $z_i$ is an arbitrary depth level and $R_0$ is the reflection response of the medium below $z_i$ (Charebt, 1985; Wapenaar et al., 2008; Amundsen, 2001). In this expression $R_0(z_i, z_i, t)$...
Marchenko imaging strategies

Figure 1: Correlation-imaging with (a) the up- and down-going retrieved Green’s functions $G^-$ and $G^+$ (b) the first arrival of the down-going Green’s function with the up-going Green’s function $G^-_f$, for the 1D model.

is the reflection response for sources and receivers at $z_i$, with the medium above $z_i$ being homogeneous. The image of the subsurface is $R_0(z_i, z_i, t = 0)$, the reflection response $R_0$ at zero time. Intuitively, for a source and a receiver coincident at an interface $z_i$, the zero time response $R_0(z_i, z_i, t = 0)$ at that location is the contribution to the image corresponding to the interface. Similarly, in the absence of an interface at $z_i$, the contribution of $R_0(z_i, z_i, t = 0)$ at zero time is zero.

We begin with correlation imaging. The contribution at the zero-lag correlation of two 1D signals is due to kinematically similar events in the signals. For this reason standard imaging techniques, for example Reverse Time Migration (RTM), generally use the correlation at zero lag as the contribution to the image (Baysal et al., 1983; McMechan, 1989; Whitmore, 1983). However, unlike the Marchenko wavefields, the fields used in conventional RTM assumes the Born approximation and therefore RTM has artifacts in the presence of multiples (Glogovsky et al., 2002; O’Brien and Gray, 1996). In the time domain, the zero-lag correlation-imaging condition in 1D for the retrieved up- and down-going Green’s function is $C(z_i, z_0, t = 0) = \int_{-\infty}^{\infty} G^-(z_i, z_0, t) G^+(z_i, z_0, t) dt$. This equation means that the correlation image is the time integral of the up- and down-going Green’s functions. We apply the correlation imaging condition at each virtual receiver location for the associated $G^+$ and $G^-$. The corresponding image is shown in Figure 1a.

In the frequency domain, the reflection response $R_0$ (equation 1) in 1D is a deconvolution $R_0(z_i, z_i, \omega) = G^-(z_i, \omega) G^+(z_i, \omega)^* \left(\frac{G^+(z_i, \omega)}{|G^+(z_i, \omega)|} + \epsilon\right)^{-1}$, where $\ast$ represents the complex conjugate and $\epsilon$ is a regularization parameter to avoid division by zero (Clayton and Wiggins, 1976). The deconvolution imaging condition is $R_0$ at zero time at the location of the Green’s function virtual receiver $z_i$. Using equation 1, we apply the deconvolution-imaging condition to the $G^+$ and $G^-$ for the 1D model. The corresponding image is shown in Figure 2a.

For the 1D model the reflection coefficient of the first layer at 1.5 km is 0.33 while the second interface at 2.2 km is 0.38. Comparing the parameters for the actual model to the correlation and deconvolution images in Figure 2 shows that: 1) the deconvolution image obtains the correct reflection coefficients at the two interfaces but the correlation image does not 2) the two interfaces are correctly positioned at 1.5 km and 2.2 km, respectively; 3) as we exactly solve for $R_0$ in 1D by deconvolution, there are no false interfaces in Figure 2a due to kinematically similar events in $G^+$ and $G^-$ at 0.7 km compared to the correlation image in Figure 1a 4) the relative amplitudes of the interfaces in the correlation image are incorrect.

The presence of the false interface at 0.7 km is due to a free surface multiple in $G^+$ interacting with the up-going reflection from the second layer in $G^-$ at approximately 0.7 km. At this depth, $G^+$ and $G^-$ have kinematically similar events and hence an incorrect contribution to the correlation image. There is a weak artifact in the deconvolution image at 0.7 km (see Figure 2a); however these relatively negligible events, compared to the events at the interface, are due to numerical errors in the deconvolution and the finite recording time of the reflection response at the surface.

We also propose using the first arrival of $G^+$ (defined as $G^+_f$) and $G^-$ for imaging the subsurface. A smooth version of the velocity model can be used, in general, to mute $G^+$ to get its first arrival, $G^+_f$. We follow the correlation and deconvolution imaging procedures for imaging with $G^+_f$ and $G^-$. Figures 1b and 2b shows the corresponding correlation image and deconvolution image, respectively. Note that the correlation image, Figure 1b, has no false interfaces around 0.7 km compared to Figure 1a, because $G^+_f$ does not include any multiples. (The free-surface multiple no longer exist in $G^+_f$ and therefore does not contribute to a false interface.) However, the amplitudes of the reflectors for the correlation image still do not match the true reflectivity even though the image do not include false interfaces. The deconvolution image, Figure 2b, matches the true reflectivity of the model (solid black line), despite windowing $G^+$ with $G^+_f$. We do not get the artifacts at 0.7 km in Figure 2b compared to Figure 2a (deconvolution image with $G^+$ and $G^-$). Therefore imaging with $G^+_f$ and $G^-$ removes the false interface at 0.7 km and gives the correct reflectivity of the subsurface. However, we do not reconstruct the correct redatumed response $R_0(z, z, t)$. Since $G^+_f$ excludes the multiples in $G^+$ and we form the image by zero-time correlation, the multiples in $G^-$ arrive too late to contribute to this corre-
Marchenko imaging strategies

2D STRATEGIES FOR IMAGING

The equations that govern imaging with the retrieved Green’s functions in multidimensions are similar to the imaging equations in 1D, except in 2D they have an additional horizontal space variable. The mathematics of Marchenko imaging using the correlation and MDD imaging condition are covered in detail by Wapenaar et al. (2014), for this reason we show the resulting Marchenko images from these imaging conditions. Note that our 1D analysis can be extended to multidimensions, so we will not repeat the imaging analysis in the previous section (where the 1D imaging section rigorously analyzed Marchenko imaging with different imaging conditions) but instead, we compare Marchenko imaging with conventional RTM. We utilize the velocity and density models in Figures 3a and 3b, respectively, to compute the reflection response R at the surface. The reflection response includes primaries, internal and free-surface multiples. We use the reflection response at the surface as inputs for all imaging examples in this section. Our goal is to image a target area in the subsurface, which is enclosed by the box in Figure 3a.

We show the image we obtain from RTM in Figure 4a. We construct the RTM image by evaluating the correlation of the back propagated reflection response and forward propagated source function in the smooth velocity model at zero time and zero-offset. We call this the RTM correlation imaging condition; which is different from the correlation imaging condition previously mentioned that uses $G^+$ and $G^-$. The RTM image in Figure 4a is a zoomed-in version of the entire image at the target area. The free-surface multiples generated by the syncline above the target area, as well as internal multiples, contaminate the image in Figure 4a.

We now investigate Marchenko imaging of the target area in Figure 3a with the following imaging conditions: 1) correlation, and 2) multidimensional deconvolution. We compare the images generated by these imaging conditions constructed with either $G^+$ and $G^-$ or with $G^+_f$ and $G^-$ (Figure 4b). Note that the associated Marchenko images are constructed from the same inputs as RTM, i.e., the reflection response at the surface and a smooth version of the velocity model. The correlation imaging with the retrieved Green’s functions are shown in Figure 4b. This figure is obtained by computing the Green’s function $G^\pm(x',x,t)$ at the surface for virtual receivers, at intervals of 4 m, in the target area; the image is the superposition of the correlation imaging condition at each Green’s function virtual receiver location. In the Marchenko image in Figure 4b the reflectors are clearly discernible and match the interfaces in the target area in Figure 3a. In the imaging box the artifacts from the free-surface and internal multiples are no longer visible compared to the RTM image in Figure 4a.

Solving equation 1 for $R_0$ in 1D requires deconvolution. However, in higher dimension, we solve for $R_0$ by MDD (Wapenaar et al., 2014). The MDD imaging condition is $R_0(x_1,x,t = 0)$ (the reflection response $R_0$ at zero-offset and at time = zero seconds). To construct the image, we compute $R_0(x_1,x,t = 0)$ at every sampled point in the image. The Marchenko images constructed with correlation and MDD yield similar results; however, as a more instructive approach to compare these images, we show a trace at $x_1 = (-40)$ m below 1 km for each of the corresponding images (see Figure 5a). The traces in Figure 5a show that 1) MDD matches the true reflectivity better than the correlation imaging conditions, 2) the events in the traces (MDD and correlation) correspond to the interfaces in the actual model at the right locations. The true reflectivity trace in Figure 5a is constructed by computing the reflection coefficients at zero offset at $x_1 = (-40)$ m below 1 km, then convolving this trace with the Ricker wavelet used in finite difference modeling of the reflection response at the surface.

We use the $G^+$ and $G^-$ at virtual receivers $x'_1 = (-2$ to $2, 1)$ km to compute the redatumed reflection response $R_0$ by MDD (Wapenaar et al., 2014) and we use this response to image the subsurface using standard imaging algorithms. We perform RTM using the redatumed response $R_0(x_1,x'_1,t)$ at $x_1 = (-2$ to $2, 1)$ km to image the target area (see Figure 4c). The RTM correlation imaging condition is used to construct the redatumed RTM in Figure 4c and the RTM with surface recordings in Figure 4a. In Figure 4c the artifacts are drastically reduced compared to the RTM image in Figure 4a. Specifically,
the multiples from the syncline structure are not present in the image in Figure 4c using the redatumed response compared to the RTM in Figure 4a. This reduction in artifacts is a result of the fact that the redatumed reflection response $R_0$ only includes the reflections below 1 km. The redatumed reflection response still, however, includes internal multiples from the interfaces below the redatuming depth. Therefore, the redatumed RTM image does in fact have artifacts from such internal multiples, for instance at $z=1.68$ km in Figure 4c, but they are significantly weaker than the reflections caused by the overburden, i.e. the syncline reflections (see Figure 4a).

We performed correlation and MDD imaging condition using $G_f^+$ and $G_f^-$; the images are the same as the images constructed with $G^+$ and $G^-$ using the same imaging conditions for our 2D examples. Figure 5b shows a comparison between the true reflectivity, a trace from the MDD imaging with $G^+$ and $G^-$, and MDD imaging with $G_f^+$ and $G_f^-$. As expected from the 1D imaging section, imaging with $G^+$ and $G^-$ or imaging with $G_f^+$ and $G_f^-$ gives similar contributions at the interfaces. However, similar to the 1D imaging section, using $G_f^+$ and $G_f^-$ does not give the correct redatumed response, and hence cannot be used to create an image below the redatuming depth.

DISCUSSION

The multidimensional deconvolution imaging condition applied to $G^+$ and $G^-$ yields a good match with the true reflectivity and minimizes false interfaces as MDD is the theoretically accurate way to solve for the image compared to the other methods in this paper. Note that $G^+$ and $G^-$ must either have all the multiples included or both $G^+$ and $G^-$ must be truncated in such a way that they ($G^+$ and $G^-$) include the same order of multiples to have the multiples removed in the imaging step. (Failure to include the same order of multiples in $G^+$ and $G^-$ creates false interfaces in the imaging step.) However in practice, it is not feasible to know if we have the same order of multiples in $G^+$ and $G^-$. For this reason, it is more advantageous to use the first arrival of $G_f^+$ and $G_f^-$ as we do not need to match the order of multiples in the up- and down-going fields while still matching the true reflectivity and avoiding false interfaces.

CONCLUSION

Even though Marchenko imaging reduces the artifacts caused by multiples compared to standard imaging algorithms, these artifacts are still present. Theoretically, the correct procedure to image with the retrieved Green’s functions is MDD and therefore best matches the correct image of the subsurface compared to other Marchenko imaging conditions. However, instead of using $G^+$ and $G^-$, Marchenko imaging with the first arrival of the down-going Green’s function $G_f^-$ and the associated up-going Green’s function $G_f^+$, removes these artifacts corresponding to false interfaces. Despite the fact that $G_f^-$ does not contain the reflection events, the resulting MDD image better matches the true reflectivity of the model compared to standard imaging or Marchenko imaging with correlation or deconvolution. Note that since only the primaries contribute to the construction of the image while the multiples are implicitly removed in the inversion process to produce the image, it suffices to only use $G_f^+$ and $G^-$ compared to $G^+$ and $G^-$ in the imaging. Importantly, the inputs for Marchenko imaging are exactly the same as most standard imaging techniques; a smooth version of the velocity and the reflection response at the surface. Unlike standard imaging techniques, in Marchenko imaging, we do not need to remove the free-surface or internal multiples from the reflection response, as the Marchenko equations in this paper properly handle these multiples.

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Beyond Marchenko – Obtaining virtual receivers and virtual sources in the subsurface
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SUMMARY

By solving the Marchenko equations, the Green’s function can be retrieved between a virtual receiver in the subsurface to points at the surface (no physical receiver is required at the virtual location). We extend the idea of these equations to retrieve the Green’s function between any two points in the subsurface, i.e., between a virtual source and a virtual receiver (no physical source or physical receiver is required at either of these locations). This Green’s function is called the virtual Green’s function and includes all the primaries, internal and free-surface multiples. Similar to the Marchenko Green’s function, we require the reflection response at the surface (single-sided illumination) and an estimate of the first arrival travel time from the virtual location to the surface.

INTRODUCTION

We propose a method to retrieve the Green’s function between two points in the subsurface of the Earth. We call these two points a virtual source and a virtual receiver pair. To retrieve the Green’s function at a virtual receiver for a virtual source we require neither a physical source nor a physical receiver at the virtual source and receiver. The requirements for the retrieval of this Green’s function is the reflection response for physical sources and physical receivers at the surface (single-sided illumination) and a smooth version of the velocity model (no small-scale details of the model are necessary). For brevity we define this Green’s function i.e., the response of a virtual source recorded at a virtual receiver, as the Virtual Green’s function. We label the method of retrieving the Virtual Green’s function as the modified Marchenko method.

Similar ideas of retrieving the Green’s function between two points have been proposed in seismic interferometry (Wapenaar, 2004; Curtis et al., 2006; Snieder et al., 2007; Bakulin and Calvert, 2006; van Manen et al., 2006; Curtis et al., 2009; Curtis and Halliday, 2010) and in the Marchenko method (Broggini et al., 2012; Broggini and Snieder, 2012; Wapenaar et al., 2013; Slob et al., 2014; Wapenaar et al., 2014; Singh et al., 2015, 2016). However, these methods (interferometry and Marchenko method) have more restrictions in the source-receiver geometry, as discussed later, for the accurate retrieval of the Green’s function than our proposed method (modified Marchenko method).

In seismic interferometry, we create virtual sources at locations where there are physical receivers. We also require a closed surface of sources to adequately retrieve the Green’s function. Unlike interferometry, a physical receiver or physical source is not needed by our modified Marchenko method to create either a virtual source or a virtual receiver and we only require single-sided illumination (a closed surface of sources not needed). The Green’s function retrieved by the Marchenko equations is the response to a virtual source in the subsurface recorded at physical receivers at the surface (Broggini et al., 2012; Broggini and Snieder, 2012; Wapenaar et al., 2013; Slob et al., 2014; Wapenaar et al., 2014; Singh et al., 2015, 2016). The Marchenko retrieved Green’s function requires neither a physical source nor a physical receiver at the virtual source location in the subsurface.

Our algorithm retrieves the Green’s function (both up- and down-going at the receiver) for virtual sources and virtual receivers. The Marchenko-retrieved Green’s functions are limited to virtual sources in the subsurface recorded at the surface but the Modified Marchenko method (our Work) is not restricted to recording on the surface for each virtual source. In our method, the response of the virtual source can be retrieved for a virtual receiver anywhere in the subsurface.

Wapenaar et al. (2016) has proposed similar work to ours, but their approach retrieves (1) the two-way virtual Green’s function while our work retrieves the up- and down-going (one-way) virtual Green’s function, the summation of these one-way Green’s function gives the two-way Green’s function, and (2) the homogeneous Green’s function while we retrieve the causal Green’s function.

We discuss in this paper the theory of retrieving the virtual Green’s function. Our numerical examples are split into two sections (1) A verification of our algorithm in 1D (2) A 2D numerical example of the virtual Green’s function constructed in such a way that we create a wavefield with all the reflections and first arrivals from a virtual source. This last numerical example is complicated since the discontinuities in the density and the velocity are at different locations.

THEORY

To retrieve the Green’s function from a virtual receiver in the subsurface for sources on the surface, one solves the Marchenko equations. The retrieval only requires the reflection response at the surface and an estimate of the first arrival travel-time from the virtual receiver to the surface. The retrieved Green’s function can either include free-surface multiples (Singh et al., 2015, 2016) or exclude these multiples (Broggini et al., 2012; Broggini and Snieder, 2012; Wapenaar et al., 2013; Slob et al., 2014; Wapenaar et al., 2014). In addition to the retrieved Green’s function, the Marchenko equations also give us the one-way focusing functions. These functions are outputs from the Marchenko equations that exist at the acquisition level ∂Dt (acquisition surface) and focus on an arbitrary depth level ∂Dt at t = 0 (time equal zero).
The focusing functions are auxiliary wavefields that reside in a truncated medium that has the same material properties as the actual inhomogeneous medium between \( \partial D_0 \) and \( \partial D_2 \) and that is homogeneous above \( \partial D_0 \) and reflection-free below \( \partial D_2 \) (Slob et al., 2014). Therefore, the boundary conditions on \( \partial D_0 \) and \( \partial D_2 \) in the truncated medium, where the focusing function exists, are reflection-free. Our algorithm moves the sources of the Green’s function retrieved by Marchenko equations from the surface into the subsurface at a virtual point with the help of the focusing function.

In this paper, the spatial coordinates are defined by their horizontal and depth components; for instance \( \mathbf{x}_0 = (\mathbf{x}_{H,0}, x_{3,0}) \), where \( \mathbf{x}_{H,0} \) stands for the horizontal coordinates at a depth \( x_{3,0} \). Superscript (+) refers to down-going waves and (−) to up-going waves at the observation point \( \mathbf{x} \). Additionally, wavefield quantities with a subscript 0 (e.g., \( R_0 \)) indicates that no free-surface is present. One-way reciprocity theorems of the convolution and correlation type are used to relate up- and down-going fields at arbitrary depth levels to each other in different wave states (Wapenaar and Grimbergen, 1996).

The correlation reciprocity theorem is based on time reversal invariance of our wavefields, which implicitly assumes that the medium is lossless. Since we assume the wavefields can be decomposed into up- and down-going waves, we ignore evanescent waves.

Wave state A is defined for the truncated medium where the focusing functions reside. The one-way wavefields for wave state A that focus at \( \mathbf{x}_f' \) (above \( \mathbf{x}_f \)) are given in Table 1.

The Green’s functions in the actual medium are defined as wave state B. The one-way wavefields for wave state B, the actual medium, for a source at \( \mathbf{x}_f \) are given in Table 1. We substitute the one-way wavefields described in Table 1 into the reciprocity theorems and use the sifting property of the delta function to yield

\[
G^-(\mathbf{x}_1', \mathbf{x}_J', \omega)^* = \int_{-\infty}^{\infty} [rG^-(\mathbf{x}_0, \mathbf{x}_J', \omega)] f_1^+(\mathbf{x}_0, \mathbf{x}_J', \omega) d\mathbf{x}_0,
\]

where \( r \) denotes the reflection coefficient of the free surface (in the examples shown in this paper \( r = -1 \)).

Equations 1 and 2 yield the up- and down-going virtual Green’s functions, respectively, for a virtual receiver at \( \mathbf{x}_f' \) and a virtual source at \( \mathbf{x}_f'' \) in the subsurface. Note that for the total Green’s function, we are not limited to the source \( \mathbf{x}_f'' \) being below the receiver \( \mathbf{x}_f' \) since by reciprocity, \( G(\mathbf{x}_1', \mathbf{x}_J', \omega) = G(\mathbf{x}_J', \mathbf{x}_1', \omega) \). To compute the up- and down-going virtual Green’s function in equations 1 and 2, we require 1) the Green’s function \( G^-(\mathbf{x}_0, \mathbf{x}_J', \omega) \) at the surface \( \mathbf{x}_0 \) for a focal point at \( \mathbf{x}_J' \) and 2) the focusing function \( f_1^+(\mathbf{x}_0, \mathbf{x}_J', \omega) \) at the surface \( \mathbf{x}_0 \) for a virtual source at \( \mathbf{x}_J' \). We retrieve both these functions by solving the Marchenko equations which requires the reflection response (including free-surface multiples) as input (Singh et al., 2015, 2016). Note that these Green’s functions \( G^-(\mathbf{x}_0, \mathbf{x}_J', \omega) \) include the primary, internal, and free-surface multiple reflections of the actual medium.

We can also retrieve the virtual Green’s function which does not include free-surface multiples by simply setting the reflection coefficient at the free-surface \( r \) to zero in equations 1 and 2. Thus, the equation to retrieve the virtual Green’s function without the presence of a free surface is

\[
G_0^-(\mathbf{x}_1', \mathbf{x}_J', \omega)^* = \int_{-\infty}^{\infty} G_0^-(\mathbf{x}_0, \mathbf{x}_J', \omega) f_1^+(\mathbf{x}_0, \mathbf{x}_J', \omega) d\mathbf{x}_0,
\]

\[
G_0^+(\mathbf{x}_J', \mathbf{x}_f', \omega)^* = -\int_{-\infty}^{\infty} G_0^+(\mathbf{x}_0, \mathbf{x}_J', \omega) f_0^- (\mathbf{x}_0, \mathbf{x}_J', \omega) d\mathbf{x}_0,
\]

where \( G_0^+(\mathbf{x}_J', \mathbf{x}_f', \omega) \) is the up- and down-going Green’s function without free-surface multiples for a virtual receiver at \( \mathbf{x}_J' \) and virtual source at \( \mathbf{x}_f' \).

**NUMERICAL EXAMPLES**

The first example illustrates the retrieval of the virtual Green’s function with the free-surface reflections (Figure 1) for the 1D model given in Figure 2 with the virtual source and receiver shown by the blue and red dots, respectively. This example also contains variable density, with discontinuities at the same depth as the velocity model, with densities ranging from 1 g cm\(^{-3}\) to 3 g cm\(^{-3}\). As shown in Figure 1, there is an almost perfect match between the modeled Green’s function and the retrieved virtual Green’s function. The 1D numerical example have perfect aperture, hence, the 1D examples almost perfectly match the retrieved virtual Green’s function to the modeled Green’s function. Note that to retrieve the virtual Green’s function in Figure 1 we only use the reflection response at the surface.

A fair question to ask is: why not use interferometry to cross-correlate the Green’s function at a virtual receiver and at virtual source to get the virtual Green’s function between the virtual source and the receiver? This interferometric method will not retrieve the virtual Green’s function when we only have a
source at the surface because interferometry requires sources on both sides of the receiver. In Figure 3 (red line), we show the interferometric Green’s function, (cross-correlation of the Green’s functions from the virtual source and receiver to the surface), for the same model (see Figure 1) with the same virtual source $x''_i$ at depth 1.75 km and recording at the virtual receiver $x'_j$ at depth 0.75 km for the model in Figure 2 with a free surface. The modeled Green’s function is superimposed on it which also includes the free-surface multiples (black line).

Our algorithm allows us to place virtual sources and virtual sources in any target location in the subsurface. For our numerical example, we retrieve the virtual Green’s function $G(x'_i, x''_j, t)$, Figure 5, where $x'_i$ are the virtual receivers populating the target location at every 32 m (black box in Figure 4a) and $x''_j = (0, 0.7) km$ is the virtual source (black dot in Figure 4a). In Figure 5 notice:

1. In panel b, the first arrival from the virtual source $x''_j = (0, 0.7) km$ and the reflection from the bottom velocity layer.
2. In panels c and d, the inability of our algorithm to handle the horizontal propagating energy of the first arrival from the virtual source, hence the dimming on the sides of the first arrival of the virtual Green’s function. To retrieve near-horizontally propagating events (in this case, these waves are not evanescent) especially in the first arrival of the virtual Green’s function, we require a much larger aperture than is used in this example. Note that the later arriving up- and down-ward propagating waves are retrieved accurately at the depth of the virtual source $x''_j = (0, 0.7) km$ in Figure 5, panel d and e, since the reflections are purely up- and down-going.
3. In panels c and d, we do however, retrieve the reflections from the density layer (pink line in Figure 5) although we did not use any explicit information of the density model in our numerical retrieval of the virtual Green’s function.
4. In panel f, a free-surface multiple is present. As expected, there is a polarity change of the free surface multiple compared to the incident wave at the top of panel e due to the interaction of this wave in panel e with the free surface.
5. In panel h, we obtain the up-going reflections caused by the free-surface multiple interacting with the velocity and density layer.

In our algorithm, we evaluate an integral over space us-
Beyond Marchenko

Figure 4: Synthetic model (a) velocity model with velocities ranging from 2.0 to 2.4 km/s (b) one-interface density model with densities ranging from 2.0 to 3.0 g/cm³. The dot shows the position of the virtual source for the virtual Green’s function and the black box is the target zone where we place virtual receivers.

Figure 5: Snapshots of the virtual Green’s function \( G(x_j', x_i'') \) with virtual sources \( x_i'' = (0.0, 0.7) \) km and virtual receivers \( x_j' \) populating the target box in Figure 4a. The dotted lines represent the velocity interface (blue) and the density interface (magenta).

DISCUSSION

The theory of the virtual Green’s function is based on Marchenko equations and uses the Marchenko solutions as well; hence, the virtual Green’s function also suffers from the shortcomings and requirements of the Marchenko retrieved Green’s function that are described elsewhere (Broggini et al., 2012; Broggini and Snieder, 2012; Wapenaar et al., 2013; Slob et al., 2014; Wapenaar et al., 2014; Singh et al., 2015, 2016).

For the simple 2D model, the discontinuities and dip in the velocity and density are different. However, we retrieve the two-way and one-way wavefield of the virtual Green’s function without any knowledge of the density model and small-scale details in the velocity model. Figure 5 shows reflections from the density interface (middle interface in Figure 5), even though no density information was included in our algorithm. We retrieve these reflections because the density information is embedded in the reflection response recorded at the surface and the Marchenko equations are able to retrieve the density reflections from this response.

CONCLUSION

We can retrieve the Green’s function between two points in the subsurface with single-sided illumination. Generally, interferometry gives inaccurate Green’s functions for illumination from above (single-sided) because we do not have the illumination contributions from below. However, the Marchenko equations can be thought of as the mechanism to obviate the need for illumination from below to retrieve the virtual Green’s function. The removal of the requirement for illumination from below (for interferometry) comes from the use of the focusing function, a solution to the Marchenko equations. The events in the focusing function only depend on the truncated medium and this function is solved using illumination only from above. In this paper, we explore this single-side illumination advantage of the focusing function to avoid the illumination from below to retrieve the virtual Green’s function.

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Feasibility of waveform inversion in acoustic orthorhombic media
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SUMMARY

3D waveform inversion (WI) for anisotropic media is highly challenging due to its computational cost, large model space, and trade-offs between the model parameters. Here, we explore the feasibility of 3D waveform inversion for orthorhombic media in the acoustic approximation. A separable form of the wave equation is implemented using the pseudospectral method. The pseudospectral extrapolator is stable and produces kinematically accurate pure-mode P-wavefields with an acceptable computational cost. To build the initial long-wavelength model for waveform inversion, we use the envelope-based misfit functional, which alleviates the reliance of WI on low-frequency data. The WI gradients are derived for both the conventional data-difference and the envelope-based objective functions. Testing on a modified SEG/EAGE overthrust model illustrates the performance of the developed wavefield-extrapolation and gradient-computation algorithms for realistic orthorhombic media.

INTRODUCTION

Most existing waveform-inversion techniques are designed to recover just P-wave velocity due to the high computational cost and the intrinsic nonlinearity of the inverse problem. Recently, WI has been extended to both acoustic and elastic transversely isotropic models with a vertical symmetry axis (VTI) (Gholami et al., 2011; Kamath and Tsvankin, 2016). Transverse isotropy, however, cannot describe many subsurface formations that exhibit orthorhombic symmetry due to the influence of aligned fractures and nonhydrostatic stresses. Orthorhombic models have been successfully used in processing of wide-azimuth reflection and VSP data and fracture characterization (Tsvankin, 1997; Tsvankin and Grechka, 2011). In this paper, we focus on acoustic orthorhombic models described by a simplified wave equation that preserves the P-wave kinematics (Alkhalifah, 1998, 2000).

Two categories of methods have been proposed to model P-wave propagation in anisotropic media: coupled systems and mixed-domain wavefield extrapolators. The coupled systems have been originally introduced for TI media (Fletcher et al., 2009; Fowler et al., 2010) and can be extended to orthorhombic symmetry. However, common problems with the coupled systems are the existence of shear-wave “artifacts” and the ambiguity in the physical interpretation of the auxiliary wavefield variables. Here, we implement waveform inversion in orthorhombic media using a mixed-domain acoustic wavefield extrapolator.

To improve long-wavelength models at early stages of waveform inversion, it is common to use multiscale methods (Bunks et al., 1995). However, conventional seismic acquisition cannot provide ultra-low-frequency (down to 1–2 Hz) data, which are critical for constraining long-wavelength parameter fields. The envelope-based misfit functional (Wu et al., 2014; Luo and Wu, 2015) can improve the convergence of WI without low-frequency data. We obtain the WI gradients for both the data difference and squared envelope misfit functions and apply the algorithm to an overthrust orthorhombic model.

WAVEFIELD SIMULATOR

The starting point for deriving pure-mode mixed-domain wavefield extrapolators is the dispersion relation of the corresponding wave mode. These extrapolators satisfy a general equation of the form

\[ \partial_{tt} u(k, t) + \Phi(x, k) u(k, t) = 0, \]

where \( u(k, t) \) denotes the scalar wavefield variable in the time-wavenumber domain, \( k \) is the wave vector, \( \partial_{tt} \) is the second time-derivative operator, and \( \Phi(x, k) \) is a linear operator defined in the mixed (spatial and wavenumber) domain; the source term in equation 1 is ignored. In isotropic media, the mixed-domain operator \( \Phi \) reduces to

\[ \Phi(x, k) = v^2(x) |k|^2, \]

where \( v(x) \) is the velocity. If the model is anisotropic, the mixed-domain operator for a certain mode can be obtained from the corresponding dispersion relations using the Christoffel equation. The dispersion relations for TI and orthorhombic media can be solved analytically because they yield quadratic and cubic equations, respectively.

Note that for a spatially invariant operator \( \Phi(x, k) = \Phi(k) \), equation 1 reduces to a system of ordinary differential equations with the free variable \( t \), which has the formal solution

\[ u(k, t \pm \Delta t) = e^{\pm i \sqrt{\Phi(k)} \Delta t} u(k, t). \]

Adding the outgoing and incoming solutions of equation 3, one arrives at the time-stepping formula:

\[ u(x, t + \Delta t) + u(x, t - \Delta t) = \mathcal{F}^{-1}[2 \cos(\sqrt{\Phi} \Delta t) \mathcal{F}[u(x, t)]] , \]

where \( \mathcal{F}[] \) and \( \mathcal{F}^{-1}[] \) denote the forward and inverse Fourier transforms, respectively. When the mixed-domain operator \( \Phi(x, k) \) varies in space, the time-stepping formula 4 provides only an approximate solution to equation 1. Solving equation 4 is time-consuming because the number of inverse FFT’s is equal to the number of the spatial grid points. One way to speed up the computation is to recognize the cosine term as a low-rank matrix and decompose it into two separate submatrices in the space- and wavenumber-domain (Fomel et al., 2013b).

Alternatively, we can use a technique called the generalized pseudospectral method, which approximates the derivatives using global basis functions, rather than local finite-differences.
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First, the cosine term in equation 4 is expanded in a linear Taylor series:

\[
\cos \left( \sqrt{\Phi} \Delta t \right) = 1 - \frac{1}{2} (\Delta t)^2 \Phi ,
\]

(5)

The time-stepping formula then becomes:

\[
u(x, t + \Delta t) + u(x, t - \Delta t) = 2u(x, t) - (\Delta t)^2 \mathcal{F}^{-1} \left( \Phi \mathcal{F} [u(x, t)] \right) .
\]

(6)

However, application of equation 6 is still hampered by the fact that the operator \( \Phi(x, k) \) varies spatially and appears inside the inverse Fourier transform. Therefore, the mixed-domain operator must be represented in separable form in order to use fast Fourier transforms:

\[
\Phi(x, k) = \sum_i f_i(x) g_i(k) ,
\]

(7)

where \( f_i(x) \) and \( g_i(k) \) are the pure spatial- and wavenumber-domain operators, respectively.

For acoustic orthorhombic medium with the symmetry planes that coincide with the Cartesian coordinate planes, the separable mixed-domain operator can be derived as (Fowler and Lapilli, 2012):

\[
\Phi(x, k) = \sum_{i=1}^3 \left( \frac{V_{p1}^2}{V_r^2} k_i - \frac{V_{p2}^2 V_{p3}^2 V_{p1}^2}{V_r^2} k_i k_2 k_3 \right) ,
\]

(8)

where \( V_r \) is a reference velocity, \( V_{p1} \) (i = 1, 2, 3) are the P-wave velocities in the coordinate directions, and

\[
V_{pa1} = V_{nm01} V_{p3} ,
V_{pa2} = V_{nm02} V_{p3} ,
V_{pa3} = V_{nm03} V_{p1} ,
\]

(9)

\( V_{nm01} \) (i = 1, 2, 3) are the P-wave NMO velocities. The velocities \( V_{nm01} \) and \( V_{nm02} \) are measured in the \( x_1 \)- and \( x_2 \)-directions, respectively, above a horizontal orthorhombic layer. They can be expressed in Tsvankin’s (1997) notation as follows:

\[
V_{nm01} = V_{nm02} = V_{p0} \sqrt{1 + 2\delta^{(2)}},
V_{nm02} = V_{nm03} = V_{p0} \sqrt{1 + 2\delta^{(1)}},
\]

(10)

(11)

where \( \delta^{(1)} \) and \( \delta^{(2)} \) are the anisotropy coefficients in the \( [x_2, x_3] \)- and \( [x_1, x_3] \)- planes, respectively. The velocity \( V_{nm03} \) is defined by Fowler and Lapilli (2012) in a similar fashion:

\[
V_{nm03} = V_{nm03} = V_{p1} \sqrt{1 + 2\delta^{(3)}},
\]

(12)

where \( \delta^{(3)} \) corresponds to the \( [x_1, x_2] \)-plane (Tsvankin, 1997).

Introducing the parameters \( V_{pa1} \) makes the mixed-domain operator symmetric with respect to the coordinate indices. An important component of the numerical modeling is boundary conditions. For pseudospectral simulators, absorbing boundary conditions can be implemented by adding an exponentially decaying term to the wavefield, which leads to the following time-stepping formula:

\[
u(x, t + \Delta t) + e^{-2\alpha(x)} u(x, t - \Delta t) = e^{-\alpha(x)} \left\{ 2u(x, t) - (\Delta t)^2 \mathcal{F}^{-1} \left[ \Phi \mathcal{F} [u(x, t)] \right] \right\} ,
\]

(13)

where \( \alpha(x) \) is the damping profile with nonzero values on the boundary.

INVERSION GRADIENTS

Medium parameterizations

Tsvankin (1997, 2012) shows that all kinematic signatures of P-wave in orthorhombic media are fully controlled by the vertical velocity \( V_{p0} \) and five anisotropy parameters \( (\delta^{(1)}, \delta^{(2)}, \delta^{(3)}) \) defined in the symmetry planes. The optimal choice of parameterization is crucial in obtaining accurate inversion results (Masmoudi and Alkhalifah, 2015; Kammath and Tsvankin, 2016). Here, we compute the gradients of the WI objective function using the parameters \( V_{p1}^2, V_{p2}^2, V_{p3}^2, V_{nm01}^2, V_{nm02}^2, \) and \( V_{nm03}^2 \). Our choice of parameterization is based primarily on the convenience in computing the gradients. Analysis of the radiation patterns can help in choosing optimal parameter sets for specific acquisition geometries and inversion scenarios.

Misfit functionals

Waveform inversion is performed by minimizing a certain misfit functional (objective function). Two types of misfit functionals are considered here: the classical \( \ell_2 \)-norm data difference and the \( \ell_2 \)-norm squared envelope difference. The \( \ell_2 \)-norm data difference is defined as:

\[
J_{\text{dat}} = \frac{1}{2} \sum_{i \in \Gamma} \| d_i - d_{oi} \|^2 ,
\]

(14)

where the subscript \( i \) denotes the data coordinate, \( \Gamma \) is an index set for the data coordinates, and \( d_i \) and \( d_{oi} \) are the modeled and observed discrete-time data (respectively) for a given source-receiver pair that belongs to \( \mathbb{R}^{Nt} \). The envelope-based functional is given by (Wu et al., 2014):

\[
J_{\text{env}} = \frac{1}{2} \sum_{i \in \Gamma} \| e_i - e_{oi} \|^2 ,
\]

(15)

where \( e_i \) and \( e_{oi} \) are the envelopes of the modeled and observed discrete data, respectively. The envelope function of a continuous-time signal \( d(t) \) is defined as:

\[
e(t) = \sqrt{d^2(t) + H[d]^2(t)} ,
\]

(16)

where \( H[d](t) \) is the Hilbert transform of the signal. Misfit functionals produce different adjoint-source functions used for modeling the adjoint variables. For the \( \ell_2 \)-norm data difference, the adjoint source function is

\[
f^a = d - do .
\]

(17)
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The adjoint source function for the envelope-based misfit functional is

\[ f^a = 2 \{ \Delta e^2 \circ d - H \left( \Delta e^2 \circ Hd \right) \}, \]  

where \[ \Delta e^2 = e^2 - eo^2 \]  
is the squared envelope difference. The symbol \( \circ \) denotes the Hadamard (Schur) product (Davis, 1962) and \( H \) is the Hilbert-transform matrix.

**WI Gradients**

To compute the gradients of the objective function using the adjoint-state method (Tromp et al., 2005; Plessix, 2006), one augments the misfit functional as:

\[ \chi = J - \langle \lambda, F \rangle, \]  

where the symbol \( \langle \cdot, \cdot \rangle \) denotes the inner product in the \( L_2 \)-space (to which the state and adjoint variables belong), and \( F \) is the discretized state equation. The adjoint variable \( \lambda \) satisfies the discretized adjoint equation. The gradients of the original misfit functional are then derived by setting the derivatives of the augmented misfit functional (equation 20) to zero:

\[ \frac{\partial \chi}{\partial m} = \frac{\partial J}{\partial m} - \left( \lambda, \frac{\partial F}{\partial m} \right) = 0. \]  

Substituting the parameters \( V_{P1}^2, V_{P2}^2, V_{P3}^2, V_{nmo1}^2, V_{nmo2}^2, \) and \( V_{nmo3}^2 \) for \( m \) yields

\[ \frac{\partial J}{\partial (V_{P1}^2)} = \left( \lambda, \left( k_1^2 - \frac{V_{P1}^2}{V_r} \right) \frac{k_2^4 k_3^4}{k^2} \right) u. \]  

\[ \frac{\partial J}{\partial (V_{P2}^2)} = \left( \lambda, \left( k_2^2 - \frac{V_{P2}^2}{V_r} \right) \frac{k_1^4 k_3^4}{k^2} \right) u. \]  

\[ \frac{\partial J}{\partial (V_{P3}^2)} = \left( \lambda, \left( k_3^2 - \frac{V_{P3}^2}{V_r} \right) \frac{k_1^4 k_2^4}{k^2} \right) u. \]  

\[ \frac{\partial J}{\partial (V_{nmo1}^2)} = \left( \lambda, \left( \frac{V_{P1}^2}{V_r} \right) \frac{k_2^4 k_3^4}{k^2} \right) u. \]  

\[ \frac{\partial J}{\partial (V_{nmo2}^2)} = \left( \lambda, \left( \frac{V_{P3}^2}{V_r} \right) \frac{k_1^4 k_3^4}{k^2} \right) u. \]  

\[ \frac{\partial J}{\partial (V_{nmo3}^2)} = \left( \lambda, \left( \frac{V_{P1}^2}{V_r} \right) \frac{k_1^4 k_2^4}{k^2} \right) u. \]

**NUMERICAL EXAMPLE**

The algorithm is tested on a modified 3D SEG/EAGE overthrust model. The medium parameters are obtained by heuristically scaling the P-wave velocity to ensure that each parameter varies within a reasonable range (Figure 1). The initial models for all velocities linearly increase with depth and contain little structural information (Figure 2).
First, the data are generated for a single source and recorded by a receiver array. Figure 3a displays a random selection of 16 traces from the predicted (red) and observed (green) data sets. Because the initial model significantly deviates from the actual one, the predicted synthetic data are cycle-skipped compared to the observed data, which implies that the objective function is highly nonlinear. A direct use of this data difference as the residual will guide the optimization search toward a local minimum.

For comparison, we compute the envelopes and squared envelopes of the predicted and observed data (Figures 3b and 3c). Although the envelope functions seem to be cycle-skipped as well, they look much simpler than the original data. Intuitively, the inversion operating with the envelope functions should be better posed. Figure 4 shows the frequency content of the predicted and observed data, with the spectra averaged over the traces for all receivers. The lack of low frequencies in these spectra is one of the reasons for the wavenumber gaps in the inversion results. In contrast, the envelope functions shift the spectra toward lower frequencies (yellow and green lines in Figure 4), which indicates that the \( \ell_2 \)-norm envelope misfit functional could be used to either generate an accurate long-wavelength initial model for WI or to update the model during early iterations.

Next, we apply both misfit functionals to compute the gradients for the model in Figure 1. The data are generated for 25 shots (red dots) in Figure 2 at every grid point on the horizontal surface. Figures 5 and 6 display the gradients obtained using the data-difference and squared envelope misfit functionals. Both gradients have substantial values only in the shallow part of the model because the initial velocity fields are quite smooth and most of the modeled energy represents diving waves. The gradients computed with the data-difference functional contain higher-wavenumber information, which should generally be avoided during the early stages of WI. In contrast, the gradients produced by the squared envelope misfit functional are more smooth and have a lower-wavenumber content, which should help in updating long-wavelength macro models for later iterations of WI.

CONCLUSIONS

We studied the feasibility of 3D waveform inversion for acoustic orthorhombic media. Wavefield simulation is carried out with the mixed-domain extrapolator using generalized pseudospectral methods. The choice of wavefield extrapolator is based on its efficiency and the convenience of gradient computation. The algorithm was applied to generate the inversion gradients for an orthorhombic version of the SEG/EAGE overthrust model. The results show that the envelope based misfit functional, which does not require acquisition of low frequencies, can be employed for model updating at initial iterations of WI.

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