Anisotropy signature in P-wave extended images for VTI media

Vladimir Li and Ilya Tsvankin (CWP) and Tariq Alkhalifah (King Abdullah University of Science and Technology)

Summary

Extended images obtained from reverse-time migration (RTM) contain information about the accuracy of the velocity field and subsurface illumination at different incidence angles. Here, we evaluate the influence of errors in the anisotropy parameters on the residual moveout (RMO) in P-wave extended images obtained with RTM for VTI (transversely isotropic with a vertical symmetry axis) media. Assuming the actual spatial distribution of the zero-dip normal-moveout velocity, we analyze extended images computed with distorted fields of the parameters $\eta$ and $\delta$. Differential semblance optimization (DSO) and stack-power criteria are employed to study the sensitivity of focusing to the anisotropy parameters. The results show that the signature of $\eta$ is dip-dependent, whereas errors in $\delta$ cause defocusing in extended images only if that parameter varies laterally. We also obtain and analyze the gradients of the DSO objective function with respect to the anisotropy parameters. The results of this work provide the foundation for anisotropic wavefield tomography operating with extended images.
Introduction

The extended imaging condition retains information about the wavefield directionality and angle-dependent reflector illumination by preserving the spatial and/or temporal correlation lags in the output. For example, one can obtain space-lag (Rickett and Sava, 2002) or time-lag (Sava and Fomel, 2006) extended common-image gathers (CIG), which are computed at fixed horizontal coordinates.

Wavefield tomography based on minimizing the residual energy at nonzero lags in the extended domain has recently attracted considerable attention in the literature (Yang and Sava, 2011; Li et al., 2014). The defocusing of energy can be quantified using differential semblance optimization (DSO) (Symes and Carazzone, 1991) and/or a measure of stack power (Chavent and Jacewitz, 1995). The gradients of the corresponding objective function can be efficiently found by applying the adjoint-state method (ASM) (Plessix, 2006).

Velocity analysis in the extended image domain has significant potential for anisotropic velocity model building in structurally complex areas. P-wave kinematics in VTI media is controlled by the vertical velocity $V_{P0}$ and Thomsen parameters $\epsilon$ and $\delta$. An alternative parameter set includes the normal-moveout velocity for a horizontal interface ($V_{nmo} = V_{P0}\sqrt{1 + 2\delta}$), the anellipticity parameter $\eta = (\epsilon - \delta)/(1 + 2\delta)$, and $\delta$. Sava and Alkhalifah (2012) study the $\eta$-signature in extended image domain for TI media and conclude that $\eta$-errors cause consistent “V”-shape defocusing for horizontal reflectors regardless of the complexity of the $V_{nmo}$-field.

Here, we evaluate the defocusing in space-lag CIGs caused by errors in $\eta$ and $\delta$ for VTI models with curved interfaces and laterally varying $\delta$-fields. We also obtain the gradient of the DSO objective function with respect to the parameter $\eta$ and present the corresponding sensitivity kernels.

Methodology

Alkhalifah and Tsvankin (1995) demonstrate that P-wave reflection moveout and time-domain processing for a laterally homogeneous VTI medium above the target horizon are governed by $V_{nmo}$ and $\eta$. In the case of a horizontal VTI layer, $\eta$ controls the nonhyperbolic (long-offset) portion of the P-wave moveout (Tsvankin, 2012). For a dipping reflector beneath VTI media, the P-wave NMO velocity depends on both $V_{nmo}$ and $\eta$; if $\eta > 0$ (typical case), $V_{nmo}$ increases much faster with dip compared to elliptical ($\epsilon = \delta$) or purely isotropic models.

P-wave moveout and time-domain processing still depend on just $V_{nmo}$ and $\eta$ even when these parameters vary laterally above the target horizon, but $\delta$ changes only with depth (Alkhalifah et al., 2001). However, if $\delta$ is laterally variable, P-wave traveltimes become sensitive to all three relevant parameters - $V_{nmo}$, $\eta$, and $\delta$ (or $V_{P0}$, $\epsilon$, and $\delta$) (Alkhalifah and Tsvankin, 1995; Tsvankin, 2012).

Parameter estimation in VTI media is often accomplished by applying ray-based reflection tomography. Ray theory, however, may break down for complicated structures and should be replaced with wave-equation-based methods. Inexpensive and kinematically accurate reconstruction of P-wavefields in TI models can be achieved by solving a system of two second-order coupled equations where the shear-wave vertical velocity $V_{S0}$ is set to zero (Fowler et al., 2010; Duvence and Bakker, 2011). The 2D version of the formulation proposed by Fowler et al. (2010) can be written as:

$$
\frac{\partial^2 p}{\partial t^2} = V_{hor}^2 \frac{\partial^2 p}{\partial x^2} + V_{P0}^2 \frac{\partial^2 q}{\partial z^2}, \\
\frac{\partial^2 q}{\partial t^2} = V_{nmo}^2 \frac{\partial^2 p}{\partial x^2} + V_{P0}^2 \frac{\partial^2 q}{\partial z^2},
$$

where $V_{hor} = V_{nmo}\sqrt{1 + 2\eta} = V_{P0}\sqrt{1 + 2\epsilon}$ is the P-wave horizontal velocity. Both the $p$- and $q$-components contain a wavefield with accurate P-wave kinematics and a shear-wave artifact caused by eliminating $V_{S0}$. One way to remove the S-wave artifact, used here, is to place sources and receivers in a purely isotropic or elliptical ($\epsilon = \delta$, $\eta = 0$) medium (Duvence and Bakker, 2011).

The derivatives of the objective function with respect to the model parameters can be efficiently computed with the adjoint-state method. The adjoint wavefields needed to obtain the gradient of the DSO objective function $J_{DSO}$ are found by solving a system of equations adjoint to equations 1. The amplitude and spatial distribution of the adjoint sources are defined by the residual energy in the extended
images. The $\eta$-gradient is given by:

$$
\frac{\partial J_{DSO}}{\partial \eta} = \int \int 2 \left( \frac{\partial \lambda_s}{\partial x} \right) V_{nmo}^2 \frac{\partial u_s}{\partial x} \, ds \, dt + \int \int 2 \left( \frac{\partial \lambda_r}{\partial x} \right) V_{nmo}^2 \frac{\partial u_r}{\partial x} \, ds \, dt,
$$

where $u_s$ and $u_r$ are the source- and receiver-side forward wavefields, and $\lambda_s$ and $\lambda_r$ are the source- and receiver-side adjoint wavefields, respectively. One can notice that, for the chosen parameterization, the $\eta$-gradient involves the lateral derivatives and depends on the background value of $V_{nmo}$, which indicates the well-known trade-off between $V_{nmo}$ and $\eta$ in the horizontal direction.

**Signature of $\eta$ and $\delta$ in extended images**

Here, we analyze how the anisotropy parameters $\eta$ and $\delta$ influence the residual moveout in RTM extended images for a modified segment of the BP 2007 TTI model with an anticline structure (Figure 1). The model, which includes a tilted symmetry axis, is simplified as follows:

- The symmetry-axis tilt is removed to make the model VTI.
- The original $V_{nmo}$-field is smoothed, and only the two strongest reflectors are retained to avoid reflections from multiple interfaces.
- The parameter $\eta$ is taken to be constant ($\eta = 0.15$) throughout the model.

The spatially varying $\delta$-field in the original BP model is left unchanged, and density is assumed to be constant. Sources and receivers are located at the surface, and the near-surface layer is taken to be isotropic to suppress the shear-wave artifact. We obtain RTM space-lag CIGs for $\eta$-values ranging from 0 to 0.3 with a 0.05 increment. The signature of $\eta$ in space-lag CIGs computed with the actual $\delta$-field for the dipping interface segments deviates from the “V”-shape and resembles the residual caused by an inaccurate velocity model for isotropic media (Figure 2d, f). This is explained by the fact that for dipping reflectors $\eta$ changes the NMO velocity and, therefore, conventional-spread moveout. Repeating the test with the erroneous $\delta = 0$ shows that for subhorizontal reflector segments the signature of $\eta$ maintains the “V”-shape even if $\delta$ is incorrect. As expected, the residual moveout (RMO) due to the $\eta$-errors in space-lag CIGs for dipping reflector segments does not have the “V”-shape.

Figure 3 demonstrates that if $\delta$ is distorted, the extrema of the DSO and stack-power objective functions
Figure 2 Space-lag CIGs computed with (a, d) $\eta = 0$, (b, e) $\eta = 0.15$ (actual value), and (c, f) $\eta = 0.3$ at: (a, b, c) $x = 2.1$ km and (d, e, f) $x = 3.5$ km.

Figure 3 Influence of $\eta$ on the DSO (solid) and stack-power (dashed) objective functions calculated from space-lag extended images. The images are obtained with (a) the actual $\delta$-field and (b) $\delta = 0$.

are shifted toward lower $\eta$-values (close to 0.1). Indeed, the laterally varying $\delta$ influences the focusing in extended images, and, therefore, the shape of the DSO and stack-power objective functions computed as a function of $\eta$. However, because in this model the lateral variation in $\delta$ is relatively mild, that parameter does not change the shape of RMO caused by errors in $\eta$ for both subhorizontal and dipping interface segments.

Gradient of DSO objective function

Here, we discuss the $\eta$-gradient for a model that includes a horizontal interface beneath a homogeneous VTI layer. First, we compute the gradient for the trial $\eta = 0$ using a single source-receiver pair and a single image point (Figure 4). These gradients are often referred to as sensitivity kernels, which describe the spatial distribution of the parameter update for given acquisition geometry. Figure 5 shows the $\eta$-gradients computed using sources and receivers uniformly distributed at the surface. The gradient changes sign depending on the sign of the error in $\eta$. One can notice that even for the actual $\eta$-field, the gradient does not go to zero (Figure 5b) because imaging with the actual velocity model still produces residual energy at nonzero lags due to the aperture limitations. We are currently implementing the gradients with respect to the VTI parameters in a DSO-based inversion algorithm.

Conclusions

We presented a study of the anisotropy signature in RTM extended images for VTI models with curved interfaces and laterally varying $V_{nmo}$- and $\delta$-fields. The residual moveout due to errors in $\eta$ maintains
Gradients for the homogeneous VTI model from Figure 4 computed with different values of $\eta$: (a) $\eta = 0$, (b) $\eta = 0.15$ (actual value), and (c) $\eta = 0.3$.

Figure 5

Acknowledgments
This work was supported by the sponsors of the Center for Wave Phenomena (CWP) and competitive research funding from the King Abdullah University of Science and Technology (KAUST), Saudi Arabia.

References