Gradients for VTI acoustic wavefield tomography

Vladimir Li\textsuperscript{1}, Hui Wang\textsuperscript{1}, Ilya Tsvankin\textsuperscript{1}, Esteban Díaz\textsuperscript{1} & Tariq Alkhalifah\textsuperscript{2}

\textsuperscript{1}Center for Wave Phenomena, Colorado School of Mines
\textsuperscript{2}King Abdullah University of Science and Technology
Anisotropic wavefield extrapolation

Differential

\[
\frac{\partial^2 p}{\partial t^2} = V_{\text{hor}}^2 \frac{\partial^2 p}{\partial x^2} + V_{P0}^2 \frac{\partial^2 q}{\partial z^2}
\]

\[
\frac{\partial^2 q}{\partial t^2} = V_{\text{nmo}}^2 \frac{\partial^2 p}{\partial x^2} + V_{P0}^2 \frac{\partial^2 q}{\partial z^2}
\]

\[
V_{\text{hor}} = V_{\text{nmo}} \sqrt{1 + 2\eta} = V_{P0} \sqrt{1 + 2\varepsilon}
\]

(Fletcher et al., 2009)

Integral

\[
U(x, t \pm \Delta t) = \int \hat{U}(k, t) e^{\pm i \phi(x, k, \Delta t)} d\mathbf{k}
\]

(Fowler and Lapilli, 2012)

(Fomel et al., 2013)
VTI medium with $\eta > 0$

Differential

Integral
Data-domain objective function

\[ J = \frac{1}{2} \| d^{\text{cal}} - d^{\text{obs}} \|^2 \]
Data-domain gradient

\[ \frac{\partial J}{\partial \varepsilon} = \mathcal{F}(u, a, V_{\text{hor}}, \varepsilon, k_z^2) \]

\[ \frac{\partial J}{\partial \eta} = \mathcal{F}(u, a, V_{\text{hor}}, \eta, k_z^2, k_x^2) \]

\[ \frac{\partial J}{\partial V_{\text{hor}}} = \mathcal{F}(u, a, V_{\text{hor}}, \varepsilon, \eta, k_z^2, k_x^2) \]

\( u \) – forward wavefield

\( a \) – adjoint wavefield
Model 1
$\varepsilon$-gradients with differential operator

$\varepsilon_{\text{peak}} = 0$

$\varepsilon_{\text{peak}} = 0.3$
\( \varepsilon \)-gradients with differential operator

\[ \varepsilon_{\text{peak}} = 0 \]

\[ \varepsilon_{\text{peak}} = 0.3 \]
\( \epsilon \)-gradients with integral operator

\[ \epsilon_{\text{peak}} = 0 \]

\[ \epsilon_{\text{peak}} = 0.3 \]
Image-domain criteria

<table>
<thead>
<tr>
<th>Surface-offset CIGs</th>
<th>Angle-domain CIGs</th>
<th>Extended CIGs</th>
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<tbody>
<tr>
<td>$h$</td>
<td>$\theta$</td>
<td>$\lambda_x$</td>
</tr>
<tr>
<td>flatness</td>
<td>flatness</td>
<td>focusing at zero lag</td>
</tr>
<tr>
<td>$z$</td>
<td>$z$</td>
<td>$z$</td>
</tr>
</tbody>
</table>
Extended-domain objective function

\[ J = \frac{1}{2} \| P(\lambda) I(x, \lambda) \|^2 \]
Extended-domain objective function

\[ J = \frac{1}{2} \| P(\lambda) I(x, \lambda) \|^2 \]

model update

Inaccurate

Accurate
Extended-domain objective function

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Extended-domain objective function

\[ J = \frac{1}{2} \| P(\lambda) I(x, \lambda) \|^2 \]

model update

Inaccurate

Accurate
Penalty operators

\[ P(\lambda) = |\lambda x | \]

\[ P(\lambda) = \delta(\lambda x) \]

\[ P(\lambda) = \exp(-\alpha \lambda^2 x) \]
Penalty operators

\[ P(\lambda) = |\lambda x| \]
Penalty operators

\[ P(\lambda) = |\lambda_x| \]

\[ P(\lambda) = \delta(\lambda_x) \]
Penalty operators

\[ P(\lambda) = |\lambda_x| \]

\[ P(\lambda) = \delta(\lambda_x) \]

\[ P(\lambda) = \exp(-\alpha \lambda_x^2) \]
Images with $P(\lambda)$ applied
Images with $P(\lambda)$ applied

$$P(\lambda) = |\lambda_x|$$
Images with $P(\lambda)$ applied

$P(\lambda) = |\lambda_x|$

$P(\lambda) = \delta(\lambda_x)$
Images with $P(\lambda)$ applied

\[ P(\lambda) = |\lambda_x| \]
\[ P(\lambda) = \delta(\lambda_x) \]
\[ P(\lambda) = \exp(-\alpha \lambda_x^2) \]
Image-domain gradient

\[
\frac{\partial J}{\partial \delta} = \mathcal{F}(u_i, a_i, V_{nmo}, \delta, k_z^2)
\]

\[
\frac{\partial J}{\partial \eta} = \mathcal{F}(u_i, a_i, V_{nmo}, k_z^2, k_x^2, k_x^4)
\]

\[
\frac{\partial J}{\partial V_{nmo}} = \mathcal{F}(u_i, a_i, V_{nmo}, \eta, \delta, k_z^2, k_x^2, k_x^4)
\]

\[i = s(\text{source}), \ r(\text{receiver})\]
Model 2 ($V_{nmo}$)

\[ \eta = \delta = 0.15 \]
Extended images

$\eta = 0$

$\eta = 0.15$

$\eta = 0.3$
Extended images

\[ \eta = 0 \]

\[ \eta = 0.15 \]

\[ \eta = 0.3 \]
Extended images

\[ \eta = 0 \]

\[ \eta = 0.15 \]

\[ \eta = 0.3 \]
DSO gradients for $\eta$

$\eta = 0$

$\eta = 0.3$
DSO gradients for $\eta$

$\eta = 0$

$\eta = 0.3$
Partial image-power gradients for $\eta$

$\eta = 0$

$\eta = 0.3$
Partial image-power gradients for $\eta$

$\eta = 0$

$\eta = 0.3$
Shot gather at $x = 4$ km
Extended images

$V_{\text{nmo}} = 1.8 \text{ km/s}$

$V_{\text{nmo}} = 2 \text{ km/s}$

$V_{\text{nmo}} = 2.2 \text{ km/s}$
Extended images with DSO penalty

\[ \lambda_x \quad (\text{km}) \]

\[ \begin{array}{c}
\text{z (km)} \\
0 & 1 & 2 & 3 & 4
\end{array} \]

\[ V_{nmo} = 1.8 \text{ km/s} \]

\[ \lambda_x \quad (\text{km}) \]

\[ \begin{array}{c}
\text{z (km)} \\
0 & 1 & 2 & 3 & 4
\end{array} \]

\[ V_{nmo} = 2 \text{ km/s} \]

\[ \lambda_x \quad (\text{km}) \]

\[ \begin{array}{c}
\text{z (km)} \\
0 & 1 & 2 & 3 & 4
\end{array} \]

\[ V_{nmo} = 2.2 \text{ km/s} \]
Extended images with partial image-power penalty

\[ V_{nmo} = 1.8 \text{ km/s} \]

\[ V_{nmo} = 2 \text{ km/s} \]

\[ V_{nmo} = 2.2 \text{ km/s} \]
Objective functions
Partial image-power gradients for $V_{nmo}$

$V_{nmo} = 1.8 \text{ km/s}$

$V_{nmo} = 2.2 \text{ km/s}$
Partial image-power gradients for $V_{nmo}$

\[ V_{nmo} = 1.8 \text{ km/s} \]

\[ V_{nmo} = 2.2 \text{ km/s} \]
Summary

- integral extrapolation operator:
  - cleaner gradients (no S-wave artifact)
  - stable for $\eta < 0$
  - scalar wavefields

- partial image-power more robust than DSO
Future work

- inversion for three VTI parameters
- image-guided regularization
- extension to TTI media
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