Preconditioned Least-Squares RTM with Matching Filters

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Goal: making LSRTM cheaper
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Filtering – cost = 3 migrations

LSRTM – cost = 30 migrations
Approximate the inverse Hessian with 3-D non-stationary matching filters
Previous work

• Guitton, 2004, “Amplitude and kinematic corrections of migrated images for nonunitary imaging operators”, 69(4), 1017-1024
  – Operator: 2-D 1-way, DSR operator, written by Paul Sava (buggy 😊)
  – Filters: Non-stationary 2-D
  – Final product: 2-D filtered images

• Today:
  – Operator: RTM
  – Filters: Non-stationary 3-D
  – Final products: 3-D filtered images and/or preconditioned LSRTM images
Filter estimation workflow

1. Compute a 1\textsuperscript{st} RTM image from the data

\[ m_1 = L'd \]

2. Compute a 2\textsuperscript{nd} RTM image from the demigrated data

\[ m_2 = L'Lm_1 \]

3. Estimate a bank of NS-filters \( \mathbf{a} \) to minimize

\[ f(\mathbf{a}) = \| M_2 \mathbf{a} - m_1 \|_2^2 \]
Relationship of filters $a$ with Hessian $H=L'L$

• Estimate a bank of NS-filters $a$ to minimize

$$f(a) = \| M_2a - m_1 \|^2_2 \Rightarrow \tilde{a} = (M'_2M_2)^{-1} M'_2m_1$$

• Filters: Least-squares estimate of $H$
  – Conditioning issues ($\text{cond}(HH) > \text{cond}(H)$)
  – Low-rank approximation
  – Depend on $m_1$
Two ways to use the filters

1. Filtering of RTM image only \( m_{RTM} \ast \tilde{a} \)
2. Preconditioning of LSRTM

**Standard CG**

\[
\begin{align*}
r & \leftarrow Lm_0 - d \\
\text{do iter} & \\
\Delta m & \leftarrow L' r \\
\Delta r & \leftarrow L \Delta m \\
\text{update-cg}(\Delta m, \Delta r, m, r) & \\
\text{end do}
\end{align*}
\]
Benefits of filters

• Are simple and cheap to estimate
• Cost of at least 2 migrations, at most 3 (depend on m1)
• Work with any imaging operator and imaging condition
  – Will show you examples with LSRTM and energy norm (D. Rocha)
  – Will show you examples with VTI pseudo-acoustic operator (V. Li)
Synthetic example – small SEAM 3D

**Velocity model**

```
5 km/s
4.5
1.5
3.4
2.8
```

**“Reflectivity” model**

```
0.6 km/s
5
3.4
2.8
```

x (km)  y (km)  z (km)
Filter estimation workflow

1. Compute a 1st RTM image from the data $m_1 = L'd$
Filter estimation workflow

2. Compute a 2\textsuperscript{nd} RTM image \( m_2 = L'Lm_1 \)
Filter estimation workflow

3. Estimate a bank of NS-filters $a : f(a) = \| M_2 a - m_1 \|_2^2$

$\hat{m}_1 = M_2 a$
Filter estimation workflow

1. Compute a 1\textsuperscript{st} RTM image from the data \( \mathbf{m}_1 = \mathbf{L}' \mathbf{d} \)
Two ways to use the filters

1. Filtering of RTM image only \( m_{RTM} \ast \hat{a} \)
2. Preconditioning of LSRTM

\[
\begin{align*}
\text{Standard CG} & \\
\text{do} & \text{ iter} \\
\Delta m & \leftarrow L'rL'r \\
\Delta r & \leftarrow L\Delta m \\
\text{update-cg}(\Delta m, \Delta r, m, r) & \\
\text{end do}
\end{align*}
\]
Two ways to use the filters: To Filter

**m_{RTM} \ast \hat{a}**

“Reflectivity” model
Two ways to use the filters: To Filter

"Reflectivity" model
Two ways to use the filters: To Filter

Filtered RTM image

ce number
Two ways to use the filters: To Filter
Two ways to use the filters: To Precondition

2. Preconditioning of LSRTM

\[
\begin{align*}
\mathbf{r} &\leftarrow \mathbf{L}m_0 - \mathbf{d} \\
\text{do iter} & \\
\Delta \mathbf{m} &\leftarrow \mathbf{L'} \mathbf{r} \\
\Delta \mathbf{r} &\leftarrow \mathbf{L} \Delta \mathbf{m} \\
\text{update-cg}(\Delta \mathbf{m}, \Delta \mathbf{r}, \mathbf{m}, \mathbf{r}) & \\
\text{end do}
\end{align*}
\]
Two ways to use the filters: To Precondition

Preconditioned LSRTM 5 iterations

"Reflectivity" model
Two ways to use the filters: To Precondition

LSRTM 15 iterations

Two ways to use the filters: “Reflectivity” model

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Two ways to use the filters: To Filter

$m_{RTM} \ast \hat{a}$

“Reflectivity” model
Comparisons LSRTM/Precond-LSRTM

LSRTM 15 iterations

Preconditioned-LSRTM 5 iterations

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Observations

• Filtering gets most of job done
• Preconditioning with matching filters works
  – Effective amplitude compensation
• Outstanding issues with preconditioned LSRTM
  – Convergence stuck in 2 iterations
    • Filters depend on $m_1$
    • Filters don’t capture the whole effect of the inverse Hessian
• LSRTM works well in low illumination areas
Recommendations

• For a 75% solution, filtering works well
• For a 90% solution, preconditioning accelerates convergence – Iterate twice with the filters (no benefits beyond)
• For a 100% solution, switching to standard LSRTM is advised
Field data example 1 (results from Daniel Rocha)

Cardamom OBN survey
3D LSRTM with the Energy Norm Application on Cardamom dataset

Prec. LSRTM

LSRTM

normalized OF

iteration

0 1 2 3 4 5 6 7 8 9

0.7 0.8 0.9 1
3D Field data example – D. Rocha

RTM
3D Field data example – D. Rocha

LSRTM
Precond. LSRTM/LSRTM
Field data example 2

North Sea - Volve Field - OBC survey
3D Field data example – North Sea

RTM
Precond. LSRTM 2 iterations

3D Field data example – North Sea
3D Field data example – North Sea

Precond. LSRTM (2 iter)+LSRTM (5 iter)
3D Field data example – North Sea

LSRTM 15 iterations
3D Field data example – North Sea

Issues affecting convergence:
- Inaccuracies in the velocity model
- Data processing (PP/PS separation)
- Noise in the data ➔ L1 norm

LSRTMs

Prec. LSRTM  LSRTM
Observed data

Modeled data with LSRTM
Field data example 3

Colorado- Wattenberg base survey
Filtered RTM

Y (kft)
216 2218 2220 2222
Precond. LSRTM (2) + LSRTM (5)

\[ Y \text{ (kft)} \]

14 2216 2218 2220 2222
RTM
Precond. LSRTM (2) + LSRTM (5)
Observed data
Modeled data from filtered image
Modeled data from Precond. LSRTM + LSRTM
Observed data
Residual from filtered image
Residual from Precond. LSRTM + LSRTM
Convergence

- Precond. LSRTM+LSRTM
- LSRTM
Conclusions (1)

• Matching filters (MFs) yield a good approximation of inverse Hessian
• MFs can be used for preconditioning
  – Need to switch to regular inversion after two iterations
• Cost saving can be a factor 2 to 3 compare to LSRTM, more if only the filtering option is used
• MFs can be used with ANY imaging operator, with/without gathers
• MFs could be very be useful for Reflection Waveform Inversion
Conclusions (2)

• A possible improvement:
  – Try different starting images $m_1$: Would spikes work? Would horizontal planes work?

• Unanswered questions:
  – What are the effects of velocity errors and noise
  – How can we improve the filters so that the precond. LSRTM is efficient beyond two iterations ($m_1$)?
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• Morgan Brown, NEOS, for building the Wattenberg model

* The views expressed in this presentation are the views of the authors and do not necessarily reflect the views of Statoil ASA and the Volve field license partners.
Volve 3D P-wave velocity model
Wattenberg velocity model
X = 290 kft
Filtered RTM

Y (kft) 216 2218 2220 2222

X = 290 kft
\[ X = 290 \text{ kft} \]
$X = 290 \text{ kft}$
Filtered RTM

\[ Y \text{ (kft)} \]

\begin{align*}
18 & \quad 2220 & \quad 2222 \\
\end{align*}

\[ X = 290 \text{ kft} \]
X = 290 kft
$X = 290 \text{ kft}$
Filtered RTM

$X = 290\text{ kft}$

$Y (\text{kft})$

$\begin{array}{c}
6 \\
2218 \\
2220 \\
2222 \\
\end{array}$
LSRTM

Y (kft) 216 2218 2220 2222

X = 290 kft