Waveform inversion for microseismic velocity analysis and event location in VTI media

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Previous work

- 2D inversion for VTI media
- source and medium parameters
- adjoint-state method
- based on finite-differences
Elastic wave equation

\[ \rho \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial}{\partial x_j} \left( c_{ijkl} \frac{\partial u_k}{\partial x_l} \right) = -M_{ij} \frac{\partial [\delta(x - x^s)]}{\partial x_j} S(t) \]

- \text{density}
- \text{stiffness tensor}
- \text{moment tensor}
- \text{source function}
Source parameters

\[ \mathbf{m} = \{x_1^s, x_3^s, t_0, M_{11}, M_{13}, M_{33}, M_{12}, M_{23}\} \]

SH-waves
VTI parameterization

\[ V_{\text{hor}} = V_{P0} \sqrt{1 + 2\varepsilon} \]

\[ \eta = \frac{\varepsilon - \delta}{1 + 2\delta} \]

\[ \varepsilon \]

\[ V_{S0} \]  
(Alkhalifah and Plessix, 2014; Kamath et al., 2016)
Methodology

\[ \mathcal{F}(m) = \frac{1}{2} \sum_{s, r} \| u_{\text{pre}}(m) - u_{\text{obs}} \|^2 \]

**Gradient:** Adjoint-state method

L-BFGS method (Byrd et al., 1995)
Inversion for event location and VTI parameters
Data rotation

Source
Data rotation

\[ x_2 \]

\[ x_1 \]

\[ u_{\parallel} \]

\[ u_{\perp} \]

\[ u_1 \]

\[ u_2 \]

\[ \theta \]

Source
Radial and transverse components

\[
\begin{bmatrix}
    u_{\parallel} \\
    u_{\perp} \\
    u_3
\end{bmatrix} =
\begin{bmatrix}
    \cos \theta & \sin \theta & 0 \\
    -\sin \theta & \cos \theta & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    u_1 \\
    u_2 \\
    u_3
\end{bmatrix}
\]
Vertical plane
Methodology: sequential inversion

- inversion stages
- each stage:
  - iterative event location
  - iterative velocity analysis
Synthetic example

- error in initial locations: ± 20 m
- initial model obtained by smoothing
- five inversion stages
- dislocation-type sources
Actual and initial parameters

$V_{\text{hor}}$  
$V_{S0}$  
$\eta$  
$\varepsilon$

(m/s)  
(m/s)  

Inverted (after stage 1), actual, and initial parameters

\[ V_{\text{hor}} \]

\[ V_{S0} \]

\[ \eta \]

\[ \varepsilon \]
Inverted (after stage 5), actual, and initial parameters

$V_{\text{hor}}$, $V_{S0}$, $\eta$, $\varepsilon$

(m/s) (m/s)
Inverted $V_{\text{hor}}$
Source coordinates

**Horizontal**

![Horizontal graph with stage number on the x-axis and $x_1$ (m) on the y-axis.](image)

**Vertical**

![Vertical graph with stage number on the x-axis and $x_3$ (m) on the y-axis.](image)
Influence of noise
Data with Gaussian noise (S/N ≈ 11)
Actual and initial parameters

\[ V_{\text{hor}} \]

\[ V_{S0} \]

\[ \eta \]

\[ \varepsilon \]
Inverted, actual, and initial parameters (at 600 m)

\[ V_{\text{hor}} \]

\[ V_{S0} \]

\[ \eta \]

\[ \varepsilon \]
Extension to 3D
3D pseudospectral modeling

- less numerical dispersion
- arbitrary anisotropy (Sun et al., 2016)
Initial model

$V_{\text{hor}}$ (m/s)
Actual and initial parameters

$V_{\text{hor}}$  
$V_{S0}$  
$\eta$  
$\varepsilon$

$x_3$ (m)  
$x_3$ (m)  
$x_3$ (m)  
$x_3$ (m)

(m/s)  
(m/s)  
(m/s)  
(m/s)
Inverted $V_{\text{hor}}$ and $V_{S0}$
Inverted $\eta$ and $\varepsilon$
Inverted, actual, and initial parameters

$V_{\text{hor}}$  

$V_{S0}$  

$\eta$  

$\varepsilon$  

$x_3$ (m)  

$x_3$ (m)  

$x_3$ (m)  

$x_3$ (m)  

(m/s)  

(m/s)  

(m/s)  

(m/s)
Initial model
Inverted $V_{\text{hor}}$ (30 events)
Inverted $V_{\text{hor}}$ (10 events)
Inverted $V_{\text{hor}}$ (30 events)
Inverted, actual, and initial parameters

\( V_{\text{hor}} \)

\( V_{S0} \)

\( \eta \)

\( \varepsilon \)
Displacement for initial model

Well 1

Well 2
Displacement for inverted model

Well 1

Well 2
Observed displacement

Well 1

Well 2
Summary

- WI for event location and VTI parameters
- sequential inversion, multiple stages
- extension to 3D VTI media
- velocity inversion with three wells
Ongoing and future work

- 3D event location
- inversion for orthorhombic media
- application to field data
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