Stacking-velocity tomography in tilted orthorhombic media

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Methodology

• modes: PP, $S_1 S_1, S_2 S_2$ (PP + PS = SS)

• input: NMO ellipses, zero-offset times, reflection time slopes

• model: homogeneous layers with plane/curved boundaries

• inversion for: interval medium parameters, interfaces

Developed for TTI and ORTH with horizontal symmetry plane (Grechka et al., 2005)
NMO ellipse

\[ V_{nmo}^{-2}(\alpha) = W_{11} \cos^2 \alpha + 2W_{12} \sin \alpha \cos \alpha + W_{22} \sin^2 \alpha \]

(Grechka and Tsvankin, 1998)

• single layer: \( W_{ij} = f(p_1, p_2, q) \)

\((p_1, p_2, q)\): slowness vector of zero-offset ray

\( q(p_1, p_2) \) from Christoffel equation

• layered model: Dix-type averaging (tracing zero-offset ray)
Stacking-velocity tomography (SVT)

Input data for each event:

- $\tau_0, p_1, p_2$
- $W_{11}, W_{12}, W_{22}$
Workflow

trial model (interval parameters) → $p_1, p_2, \tau_0$

zero-offset ray → reflector

effective NMO ellipses $W_{ij}$ → misfit in $W_{ij}$ and depths
Objective function

\[ \mathcal{F}(\mathbf{m}) = \| 1 - \frac{W^{\text{cal}}(\mathbf{m})}{W^{\text{obs}}} \| + \alpha \| 1 - \frac{z_i(\mathbf{m})}{z_j(\mathbf{m})} \| \]
Inversion

- Gauss-Newton optimization
- Fréchet derivatives
- feasibility depends on model assumptions
Orthorhombic medium

parameters: $V_{P0}$, $V_{S0}$, $\epsilon^{(1,2)}$, $\delta^{(1,2,3)}$, $\gamma^{(1,2)}$
Tilted orthorhombic (TOR) medium

parameters: $V_{P0}$, $V_{S0}$, $\epsilon^{(1,2)}$, $\delta^{(1,2,3)}$, $\gamma^{(1,2)}$, $\beta_1$, $\beta_2$, $\beta_3$
Layered TOR medium

- interval parameters:
  \( V_{P0}, V_{S0}, \epsilon^{(1,2)}, \delta^{(1,2,3)}, \gamma^{(1,2)}, \beta_1, \beta_2, \beta_3 \)

- interfaces: polynomial coefficients
Data vector

\[ d(Q, Y) = \{ \tau_0, Q(Y), p_1, Q(Y), p_2, Q(Y), W_{11}, Q(Y), W_{12}, Q(Y), W_{22}, Q(Y) \} \]

\[ Q = P, S_1, S_2 \]

\[ Y: \text{CMP location} \]
Plane dipping reflector
Data vector for single TOR layer

• $p_1/p_2 = \tan \psi$ for all three modes

• only 4 out of 6 in $(p_{1,Q}, p_{2,Q})$ are independent

• only one $\tau_{0,Q}$ is independent (Grechka et al., 2005)
Feasibility for single TOR layer

- 14 measurements:
  \( \tau_0, P, p_1, p_1, s_1, p_1, s_2, p_2, P \)
  \( W_{11, Q}, W_{12, Q}, W_{22, Q} \)

- 15 parameters:
  \( V_{P_0}, V_{S_0}, \epsilon^{(1,2)}, \delta^{(1,2,3)}, \gamma^{(1,2)}, \beta_1, \beta_2, \beta_3, D, \phi, \psi \)
Assumption:

Reflector coincides with symmetry plane $[x_1, x_2]$

- $\beta_2 = \psi$, $\beta_3 = \phi$
- $\delta^{(3)}$ has no influence on $W_{ij}$
Additional assumption:

Dip plane coincides with symmetry plane \([x_1, x_3]\)

- \(\beta_1 = \psi\)
- \(W_{ij,Q}\) has two independent elements
Analytic solution

- strike line:

\[ V_{nmo,P}^{(1)} = V_{P0} \sqrt{1 + 2\delta^{(1)}} \]

\[ V_{nmo,S1}^{(1)} = V_{S1} \sqrt{1 + 2\sigma^{(1)}} \]

\[ V_{nmo,S2}^{(1)} = V_{S2} \sqrt{1 + 2\gamma^{(1)}} = V_{S1} \sqrt{1 + 2\gamma^{(2)}} \]

\((V_{S2} = V_{S0})\)
Analytic solution

• dip line:

\[ V_{nmo,P}^{(2)} = V_{P0} \sqrt{1 + 2\delta^{(2)}} / \cos \phi \]

\[ V_{nmo,S1}^{(2)} = V_{S1} \sqrt{1 + 2\gamma^{(2)}} / \cos \phi = V_{nmo,S2}^{(1)} / \cos \phi \]

\[ V_{nmo,S2}^{(2)} = V_{S2} \sqrt{1 + 2\sigma^{(2)}} / \cos \phi \]

• \( p_{1,Q} = \sin \phi / V_{Q0} \Rightarrow V_{P0}, V_{S1}, V_{S2} \)

• \( D = V_{P0} \tau_{0,P} \)
P-waves only

- $V_{nmo,P}^{(1)} = V_{P0} \sqrt{1 + 2\delta^{(1)}}$
- $V_{nmo,P}^{(2)} = V_{P0} \sqrt{1 + 2\delta^{(2)}} / \cos \phi$

- model vector: $V_{P0}, \delta^{(1,2)}, \phi, D$

- data vector: $\tau_0, p_1, V_{nmo,P}^{(1,2)}$

- need one constraint (e.g., vertical velocity)
P-wave inversion (known vertical velocity)
Arbitrary dip-plane orientation

\[ x_3 \]
\[ x_1 \]
\[ x_2 \]
Feasibility

• $\beta_2 = \psi$, $\beta_3 = \phi$

• 14 measurements: $\tau_0, P, p_1, P_1, s_1, p_1, s_2, p_2, P, W_{ij}, Q$

• 12 parameters: $V_{P0}, V_{S0}, \epsilon^{(1,2)}, \delta^{(1,2)}, \gamma^{(1,2)}, \beta_1, D, \phi, \psi$
Objective function
Inversion results

![Graph showing inverted and actual data](image-url)
Inversion with noise (1% for $\tau_0$, 2% for $p_{1,2}$, 2% for $V_{nmo}$)
Inversion with noise (1% for $\tau_0$, 5% for $p_{1,2}$, 3% for $V_{nmo}$)
Inversion with noise \((1\% \text{ for } \tau_0, 5\% \text{ for } p_{1,2}, 3\% \text{ for } V_{nmo})\) (vertical velocity known)
Summary

- extension of multicomponent SVT to tilted ORTH media

- single layer:
  - generally underdetermined
  - feasible if reflector coincides with symmetry plane
  - P-waves: need additional constraints
Ongoing work

- layered TOR medium
- horizontal + dipping reflectors
- curved interfaces
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