Retrieving the potential field response from cross-correlations

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ABSTRACT
We show that the two-point cross-correlation of potential-field recordings is equal to the Green's function between the two points. This holds under the condition that spatially and temporally uncorrelated noise sources exist throughout the volume. They should have a known amplitude spectrum and their correlated strengths should be equal to the real part of the dissipative medium property function. Natural fluctuations, such as thermal noise, may occur that satisfy the necessary conditions. When these fluctuations are random deviations from a state of thermal equilibrium the fluctuation-dissipation theorem determines these external sources. This allows for Green's function retrieval for all types of fields that satisfy a similar quasi-static field equation. Under equilibrium conditions, possible downhole reservoir applications include virtual source DC electric resistivity measurements, fluid flow measurements and local temperature estimations.

Key words: Green's function retrieval, potential fields, thermal fluctuations, down hole.

1 INTRODUCTION

The notion of interferometry to mean the extraction of transmission or reflection responses from passive recordings, and use of the newly constructed data for imaging the surroundings has been known for more than 20 years (Scherbaum, 1987; Buckingham *et al.*, 1992). During the past eight years many interferometric methods have been developed for random fields and for controlled-source data. Many of the underlying theories have in common that the medium is assumed to be lossless. The main reason for this underlying assumption is that the wave equation in lossless media is invariant for time-reversal. For an overview of the theory of seismic interferometry or Greens function retrieval and its applications to passive as well as controlled-source data, we refer to a reprint book of Wapenaar *et al.* (2008), which contains a large number of papers on this subject.

It has been shown (Snieder, 2006; Snieder, 2007; Weaver, 2008) that a volume distribution of uncorrelated noise sources, with source strengths proportional to the dissipation parameters of the medium, precisely compensates for the energy losses. This approach holds both for waves in dissipative media and for pure diffusion processes. Recently Wapenaar *et al.* (2006) and Snieder *et al.* (2007) showed that interferometry by cross-correlation, including its extensions for wave fields and diffusive fields in dissipative media, can be represented in a unified form. This naturally leads to the question of whether potential fields can also be retrieved by cross-correlation of noise measurements. Potential fields are the late-time limits of quasi-static fields. Low frequency induced polarization methods use quasi-static electric fields. In practice potential field values are obtained by integrating measured field values. This is done by sampling the field at a certain rate and averaging the results over a predetermined time window. This average value is then stored as the value corresponding to the potential field. Therefore all potential field measurements involve a frequency bandwidth within which the potential field is measured. That is why there can be measurable effects of temporal fluctuations in a potential field.

Physical mechanisms for the noise sources were not identified in the previous studies on field correlations in dissipative media. Here we find that when these noise sources are caused by thermal fluctuations, the cross-correlation functions of these thermal noise sources are completely described by the fluctuation-dissipation theorem (FDT) (Callen & Welton, 1951). We first describe the principle using the quasi-static electric-field approximation to Maxwell's equations. Electric fluctuations at the microscopic level are caused by thermal motion of electrically charged micro-particles, such as electrons and ions. At the macroscopic level these correspond to fluctuations in the electric field, which then act as the source (Landau & Lifshitz, 1960). We derive an identity for the real part and the imaginary part of the Green's function between two points from cross-correlations of field fluctuations at the same two points that are caused by sources distributed throughout the volume. We then identify thermal noise sources as possible sources satisfying all necessary requirements and show the relation with Brownian motion and the Johnson-Nyquist electric circuit noise. We then extend these findings to measurements both in a macroscopic piecewise-continuous open medium of infinite extent and a partially closed medium, such as when the surface of the earth is included. We generalize this analysis to hold for all quasi-static fields satisfying an equation similar to that of the quasi-static electric field, subsurface fluid flow being an example. Finally, we discuss some possible downhole applications for reservoir characterization and production management.

2 QUASI-STATIC ELECTRIC FIELD EQUATIONS AND INTERACTIONS

When the time variations in the magnetic field are negligible, the electric field is curl-free. This implies the electric field can be written as the gradient of a space-time dependent scalar electric potential function. This is known as the quasi-static electric field. The macroscopic space-time quasi-static electric field is determined by the scalar electric field potential $V(\boldsymbol{r},t)$, the total electric current density $\boldsymbol{J}(\boldsymbol{r},t)$, the total anisotropic electric-conductivity 3×3 tensor time-convolution operator, $\boldsymbol{\sigma}(\boldsymbol{r},t)$, which includes the time derivative of the permittivity, the external electric-charge injection or extraction rate $\dot{q}^e(\boldsymbol{r},t)$, and the external electric field strength, $\boldsymbol{E}^e(\boldsymbol{r},t)$. We define the time-Fourier transform of a space-time-dependent quantity as $\hat{f}(\boldsymbol{r},\omega) = \int \exp(\mathrm{i}\omega t) f(\boldsymbol{r},t)\mathrm{d}t$, where i is the imaginary unit and ω denotes angular frequency. The frequency-domain total conductivity is given by $\hat{\boldsymbol{\sigma}}(\boldsymbol{r},\omega)$. The effects of all possible time-relaxation mechanisms are incorporated in the frequency-dependent complex-valued conductivity tensor. The reciprocal of the conductivity tensor is the resistivity tensor $\hat{\boldsymbol{\zeta}}$, such that $\hat{\boldsymbol{\zeta}}\hat{\boldsymbol{\sigma}} = \boldsymbol{I}$, where $\boldsymbol{I}$ denotes the 3×3 identity matrix. Starting from Maxwell's equations the quasi-static electric field equations are written as

$$\nabla \cdot \hat{\boldsymbol{J}} = \hat{\dot{q}}^e, \tag{1}$$

$$\nabla \hat{V} + \hat{\boldsymbol{\zeta}} \cdot \hat{\boldsymbol{J}} = -\hat{\boldsymbol{E}}^e, \tag{2}$$

where the resistivity matrix, $\hat{\boldsymbol{\zeta}}$, is symmetric $\hat{\boldsymbol{\zeta}} = \hat{\boldsymbol{\zeta}}^T$, and $\hat{\boldsymbol{\zeta}} \cdot \hat{\boldsymbol{J}}$ stands for

$$(\hat{\boldsymbol{\zeta}} \cdot \hat{\boldsymbol{J}})_k = \sum_{m=1}^{3} \hat{\zeta}_{km} \hat{J}_m, \qquad \text{for } k = 1,2,3, \tag{3}$$

m and k denoting the three Cartesian vector components. These field equations can be combined into a single equation given by

$$\nabla \cdot (\hat{\boldsymbol{\sigma}} \cdot \nabla \hat{V}) = -\hat{I}, \tag{4}$$

where the source term is the effective electric current that is injected or extracted; it is given by

$$\hat{I}(\boldsymbol{r},\omega) = \hat{\dot{q}}^e(\boldsymbol{r},\omega) + \nabla \cdot (\hat{\boldsymbol{\sigma}}(\boldsymbol{r},\omega) \cdot \hat{\boldsymbol{E}}^e(\boldsymbol{r},\omega)), \tag{5}$$

where $\nabla \cdot (\hat{\boldsymbol{\sigma}} \cdot \hat{\boldsymbol{E}}^e)$ stands for

$$\nabla \cdot (\hat{\boldsymbol{\sigma}} \cdot \hat{\boldsymbol{E}}^e) = \sum_{k=1}^{3}\sum_{m=1}^{3} \frac{\partial}{\partial x_k} \hat{\sigma}_{km} \hat{E}^e_m. \tag{6}$$

Henceforth the dependency on frequency is omitted from function arguments. For Green's functions we use point sources $\hat{I}(\boldsymbol{r}) = \delta(\boldsymbol{r}-\boldsymbol{r}')$, and the electric potential becomes a Green's function, $\hat{V}(\boldsymbol{r}) = \hat{G}(\boldsymbol{r},\boldsymbol{r}')$, which satisfies

$$\nabla \cdot [\hat{\boldsymbol{\sigma}}(\boldsymbol{r}) \cdot \nabla \hat{G}(\boldsymbol{r},\boldsymbol{r}')] = -\delta(\boldsymbol{r}-\boldsymbol{r}'). \tag{7}$$

Two source types are given in equation (5) and we can define two distinct Green's functions

$$\hat{G}^{Vq}(\boldsymbol{r},\boldsymbol{r}') \stackrel{\text{def}}{=} \hat{G}(\boldsymbol{r},\boldsymbol{r}'), \tag{8}$$

$$\hat{\boldsymbol{G}}^{VE}(\boldsymbol{r},\boldsymbol{r}') = \hat{G}(\boldsymbol{r},\boldsymbol{r}')\nabla' \cdot \hat{\boldsymbol{\sigma}}(\boldsymbol{r}'), \tag{9}$$

where ∇' acts on $\boldsymbol{r}'$. For a distributed external electric field, occupying the domain $\mathbb{D}$, the electric potential is given by

$$\hat{V}^E(\boldsymbol{r}) = \int_{\boldsymbol{r}'\in\mathbb{D}} \hat{G}(\boldsymbol{r},\boldsymbol{r}')\nabla' \cdot [\hat{\boldsymbol{\sigma}}(\boldsymbol{r}') \cdot \hat{\boldsymbol{E}}^e(\boldsymbol{r}')]\mathrm{d}^3\boldsymbol{r}'. \tag{10}$$

2.1 Global field interactions

Two different states, labeled A and B, are considered. The states can differ in their source mechanisms, their medium parameters, and their spatial and temporal locations. Here we take $\hat{\boldsymbol{\sigma}}_A = \hat{\boldsymbol{\sigma}}_B = \hat{\boldsymbol{\sigma}}$, and we assume that $\hat{\boldsymbol{\sigma}}$ is symmetric. Equation (4) for state B can be pre-multiplied with the electric potential $\hat{V}_A$, resulting in

$$\hat{V}_A \nabla \cdot \hat{\boldsymbol{\sigma}} \cdot \nabla \hat{V}_B = -\hat{V}_A \hat{I}_B, \tag{11}$$

and then be integrated over an arbitrary bounded spatial domain $\mathbb{D}$, with outer boundary $\partial\mathbb{D}$, which has a continuous outward pointing unit normal vector $\boldsymbol{n}^T = \{n_1, n_2, n_3\}$, to give

$$\int_{\mathbb{D}} \hat{V}_A \hat{I}_B \mathrm{d}^3\boldsymbol{r} = \int_{\mathbb{D}} (\nabla \hat{V}_A) \cdot \hat{\boldsymbol{\sigma}} \cdot (\nabla \hat{V}_B) \mathrm{d}^3\boldsymbol{r} - \oint_{\partial\mathbb{D}} \hat{V}_A \boldsymbol{n} \cdot \hat{\boldsymbol{\sigma}} \cdot \nabla \hat{V}_B \mathrm{d}^2\boldsymbol{r}. \quad (12)$$

To arrive at equation (12), integration by parts and Gauss' divergence theorem has been used in the integral containing the divergence operator. The resulting boundary integral runs over the outer boundary, where continuity conditions or explicit conditions of the Dirichlet and/or Neumann types are assumed to apply. In the latter case the boundary integral vanishes. Similarly, equation (4) for state A can be multiplied with $\hat{V}_B$ and integrated over the domain $\mathbb{D}$, resulting in

$$\int_{\mathbb{D}} \hat{V}_B \hat{I}_A \mathrm{d}^3\boldsymbol{r} = \int_{\mathbb{D}} (\nabla \hat{V}_A) \cdot \hat{\boldsymbol{\sigma}} \cdot (\nabla \hat{V}_B) \mathrm{d}^3\boldsymbol{r} - \oint_{\partial\mathbb{D}} \hat{V}_B \boldsymbol{n} \cdot \hat{\boldsymbol{\sigma}} \cdot \nabla \hat{V}_A \mathrm{d}^2\boldsymbol{r}. \quad (13)$$

Notice that the volume integral in the right-hand side of equation (12) is equal to the volume integral in the right-hand side of equation (13).

We now use delta functions for the sources, $\hat{I}_{A,B} = \delta(\boldsymbol{r} - \boldsymbol{r}_{A,B})$, and the potentials become Green's functions as defined by $\hat{V}_{AB} = \hat{G}(\boldsymbol{r}, \boldsymbol{r}_{A,B})$. For these sources and fields equations (12) and (13) become

$$\hat{G}(\boldsymbol{r}_B, \boldsymbol{r}_A) = \int_{\mathbb{D}} (\nabla \hat{G}(\boldsymbol{r}, \boldsymbol{r}_A)) \cdot \hat{\boldsymbol{\sigma}} \cdot (\nabla \hat{G}(\boldsymbol{r}, \boldsymbol{r}_B)) \mathrm{d}^3\boldsymbol{r} - \oint_{\partial\mathbb{D}} \hat{G}(\boldsymbol{r}, \boldsymbol{r}_A) \boldsymbol{n} \cdot \hat{\boldsymbol{\sigma}} \cdot \nabla \hat{G}(\boldsymbol{r}, \boldsymbol{r}_B) \mathrm{d}^2\boldsymbol{r}, \quad (14)$$

$$\hat{G}(\boldsymbol{r}_A, \boldsymbol{r}_B) = \int_{\mathbb{D}} (\nabla \hat{G}(\boldsymbol{r}, \boldsymbol{r}_A)) \cdot \hat{\boldsymbol{\sigma}} \cdot (\nabla \hat{G}(\boldsymbol{r}, \boldsymbol{r}_B)) \mathrm{d}^3\boldsymbol{r} - \oint_{\partial\mathbb{D}} \hat{G}(\boldsymbol{r}, \boldsymbol{r}_B) \boldsymbol{n} \cdot \hat{\boldsymbol{\sigma}} \cdot \nabla \hat{G}(\boldsymbol{r}, \boldsymbol{r}_A) \mathrm{d}^2\boldsymbol{r}. \quad (15)$$

Here we have assumed that both points $\boldsymbol{r}_{A,B}$ are inside $\mathbb{D}$.

Subtracting equation (14) from (15) gives

$$\hat{G}(\boldsymbol{r}_A, \boldsymbol{r}_B) - \hat{G}(\boldsymbol{r}_B, \boldsymbol{r}_A) = \oint_{\partial\mathbb{D}} \Big(\hat{G}(\boldsymbol{r}, \boldsymbol{r}_A) \boldsymbol{n} \cdot \hat{\boldsymbol{\sigma}} \cdot \nabla \hat{G}(\boldsymbol{r}, \boldsymbol{r}_B) - \hat{G}(\boldsymbol{r}, \boldsymbol{r}_B) \boldsymbol{n} \cdot \hat{\boldsymbol{\sigma}} \cdot \nabla \hat{G}(\boldsymbol{r}, \boldsymbol{r}_A) \Big) \mathrm{d}^2\boldsymbol{r}. \quad (16)$$

For two fixed locations, $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$, the left-hand side of equation (16) is fixed, independent of the choice of $\mathbb{D}$, therefore the right-hand side is also fixed and independent of the choice of $\mathbb{D}$. The right-hand side of equation (16) is therefore independent of the size and shape of $\mathbb{D}$ as long as the points $\boldsymbol{r}_{A,B}$ are both inside the volume. When the volume is taken as infinite space the surface integral goes to zero. This is because the Green's function is proportional to the inverse of distance, while its gradient is proportional to the inverse of distance squared; hence, the product goes to zero proportionally to inverse distance cubed and we sum over a spherical surface proportional to only distance squared. Combining this with the value of the left-hand side that is independent of choice of $\mathbb{D}$, implies that the boundary integral is zero for any bounded volume, even when the medium is arbitrarily heterogeneous outside $\mathbb{D}$. This establishes the well-known source-receiver reciprocity relation $\hat{G}(\boldsymbol{r}_A, \boldsymbol{r}_B) = \hat{G}(\boldsymbol{r}_B, \boldsymbol{r}_A)$, and also leads to the relation

$$\oint_{\partial\mathbb{D}} \hat{G}(\boldsymbol{r}, \boldsymbol{r}_A) \boldsymbol{n} \cdot \hat{\boldsymbol{\sigma}} \cdot \nabla \hat{G}(\boldsymbol{r}, \boldsymbol{r}_B) \mathrm{d}^2\boldsymbol{r} = \oint_{\partial\mathbb{D}} \hat{G}(\boldsymbol{r}, \boldsymbol{r}_B) \boldsymbol{n} \cdot \hat{\boldsymbol{\sigma}} \cdot \nabla \hat{G}(\boldsymbol{r}, \boldsymbol{r}_A) \mathrm{d}^2\boldsymbol{r}, \quad (17)$$

which remains valid when both $\boldsymbol{r}_{A,B}$ are outside the volume, because then the left-hand side of equation (16) is always zero. Note that each integral in equation (17) vanishes when the volume is taken as infinite space.

3 GREEN'S FUNCTION RETRIEVAL

Taking the complex-conjugate state for state A is equivalent to taking the time-reversed causal state in the time domain. Using this in equations (12) and (13) leads to expressions involving cross-correlations of quantities in the time-domain. Using this in the frequency-domain and using source-receiver reciprocity, we directly obtain the global interactions by taking the complex-conjugate Green's function for state A in equations (14) and (15), as

$$\hat{G}^*(\boldsymbol{r}_B, \boldsymbol{r}_A) = \int_{\mathbb{D}} (\nabla \hat{G}^*(\boldsymbol{r}, \boldsymbol{r}_A)) \cdot \hat{\boldsymbol{\sigma}} \cdot (\nabla \hat{G}(\boldsymbol{r}, \boldsymbol{r}_B)) \mathrm{d}^3\boldsymbol{r} - \oint_{\partial\mathbb{D}} \hat{G}^*(\boldsymbol{r}, \boldsymbol{r}_A) \boldsymbol{n} \cdot \hat{\boldsymbol{\sigma}} \cdot \nabla \hat{G}(\boldsymbol{r}, \boldsymbol{r}_B) \mathrm{d}^2\boldsymbol{r}, \quad (18)$$

$$\hat{G}(\boldsymbol{r}_B, \boldsymbol{r}_A) = \int_{\mathbb{D}} (\nabla \hat{G}^*(\boldsymbol{r}, \boldsymbol{r}_A)) \cdot \hat{\boldsymbol{\sigma}}^* \cdot (\nabla \hat{G}(\boldsymbol{r}, \boldsymbol{r}_B)) \mathrm{d}^3\boldsymbol{r} - \oint_{\partial\mathbb{D}} \hat{G}(\boldsymbol{r}, \boldsymbol{r}_B) \boldsymbol{n} \cdot \hat{\boldsymbol{\sigma}}^* \cdot \nabla \hat{G}^*(\boldsymbol{r}, \boldsymbol{r}_A) \mathrm{d}^2\boldsymbol{r}. \quad (19)$$

where the superscript $*$ denotes complex conjugation. We extend the domain $\mathbb{D}$ to three-dimensional infinite space $\mathbb{R}^3$ and use the fact that in that case each surface integral goes to zero. We then obtain two integral relations for the Green's function directly as

$$\hat{G}^*(\boldsymbol{r}_B, \boldsymbol{r}_A) = \int_{\mathbb{R}^3} (\nabla \hat{G}^*(\boldsymbol{r}_A, \boldsymbol{r})) \cdot \hat{\boldsymbol{\sigma}} \cdot (\nabla \hat{G}(\boldsymbol{r}_B, \boldsymbol{r})) \mathrm{d}^3\boldsymbol{r}, \quad (20)$$

$$\hat{G}(\boldsymbol{r}_B, \boldsymbol{r}_A) = \int_{\mathbb{R}^3} (\nabla \hat{G}^*(\boldsymbol{r}_A, \boldsymbol{r})) \cdot \hat{\boldsymbol{\sigma}}^* \cdot (\nabla \hat{G}(\boldsymbol{r}_B, \boldsymbol{r})) \mathrm{d}^3\boldsymbol{r}. \quad (21)$$

Interestingly, the only difference between the right-hand sides of equations (20) and (21) is in the complex conductivity. By adding and subtracting equations (20) and (21), we obtain

$$\Im\{\hat{G}(\boldsymbol{r}_B, \boldsymbol{r}_A)\} = - \int_{\mathbb{R}^3} (\nabla \hat{G}^*(\boldsymbol{r}_A, \boldsymbol{r})) \cdot \Im\{\hat{\boldsymbol{\sigma}}\} \cdot (\nabla \hat{G}(\boldsymbol{r}_B, \boldsymbol{r})) \mathrm{d}^3\boldsymbol{r}, \quad (22)$$

and

$$\Re\{\hat{G}(\boldsymbol{r}_B,\boldsymbol{r}_A)\} = \int_{\mathbb{R}^3} (\nabla\hat{G}^*(\boldsymbol{r}_A,\boldsymbol{r}))\cdot\Re\{\hat{\boldsymbol{\sigma}}\}\cdot(\nabla\hat{G}(\boldsymbol{r}_B,\boldsymbol{r}))\mathrm{d}^3\boldsymbol{r}. \quad (23)$$

Equations (22) and (23) show that either the real or imaginary parts of the Green's function between $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$ is obtained from products of the complex-conjugate Green's function measured at $\boldsymbol{r}_A$ and Green's functions measured at $\boldsymbol{r}_B$. Both are caused by external electric-field point sources at $\boldsymbol{r}$, whose strengths depend on either the real or imaginary parts of the conductivity. Integration over all sources in $\mathbb{R}^3$ finally gives the Green's function. To see the application of the expression in an experiment we consider an external electric-field source occupying infinite space. According to equation (10) we can write for a receiver at $\boldsymbol{r}_A$ and a source at $\boldsymbol{r}$ as

$$\hat{V}^E(\boldsymbol{r}_A) = \int_{\boldsymbol{r}\in\mathbb{R}^3} \hat{G}(\boldsymbol{r}_A,\boldsymbol{r})\nabla\cdot\hat{\boldsymbol{\sigma}}(\boldsymbol{r})\cdot\hat{\boldsymbol{E}}(\boldsymbol{r})\mathrm{d}^3\boldsymbol{r}, \quad (24)$$

and we can use one integration by parts to obtain

$$\hat{V}^E(\boldsymbol{r}_A) = -\int_{\boldsymbol{r}\in\mathbb{R}^3} [\nabla\hat{G}(\boldsymbol{r}_A,\boldsymbol{r})]\cdot\hat{\boldsymbol{\sigma}}(\boldsymbol{r})\cdot\hat{\boldsymbol{E}}(\boldsymbol{r})\mathrm{d}^3\boldsymbol{r}. \quad (25)$$

This means we can write the vector Green's function corresponding to this electric-potential field as

$$\hat{\boldsymbol{G}}^{VE}(\boldsymbol{r}_A,\boldsymbol{r}) = -[\nabla\hat{G}(\boldsymbol{r}_A,\boldsymbol{r})]\cdot\hat{\boldsymbol{\sigma}}(\boldsymbol{r}). \quad (26)$$

To be able to substitute this expression in equations (22) and (23) we need to rewrite the imaginary and real parts of the conductivity tensor as

$$\Re\{\hat{\boldsymbol{\sigma}}\} = \frac{1}{2}(\hat{\boldsymbol{\sigma}}^* + \hat{\boldsymbol{\sigma}}) = \frac{1}{2}\hat{\boldsymbol{\sigma}}^*\cdot(\hat{\boldsymbol{\zeta}} + \hat{\boldsymbol{\zeta}}^*)\cdot\hat{\boldsymbol{\sigma}} = \hat{\boldsymbol{\sigma}}^*\cdot\Re\{\hat{\boldsymbol{\zeta}}\}\cdot\hat{\boldsymbol{\sigma}}, \quad (27)$$

$$\Im\{\hat{\boldsymbol{\sigma}}\} = \frac{1}{2\mathrm{i}}(\hat{\boldsymbol{\sigma}} - \hat{\boldsymbol{\sigma}}^*) = \frac{1}{2\mathrm{i}}\hat{\boldsymbol{\sigma}}^*\cdot(\hat{\boldsymbol{\zeta}}^* - \hat{\boldsymbol{\zeta}})\cdot\hat{\boldsymbol{\sigma}} = -\hat{\boldsymbol{\sigma}}^*\cdot\Im\{\hat{\boldsymbol{\zeta}}\}\cdot\hat{\boldsymbol{\sigma}}. \quad (28)$$

Using these results we can express equations (22) and (23) in terms of the vector Green's function for the electric-potential caused by an external electric field as

$$\Im\{\hat{G}(\boldsymbol{r}_B,\boldsymbol{r}_A)\} = \int_{\mathbb{R}^3} [\hat{\boldsymbol{G}}^{VE}(\boldsymbol{r}_A,\boldsymbol{r})]^*\cdot\Im\{\hat{\boldsymbol{\zeta}}\}\cdot\hat{\boldsymbol{G}}^{VE}(\boldsymbol{r}_B,\boldsymbol{r})\mathrm{d}^3\boldsymbol{r}, \quad (29)$$

$$\Re\{\hat{G}(\boldsymbol{r}_B,\boldsymbol{r}_A)\} = \int_{\mathbb{R}^3} [\hat{\boldsymbol{G}}^{VE}(\boldsymbol{r}_A,\boldsymbol{r}))]^*\cdot\Re\{\hat{\boldsymbol{\zeta}}\}\cdot\hat{\boldsymbol{G}}^{VE}(\boldsymbol{r}_B,\boldsymbol{r})\mathrm{d}^3\boldsymbol{r}. \quad (30)$$

Two other equally valid expressions for the real and imaginary parts of the Green's function are obtained by adding and subtracting equations (14) and (18),

$$\Im\{\hat{G}(\boldsymbol{r}_B,\boldsymbol{r}_A)\} = \int_{\mathbb{R}^3} \Im\{\nabla\hat{G}^*(\boldsymbol{r}_A,\boldsymbol{r})\}\hat{\boldsymbol{\sigma}}(\nabla\hat{G}(\boldsymbol{r}_B,\boldsymbol{r}))\mathrm{d}^3\boldsymbol{r}, \quad (31)$$

$$\Re\{\hat{G}(\boldsymbol{r}_B,\boldsymbol{r}_A)\} = \int_{\mathbb{R}^3} \Re\{\nabla\hat{G}^*(\boldsymbol{r}_A,\boldsymbol{r})\}\hat{\boldsymbol{\sigma}}(\nabla\hat{G}(\boldsymbol{r}_B,\boldsymbol{r}))\mathrm{d}^3\boldsymbol{r}. \quad (32)$$

We can either take the real and imaginary parts of the conductivity as a weight for the cross-correlations of measurements or take the real and imaginary parts of the measurements at $\boldsymbol{r}_A$ for the cross-correlations and use the complex conductivity as a weight.

4 FIELD FLUCTUATIONS

In the time-domain the Green's matrix is causal, hence $G(\boldsymbol{r}_B,\boldsymbol{r}_A,t) = 0$ for $t < 0$, and the time-reversed Green's function is time-reversed causal, hence $G(\boldsymbol{r}_A,\boldsymbol{r}_B,-t) = 0$ for $-t > 0$. For this reason either $G(\boldsymbol{r}_B,\boldsymbol{r}_A,t)$ or $G(\boldsymbol{r}_A,\boldsymbol{r}_B,-t)$ can be retrieved from the left-hand sides of equations (29)-(30). In the zero-frequency limit the imaginary part of the Green's function goes to zero, because the imaginary part of $\hat{\boldsymbol{\sigma}}$ goes to zero when frequency goes to zero; then only equation (30) can be used to retrieve the Green's function.

Apart from the usefulness of equations (29)-(30) for modeling and inversion (Wapenaar, 2007), and for validation of numerical codes, here the primary interest is in possible applications of remote sensing without active sources.

4.1 Fluctuations about a thermodynamic equilibrium state

If the system is in thermal equilibrium, then spontaneous fluctuations will occur for which the ensemble average cross-correlations of the equivalent source densities are determined by the fluctuation dissipation theorem (FDT) (Callen & Welton, 1951). The FDT is a generalization of two historic observations and explanations. The first is the original observation on Brownian motion by Einstein (1905) in which he noted that the random forces that cause the erratic motion of a particle in Brownian motion, would also cause drag if the particle were pulled through the fluid. From this analysis it followed that the average of the frictional force squared is equal to the diffusion constant, D with $D = k_BT\mu$. Here, μ is the mobility of the particles, k_B is Boltzmann's constant, and T is temperature. The second is an observation by Johnson and its explanation by Nyquist (Johnson, 1928; Nyquist, 1928), now known as the Johnson-Nyquist experiment regarding thermal noise in an electric R, C circuit, where R is the constant resistance of the circuit and C its constant capacitance. Johnson found that the square of the measured voltage over the circuit was proportional to the resistance of the circuit, given by $\langle V^2\rangle = 2k_BTR/\pi$, where the brackets mean the measurement would be averaged over all times. In practice the measurement is carried out with digital equipment, by averaging sampled values over a finite time window. The sampling rate defines the maximum frequency and the time-window defines the minimum frequency of the experiment. This implies that the measurement can be written in the frequency-domain as

$$\int_{\omega=\omega_{\min}}^{\omega_{\max}} \hat{V}(\omega)\hat{V}^*(\omega)\mathrm{d}\omega = \frac{2}{\pi}k_B T R(\omega_{\max}-\omega_{\min}). \quad (33)$$

This is under the assumption that the resistance is a constant for all frequencies involved and the capacitance plays no role. When the total resistance varies as a function of frequency, equation (33) is modified to

$$\int_{\omega=\omega_{\min}}^{\omega_{\max}} \hat{V}(\omega)\hat{V}^*(\omega)\mathrm{d}\omega = \frac{2}{\pi}k_B T \int_{\omega=\omega_{\min}}^{\omega_{\max}} \Re\{\hat{Z}(\omega)\}\mathrm{d}\omega, \quad (34)$$

where $\hat{Z}$ denotes the impedance of the circuit. In the case of Johnson's experiment the impedance is given by $\hat{Z} = (R^{-1} - \mathrm{i}\omega C)^{-1}$, C being the capacitance of the circuit, and $\Re\{\hat{Z}\} = R/(1+(\omega RC)^2)$; hence, for a non-conductive circuit, $R \to \infty$ leads to $\Re\{\hat{Z}\} \to 0$.

These results were later generalized by Callen and Welton (1951) in the FDT. Using these observations in the general form of FDT for piecewise continuous macroscopic systems, we find that when a scalar observable $\hat{V}$ is related to a field $\hat{I}$ through a linear response function $\hat{R}$ as $\hat{V} = \hat{I}\hat{R}$, the power spectrum of the observable is proportional to the real part of the linear response function $\Re\{\hat{R}\}$. For a general linear medium and the frequency dependent conductivity being a piecewise continuous function of position we can use the same theorem for thermal fluctuations of the electric field vector to satisfy (Landau & Lifshitz, 1960) the relation that is written in vector components as

$$\int_{\boldsymbol{r}'\in\mathbb{R}^3} \hat{E}_k(\boldsymbol{r})\hat{E}_r^*(\boldsymbol{r}')\mathrm{d}^3\boldsymbol{r}' = \frac{2}{\pi}k_B T\Re\{\hat{\zeta}_{kr}(\boldsymbol{r})\}. \quad (35)$$

Only equation (30) can be used for Green's function retrieval using this thermal fluctuational field, because the right-hand side of equation (35) contains the real part of the electric resistivity matrix, which is contained in equation (30). By writing the observation of electric-potential at $\boldsymbol{r}_A$ or $\boldsymbol{r}_B$ as a response to a thermal noise source in accordance with equation (5) as

$$\hat{V}(\boldsymbol{r}_A) = \int_{\boldsymbol{r}\in\mathbb{R}^3} \hat{\boldsymbol{G}}^{VE}(\boldsymbol{r}_A,\boldsymbol{r})\cdot\hat{\boldsymbol{E}}(\boldsymbol{r})\mathrm{d}^3\boldsymbol{r}, \quad (36)$$

$$\hat{V}(\boldsymbol{r}_B) = \int_{\boldsymbol{r}'\in\mathbb{R}^3} \hat{\boldsymbol{G}}^{VE}(\boldsymbol{r}_B,\boldsymbol{r}')\cdot\hat{\boldsymbol{E}}(\boldsymbol{r}')\mathrm{d}^3\boldsymbol{r}', \quad (37)$$

and using equation (35) in the product of $\hat{V}^*(\boldsymbol{r}_A)$ and $\hat{V}(\boldsymbol{r}_B)$ results in

$$\hat{V}(\boldsymbol{r}_A)^*\hat{V}(\boldsymbol{r}_B) = \int_{\boldsymbol{r}\in\mathbb{R}^3}\int_{\boldsymbol{r}'\in\mathbb{R}^3} [\hat{\boldsymbol{G}}^{VE}(\boldsymbol{r}_A,\boldsymbol{r})]^*\cdot\hat{\boldsymbol{E}}^*(\boldsymbol{r}) \hat{\boldsymbol{E}}(\boldsymbol{r}')\cdot\hat{\boldsymbol{G}}^{VE}(\boldsymbol{r}_B,\boldsymbol{r}')\mathrm{d}^3\boldsymbol{r}'\mathrm{d}^3\boldsymbol{r},$$

$$= \frac{2}{\pi}k_B T\int_{\mathbb{R}^3} [\hat{\boldsymbol{G}}^{VE}(\boldsymbol{r}_A,\boldsymbol{r})]^*\cdot\Re\{\hat{\zeta}\}\cdot\hat{\boldsymbol{G}}^{VE}(\boldsymbol{r}_B,\boldsymbol{r})\mathrm{d}^3\boldsymbol{r}, \quad (38)$$

which is equal to the right-hand side of equation (30), apart from the energy factor $2k_BT/\pi$. This gives the desired Green's function retrieval formulation from cross-correlation of the potential measurements between $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$

$$\hat{V}^*(\boldsymbol{r}_A)\hat{V}(\boldsymbol{r}_B) = \frac{2}{\pi}k_B T\Re\{\hat{G}(\boldsymbol{r}_A,\boldsymbol{r}_B)\}. \quad (39)$$

Snieder (2006) was the first to propose a similar expression for the scalar diffusion equation. The Green's function in the right-hand side of equation (39) is the electric potential at $\boldsymbol{r}_A$ due to a current injection at $\boldsymbol{r}_B$. This is an observable for actual dipole current sources and potential differences measured in the field, which are easily obtained by combining different observations.

When the medium is not dissipative everywhere in $\mathbb{R}^3$, the non-dissipative part of the medium can be excluded to allow the boundary to run at the intersection of the dissipative and non-dissipative domain. Under quasi-static electric field conditions, the earth surface is such an interface, and Dirichlet conditions apply to the normal component of the electric current, so that the boundary integrals present in equations (18) and (19) are still zero and equation (23) remains valid in the reduced volume where the medium is dissipative. This implies that the electric response of the earth as a heterogeneous half space can be determined by cross-correlations of thermal fluctuational noise recordings. If other source mechanisms are present, these thermal fluctuations might be too small to be measured, although increasing the measurement time enhances the signal-to-noise ratio in the correlation results.

4.2 Strengths of electric thermal noise signals

The expected strength of the thermal noise fields depends on the thermal energy. Boltzmann's constant is $k_B \approx 1.4\times10^{-23}$ J/K, and for a room temperature of $T \approx 300$ K, we find $k_BT \approx 4.2\times10^{-21}$ J. The total energy delivered between two electrodes is equal to the energy dissipated in the medium by the resistance of the medium, which is related to the location of the two electrodes. This results in $|V|^2/R = 2k_BT\Delta\omega/\pi$, where R is the apparent resistance of the medium and $\Delta\omega$ is the angular frequency bandwidth of the measurement. The energy depends on the bandwidth of the instrument used in the experiment; we assume a frequency bandwidth of $\Delta f = 10kHz$, and use $\Delta\omega = 2\pi\Delta f$, f being natural frequency. The power is $4k_BT\Delta f \approx 1.7\times10^{-16}$ Watt. If the apparent resistance of the measurement is 50 Ω, the mean fluctuation of the squared electric potential difference between these two points is $|V|^2 \approx 0.85\times10^{-14}$ which corresponds to measuring a voltage difference between the two electrodes of $V \approx 90n$V. This is a small number, but certainly within the measurable range of modern equipment. Furthermore, in our estimate we have not made use of any amplification of the recorded signal. If we have a high quality amplifier that has a linear behavior over the whole frequency bandwidth, the signal can be increased and the only concern is the noise in the amplifier. The bandwidth of the measurement can be increased, which will result in a stronger signal because the mean energy of the signal is proportional to the bandwidth. We can, of course, measure over periods longer than one second and average the values that are measured over one second. Finally, the apparent resistance of a measurement is given by $V/I = R_{\mathrm{app}} = (4\pi\sigma_{\mathrm{app}}|\boldsymbol{r}-\boldsymbol{r}'|)^{-1}$, where σ_{app} is the apparent conductivity of an equivalent homogeneous infinite medium, $\boldsymbol{r}$ is the position where the potential is measured and $\boldsymbol{r}'$ is the position where the current is injected. For the assumed 50Ω apparent resistance in our example, and a distance between the two electrodes of $|\boldsymbol{r}-\boldsymbol{r}'| = 1$ m, the apparent conductivity is $\sigma\mathrm{app} = 1.6$ mS/m. For larger values of the apparent conductivity the distance between the two electrodes should be proportionally smaller than 1 m, if the measurement threshold is 90 nV. By varying the distances between $\boldsymbol{r}$ and $\boldsymbol{r}'$, given an apparent conductivity, we can always find electrode pairs that will lead to a high quality virtual measurement from cross-correlations of thermal noise potential difference recordings.

5 GENERALIZATION TO OTHER POTENTIAL FIELD METHODS

We have established the Green's function retrieval formalism for the quasi-static electric field. We have shown that thermal fluctuations lead to correlations of the electric potential that are proportional to the real part of the Green's function, with the thermal energy as the proportionality factor. This was obtained using the fluctuation-dissipation theorem, which is valid for any linear system. This result can now be used to generalize the concept to any other type of potential field method.

The general equation can be written as

$$\nabla\cdot(\hat{\boldsymbol{\chi}}\nabla\hat{\phi}) = -\nabla\cdot\hat{\boldsymbol{s}}, \tag{40}$$

which is equal to equation (4) with $\hat{\boldsymbol{\chi}} = \hat{\boldsymbol{\sigma}}$, $\hat{\phi} = \hat{V}$ and $\hat{\boldsymbol{s}} = \hat{\boldsymbol{\sigma}}\cdot\hat{E}^e$. For this general equation the Green's function retrieval from cross-correlations of thermal fluctuations is given by

$$\Re\{\hat{G}(\boldsymbol{r}_B,\boldsymbol{r}_A)\} = \int_{\mathbb{R}^3}[\nabla\hat{G}^*(\boldsymbol{r}_A,\boldsymbol{r})]\cdot\Re\{\hat{\boldsymbol{\chi}}\}\cdot[\nabla\hat{G}(\boldsymbol{r}_B,\boldsymbol{r})]\mathrm{d}^3\boldsymbol{r}. \tag{41}$$

FDT gives the general correlations of the vector source term as

$$\int_{\mathbb{R}^3}\hat{\boldsymbol{s}}(\boldsymbol{r})\hat{\boldsymbol{s}}(\boldsymbol{r}')\mathrm{d}\boldsymbol{r}' = \frac{2}{\pi}k_BT\Re\{\hat{\boldsymbol{\chi}}(\boldsymbol{r})\}. \tag{42}$$

For example, for subsurface flow problems we have $\hat{\boldsymbol{\chi}} = \hat{\boldsymbol{\kappa}}$, $\hat{\boldsymbol{\kappa}}$ being the hydraulic conductivity, $\hat{\phi} = \hat{h}$, being the hydraulic head and $\nabla\cdot\hat{\boldsymbol{s}} = \hat{q}$ being the source or sink. Thermal fluctuations of the hydraulic head lead in this case to correlations of the gradients of the hydraulic heads being proportional to the real part of the hydraulic conductivity.

All fields that satisfy equation (40) can be used when fluctuational noise sources exist, either thermal or of another origin. In the latter situation the factor $2k_BT/\pi$ must be replaced in equation (42) with an appropriate measure of the noise. The only important property is that the noise sources should be uncorrelated in space and time, relative to the resolution scale of the measurement.

Other possible fields that satisfy equation (40) are quasi-stationary mass diffusion and, when equation (40) is modified to allow for vector fields, linearized momentum transport. Those fields can also be used to retrieve the system's Green's function from cross-correlations of fluctuational noise recordings. Other possible extensions are electro-static fields, gravitational fields, quasi-static magnetic fields and heat conduction fields. For these and conceivable other potential fields thermal equilibrium fluctuations may not be the source of the noise, but other suitable causes of noise may exist. These results can be applied to other type of fields, like diffusive fields and wave fields, see e.g. Snieder *et al.* (2009).

6 DISCUSSION ON POSSIBLE OILFIELD APPLICATIONS

The two examples of quasi-static (or DC) electric field measurements and subsurface flow problems are potentially of particular interest for applications in hydrocarbon exploration and time lapse monitoring of the local changes caused by production. Especially, downhole situations can possibly be regarded as in (near) thermal equilibrium over the measurement time windows, because they are far away from cultural noise generated at the surface and are much less dependent on the daily thermal cycles caused by solar radiation and nightly cooling. This implies that downhole equipment would need to consist of measurement devices only; no power is required to generate the fields. By measuring time series of the electric potential differences between four electrodes in a single hole or cross hole, two can be used as a virtual electric dipole source, while the other two as the measurement electrodes. Repeating this for all available electrodes, full coverage can be reached where any electrode pair in the configuration can be used as a virtual electric dipole source and any other electrode pair can be used to measure the resulting electric potential difference. The technical merits of such passive measurements and the specifications of the measurement set up for a laboratory experiment are detailed in Johnson (1928). We have also shown that the static limit of a DC-electric measurement can be obtained from thermal fluctuations.

During pauses in production, e.g., after a production cycle, it would perhaps be possible to measure fluid flow in the reservoir, thereby obtaining information as if any location of a measurement is a virtual injection or production point. The response to such a virtual source or sink is obtained from cross-correlating recordings of the head differences at any two measurement locations in a well or between two wells. This could then be used to gain a better insight in the reservoir flow properties and in the changes in these properties over sufficiently large production windows.

A third potentially attractive application is the possibility of measuring downhole temperatures using cross-correlations of electric noise measurements. The cross-correlation of the local electric potential difference between two electrodes is equal to the product of the local temperature and the Green's function between the electrodes, which is an apparent resistance. When an active measurement is performed between the same two electrodes to obtain the local apparent resistance value corresponding to that measurement configuration, the ratio of the noise measurement and the active measurement directly gives the temperature scaled by Boltzmann's constant and $2/\pi$. This can be a possible solution for the downhole temperature measurement at many locations inside or outside a well. The practical applicability depends on the strength of the thermal noise signal. This should be such that the necessary time windows, over which the signal should be averaged, are short enough to assume a relatively constant temperature. Calibration and testing of the method under various circumstances could be achieved by simply combining this procedure with independent temperature measurements in or near a well. Combining passive and active measurements at many receiver locations across wells could be used to obtain an estimate of the volumetric temperature distribution. Specimen temperature has been estimated in the lab on an aluminum cylinder from cross-correlations of acoustic thermal noise recordings and calibrating them to active recordings (Weaver & Lobkis, 2003).

Under the assumption of subsurface steady state and thermal equilibrium conditions all three proposed possible applications can have some merit for measurement-and-control type reservoir management systems. More applications are deemed possible.

7 CONCLUSIONS

We have derived interferometric relations to obtain the Green's function of a quasi-static field between two points from the two-point correlation of noise measurements. This has led to representations for the imaginary part and for the real part of the Green's function. The static potential function is the late-time limit of the quasi-static field and is a real function. For this reason the representation for the real part is of interest. We have shown the relationship between the autocorrelation of the electric potential over an electric circuit and the resistance of the circuit. This is extended by the fluctuation-dissipation theorem to arbitrary linear dissipative systems and can be used for Green's function retrieval. We have shown that thermal fluctuations of electric fields have volume-integrated correlations equal to the real part of the local resistivity times the thermal energy. This directly leads to the identity for the cross-correlation of thermal noise measurements at two locations being equal to the Green's function between those two locations.

Based on the analysis for the quasi-static electric field and the general form of the fluctuation-dissipation theorem, we have generalized the quasi-static field Green's function retrieval formulation to any linear system satisfying a similar modified Laplace equation. Subsurface fluid flow was given as an example. According to the fluctuation-dissipation theorem, thermal noise gives the correlation functions of the sources that are required by the Green's function retrieval formulation from the reciprocity theorem.

Finally, we have proposed three possible downhole applications in reservoir environments. Full coverage DC electric resistivity measurements can be generated from correlations of electric noise recordings. A volumetric distribution of flow properties can be determined from correlations of thermal noise measurements of the hydraulic head at all possible well locations. Downhole temperature can be computed from the ratio of cross-correlation of electric potential thermal noise recordings to active resistivity measurements between the same electrodes. These applications depend on the assumption that thermal noise is the major source of fluctuations and requires that the system is in steady-state and in thermal equilibrium.

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