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On the phase of the apparent attenuation operator in the convolutional model of the seismic trace

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ABSTRACT

Short-period multiples and transmission losses can be included in the convolutional model of the seismic trace through the operator R_m/R, where R is the spectrum of the primary reflectivity series and R_m is the spectrum of the reflection impulse response of the medium. When short-period multiples are weak, or even moderate, the properties of this operator are similar to those of intrinsic absorption, so the two can be combined into a single "effective attenuation" operator in the convolutional model of the trace. This can no longer be done when short-period multiples are strong, because the apparent attenuation operator R_m/R becomes non-minimum phase. The problem is actually caused by surface-related multiples – without them R_m/R is always minimum-phase. Surface-related multiples don't change significantly the phase of the operator, but whiten its power spectrum (reduce its high-frequency deficit). The whitening can be significant in strong reflectivities, i.e., for media in which the apparent attenuation due to short-period multiples is strong. In such cases, the minimum-phase equivalent of the whitened spectrum doesn't predict well the phase of the operator. This has a direct bearing to wavelet estimation from seismic traces which contain surface-related multiples – wavelet estimates derived under the minimum phase assumption tend to underestimate the time delay of the wavelet. A bit extreme yet realistic example shows that the phase error can reach up to 45 degrees over the low third of the seismic frequency band. It can be corrected on the basis of borehole information.

Key words: layered media, attenuation, scattering, Q, seismic deconvolution, seismic spectra, reflection seismology

1 INTRODUCTION

Does the stratigraphic filter due to thin horizontal layering act identically to intrinsic absorption on the seismic trace? People working on deconvolution would like the answer to be "yes" because then the two effects can be combined into a single "effective attenuation" operator in the convolutional model of the trace. People interested in absorption estimation would be happy the answer to be "no", hoping the differences may help them separate the intrinsic absorption (which carries information about lithology and reservoir conditions) from scattering effects.

Experience and theory tell us the two filters are very similar. It is well known that the earth reflectivity is "blue", i.e., it's rich in high frequencies over the seismic frequency band (Walden & Hosken, 1985; Saggaf & Robinson, 2000). It's also well known that an impulse transmitted through such a blue sequence of reflectors is tailed by short-period multiples with a red spectrum, i.e., the transmission response of the layered elastic earth is dispersive and high-frequency deficient (O'Doherty & Anstey, 1971). It is minimum-phase, too (Banik *et al.*, 1985). In this sense, the transmission through a stack of elastic thin layers is very similar to the transmission through a homogeneous but absorbing slab. That's how the famous statement that short-period multiples cause apparent attenuation originated. The seismic trace however is more complicated – it comes from a reflection experiment over a half-space, bounded by a free surface. That's why the question posed at the beginning should be approached with care. In this paper I don't aim at a thorough comparison between the intrinsic and apparent attenuation operators – I only investigate whether the two can be combined for the purposes of wavelet estimation and deconvolution. First I derive the convolutional operator accounting for short-period multiples and transmission losses in a most general form. Then I compare its spectral properties to those of intrinsic absorption. I focus on the phase spectrum and show that the apparent attenuation operator along a trace containing surface-related multiples may be significantly non-minimum phase in media characterized by strong reflectivities.

2 CONVOLUTIONAL MODEL OF THE TRACE

Consider a horizontally layered medium. Let r_m denotes its elastic impulse response. Then, the (noise-free) seismic trace can be written as $w_0 * r_m$, where the star means convolution and w_0 is some basic wavelet accounting for source, receiver, and instrumentation signatures, as well as anelasticity. On the other hand, the conventional convolutional model of the trace is $w * r$ where r is the primary reflectivity series and w is a wavelet. In order that both representations might be equivalent, the wavelet w should account for transmission losses and multiples. Its relation to the basic wavelet w_0 is easily seen in the frequency domain:

$$W_0 \, R_m \;=\; W_0 \, \frac{R_m}{R} \, R \;=\; W \, R \,, \qquad (1)$$

where the capital letters stand for the Fourier transforms of the respective time series. This simple consideration shows that transmission losses and multiples can be included in the convolutional model of the trace through the operator $R_m(\omega)/R(\omega)$. This is what I call the "apparent attenuation" operator. If it has the same properties as the intrinsic absorption operator, namely:

- exponential decay with frequency
- minimum-phase

the two can be combined into an "effective attenuation" operator in the wavelet model.

The former condition, although not met exactly, is reasonable well satisfied in practice (Appendix B).

The minimum-phase property is under investigation in this paper – first, from a theoretical point of view and then, through a synthetic example.

3 THE OPERATOR R_m/R

Weak-reflectivity approximations of R_m for various receiver locations are given in Appendix A. Despite that my analysis is not limited to the weak-reflectivity case (on the contrary!), they are useful in predicting when the minimum-phase property of R_m/R might fail.

3.1 Earth model without a free surface

Consider first a trace free of surface-related multiples. As it follows from eq. (A4), in a short time-window starting at time T, the operator R_m/R is *minimum-phase* and has an amplitude spectrum

$$|R_m/R| \;\approx\; e^{-\,|R|^2\,T} \,, \qquad (2)$$

where $|R|^2$ is the power spectrum of the primary reflectivity series.

An underlying assumption in eq. (2) is that most of the energy arriving at time T comes from the depth reached by the ballistic arrival in that time, i.e., ray-paths trapped above the maximum depth of penetration are ignored. In strong reflectivities, however, the shallow ray-paths may contribute significantly to the signal at time T. Potentially, this might violate the minimum-phase property of R_m/R. The robustness of the minimum-phase property in the strong reflectivity case is tested in the next section.

Establishing that R_m/R is minimum-phase in the absence of surface-related multiples is promising but not sufficient – the earth surface (a free surface) has a very strong influence on the seismic trace and must be taken into account.

3.2 Earth model with a free surface

Let $(R_m/R)_0$ denote the operator discussed above for the model without a free surface, and let $(R_m/R)_1$ be that for a model with a free surface. Then, as seen from eq. (A5),

$$\left(\frac{R_m}{R}\right)_1 \approx \left(\frac{R_m}{R}\right)_0 \left[1 + R(\omega) + R^2(\omega) + ...\right] \,, \qquad (3)$$

Obviously, $(R_m/R)_1$ is not minimum-phase since the primary reflectivity terms in the brackets of eq. (3) have random phases. However, the phase distortions will occur mainly at high frequencies where $|R|$ is large. At low frequencies, where most of the power of R_m/R is, the phase of $(R_m/R)_1$ will be almost the same as that of $(R_m/R)_0$.

On the other hand, the amplitude spectrum $|R_m/R|_1$ will have a smaller slope than $|R_m/R|_0$. This is easily seen from eqs. (2) and (3) – since the primary reflectivity spectrum R is blue, $|R_m/R|_0$ is high-frequency deficient; surface-related multiples (the R-terms in the brackets of eq. (3)) partially compensate that high-frequency deficit. This whitening effect is more pronounced in strong reflectivities (even-though they are not well described by eqs. (2) and (3)) and at low frequencies.

To summarize, surface-related multiples don't change much the phase of R_m/R but reduce the slope of its amplitude spectrum. Therefore, the minimum-phase equivalent of $|R_m/R|_1$ will underestimate the time delay of the apparent attenuation operator. The error

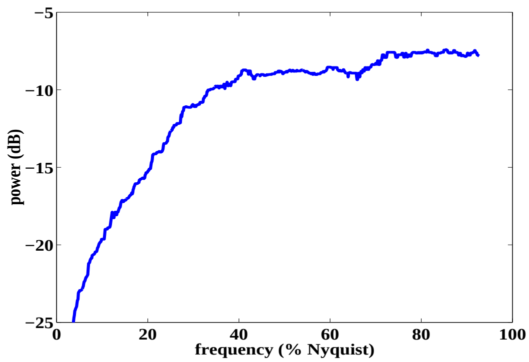

Figure 1. Power spectrum of the primary reflectivity estimated from 100 realizations and smoothed by a 15% median filter.

will grow with time because, while $|R_m/R|_1$ is stationary (Mateeva, 2001; White *et al.*, 1990eqs. (7)–(8)), $|R_m/R|_0$ gets increasingly high-frequency deficient with time, i.e., the minimum-phase equivalents of $|R_m/R|_0$ and $|R_m/R|_1$ become more and more different.

To test the phase properties of the stratigraphic filter, a strong-reflectivity example is considered below.

4 EXAMPLE

4.1 Data

The synthetic reflectivity in this example is similar to that of Well 8 from the papers of Walden and Hosken (1985, 1986). It has the following statistical properties:

- mean $= -0.0002$, standard deviation ≈ 0.13;
- reflection coefficient magnitudes follow (approximately) a Laplacian distribution;
- the reflectivity series is an ARMA(1,1) process with an autoregressive parameter $\theta = 0.9$ and a moving average parameter $\phi = 0.3$; this gives rise to the blue spectrum depicted in Figure 1.

To enhance the spectral analysis reliability, I generated 100 realizations of the primary reflectivity series, 1024 samples each. Then, assuming a horizontally layered elastic medium, I computed the corresponding 100 normal-incidence impulse responses with and without surface-related multiples. Each time series (1024 samples) was appended by 1024 zeroes to assure causality. The mean of each series was removed and a 20% cosine taper applied before FFT. The resulting estimates of $R(\omega)$, $(R_m(\omega)/R(\omega))_1$, and $(R_m(\omega)/R(\omega))_0$ were averaged over the 100 realizations to reduce variability. In addition, the power and phase estimates presented in the figures of this paper were mildly smoothed by a 10% to 15% median filter (a 101- or 151-sample filter applied to 1024-sample spectra).

The inverse-of-inverse method was used to compute minimum-phase equivalents.

4.2 Assumptions test

The phase of $(R_m/R)_1$ can be modeled on the basis of two assumptions:

- $(R_m/R)_0$ is minimum-phase
- $\arg{(R_m/R)_1} \approx \arg{(R_m/R)_0}$

As explained in section 3.1, $(R_m/R)_0$ is approximately minimum-phase in weak reflectivities and over a small time window. The reflectivity in this example is about as strong as it ever gets. Moreover, the spectral estimates are based on a very long time-window, in which $(R_m/R)_0$ is not stationary, and thus, the time-averaged estimate may not be minimum-phase*. Nevertheless, the observed phase of $(R_m/R)_0$ is quite close to the minimum phase (Figure 2a). Tests with weaker reflectivities (not shown here) suggest that the deviations from the minimum phase are more likely due to the long time window than to the reflectivity strength (contributions from ray-paths trapped in the shallow).

A test of the cruder assumption that surface-related multiples do not change significantly the phase of R_m/R is shown in Figure 2b. As expected, the deviations are larger at higher frequencies, though near the Nyquist they diminish because both phase spectra go to zero (characteristic behavior of a phase spectrum which is a Hilbert transform of a power spectrum).

Overall, these tests suggest it might be reasonable to model the phase of $(R_m/R)_1$ by the minimum phase derived from $|R_m/R|_0$ even under most unfavorable conditions.

4.3 Modeling the phase of $(R_m/R)_1$

Figure 4a shows the error in the phase of the apparent attenuation operator derived under the minimum-phase assumption in the presence of surface-related multiples. The error reaches up to 45 degrees over the lower one-third of the spectrum which includes the frequency band of maximum trace power.

The minimum-phase spectrum in Figure 4a is based on the amplitude spectrum $|R_m/R|_1$ measurable from the seismic trace. If we could base it on $|R_m/R|_0$ instead, the phase error would be greatly reduced, as speculated in section 3 and demonstrated in Figure 4b.

5 DISCUSSION AND CONCLUSIONS

Even though the fit in Figure 4b is not perfect, it is better than that in Figure 4a. Most important is the improvement over the low-frequency end because that is where the power of the trace is concentrated. The phase deviations at high frequencies were expected –

*Since the sum of different minimum-phase operators is not minimum-phase.

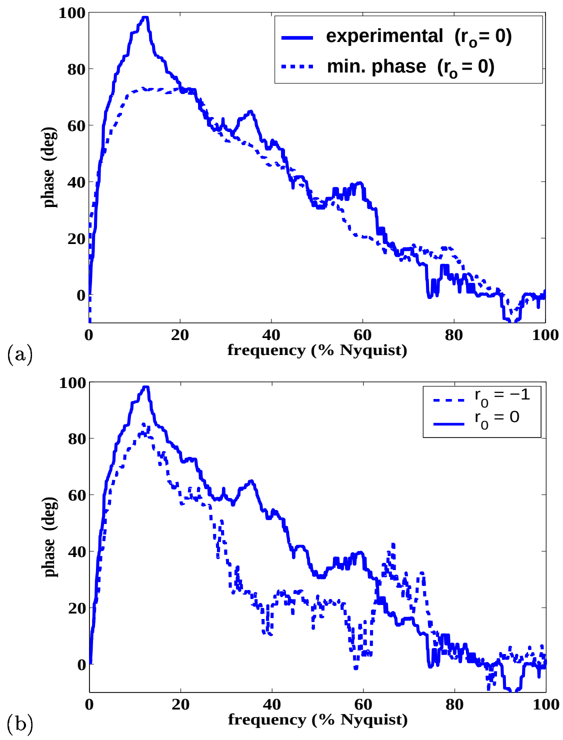

Figure 2. Assumptions test: (a) the minimum-phase property of R_m/R for a model with an absorbing surface ($r_0 = 0$); (b) free surface influence on the phase of R_m/R.

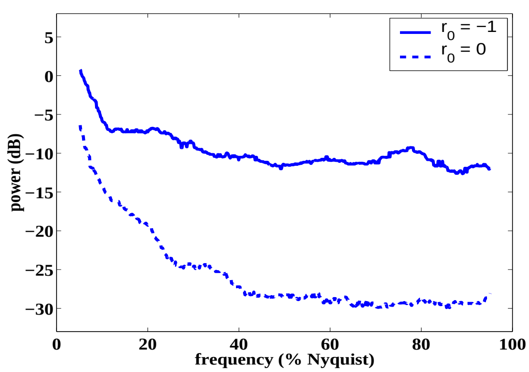

Figure 3. Power spectrum of R_m/R with and without surface-related multiples.

they are a direct consequence of the random phase of the primary reflectivity series and the assumption that $arg(R_m/R)_1 \approx arg(R_m/R)_0$.

To achieve the phase match of Figure 4b we need an estimate of $|R_m/R|_0$. The most obvious but hardly practical way to go is to suppress any surface-related multiples before inverting the trace spectrum for the wavelet parameters.

A better approach would be to exploit borehole information. For example, use well-logs to assess the impulse response of the medium, both with and without

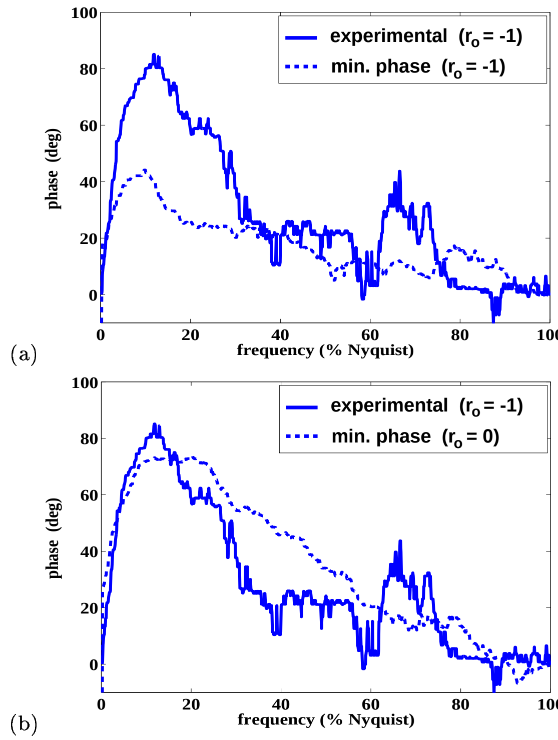

Figure 4. Modeling the phase spectrum of R_m/R: (a) the minimum-phase spectrum computed from the observed amplitude spectrum $|R_m/R|_1$ underestimates the true phase of $(R_m/R)_1$; (b) suggested fix: use the minimum-phase equivalent of $|R_m/R|_0$ instead – fits better the true phase but requires information on reflectivity (well-logs).

surface-related multiples. Then compute the difference between the minimum phase corresponding to $|R_m/R|_0$ and that, corresponding to $|R_m/R|_1$ and add it to the minimum phase corresponding to the "effective Q" measured on the trace. Preferably, do this for several time windows (the phase correction increases with time) and take offset into account in the forward modeling.

Still another possibility is to compare the wavelet model produced with the minimum-phase assumption to corridor-stacked VSP data (supposedly zero-phase) to assess the phase correction needed.

Finally, it should be pointed out that the phase-correction gets significant only in very strong reflectivities[†]. The standard assumption in seismic data processing that the stratigraphic filter is minimum-phase works well for reflectivities with up to a moderate strength, because for them the spectral slope of $|R_m/R|_0$ is practically the same as that of $|R_m/R|_1$ (Figure 5).

[†]Unless the reflectivity is white. However, strong reflectivities in the earth are necessarily blue – that's the only way to have a large number of large reflection coefficients while keeping the acoustic impedance within certain geologically-feasible bounds.

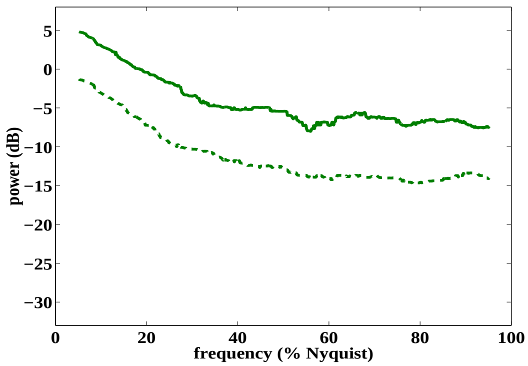

Figure 5. Power spectrum of R_m/R with (solid) and without (dashed) surface-related multiples for a primary reflectivity series with a twice smaller standard deviation than that in the main example. The two spectra have virtually the same slope.

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APPENDIX A: REFLECTION AND TRANSMISSION RESPONSE OF A LAYERED MEDIUM

Consider the normal incidence impulse response of a horizontally layered elastic medium.

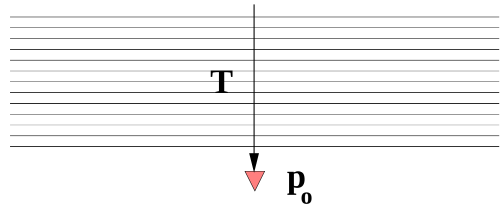

Figure A1. Transmission through a stack of layers

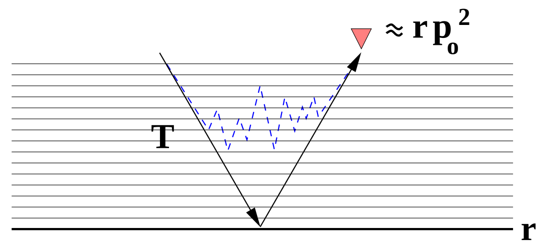

Figure A2. Two-way transmission through a stack of layers: ray-paths trapped in the shallow subsurface (dash-line) are ignored.

- *Transmission through a stack of layers (Figure A1):*

Near the direct arrival time T, the transmission response p_0 is *minimum-phase*, with an amplitude spectrum

$$|p_0| = e^{-R^2(\omega)\,T}\,, \tag{A1}$$

where $R^2(\omega)$ is the reflectivity power spectrum (O'Doherty & Anstey, 1971; Banik *et al.*, 1985).

- *Reflector r below a stack of layers (Figure A2):*

In a small window after the two-way traveltime to the reflector, the reflection impulse response p is

$$p \approx r\,p_0^2\,, \tag{A2}$$

where p_0 is the one-way transmission response already defined. Eq. (A2) is approximate because it assumes that most of the energy arriving at time $2T$ comes from the depth reached by the ballistic arrival in that time, i.e., contributions from ray-paths trapped in the shallower regions are ignored (reasonable for weak reflectivity; worsens with time). In this approximation the impulse response p is minimum-phase.

- *Buried receiver (Figure A3):*

Using the results of Banik *et al.* (1985), it's easy to see

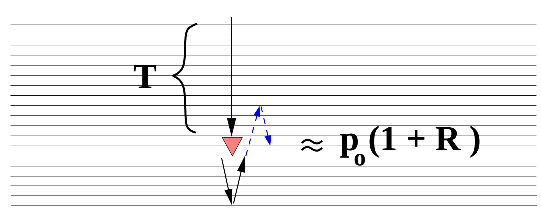

Figure A3. Signal in a buried receiver: direct arrival and primary reflections from below; multiples of the reflections from below (dash-line) are ignored.

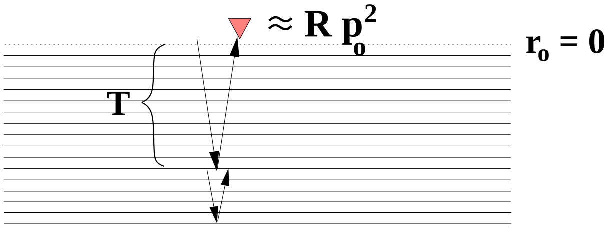

Figure A4. Signal in a surface receiver (earth surface omitted)

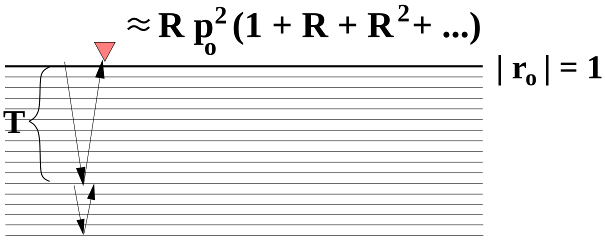

Figure A5. Signal in a surface receiver (earth surface accounted for)

that near the time T of the direct arrival the impulse response p in a buried receiver is

$$p \approx p_0 \left(1 + R(\omega)\right), \tag{A3}$$

This approximation ignores multiples of the reflections from below the receiver as well as time-changes in the down-going pulse over the considered time window.
Since the primary reflectivity $R(\omega)$ has a random phase, the signal in a down-hole receiver is not minimum-phase. However, it may appear approximately minimum-phase at low frequencies where the transmission $|p_0|$ is strong and $|R|$ is weak.

- *Surface seismogram without surface-related multiples (earth model with an absorbing surface) (Figure A4):*

$$p \approx R(\omega)\, p_0^2 \tag{A4}$$

This is approximate as in the previous two cases; not minimum-phase because $R(\omega)$ has a random phase. Eq. (A4) predicts high-frequency loss with time because of the time dependence in p_0 (eq. (A1)) and $R(\omega)$ being blue.

- *Surface seismogram with surface-related multiples (earth model with a free surface) (Figure A5):*

In a small time window, the impulse response has the form

$$p \approx R(\omega)\, p_0^2 \left[1 + R(\omega) + R^2(\omega) + ... + R^{N(T)}(\omega)\right] \tag{A5}$$

The terms proportional to powers of $R(\omega)$ in the brackets account for surface-related multiples of different orders. For a discrete (layered) medium, the highest possible order N increases with time. Surface-related multiples compensate the high-frequency loss with time that would otherwise occur along the seismogram, according to eq. (A4) (White *et al.*, 1990; Mateeva, 2001).

APPENDIX B: POWER SPECTRUM OF R_m/R

The power spectrum of R_m/R is governed by the power spectrum of the primary reflectivity series $R(\omega)$. Strictly speaking, $|R_m/R|$ is not linear on a semi-log scale (dB/Hz) but usually it can be approximated by a straight line over a large portion of the seismic frequency range. As seen in Figure 3 and Figure 5, $|R_m/R|$ tends to whiten (level off) at high frequencies but this effect may or may not be seen in seismic data, depending on geology, frequency band, length of the time window for spectral estimation, presence of surface-related multiples and background noise.

When the slope change is observable, we may need to restrict the frequencies, used for effective-Q estimation. In fact, the whitening might be beneficial – it may allow us to evaluate intrinsic absorption from the spectral slope over the band in which R_m/R is approximately frequency-independent. One could either manually divide the trace spectrum into two regions with a different slope and fit them separately for $Q_{effective}$ and $Q_{intrinsic}$, or attempt fitting a trace model of the kind

$$X(\omega) = \sigma_s W_s e^{-\frac{\omega}{Q_i}} e^{-\frac{\omega}{Q_a} H(\omega_c - \omega)} e^{-\frac{\omega_c}{Q_a} H(\omega - \omega_c)} + \sigma_n W_n \tag{B1}$$

where $X(\omega)$ is the power spectrum of the trace, $\sigma_s = const$, $\sigma_n = const$, W_s accounts for source and receiver signatures, W_n – for non-white background noise (Hart *et al.*, 2001), Q_i is the quality factor corresponding to intrinsic absorption, Q_a corresponds to apparent attenuation (short-period multiples), ω_c is the corner frequency above which R_m/R is white (or can be considered as such given the variability of the spectral estimate), and $H(.)$ is the Heaviside step-function. To reduce the number of parameters to be fit, ω_c can be set by hand and, perhaps, the ambient noise term $\sigma_n W_n$ omitted since the high-frequency "tail" of the spectrum is now included in the signal model[‡]. The attenuation at low frequencies ($\omega < \omega_c$) can be described by an effective Q given by

$$\frac{1}{Q_{eff}} = \frac{1}{Q_i} + \frac{1}{Q_a}, \tag{B2}$$

while the attenuation at high frequencies ($\omega > \omega_c$) is caused, presumably, by intrinsic absorption and described by Q_i.

[‡]Moreover, the ambient noise estimated from a window before the first arrival is typically much weaker than the source-generated "noise" later on the trace.