CWP-405

Quartic moveout coefficient: 3-D analytic description and application to tilted TI media

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ABSTRACT

Nonhyperbolic (long-spread) moveout provides essential information for a number of seismic inversion/processing applications, particularly for parameter estimation in anisotropic media. Here, we present an analytic expression for the quartic moveout coefficient A_4 that is responsible for the magnitude of nonhyperbolic moveout of pure (non-converted) modes. Our result takes into account reflection-point dispersal on irregular interfaces and is valid for arbitrarily anisotropic, heterogeneous media. All quantities needed to compute A_4 can be evaluated during the tracing of a single (zero-offset) ray, so long-spread moveout can be modeled without time-consuming multi-offset, multi-azimuth ray tracing.

The general equation for the quartic coefficient is used to study azimuthally varying nonhyperbolic moveout of P-waves in a dipping transversely isotropic (TI) layer with an arbitrary tilt ν of the symmetry axis. Assuming that the symmetry axis is confined to the dip plane, we employed the weak-anisotropy approximation to gain insight into the dependence of A_4 on the anisotropic parameters. The linearized expression for A_4 is proportional to the anellipticity coefficient $\eta \approx \epsilon - \delta$ and does not depend on the individual values of the Thomsen parameters. On the whole, the magnitude of nonhyperbolic moveout in tilted TI media above a dipping reflector usually is highest near the reflector strike, while deviations from hyperbolic moveout on the dip line are substantial only for mild dips.

The azimuthal variation of the quartic coefficient is governed by the tilt ν and dip ϕ and has a much more complicated character than the NMO ellipse. For example, in a VTI layer above a reflector with the dip $\phi > 30°$, A_4 goes to zero on two lines with different azimuths where it changes sign. If the symmetry axis is orthogonal to the reflector (this model is typical for thrust-and-fold belts), the strike-line quartic coefficient is defined by the well-known expression for a horizontal VTI layer (i.e., it is independent of dip), while dip-line A_4 is proportional to $\cos^4 \phi$ and rapidly decreases with dip. The high sensitivity of the quartic moveout coefficient to the parameter η and the tilt of the symmetry axis can be exploited in the inversion of wide-azimuth, long-spread P-wave data for the parameters of TI media.

Key words: TI media, P-waves, pure S-waves, nonhyperbolic moveout

Introduction

In conventional seismic data processing, reflection moveout of pure modes is typically assumed to be hyperbolic, at least for spreadlengths not exceeding reflector depth. However, the presence of heterogeneity (either lateral or vertical) or anisotropy causes deviations from hyperbolic moveout which sometimes cannot be ignored even for offsets-to-depth ratios smaller than unity (e.g., Al-Dajani and Tsvankin, 1998). Insufficient understanding of nonhyperbolic moveout and practical difficulties in analyzing long-offset data often forces seismic processors to mute out the nonhyperbolic portion of the moveout curve. Long-spread moveout, however, has proved useful in a number of applications, such as anisotropic parameter estimation, suppression of multiples and large-angle AVO analysis.

A detailed overview of existing results on nonhyperbolic moveout analysis in anisotropic media can be found in Tsvankin (2001). Most earlier work on the contribution of anisotropy to long-spread moveout (e.g., Hake et al., 1984; Byun and Corrigan, 1990; Muir et al., 1993) is restricted to transversely isotropic models with a vertical symmetry axis (VTI media). Tsvankin and Thomsen (1994) developed a general nonhyperbolic moveout equation based on the exact normal-moveout (NMO) velocity and the quartic moveout coefficient of the $t^2(x^2)$–function. In contrast to the conventional Taylor series, the Tsvankin-Thomsen equation converges at offsets approaching infinity, which ensures its high accuracy in the intermediate offset range (i.e., for offsets two to three times the reflector depth) important in reflection seismology. A particularly convenient form of this equation for P-waves in VTI media was suggested by Alkhalifah and Tsvankin (1995) (also see Grechka and Tsvankin, 1998a) who showed that P-wave reflection moveout, as well as other time-domain signatures, is controlled by just the NMO velocity and the anellipticity coefficient η defined as $\eta \equiv (\epsilon - \delta)/(1 + 2\delta)$; ϵ and δ are Thomsen (1986) parameters. The equation of Alkhalifah and Tsvankin (1995) has been widely used for estimating η from P-wave long-spread traveltimes and build vertically heterogeneous VTI models for time processing (e.g., Alkhalifah, 1997; Toldi et al., 1999).

The behavior of nonhyperbolic moveout becomes much more complicated if the medium is azimuthally anisotropic. Al-Dajani and Tsvankin (1998) derived the quartic moveout coefficient for TI media with a horizontal symmetry axis (HTI) and extended the Tsvankin-Thomsen equation to horizontally layered HTI media. A different method based on spherical harmonics was employed by Sayers and Ebrom (1997) to describe long-spread P-wave moveout in an azimuthally anisotropic layer. It should be emphasized that all papers listed above treat laterally homogeneous models with a horizontal symmetry plane, in which the derivation of the quartic moveout coefficient for pure (non-converted) modes is simplified by the absence of reflection-point dispersal on common-midpoint (CMP) gathers. Fomel (1994) developed a more general approach to the analytic description of nonhyperbolic moveout that accounts for reflection-point dispersal at dipping or curved interfaces. Fomel and Grechka (2001) applied this methodology to P-wave moveout in heterogeneous VTI media.

Here, we present an analytic description of P-wave long-spread moveout in TI media with an arbitrary orientation of the symmetry axis. Models with the symmetry axis tilted away from the vertical (TTI, or tilted TI media) are typical for fold-and-thrust belts such as the Canadian Foothills and sediments near the flanks of salt domes (Isaac and Lawton, 1999; Tsvankin, 1997, 2001). Non-vertical symmetry axis creates an azimuthally anisotropic model without a horizontal symmetry plane, where nonhyperbolic moveout is influenced by reflection-point dispersal. We present analytic expressions for the quartic moveout term for both horizontal and dipping reflectors and study the azimuthal dependence of nonhyperbolic moveout as a function of reflector dip and symmetry-axis orientation. Strong azimuthal variations of nonhyperbolic moveout (e.g., it completely vanishes in certain azimuthal directions) may be used to constrain anisotropic parameter-estimation algorithms operating with wide-azimuth reflection traveltimes.

Analytic description of nonhyperbolic moveout

Reflection traveltime of pure (non-converted) modes is conventionally approximated by a Taylor series expansion of the squared traveltime t^2, which is often truncated after the quartic term (Taner and Koehler, 1969):

$$t^2 = A_0 + A_2 X^2 + A_4 X^4 \,, \tag{1}$$

where X is the source-receiver offset and

$$A_0 = t_0^2, \;\; A_2 = \left. \frac{d(t^2)}{d(X^2)} \right|_{X=0} ,$$
$$A_4 = \frac{1}{2} \frac{d}{d(X^2)} \left. \left[\frac{d(t^2)}{d(X^2)} \right] \right|_{X=0} . \tag{2}$$

$t_0 = t(0)$ is the squared zero-offset traveltime, and A_2 is related to the NMO velocity as $A_2 = V_{\rm nmo}^{-2}$. The first two terms in equation (1) describe the hyperbolic part of the moveout curve, while A_4 is the quartic coefficient primarily responsible for nonhyperbolic moveout.

Although the series (1) provides a better approximation for long-spread moveout than does the conventional hyperbolic equation based on just the NMO velocity, it loses accuracy for offsets reaching 1.5-2 z (z is the reflector depth). Tsvankin and Thomsen (1994) modified equation (1) by adding a denominator to the quartic moveout term to make $t(X)$ convergent at $X \to \infty$:

$$t^2 = A_0 + A_2 X^2 + \frac{A_4 X^4}{1 + A X^2}, \tag{3}$$

where the additional coefficient A depends on the horizontal group velocity $V_{\rm hor}$,

$$A = \frac{A_4}{V_{\rm hor}^{-2} - V_{\rm nmo}^{-2}}. \tag{4}$$

Equation (3) was originally derived for VTI media, but its generic form makes it suitable for anisotropic media of any symmetry. For example, Al-Dajani and Tsvankin (1998) obtained the exact moveout parameters A_2 ($V_{\rm nmo}$) and A_4 for a horizontal HTI layer and used them to extend equation (3) to azimuthally dependent P-wave moveout in HTI media. While equation (3) is not always adequate for pure (non-converted) S-waves (Tsvankin and Thomsen, 1994), it provides a simple and numerically efficient way for modeling the reflection traveltimes of P-waves and, for relatively simple models, converted waves (Tsvankin, 2001).

For horizontally layered anisotropic media above the reflector, the effective NMO velocity (or the effective A_2) in a certain azimuthal direction can be determined from the generalized Dix formula of Grechka et al. (1999) that operates with interval NMO ellipses. Grechka and Tsvankin (2002) extended this Dix-type averaging equation for NMO ellipses to anisotropic media with arbitrary heterogeneous overburden. The velocity $V_{\rm hor}$ used in equation (4) to define the coefficient A is found by averaging the interval horizontal velocities (Tsvankin, 2001).

Therefore, the key issue in applying equation (3) is to derive the corresponding quartic moveout coefficient A_4. The dependence of A_4 on the medium parameters also yields valuable analytic insight into the properties of nonhyperbolic moveout. Below, we give a general representation of the coefficient A_4 and discuss its behavior for TI media with arbitrary symmetry-axis orientation.

General expression for the quartic moveout coefficient

Here, we present an exact expression for the quartic moveout coefficient in arbitrarily anisotropic, heterogeneous media (Figure 1). The derivation, described in detail in Appendix A, is based on expanding the two-way traveltime in a Taylor series in half-offset and applying the so-called normal-incidence-point (NIP) theorem (Chernjak and Gritsenko, 1979; Hubral and Krey, 1980; Fomel and Grechka, 2001) that helps to relate the Taylor series coefficients to the spatial derivatives of the zero-offset traveltime. The general form of the quartic moveout coefficient A_4 can be represented as (Appendix A):

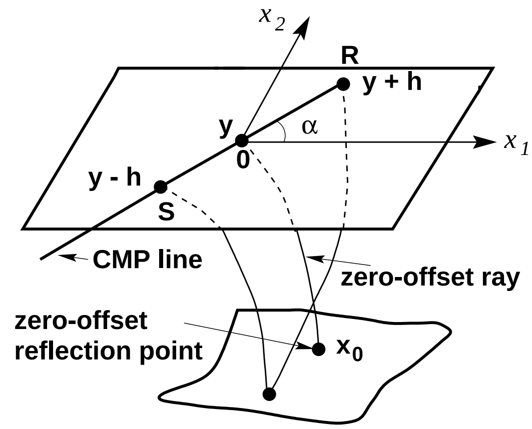

Figure 1. Reflection traveltimes from an irregular interface are recorded in a multi-azimuth CMP gather over an arbitrarily anisotropic, heterogeneous medium. The quartic moveout coefficient A_4 varies with the azimuth α of the CMP line. **h** is the half-offset vector. The derivation of the quartic coefficient in Appendix A takes into account reflection-point dispersal.

$$\begin{aligned} A_4 &= \frac{\tau_0}{48} \frac{\partial^4 \tau_0}{\partial y_p \partial y_k \partial y_m \partial y_n} L_p L_k L_m L_n \\ &- \frac{\tau_0}{16} \frac{\partial^3 \tau_0}{\partial x_i \partial y_k \partial y_m} \left(\frac{\partial^2 \tau_0}{\partial x_i \partial x_j} \right)^{-1} \\ &\frac{\partial^3 \tau_0}{\partial x_j \partial y_p \partial y_n} L_k L_m L_p L_n \\ &+ \frac{1}{16\, \tau_0^2\, V_{\rm nmo}^4(\mathbf{L})}, \end{aligned} \tag{5}$$

where $\mathbf{y}$ defines the CMP location, $\mathbf{x}$ defines the reflection point, τ_0 is the one-way zero-offset traveltime, $\mathbf{L}$ is a unit vector parallel to the CMP line and $V_{\rm nmo}(\mathbf{L})$ is the NMO velocity on the line $\mathbf{L}$. Since in our case the traveltime is recorded at the surface, $\mathbf{L} = [\cos\alpha, \sin\alpha, 0]$, where α is the azimuth of the CMP line with respect to the axis x_1.

Equation (5) was obtained without making specific assumptions about the anisotropy or heterogeneity of the model; also, it is generally valid for reflectors of irregular shape. However, our derivation assumes that the one-way zero-offset traveltime can be differentiated with respect to the spatial coordinates near the common midpoint, which is not the case, for example, in shadow zones. The Taylor series expansion for reflection traveltime may break down for models with strong lateral velocity variations (Grechka and Tsvankin, 1998b) and in the vicinity of caustics. Nonetheless, for sufficiently smooth subsurface models commonly used in seismology, equation (5) can be expected to give an accurate representation of the quartic moveout coefficient and the magnitude of nonhyperbolic moveout.

The form of the azimuthal dependence of the coefficient A_4 in equation (5) is governed by the derivatives of the zero-offset traveltime τ_0 with respect to the coordinates of the common midpoint and zero-offset reflection point. For relatively simple models, the traveltime τ_0 can be expressed explicitly as a function of $\mathbf{y}$ and $\mathbf{x}$, and the derivatives in equation (5) can be evaluated in closed form. However, if the medium is laterally heterogeneous and/or has a low anisotropic symmetry, it is convenient to express equation (5) in terms of the horizontal slowness component of the zero-offset ray (Cohen, 1998; Grechka et al., 1999). Most importantly, all derivatives in equation (5) can be evaluated using quantities computed during the tracing of the zero-offset ray.

Quartic coefficient in a homogeneous TTI layer

While equation (5) is completely general, this paper is restricted to analysis of nonhyperbolic moveout in a homogeneous TTI layer overlaying a planar dipping reflector. Furthermore, we assume that the symmetry axis is confined to the dip plane of the reflector, which is typical for dipping TI formations (e.g., shales) in fold-and-thrust belts (Isaac and Lawton, 1999) or near salt domes (Tsvankin, 1997). Hyperbolic reflection moveout and the dependence of NMO velocity on the anisotropic parameters for this model was discussed by Tsvankin (1997, 2001) and Grechka and Tsvankin (2000). Following Tsvankin (1997), we parameterize the medium by the symmetry-direction velocities of P-waves (V_{P0}) and S-waves (V_{S0}) and Thomsen's anisotropic coefficients ϵ, δ and γ specified with respect to the symmetry axis. In other words, the parameters are defined by the VTI equations in the rotated coordinate system whose x_3-axis is aligned with the axis of symmetry. The tilt ν of the symmetry axis is considered positive if the axis points towards the reflector (i.e., if the symmetry axis and the reflector normal deviate from the vertical in the same direction).

Since the dip plane of the reflector contains the symmetry axis of the overburden, it represents a vertical symmetry plane for the whole model. Therefore, the dip and strike directions of the reflector determine "the principal axes" of the azimuthally-varying quartic moveout coefficient A_4. Below, we use equation (5) to study the functional form of A_4 in a TTI layer and its dependence on the reflector dip and anisotropic parameters.

For a homogeneous medium, the zero-offset traveltime τ_0 can be expressed explicitly in terms of the CMP and reflection-point coordinates (see Appendix B). This allows us to evaluate the spatial derivatives of τ_0 and obtain the coefficient A_4 from equation (5). While the exact equation for the quartic coefficient is suitable for computational purposes, it does not provide analytic insight into the dependence of A_4 on the model parameters. As demonstrated in Appendix B, significant simplification can be achieved by applying the weak-anisotropy approximation and linearizing equation (5) in the anisotropic parameters.

Although the discussion of the weak-anisotropy results below is formally limited to P-waves, any kinematic signature of SV-waves (i.e., of the mode polarized in the plane formed by the slowness vector and the symmetry axis) for weak transverse isotropy can be obtained from the corresponding P-wave signature by making the following substitutions: $V_{P0} \to V_{S0}$, $\delta \to \sigma$, and $\epsilon \to 0$ (Tsvankin, 2001). The parameter $\sigma \equiv (V_{P0}/V_{S0})^2(\epsilon - \delta)$ is fully responsible for SV-wave velocity variations in weakly anisotropic TI media.

The linearized P-wave quartic moveout coefficient in a TTI layer can be written as (Appendix B)

$$A_4^{\mathrm{TTI}} = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\left[F(\alpha,\,\phi,\,\nu)\,+\,C\right], \tag{6}$$

where $C = 9/64$ is a constant, the function F is defined in equation (B14), t_{P0} is the two-way zero-offset traveltime, α is the azimuth of the CMP line measured from the dip plane and ϕ is the reflector dip.

Figure 2 shows that the linearized equation (6) is sufficiently close to the exact quartic coefficient for relatively small values of the anisotropic parameters. The diamonds in Figure 2 correspond to the coefficient A_4 obtained by least-squares fitting of a quartic polynomial to reflection traveltimes generated by anisotropic ray tracing. Evidently, equation (6) (solid curve) provides a good approximation to the best-fit values of A_4 for the full range of azimuths. As demonstrated by Tsvankin and Thomsen (1994), the weak-anisotropy approximation may rapidly lose its accuracy with increasing parameters ϵ and δ. However, equation (6) can still be used for qualitative analysis of nonhyperbolic moveout in tilted TI media.

Analysis of the approximate quartic coefficient

It is clear from equation (6) that regardless of the tilt of the symmetry axis and reflector dip, the P-wave quartic moveout for weak transverse isotropy is controlled by a single anisotropic parameter – the anellipticity coefficient η. If the medium is elliptical ($\eta = 0$), A_4 vanishes and reflection moveout becomes purely hyperbolic. This is a

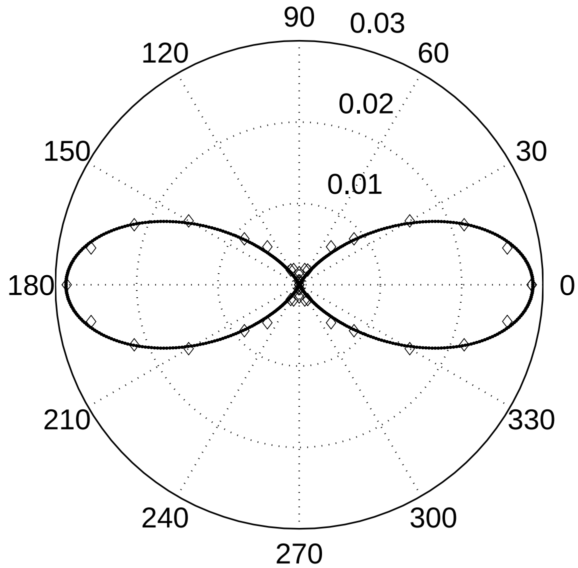

Figure 2. Accuracy of the linearized quartic moveout coefficient for a tilted TI layer. The diamonds mark values of A_4 obtained for each azimuth by fitting a quartic polynomial to the ray-traced $t^2(x^2)$–curve on the spreadlength $X_{\max} \approx z$, where $z = 1$ km is the reflector depth. The solid line is the weak-anisotropy approximation (6). The model parameters are $V_{P0} = 1$ km/s, $\epsilon = 0.1$, $\delta = 0.025$, $\phi = 0°$ and $\nu = 80°$.

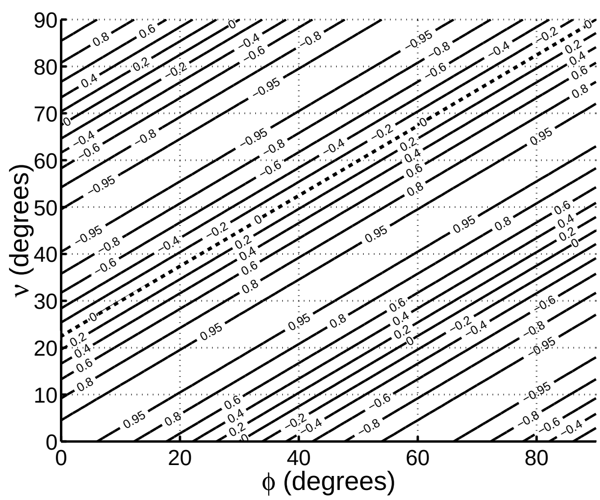

Figure 3. The factor $\cos(4\nu - 3\phi)$ plotted as a function of the tilt ν of the symmetry axis and reflector dip ϕ.

general result valid for an elliptically anisotropic layer with any strength of the anisotropy (Uren et al., 1990).

Dip and strike components of A_4

Equations (6) and (B14) can be used to find the coefficients A_4 in the dip and strike directions. On the dip line ($\alpha = 0°$),

$$A_{4,\text{dip}}^{\text{TTI}}(\phi) = -\frac{2\,\eta}{t_{P0}^2\, V_{P0}^4}\, \cos^3\phi\, \cos(4\nu - 3\phi)\,. \tag{7}$$

Note that the quartic coefficient is proportional to $\cos^3\phi$, and the magnitude of nonhyperbolic moveout has a general decreasing trend with dip [the influence of the term $\cos(4\nu - 3\phi)$ is discussed below]. Equation (7), however, becomes less accurate for near-vertical reflectors because for ϕ close to 90° several terms involving anisotropic coefficients can no longer be treated as small. Evaluation of the exact equation (5) shows that unless the symmetry axis is vertical or horizontal, A_4 for a vertical reflector ($\phi = 90°$) is relatively small but does not go to zero.

According to equation (7), the quartic moveout coefficient (and, therefore, nonhyperbolic moveout as a whole) vanishes if $\cos(3\phi - 4\nu) = 0$, or $(3\phi - 4\nu) = n\pi/2$ ($n = \pm1, \pm3, \pm5, ...$). In the special case of VTI media ($\nu = 0$), the quartic coefficient and nonhyperbolic moveout as a whole vanish for a dip of 30° (see a more detailed discussion of the VTI model below).

Figure 3 displays the function $\cos(3\phi - 4\nu)$ plotted for the angles ϕ and ν ranging from 0° to 90°. Since the argument $3\phi - 4\nu$ is a linear combination of ϕ and ν, zero values of $\cos(3\phi - 4\nu)$ are represented by straight lines. For instance, the dashed line in Figure 3 corresponds to $3\phi - 4\nu = -\pi/2$, or

$$\nu = \frac{3\phi}{4} + \frac{\pi}{8}\,. \tag{8}$$

For a fixed reflector dip, $\cos(3\phi - 4\nu)$ goes to zero for two different values of the tilt ν between 0° and 90°, which is in good agreement with the computations of analytic (NMO) and finite-spread moveout velocity in Tsvankin (1995, 2001). Hence, the absence or low magnitude of dip-line nonhyperbolic moveout in nonelliptical ($\eta \neq 0$) TTI media may be used to constrain the relationship between the reflector dip and the tilt of the symmetry axis.

Equation (7) is written in terms of reflector dip that cannot be estimated from surface reflection data unless the velocity model is known. Therefore, for purposes of anisotropic parameter estimation, it is more convenient to rewrite the quartic coefficient as a function of the horizontal component p of the slowness vector associated with the zero-offset ray (e.g., Alkhalifah and Tsvankin, 1995). The horizontal slowness component, or the ray parameter, determines the slope of reflections on zero-offset (or stacked) sections and can be measured directly from surface data.

Substituting the ray parameter $p = \sin\phi / V(\phi)$ [$V(\phi)$ is the phase velocity at the dip angle] into equation (7) yields

$$A_{4,\mathrm{dip}}^{\mathrm{TTI}}(p) = \frac{8\,\eta\,(1-y)^3}{t_{P0}^2\,[V_{\mathrm{nmo,dip}}^{\mathrm{TTI}}(0)]^4}\left[\left(y-\frac{1}{4}\right)\sqrt{1-y}\cos 4\nu + \left(y-\frac{3}{4}\right)\sqrt{y}\sin 4\nu\right], \quad (9)$$

where

$$y \equiv p^2\,[V_{\mathrm{nmo}}^{TTI}(0)]^2 \quad (10)$$

and $V_{\mathrm{nmo}}^{TTI}(0)$ is the NMO velocity from a horizontal reflector. Hence, $A_{4,\mathrm{dip}}^{\mathrm{TTI}}$ expressed as a function of p depends on three parameters: $V_{\mathrm{nmo}}^{TTI}(0)$, η and ν [or $V_{\mathrm{nmo}}^{TTI}(0)$, $\eta\cos 4\nu$ and $\eta\sin 4\nu$]. In principle, the quartic moveout coefficient can be inverted for these parameters, if accurate estimates of $A_{4,\mathrm{dip}}^{\mathrm{TTI}}$ are available for three different dips. The high level of structural complexity in overthrust areas or near salt domes in some cases may be sufficient for reconstructing the function $A_{4,\mathrm{dip}}^{\mathrm{TTI}}(p)$. However, as discussed by Grechka and Tsvankin (1998a), the trade-off between the NMO velocity and quartic moveout coefficient typically leads to a substantial uncertainty in A_4.

In anisotropic parameter estimation, nonhyperbolic moveout should be used in combination with the NMO velocity (e.g., Alkhalifah, 1997; Grechka and Tsvankin, 1998a). The dip-line P-wave NMO velocity for weakly anisotropic TTI media was given by Tsvankin (1997, 2001) as a function of dip. Rewriting his result through the ray parameter p yields

$$V_{\mathrm{nmo}}^{\mathrm{TTI}}(p) = \frac{V_{\mathrm{nmo}}^{TTI}(0)}{\sqrt{1-y}}\,[1 + f\eta\,\cos 4\nu - g\eta\,\sin 4\nu]\,, \quad (11)$$

where

$$f \equiv \frac{y}{1-y}\,(6 - 9y + 4y^2) \quad (12)$$

and

$$g \equiv \sqrt{\frac{y}{1-y}}\,(3 - 7y + 4y^2)\,. \quad (13)$$

For vertical transverse isotropy ($\nu = 0$), equation (11) reduces to the expression derived by Alkhalifah and Tsvankin (1995). Evidently, both the dip-line NMO velocity and the quartic moveout coefficient are fully governed by the same parameter combinations: $V_{\mathrm{nmo}}^{\mathrm{TTI}}(0)$, $\eta\cos 4\nu$, and $\eta\sin 4\nu$. Note, however, that according to Tsvankin (1997, 2001), the weak-anisotropy approximation for NMO velocity loses accuracy for the anisotropic coefficients reaching 0.15-0.2, and the exact V_{nmo} becomes dependent on the individual values of ϵ and δ.

Next, we analyze the strike component of the quartic moveout coefficient that can be obtained by substituting $\alpha = 90^\circ$ into equation (6):

$$A_{4,\mathrm{strike}}^{\mathrm{TTI}}(\phi) = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\,\cos^4(\phi-\nu)\,. \quad (14)$$

Both the dip and strike components of A_4 are proportional to η, but their dependencies on reflector dip ϕ and the symmetry-axis tilt ν are entirely different. Equation (14) shows that $A_{4,\mathrm{strike}}^{\mathrm{TTI}}$ goes to zero only if the symmetry axis is perpendicular to the reflector normal (i.e., the symmetry axis is confined to the reflecting plane). For example, if the reflector is vertical ($\phi = 90^\circ$), the strike-line quartic coefficient vanishes for VTI media ($\nu = 0^\circ$). Indeed, for such a model reflected rays are confined to the horizontal (isotropy) plane where velocity is independent of angle, which makes reflection moveout for any azimuth purely hyperbolic. On the whole, the dip and strike components of the quartic coefficient vanish for different combinations of ν and ϕ and, therefore, can be effectively combined in the inversion procedure.

If the symmetry axis is orthogonal the reflector ($\phi = \nu$), $A_{4,\mathrm{strike}}^{\mathrm{TTI}}$ is independent of both dip and tilt:

$$A_{4,\mathrm{strike}}^{\mathrm{TTI}} = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\,. \quad (15)$$

Equation (15) is well known for the special case of VTI media and a horizontal reflector, when $\phi = \nu = 0$ (Tsvankin and Thomsen, 1994). Another special case of interest is that of an HTI layer ($\nu = 90^\circ$) and a vertical reflector ($\phi = 90^\circ$). Since the strike line for this model is perpendicular to the symmetry axis, and reflected rays are horizontal, reflection moveout in the strike direction is identical to that for a VTI layer above a horizontal reflector.

The dip-line component of A_4 for $\phi = \nu$ is proportional to $\cos^4\nu$ [equation (7)], so it rapidly decreases with dip, while the strike-line component remains constant. Therefore, nonhyperbolic moveout from dipping reflectors for this model should be measured close to the strike direction; a more detailed discussion of the azimuthal dependence of A_4 is given below.

Azimuthal dependence of A_4

Unlike NMO velocity that has a simple elliptical azimuthal dependence (Grechka and Tsvankin, 1998b), the variation of the quartic moveout coefficient with azimuth has a much more complicated character. The nonlinear relationship between A_4 and the angles ϕ, ν and α [equation (6)] leads to multiple zeros of the function $A_4(\alpha)$ whose positions strongly depend on both dip ϕ and tilt ν. Figure 4 displays a polar plot with a typical azimuthal signature of the quartic coefficient in TTI media. Clearly, A_4 exhibits much more variability compared to the NMO ellipse, with zeros at azimuths of 38° and 142°. (The quartic coefficient and moveout signature as a whole have

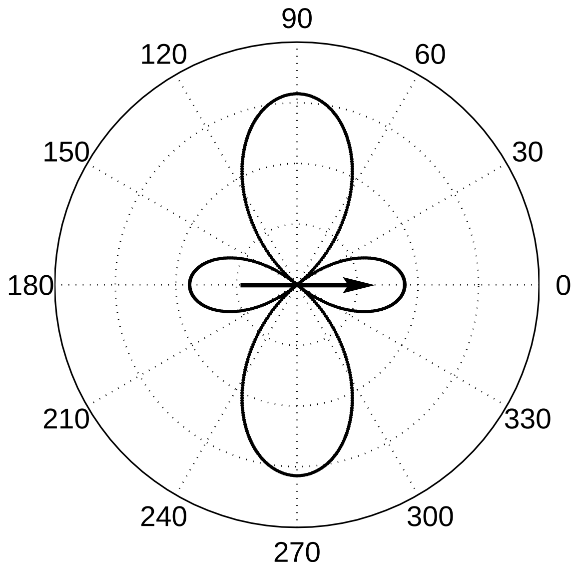

Figure 4. Azimuthally-varying quartic moveout coefficient A_4 for a TTI layer computed from equation (6). The polar radius is equal to the coefficient A_4 in the corresponding azimuthal direction (the azimuth is measured from the dip plane marked by the arrow). The reflector dip is $\phi = 15^\circ$ and the tilt of the symmetry axis is 40°; the other parameters change only the scale of the plot (intentionally undefined here).

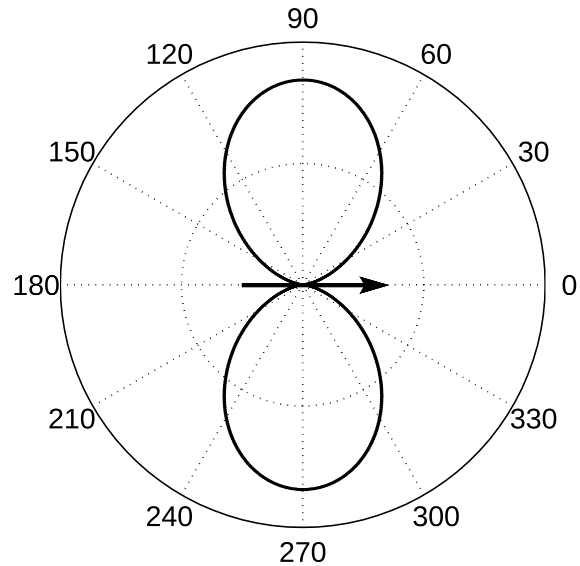

Figure 5. Azimuthally-varying coefficient A_4 for a VTI layer computed from equation (19). Reflector dip is 30°; the dip direction is marked by the arrow.

an azimuthal period of 180°.) The sign of the coefficient A_4 changes from positive near the dip direction (i.e., for the horizontally oriented lobe) to negative for the other lobe corresponding to $38^\circ < \alpha < 142^\circ$.

Evidently, the azimuthal signature of the quartic coefficient can provide useful information for anisotropic parameter estimation. In particular, the azimuthal directions of the CMP lines with vanishing A_4 depend on certain combinations of ϕ and ν and can be used to constrain the orientation of the symmetry axis [equation (6)]. The variation of the sign of A_4 with azimuth is also sensitive to both ϕ and ν.

Symmetry axis orthogonal to the reflector

Because of the complicated structure of equation (6), here we focus on several special cases of practical importance. Models with the symmetry axis orthogonal to the reflector ($\phi = \nu$) are believed to be typical for fold-and-thrust belts (e.g., the Canadian Foothills) where the anisotropy is caused by dipping TI shale layers. For TI media with $\phi = \nu$, the zero-offset ray is parallel to the symmetry axis and orthogonal to the reflector, so some features of reflection moveout are similar to those for a horizontal VTI layer. For example, Tsvankin (1995, 2001) demonstrated that if $\phi = \nu$, the dip-line NMO velocity obeys the conventional (isotropic) cosine-of-dip dependence.

The dip and strike components of the quartic coefficient for this model were discussed above. To study the azimuthal dependence of A_4, we substitute $\phi = \nu$ into equation (6) to obtain

$$A_4^{\rm TTI}(\phi = \nu) = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\,\left(1 - \sin^2\nu\cos^2\alpha\right)^2 . \qquad (16)$$

According to equation (16), the quartic coefficient goes to zero when

$$|\cos\alpha| = \frac{1}{|\sin\nu|} . \qquad (17)$$

Condition (17) can be satisfied only on the dip line ($\alpha = 0^\circ$) of a vertical reflector ($\nu = 90^\circ$, which implies a horizontal symmetry axis). Away from the dip line, the coefficient A_4 for a vertical reflector varies as

$$A_4^{\rm TTI}(\phi = \nu = 90^\circ) = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\,\sin^4\alpha . \qquad (18)$$

Equation (18) shows that A_4 rapidly decays with increasing deviation from the strike direction ($\alpha = 90^\circ$). This conclusion remains valid for arbitrary reflector dip, because the strike component for this model is independent of ν and α [equation (15)], while the dip component is proportional to $\cos^4\nu$.

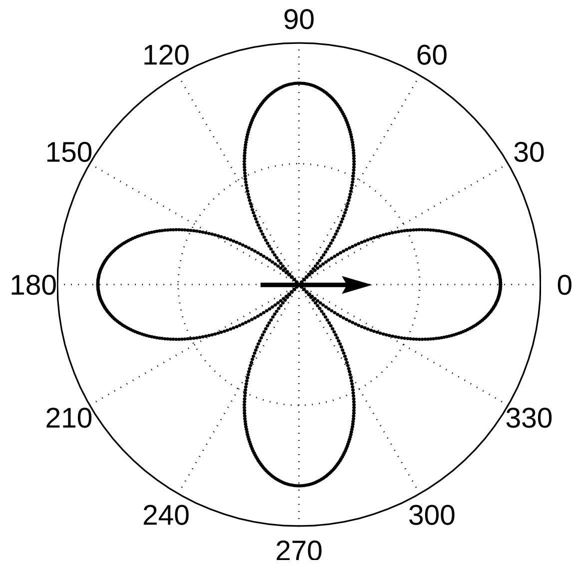

Figure 6. Same as Figure 5, but the reflector dip is 45°.

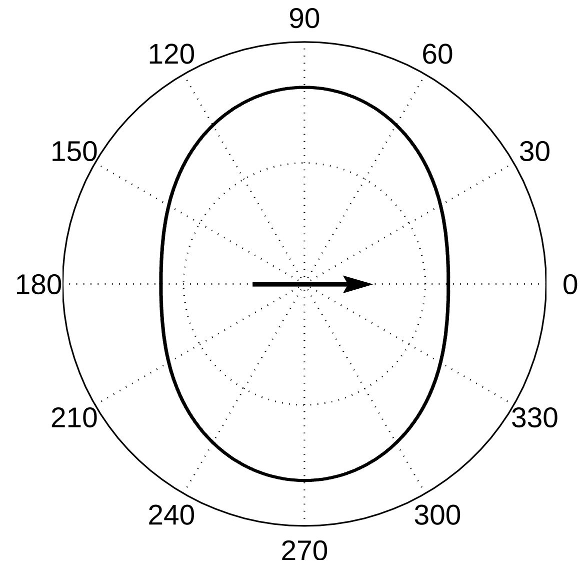

Figure 7. Same as Figure 5, but the reflector dip is 15°.

Dipping reflector beneath a VTI layer

Setting the tilt ν of the symmetry axis in equation (6) to zero yields the weak-anisotropy approximation for the quartic coefficient in VTI media:

$$A_4^{\rm VTI} = -\frac{2\,\eta\,\cos^4\phi}{t_{P0}^2\,V_{P0}^4}\,\left(1 - 4\,\sin^2\phi\,\cos^2\alpha\right)\,. \tag{19}$$

For a vertical reflector ($\phi = 90°$), A_4 vanishes regardless of the azimuth of the CMP line because reflected rays are confine to the horizontal isotropy plane where velocity is constant and moveout is purely hyperbolic. If the reflector is horizontal ($\phi = 0°$), the model as a whole is azimuthally isotropic, and the approximate A_4 is determined by the well-known expression (Tsvankin and Thomsen, 1994; Alkhalifah and Tsvankin, 1995):

$$A_4^{\rm VTI}(\phi = 0°) = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\,. \tag{20}$$

A discussion of the exact (i.e., not limited to weak anisotropy) quartic moveout coefficient of both P- and S-waves in horizontally layered VTI media can be found in Tsvankin (2001).

For a dipping reflector, the coefficient A_4 vanishes in azimuthal directions satisfying

$$|\cos\alpha| = \frac{1}{2\,\sin\phi}\,. \tag{21}$$

If the dip is equal to 30°, A_4 goes to zero only for a single azimuth $\alpha = 0°$ that corresponds to the dip plane (Figure 5). This analytic result is in good agreement with the numerical study of NMO velocity in Tsvankin (1995, 2001) who showed that the P-wave dip-line moveout approaches a hyperbola for reflector dips relatively close to 30°.

For any dip between 30° and 90°, equation (19) yields two azimuths α (plus two more azimuths different by $\pm 180°$) for which $A_4 = 0$. If the dip is equal to 45°, the quartic coefficient vanishes for azimuth $\alpha = \pm 45°$ (Figure 6). The sign of A_4 is negative near the dip plane ($-45° < \alpha < 45°$) and positive near the strike direction ($45° < \alpha < 135°$). If the dip is smaller than 30°, equation (21) does not have a solution, and $A_4 < 0$ for any azimuth (Figure 7).

Horizontal HTI layer

For a horizontal HTI layer ($\nu = 90°$ and $\phi = 0°$), equation (6) reduces to

$$A_4^{\rm HTI} = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\,\cos^4\alpha\,. \tag{22}$$

Equation (22) has the same azimuthal dependence ($\cos^4\alpha$) as the exact expression for A_4 obtained by Al-Dajani and Tsvankin (1997). In the equation given by Al-Dajani and Tsvankin (1997), however, the term multiplied with $\cos^4\alpha$ corresponds to the exact quartic coefficient in the plane that contains the symmetry axis ($\alpha = 0°$). The quartic coefficient vanishes in the isotropy plane orthogonal to the symmetry axis ($\alpha = 90°$), where reflection moveout is purely hyperbolic.

Discussion and conclusions

We have presented an exact expression for the quartic moveout coefficient A_4 valid for arbitrarily anisotropic, heterogeneous media. Unlike most existing methods, our approach does not require the model to have a horizontal symmetry plane and accounts for reflection-point dispersal on dipping or irregular interfaces. Substitution of the quartic coefficient into the general moveout equation of Tsvankin and Thomsen (1994) yields a good approximation for nonhyperbolic moveout of P-waves and, in some cases, mode-converted PS-waves in anisotropic media with realistic structural complexity.

It should be emphasized that all quantities needed to calculate the azimuthally-varying quartic coefficient can be obtained by tracing a single (zero-offset) ray. Computing the zero-offset ray is also sufficient to construct the NMO ellipse (i.e., the azimuthally varying NMO velocity) responsible for short-spread moveout (Grechka et al., 1999; Grechka and Tsvankin, 2002). Therefore, our results can be used to model azimuthally dependent long-spread moveout in a computationally efficient way, without time-consuming multi-offset, multi-azimuth ray tracing.

The general equation for A_4 was applied to study the properties of P-wave nonhyperbolic moveout in TI media with arbitrary orientation of the symmetry axis. The analysis was restricted to a homogeneous TI layer above a planar horizontal or dipping reflector; it was assumed that the symmetry axis is confined to the dip plane. To gain insight into the the dependence of the quartic moveout coefficient on the model parameters, we simplified the exact expression by linearizing it in the anisotropic parameters. The derived weak-anisotropy approximation is proportional to the anisotropic "time-processing" parameter $\eta \approx \epsilon - \delta$, so the magnitude of nonhyperbolic moveout increases as the model deviates from elliptical ($\epsilon = \delta$).

While the azimuthal dependence of A_4 is a rather complicated function of the reflector dip ϕ and the tilt ν of the symmetry axis, the expressions for the quartic coefficients in the "principal" (dip and strike) directions are relatively simple. In particular, the strike component of A_4 depends solely on the *difference* between the dip and tilt rather than on their individual values. The magnitude of the dip component is proportional to $\cos^3 \phi$, so it rapidly decreases with ϕ. For a fixed dip, the dip-line quartic coefficient vanishes for two values of the tilt between 0° and 90°.

It is well known that the azimuthal dependence of NMO velocity typically has a simple elliptical form. Our results show that the azimuthal variation of the quartic coefficient in tilted TI media may be more complicated, sometimes with A_4 vanishing in several azimuthal directions. The azimuthal positions of the zeros of the quartic coefficients and the signs of A_4 in different azimuthal sectors are largely governed by the tilt ν and reflector dip ϕ (η plays the role of a scaling coefficient). In realistic heterogeneous media, nonhyperbolic moveout is also caused by vertical and lateral velocity gradients, but anisotropy usually makes the most prominent contribution to A_4 (Alkhalifah, 1997). In particular, the azimuthal dependence of nonhyperbolic moveout over a medium containing a tilted TI layer should be well-described by the equations given in this paper.

In the important special case of the symmetry axis orthogonal to the reflector ($\phi = \nu$), the quartic coefficient goes to zero only on the dip line of a vertical reflector. For this model, the strike-line A_4 is independent of dip (and tilt) and has the same value as in VTI media, while the dip-line A_4 decreases with dip as $\cos^4 \phi$. Therefore, the magnitude of nonhyperbolic moveout for $\phi = \nu$ is significant mostly for azimuthal directions close to the reflector strike. If the medium is VTI and reflector dip is mild ($\phi < 30^\circ$), A_4 is negative for all azimuths, and its magnitude increases away from the dip direction. For a 30° dip, nonhyperbolic moveout in VTI media vanishes on the dip line, which agrees with existing numerical results (Tsvankin, 1995, 2001). If the dip exceeds 30°, A_4 goes to zero in two different azimuths that do not coincide with either strike or dip directions.

For purposes of anisotropic parameter estimation, moveout equations have to be rewritten in terms of the ray parameter p that can be determined from reflection slopes on zero-offset (or stacked) sections. The dip components of both A_4 and NMO velocity expressed through p depend on the same three parameter combinations involving η, ν and the NMO velocity from a horizontal reflector. This result and the high sensitivity of the azimuthal signature of A_4 to the symmetry-axis orientation indicate that P-wave nonhyperbolic moveout may provide valuable information for velocity analysis in TTI media. Although the trade-off between $V_{\rm nmo}$ and A_4 makes quantitative estimates of the quartic coefficient relatively unstable (Grechka and Tsvankin, 1998a), the azimuthal variation of the sign of A_4 and the directions of vanishing or small nonhyperbolic moveout should be detectable from wide-azimuth reflection data.

Acknowledgments

We thank S. Jang for reviewing the manuscript.

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APPENDIX A: Derivation of the quartic moveout coefficient

Here we develop a general analytic expression for the quartic moveout coefficient A_4 by extending the approach employed by Grechka and Tsvankin (2002) in their analysis of NMO-velocity surfaces. Suppose the traveltime of a certain pure (non-converted) reflected wave is recorded on the CMP line oriented parallel to the unit vector **L**. The coordinates of the source **S** and receiver **R** are defined by the vectors **y** - **h** and **y** + **h** (Figure 1), where **y** corresponds to the midpoint, and the half-offset vector **h** can be represented as

$$\mathbf{h} = h\mathbf{L} = h[L_1, L_2, 0]. \tag{A1}$$

The reflection point for a given source-receiver pair can be described by

$$\mathbf{x} = [x_1, x_2, z(x_1,\, x_2)]. \tag{A2}$$

The two-way traveltime can be written in terms of the one-way traveltimes τ of the downgoing and upgoing waves:

$$t(\mathbf{y}, \mathbf{h}, \mathbf{x}) = \tau(\mathbf{y} - \mathbf{h}, \mathbf{x}) + \tau(\mathbf{y} + \mathbf{h}, \mathbf{x}). \tag{A3}$$

Then the zero-offset traveltime is

$$t_0 = \tau(\mathbf{y} - \mathbf{0}, \mathbf{x}_0) + \tau(\mathbf{y} + \mathbf{0}, \mathbf{x}_0) = 2\tau(\mathbf{y}, \mathbf{x}_0) = 2\tau_0\,, \tag{A4}$$

where τ_0 is the one-way zero-offset traveltime, and $\mathbf{x}_0$ defines the zero-offset reflection point.

To obtain the quartic moveout coefficient, we expand the two-way traveltime in a Taylor series with respect to the half offset h in the vicinity of the CMP location (h=0):

$$\begin{aligned} t(h, \mathbf{L}) \;&=\; 2\tau_0 + \left.\frac{dt}{dh}\right|_{h=0} h + \left.\frac{d^2t}{dh^2}\right|_{h=0} \frac{h^2}{2\,!} \\ &+ \left.\frac{d^3t}{dh^3}\right|_{h=0} \frac{h^3}{3\,!} + \left.\frac{d^4t}{dh^4}\right|_{h=0} \frac{h^4}{4\,!} + \ldots\,. \end{aligned} \tag{A5}$$

For a pure (non-converted) reflection mode, t is an even function of h because the traveltime remains the same when the source and receiver are interchanged. Therefore, the odd derivatives of t can be dropped from equation (A5), which leads to

$$t(h, \mathbf{L}) = 2\tau_0 + \left.\frac{d^2t}{dh^2}\right|_{h=0} \frac{h^2}{2\,!} + \left.\frac{d^4t}{dh^4}\right|_{h=0} \frac{h^4}{4\,!} + \ldots\,. \tag{A6}$$

The second derivative of the traveltime was obtained by Grechka and Tsvankin (2002) as

$$\frac{d^2t}{dh^2} = \frac{\partial^2 t}{\partial h_k \partial h_m} L_k L_m + \frac{\partial^2 t}{\partial h_k \partial x_m} \frac{dx_m}{dh} L_k\,, \tag{A7}$$

where $k = 1, 2$ and $m = 1, 2$. By differentiating equation (A7) twice with respect to the half-offset h, we find the fourth derivative of t:

$$\begin{aligned} \frac{d^4t}{dh^4} \;&=\; \frac{\partial^4 t}{\partial h_p \partial h_k \partial h_m \partial h_n} L_p L_k L_m L_n \\ &+ 3\frac{\partial^3 t}{\partial x_n \partial h_k \partial h_m} \frac{\partial^2 x_n}{\partial h^2} L_k L_m \\ &+ \frac{\partial^4 t}{\partial x_p \partial h_k \partial h_m \partial h_n} \frac{\partial x_p}{\partial h} L_k L_m L_n \\ &+ 2\frac{\partial^4 t}{\partial h \partial x_n \partial h_k \partial h_m} \frac{\partial x_n}{\partial h} L_k L_m \\ &+ \frac{\partial^4 t}{\partial h \partial x_n \partial x_m \partial h_k} \frac{\partial x_n}{\partial h} \frac{\partial x_m}{\partial h} L_k \\ &+ \frac{\partial^3 t}{\partial x_n \partial x_m \partial h_k} \frac{\partial}{\partial h} \left(\frac{\partial x_n}{\partial h} \frac{\partial x_m}{\partial h}\right) L_k \\ &+ \frac{\partial^2 t}{\partial x_m \partial h_k} \frac{\partial^3 x_m}{\partial h^3} L_k \\ &+ \frac{\partial^3 t}{\partial x_n \partial x_m \partial h_k} \frac{\partial x_n}{\partial h} \frac{\partial^2 x_m}{\partial h} L_k\,. \end{aligned} \tag{A8}$$

Since not only the traveltime, but also the ray trajectory stays the same when the source and receiver are interchanged, the vector $\mathbf{x}$ is an even function of h:

$$\mathbf{x}(\mathbf{y}, h\mathbf{L}) = \mathbf{x}(\mathbf{y}, -h\mathbf{L})\,, \tag{A9}$$

and

$$\left.\frac{d\mathbf{x}}{dh}\right|_{h=0} = \left.\frac{d^3\mathbf{x}}{dh^3}\right|_{h=0} = 0\,. \tag{A10}$$

Taking equations (A10) into account, the derivative d^4t/dh^4 [equation (A8)] at the CMP location ($h = 0$) becomes

$$\begin{aligned} \left.\frac{d^4t}{dh^4}\right|_{h=0} &= \left.\frac{\partial^4 t}{\partial h_p \partial h_k \partial h_m \partial h_n}\right|_{h=0} L_p L_k L_m L_n \\ &+ 3\left.\frac{\partial^3 t}{\partial x_n \partial h_k \partial h_m}\right|_{h=0} \left.\frac{\partial^2 x_n}{\partial h^2}\right|_{h=0} L_k L_m\,. \end{aligned} \tag{A11}$$

Introducing the offset X ($X = 2h$) and the one-way traveltime τ_0 and using the results of Fomel and Grechka (2001), equation (A11) can be rewritten as

$$\begin{aligned} \left.\frac{d^4t}{dX^4}\right|_{X=0} &= \frac{1}{8}\frac{\partial^4 \tau_0}{\partial y_p \partial y_k \partial y_m \partial y_n} L_p L_k L_m L_n \\ &+ \frac{3}{8}\frac{\partial^3 \tau_0}{\partial x_n \partial y_k \partial y_m} \left(\left.\frac{\partial^2 x_n}{\partial h^2}\right|_{h=0}\right) L_k L_m\,. \end{aligned} \tag{A12}$$

To express the derivative $\partial^2 x_n/\partial h^2$ in terms of the traveltime, we use Fermat's principle expressed in the following form (Grechka and Tsvankin, 2002):

$$\frac{\partial t}{\partial x_i} = 0, \quad (i = 1, 2). \tag{A13}$$

Differentiating equation (A13) twice with respect to h yields

$$\begin{aligned} &\left.\frac{\partial^3 t}{\partial h_p \partial h_k \partial x_n}\right|_{h=0} L_p L_k \\ &+ \left.\frac{\partial^2 t}{\partial x_k \partial x_n}\right|_{h=0} \left.\frac{\partial^2 x_k}{\partial h^2}\right|_{h=0} = 0\,, \end{aligned} \tag{A14}$$

and

$$\begin{aligned} &\frac{\partial^3 \tau_0}{\partial y_p \partial y_k \partial x_n} L_p L_k \\ &+ \frac{\partial^2 \tau_0}{\partial x_k \partial x_n} \left(\left.\frac{\partial^2 x_k}{\partial h^2}\right|_{h=0}\right) = 0\,. \end{aligned} \tag{A15}$$

Combining equations (A12) and (A15), we find

$$\left.\frac{d^4t}{dX^4}\right|_{X=0} = \frac{1}{8}\frac{\partial^4\tau_0}{\partial y_p \partial y_k \partial y_m \partial y_n} L_p L_k L_m L_n - \frac{3}{8}\frac{\partial^3\tau_0}{\partial x_i \partial y_k \partial y_m}\left(\frac{\partial^2\tau_0}{\partial x_i \partial x_j}\right)^{-1} \frac{\partial^3\tau_0}{\partial x_j \partial y_p \partial y_n} L_k L_m L_p L_n \,. \tag{A16}$$

After the fourth traveltime derivative has been derived, the quartic moveout coefficient can be obtained from the Taylor series (A6). Introducing the offset $X = 2h$ into equation (A6) and squaring the first three terms of the series leads to

$$t^2(X,\mathbf{L}) \approx \left(2\tau_0 + \left.\frac{d^2t}{dX^2}\right|_{X=0}\frac{X^2}{2} + \left.\frac{d^4t}{dX^4}\right|_{X=0}\frac{X^4}{24}\right)^2. \tag{A17}$$

Keeping only the quartic and lower-order terms in X reduces equation (A17) to

$$t^2(X,\mathbf{L}) \approx A_0 + A_2X^2 + A_4X^4\,, \tag{A18}$$

where $A_0 = 4\tau_0^2$, $A_2 = 1/V_{\rm nmo}^2(\mathbf{L})$, and

$$A_4 = \frac{\tau_0}{6}\left.\frac{d^4t}{dX^4}\right|_{X=0} + \frac{1}{16\,\tau_0^2\,V_{\rm nmo}^4(\mathbf{L})}\,; \tag{A19}$$

$V_{\rm nmo}(\mathbf{L})$ is the NMO velocity on the CMP line $\mathbf{L}$ given by Grechka and Tsvankin (2002). Substituting the derivative d^4t/dX^4 from equation (A16) into equation (A19), we find the final expression for the quartic coefficient:

$$A_4 = \frac{\tau_0}{48}\frac{\partial^4\tau_0}{\partial y_p \partial y_k \partial y_m \partial y_n} L_p L_k L_m L_n - \frac{\tau_0}{16}\frac{\partial^3\tau_0}{\partial x_i \partial y_k \partial y_m}\left(\frac{\partial^2\tau_0}{\partial x_i \partial x_j}\right)^{-1} \frac{\partial^3\tau_0}{\partial x_j \partial y_p \partial y_n} L_k L_m L_p L_n + \frac{1}{16\,\tau_0^2\,V_{\rm nmo}^4(\mathbf{L})}\,, \tag{A20}$$

APPENDIX B: Weak-anisotropy approximation for the *P*-wave quartic moveout coefficient in tilted TI media

We consider a homogeneous transversely isotropic layer above a plane dipping reflector and assume that the symmetry axis (unit vector $\mathbf{c}$) lies in the dip plane (Figure B1). Without losing generality, the x_1-axis can be aligned with the dip direction, so that the vector $\mathbf{c}$ is given by

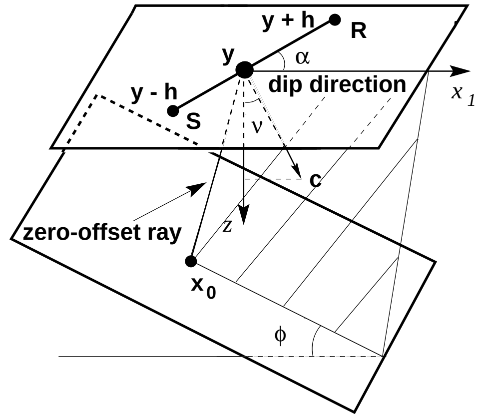

Figure B1. Reflected wave is recorded above a homogeneous TI layer with a dipping lower boundary. The symmetry axis is contained in the dip plane $[x_1, x_3]$ but may be tilted away from the vertical at an arbitrary angle ν.

$$\mathbf{c} = [\sin\nu, 0, \cos\nu]\,. \tag{B1}$$

The zero-offset ray should be confined to the dip plane that represents a vertical plane of symmetry for the whole model (Figure B1). The two-way traveltime between the common-midpoint $\mathbf{y}$ and the reflector in a homogeneous medium is simply

$$t(y_1,y_2,x_1,x_2) = \frac{2\sqrt{(x_1-y_1)^2+(x_2-y_2)^2+z^2(x_1,x_2)}}{V_g(y_1,y_2,x_1,x_2)}\,. \tag{B2}$$

Here $z(x_1,x_2)$ defines the reflecting plane, and $V_g(y_1,y_2,x_1,x_2)$ is the group velocity. Using the weak-anisotropy approximations for the P-wave group velocity and group angle in TI media (Tsvankin, 2001), V_g can be found as

$$V_g = \frac{V_{P0}}{4}\left[4 + m(-\delta - \epsilon - \eta\sin^2 a\sin^2 b - \eta\cos 2\nu(2\cos^2 b\sin^2 a + \sin^2 a\sin^2 b - 1) + \eta\cos b\sin 2a\sin 2\nu)\right], \tag{B3}$$

where we denoted

$$\sin a \equiv \frac{\sqrt{(y_1-x_1)^2+(y_2-x_2)^2}}{\sqrt{(y_1-x_1)^2+(y_2-x_2)^2+z^2}}\,, \tag{B4}$$

$$\cos a \equiv \frac{z}{\sqrt{(y_1-x_1)^2+(y_2-x_2)^2+z^2}}\,, \tag{B5}$$

$$\sin b \equiv \frac{(y_2-x_2)}{\sqrt{(y_1-x_1)^2+(y_2-x_2)^2}}\,, \tag{B6}$$

$$\cos b \equiv \frac{(y_1-x_1)}{\sqrt{(y_1-x_1)^2+(y_2-x_2)^2}}\,, \tag{B7}$$

$$m \equiv -1 - \sin^2 a\sin^2 b$$

$$-\cos 2\nu\,(-1+2\sin^2 a\cos^2 b+\sin^2 a\sin^2 b)$$
$$+\cos b\sin 2a\sin 2\nu\,. \tag{B8}$$

The parameters used in equation (B3) are introduced in the main text. Since the zero-offset traveltime needs to satisfy Fermat's principle, the minimum value of t corresponds to the coordinates of the zero-offset reflection point $x_1^{(0)}$ and $x_2^{(0)}$. This implies that the derivatives of $t(y_1, y_2, x_1, x_2)$ with respect to x_1 and x_2 should vanish at the point $[x_1^{(0)},\, x_2^{(0)}]$:

$$\left.\frac{\partial t(y_1,y_2,x_1,x_2)}{\partial x_1}\right|_{\left[x_1^{(0)},\,x_2^{(0)}\right]} = 0\,, \tag{B9}$$

$$\left.\frac{\partial t(y_1,y_2,x_1,x_2)}{\partial x_2}\right|_{\left[x_1^{(0)},\,x_2^{(0)}\right]} = 0\,. \tag{B10}$$

Equations (B9) and (B10) can be used to relate the CMP coordinates y_1 and y_2 to the coordinates $x_1^{(0)}$ and $x_2^{(0)}$ of the zero-offset reflection point. Substituting equations (B2) and (B3) into equations (B9) and (B10) and dropping quadratic and higher-order terms in the anisotropic coefficients yields

$$y_1 = z\,[\delta+\eta-\eta\cos 2(\phi-\nu)]\,\frac{\sin 2(\phi-\nu)}{\cos^2\phi} \tag{B11}$$
$$+\, z\tan\phi + x_1^{(0)}\,,$$

$$y_2 = x_2^{(0)}\,. \tag{B12}$$

Using equations (B2), (B11) and (B12) to evaluate the derivatives in equation (A20), we obtain the following linearized approximation for the P-wave quartic moveout coefficient:

$$A_4^{\rm TTI} = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\,[F(\alpha,\,\phi,\,\nu)\,+\,C]\,, \tag{B13}$$

where $C = 9/64$ is a constant, and

$$F(\alpha,\,\phi,\,\nu) = \frac{1}{128}\,[-24\cos 2\alpha$$
$$+6\cos 4\alpha + 8\cos(6\phi-4\nu)$$
$$+4\cos 2(\alpha-2\nu) - 4\cos(4\alpha-2\nu)$$
$$+24\cos 2(\phi-2\nu)$$
$$+12\cos 2(\alpha+\phi-2\nu)$$
$$+8\cos 2(\alpha+2\phi-2\nu)$$
$$+4\cos 2(\alpha+3\phi-2\nu)$$
$$+\cos 4(\alpha-\nu) + 32\cos 2(\phi-\nu)$$
$$+32\cos 4(\phi-\nu) - 16\cos 2(\alpha+\phi-\nu)$$
$$+8\cos 2\nu + 6\cos 4\nu$$
$$+\cos 4(\alpha+\nu) - 4\cos 2(2\alpha+\nu)$$
$$-16\cos 2(\alpha-\phi+\nu) + 4\cos 2(\alpha+2\nu)$$
$$+4\cos 2(\alpha-3\phi+2\nu)$$
$$+8\cos 2(\alpha-2\phi+2\nu)$$
$$+12\cos 2(\alpha-\phi+2\nu)]\,. \tag{B14}$$