2.5-D modeling and inversion/migration by the generalized Radon transform

Stig-Kyrre Foss[⋆] & Bjørn Ursin[†]

[⋆] *Guest at Center of Wave Phenomena, Colorado School of Mines from the Institute of Mathematics, NTNU, Trondheim, Norway.*
[†] *Visiting Scholar at Center of Wave Phenomena, Colorado School of Mines from the Institute of Petroleum Technology, NTNU, Trondheim, Norway.*

ABSTRACT
2.5-D modeling yields data in a seismic experiment in which source, receiver and scattering location all are in a single vertical plane in the subsurface. The wave propagation is restricted to this vertical plane beneath the acquisition line by assuming that the subsurface medium is homogeneous in the orthogonal out-of-plane direction and that the plane is a plane of anisotropic symmetry. We review 2.5-D modeling for Born volume scattering and Born-Helmhotz surface scattering. The amplitudes are corrected for 3-D wave propagation, taking into account both in-plane and out-of-plane geometrical spreading. We review and also derive some new inversion/migration formulas for the parameters in an elastic anisotropic medium using the inverse generalized Radon transform. Results for line scattering and cylindrical surface scattering are formulated as integrals over migration dip angle and scattering angle (in the case of inversion). We also give a migration formula for the energy-flux-normalized plane-wave reflection coefficient that models large-contrast reflections not treated by the Born and the Born-Helmholtz equation results.

Key words: 2.5-D modeling, 2.5-D inversion, 2.5-D migration

SYMBOLS AND NOTATION

Symbols	Description
t	time
ω	frequency
$\mathbf{x} = (x_1, x_2, x_3)$	position vector
$\mathbf{y} = (y_1, y_2, y_3)$	image point
$U(\mathbf{x}, \omega)$	scattered field
$c_{ijkl}(\mathbf{x}) = c^{(0)}_{ijkl}(\mathbf{x}) + c^{(1)}_{ijkl}(\mathbf{x})$	elastic stiffness tensor as a sum of the smooth background parameters and the perturbation, respectively
$\rho(\mathbf{x}) = \rho^{(0)}(\mathbf{x}) + \rho^{(1)}(\mathbf{x})$	density as a sum of a smoothly varying term and a perturbation, respectively
$G_{in}(\mathbf{x}^r, \omega, \mathbf{x}^s)$	Green's function in the frequency domain (subscripts explained in the main text)
$\mathbf{x}^s, \mathbf{x}^r$	source and receiver positions
$v(\mathbf{x})$	phase velocity
$V(\mathbf{x})$	group velocity
$\lvert det\, \mathbf{Q}_2\rvert^{1/2}$	relative geometrical spreading in local surface coordinates on the wavefront
(q_1, q_2)	local wavefront coordinates
$\mathbf{p} = (p_1, p_2, p_3)$	slowness vector
$\mathbf{h} = (h_1, h_2, h_3)$	polarization vector
$p = \lvert\mathbf{p}\rvert$	norm of the slowness
$A(\mathbf{x}, \mathbf{x}^s)$	amplitude for a ray at $\mathbf{x}^s$ from $\mathbf{x}$
$T(\mathbf{x}, \mathbf{x}^s)$	travel time for a ray between $\mathbf{x}$ and $\mathbf{x}^s$
$T(\mathbf{x}^r, \mathbf{x}, \mathbf{x}^s)$	two-way travel time
$\kappa(\mathbf{x}, \mathbf{x}^s)$	KMAH index accumulated for a ray between $\mathbf{x}$ and $\mathbf{x}^s$
$S(\mathbf{x})$	scattering coefficient
$\mathbf{w}(\mathbf{x}^r, \mathbf{x}, \mathbf{x}^s)$	radiation pattern vector
$\mathbf{c}^{(1)}(\mathbf{x})$	medium parameter vector
$\boldsymbol{\sigma} = (\sigma_1, \sigma_2)$	surface coordinates

$\mathbf{p}^s, \mathbf{p}^r$	slowness vector at at scattering point of the ray to source and receiver, respectively
$\mathbf{p}^m$	migration slowness
$\boldsymbol{\nu}^m = \mathbf{p}^m/p^m$	migration dip vector
ν	angle of the migration dip
$\boldsymbol{\nu}^\phi$	geological dip
θ	scattering angle
$Q_2^{\vert\vert}$	in-plane relative geometrical spreading
$Q_2^\perp$	out-of-plane relative geometrical spreading
$A^{\vert\vert}(\mathbf{x}^r, \mathbf{x}, \mathbf{x}^s)$	GRA amplitude with only in-plane geometrical spreading
$\mathcal{L}_R^\perp(\mathbf{x}^r, \mathbf{x}, \mathbf{x}^s)$	total out-of-plane geometrical spreading
R	reflection coefficient

Superscripts s and r indicate source and receiver respectively. We use the following definition of the Fourier transform of a function $f(t)$

$$F(\omega) = \int f(t)e^{i\omega t}dt.$$

INTRODUCTION

2.5-D modeling and inversion/migration considers 3-D wave propagation with source and receivers along a single line. This allows one to work with data from a reduced data set in a computationally efficient way. This is especially important in velocity analysis which gives initial estimates of the subsurface medium. As the problems often become too large to handle effectively in the full 3-D case, one usually considers slices in the earth using the 2.5-D framework. Velocity analysis and amplitude-versus-angle analysis can be done by studying common-image gathers created with formulas derived in this paper (Brandsberg-Dahl *et al.*, 1999; Brandsberg-Dahl *et al.*, 2000; Brandsberg-Dahl *et al.*, 2001).

The notion of 2.5-D modeling was introduced in seismic applications by Bleistein (1986) who considered modeling from the scalar wave equation for this case. Several authors had earlier addressed the problem as 3-D wave propagation in a 2-D medium (Červený, 1981; Goldin, 1986). The technique considers 3-D data generated and collected for source and receivers along a line on the surface in a seismic experiment. It is assumed that the parameters of the subsurface medium vary only in the vertical plane beneath the line, and for anisotropic medium this is a symmetry plane. For example it can be chosen to be one of the symmetry planes of an orthorhombic medium (Schoenberg & Helbig, 1997). The medium is assumed homogeneous in the orthogonal out-of-plane direction. We therefore confine our attention to wave propagation for which slowness and group velocity are considered to be zero in this direction. This restricts wave propagation to remain in this plane. The kinematics of the wave propagation is therefore 2-D (Ettrich *et al.*, 2001). The waves still travel in a 3-D medium and exhibit in-plane and out-of-plane geometrical spreading.

Here, we review 2.5-D modeling in a compact way and derive some new linearized 2.5-D inversion/migration formulas in elastic anisotropic media. The subsurface medium parameters are divided into known background parameters, such as density and stiffness parameters (from velocity analysis), and the unknown medium perturbations. Linearized inversion inverts for the most singular part of the unknown medium perturbations. This was done by Beylkin and Burridge (1990) for isotropic media using the generalized Radon transform (GRT). Burridge *et al.* (1998) expanded this to the anisotropic case. Most 2.5-D formulas published are valid for fluids and isotropic solids (see Stockwell (1995) for references). Geoltrain (1989) did 2.5-D Kirchhoff migration in transversely isotropic media (TI) with a vertical symmetry axis, and Sollid (2000) did 2.5-D angle migration for TI media. This paper however, uses the inverse generalized Radon transform (GRT) to derive all results in general anisotropic media under the restrictions mentioned above. Introducing a 2-D inverse GRT we derive inversion formulas for the medium pertubations and migration formulas for both 2.5-D Born and Born-Helmholtz, i.e. point scattering and surface reflection, respectively. These are valid only for small contrasts in the medium perturbations. Equivalent results for 2-D problems, found in Appendix B, are given for comparison with the 2.5-D results. 2-D versions of formulas are also needed when testing the formulas with synthetic data generated with a finite-difference modeling code, which is usually 2-D. Finally we introduce an inversion formula for the reflection coefficient where parameter contrasts are large (Ursin, 2002).

The first section of the paper starts by introducing the notation used in the rest of the paper. There, we also show that, in the case of 2.5-D, the geometrical spreading of the wave propagation factors into an in-plane and an out-of-plane part as shown by Ettrich *et al.* (2001). Starting with the single-scattering 3-D Born modeling formula (Ursin & Tygel, 1997), we proceed as in (Bleistein, 1986) to approximate the integral in the out-of-plane direction by the method of stationary phase. This reduces to the 2.5-D Born modeling equation, which models field data collected by receivers on a straight line on the Earth's surface. The wavefield is generated by a point source on that surface. Receivers in the same line on the surface observe the upward scattered waves generated by perturbations in the subsurface. The derived inversion and migration formulas benefits from are derived by a preconditioning of the data and the use of the 2-D inverse GRT (Appendix A). The entire inversion procedure is done in detail in Appendix C for this case. In the text, we also present the so-called *incomplete* GRT result (Brandsberg-Dahl *et al.*, 1999; Brandsberg-Dahl *et al.*, 2001) for the purpose of velocity analysis or amplitude-versus-angle analysis, and a migration procedure (Sollid & Ursin, 2001). All inversions in this paper are done in a similar way, but with different preconditioning of the seismic data.

We proceed considering Born-Helmholtz modeling and inversion formulas. The Born-Helmholtz modeling equation is a surface scattering equation modeling the reflected wave field from a surface reflection (De Hoop & Bleistein, 1997; Ursin & Tygel, 1997). A geological interface can be seen as a localized unknown medium perturbation along the same interface superimposed on the smooth background. By applying this in our 2.5-D Born modeling formula, we get the 2.5-D Born-Helmholtz modeling equation. The inversion result is the medium perturbation restricted to the geological interface (Stolk & De Hoop, 2002). The next section introduces a new reciprocal surface scattering modeling equation and migration formula which yields the reflection result for large contrast of the medium at the interface (Ursin, 2002). The final section discusses the implications of band-limited sources. All results derived in this paper assumes a delta function-like source. Seismic field data is generated by a band-limited wavelet. When using such data in our formulas the inversion/migration results becomes blurred (De Hoop *et al.*, 1999) resulting in wrong amplitude behavior. In the case of surface scattering inversion/migration this can be accounted for by division by a so-called stretching factor (Tygel *et al.*, 1994; Jaramillo & Bleistein, 1999; Sollid & Ursin, 2001; Ursin, 2002).

THE SINGLE SCATTERING BORN MODELING EQUATION

In the frequency domain, the Green's function G_{in} for wave propagation in an anisotropic elastic inhomogeneous medium satisfies

$$\rho(\mathbf{x})\omega^2 G_{in} + \partial_j(c_{ijkl}(\mathbf{x})\partial_l G_{kn}) = -\delta_{in}\delta(\mathbf{x}-\mathbf{x}^s), \quad i,j,k,l = 1,2,3, \quad (1)$$

where ρ is density and c_{ijkl} is the stiffness tensor. The function δ_{in} is the Kronecker delta, which gives the source term in the canonical directions, ω is frequency, $\mathbf{x} = (x_1, x_2, x_3)$ is the position vector, and $\mathbf{x}^s$ is the position of the source. We use ∂_j to indicate the derivative $\partial/\partial x_j$, and the summation convention is assumed if not otherwise is stated. The Green's function is defined in some domain $\mathcal{D}$ with homogeneous boundary conditions and thus satisfies the reciprocity relation (Aki & Richards, 1980)

$$G_{in}(\mathbf{x},\omega,\mathbf{x}^s) = G_{ni}(\mathbf{x}^s,\omega,\mathbf{x}). \quad (2)$$

In anisotropic elastic media there can exist three wave modes qP, qS1 and qS2. The geometrical ray approximation (GRA) is a high-frequency, leading order approximation of the Green's function in equation (1). Higher-order terms are smoother. The GRA gives a sum over these wave modes for rays between $\mathbf{x}^s$ and $\mathbf{x}$, where each term is of the form (Červený, 2001)

$$G_{ip}(\mathbf{x},\omega,\mathbf{x}^s) = h_i^s(\mathbf{x})A(\mathbf{x},\mathbf{x}^s)e^{i\omega T(\mathbf{x},\mathbf{x}^s)}h_p(\mathbf{x}^s), \quad (3)$$

where $T(\mathbf{x},\mathbf{x}^s)$ is the travel time along the ray between $\mathbf{x}^s$ and $\mathbf{x}$. $h_i^s(\mathbf{x})$ and $h_p(\mathbf{x}^s)$ are unit polarization vectors at each end of the ray connecting $\mathbf{x}$ and $\mathbf{x}^s$. The superscript s is to associate the polarization vector with this ray. $A(\mathbf{x},\mathbf{x}^s)$ is a complex amplitude function for the different wave modes at $\mathbf{x}$ after having traveled from $\mathbf{x}^s$. The amplitude, which becomes complex in the presence of caustics, is given by

$$A(\mathbf{x},\mathbf{x}^s) = \frac{e^{-i(\pi/2)\kappa(\mathbf{x},\mathbf{x}^s)\operatorname{sgn}\omega}}{4\pi[\rho(\mathbf{x})v^s(\mathbf{x})\rho(\mathbf{x}^s)v(\mathbf{x}^s)]^{1/2}|\det\mathbf{Q}_2(\mathbf{x},\mathbf{x}^s)|^{1/2}}, \quad (4)$$

where v^s and v are the phase velocities at $\mathbf{x}$ and $\mathbf{x}^s$, respectively. The KMAH-index, $\kappa(\mathbf{x},\mathbf{x}^s)$, takes into account the phaseshifts due to caustics the ray passes through from $\mathbf{x}^s$ to $\mathbf{x}$ (Červený, 2001; Klimeš, 1997). $|\det\mathbf{Q}_2(\mathbf{x},\mathbf{x}^s)|^{1/2}$ is the relative geometrical spreading factor determined from

$$[Q_2]_{ij}^{-1}(\mathbf{x},\mathbf{x}^s) = -\frac{\partial^2 T(\mathbf{x},\mathbf{x}^s)}{\partial q_i^s \partial q_j}, \quad i,j = 1,2, \quad (5)$$

where q_j and q_i^s are local surface coordinates on the wavefront at the points $\mathbf{x}$ and $\mathbf{x}^s$ respectively.

For 2.5-D modeling we choose q_1 to be in the (x_1,x_3)-plane and q_2 to be in the x_2-direction, normal to the plane. From the ray equations it follows that $p_2 = \partial T(\mathbf{x},\mathbf{x}^s)/\partial x_2$ is constant along a given ray, since the medium parameters do not depend on x_2. In particular, $p_2 = 0$ in the (x_1,x_3)-plane. From this it follows that

$$\frac{\partial^2 T(\mathbf{x},\mathbf{x}^s)}{\partial q_1^s \partial q_2} = \frac{\partial p_2^s(\mathbf{x})}{\partial q_1^s} = 0 \quad (6)$$

and

$$\frac{\partial^2 T(\mathbf{x},\mathbf{x}^s)}{\partial q_2^s \partial q_1} = \frac{\partial p_2(\mathbf{x}^s)}{\partial q_1} = 0. \quad (7)$$

The matrix in equation (5) thus becomes diagonal. The resulting relative geometrical spreading is

$$\mathbf{Q}_2(\mathbf{x},\mathbf{x}^s) = -\begin{bmatrix} \dfrac{1}{\dfrac{\partial^2 T(\mathbf{x},\mathbf{x}^s)}{\partial q_1^s \partial q_1}} & 0 \\ 0 & \dfrac{1}{\dfrac{\partial p_2}{\partial x_2}} \end{bmatrix} = \begin{bmatrix} Q_2^{||}(\mathbf{x},\mathbf{x}^s) & 0 \\ 0 & Q_2^{\perp}(\mathbf{x},\mathbf{x}^s) \end{bmatrix}, \quad (8)$$

where $Q_2^{||}(\mathbf{x},\mathbf{x}^s)$ and $Q_2^{\perp}(\mathbf{x},\mathbf{x}^s)$ are defined as the in-plane and out-of-plane relative geometrical spreading factors, respectively. This means that the relative geometrical spreading factor of the GRA amplitudes (4) splits into an out-of-plane and an in-plane factor

$$|\det \mathbf{Q}_2(\mathbf{x},\mathbf{x}^s)|^{1/2} = |\mathrm{Q}_2^{||}(\mathbf{x},\mathbf{x}^s)\mathrm{Q}_2^{\perp}(\mathbf{x},\mathbf{x}^s)|^{1/2}. \quad (9)$$

The inverse of the out-of-plane factor is

$$\frac{\partial p_2(\mathbf{x}^s)}{\partial x_2} = \frac{\partial p_2^s(\mathbf{x})}{\partial x_2}, \quad (10)$$

since p_2 is constant for a given ray. We note that derivatives of the slowness, polarization vector, or amplitude give higher-order, thus smoother, terms. This will become important in the derivations of high-frequency formulas in the following.

The medium parameters can be described as a sum of a smooth part, $\rho^{(0)}$ and $c_{ijkl}^{(0)}$, and a perturbation, $\rho^{(1)}$ and $c_{ijkl}^{(1)}$:

$$\rho(\mathbf{x}) = \rho^{(0)}(\mathbf{x}) + \rho^{(1)}(\mathbf{x}), \quad c_{ijkl}(\mathbf{x}) = c_{ijkl}^{(0)}(\mathbf{x}) + c_{ijkl}^{(1)}(\mathbf{x}). \quad (11)$$

The 3-D Born modeling formula is a single-scattering approximation to a seismic experiment where a source at $\mathbf{x}^s$ generates a wavefield that travels through the smooth part of the medium until it is scattered off a contrast in the medium (the perturbation) and again travels in the smooth medium to the receiver at $\mathbf{x}^r$. The wavefield is assumed to not scatter at any part of the medium apart from the scattering point. We let the GRA Green's functions in equation (3) describe the wave propagation in the smoothly varying part of the medium in equation (11); see Figure 1. The 3-D Born formula gives the scattered field U observed at $\mathbf{x}^r$. This would then be the equivalent to data collected in a seismic field experiment. We assume that we know the smoothly varying part of the medium and wish to find the medium perturbations. In the frequency domain this becomes the m-direction of the scattered field at receiver position $\mathbf{x}^r$ due to a n-directional source at $\mathbf{x}^s$ is (see Ursin & Tygel (1997) for a detailed derivation)

$$U_{mn}(\mathbf{x}^r,\omega,\mathbf{x}^s) \approx \omega^2 \int_{\mathcal{D}} h_m(\mathbf{x}^r)\rho^{(0)}(\mathbf{x})A(\mathbf{x}^s,\mathbf{x})A(\mathbf{x},\mathbf{x}^r) \cdot S(\mathbf{x})e^{i\omega T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)}h_n(\mathbf{x}^s)d\mathbf{x}, \quad (12)$$

where the two-way traveltime is

$$T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s) = T(\mathbf{x}^s,\mathbf{x}) + T(\mathbf{x},\mathbf{x}^r), \quad (13)$$

and the scattering coefficient is

$$S(\mathbf{x}) = \mathbf{w}^T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)\mathbf{c}^{(1)}(\mathbf{x}). \quad (14)$$

We define the medium perturbations by the 22 × 1-component matrix (Burridge *et al.*, 1998)

$$\mathbf{c}^{(1)}(\mathbf{x}) = \left\{ \frac{\rho^{(1)}(\mathbf{x})}{\rho^{(0)}(\mathbf{x})}, \frac{c_{ijkl}^{(1)}(\mathbf{x})}{\rho^{(0)}(\mathbf{x})v_\circ^s(\mathbf{x})v_\circ^r(\mathbf{x})} \right\}, \quad (15)$$

where $v_\circ^s$ and $v_\circ^r$ are local phase velocities averaged over all phase angles. These are introduced for numerical purposes so that the matrix has components of similar size (Burridge *et al.*, 1998). The indices of $c_{ijkl}^{(1)}$ cover all the 21 independent indices of the stiffness tensor in general anisotropic media (Tsvankin, 2001). With higher symmetry, such as isotropy, the matrix reduces accordingly (Beylkin & Burridge, 1990). The radiation pattern is defined similarly as a 22 × 1-matrix by

$$\mathbf{w}(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s) = \left\{h_m^s(\mathbf{x})h_m^r(\mathbf{x}), [h_i^s(\mathbf{x})p_j^s(\mathbf{x})h_k^r(\mathbf{x})p_l^r(\mathbf{x})]v_\circ^s(\mathbf{x})v_\circ^r(\mathbf{x})\right\}, \quad (16)$$

and $\mathbf{p}^s = (p_1^s, p_2^s, p_3^s)$ and $\mathbf{p}^r = (p_1^r, p_2^r, p_3^r)$ are the slowness vectors at the point $\mathbf{x}$; of the rays from the source $\mathbf{x}^s$ and receiver $\mathbf{x}^r$, respectively. The indices correspond to those of the stiffness tensor in the medium perturbation $\mathbf{c}^{(1)}$. All parts of the Born integral except the unknown medium perturbations are obtained from the respective GRA Green's functions and are thus calculated in the background medium. Given the incident field described by the polarization and slowness vectors, the scattered field is calculated from Snell's law (Chapman, 1999). The angle between the incoming field and the reflected field (given by the slowness vectors) is called the scattering angle, $\theta \in [0,\pi\rangle$, defined by

$$\cos\theta = \frac{\mathbf{p}^s \cdot \mathbf{p}^r}{|\mathbf{p}^s||\mathbf{p}^r|}. \quad (17)$$

The domain of the scattering angle is denoted E_θ. We also define the migration slowness as the sum of the two vectors at the scattering point; $\mathbf{p}^m = \mathbf{p}^s + \mathbf{p}^r$. The unit vector $\boldsymbol{\nu}^m = \mathbf{p}^m/|\mathbf{p}^m|$ is called the migration dip. We can parametrize this unit vector by the migration dip angle, ν, as $\boldsymbol{\nu}^m = (\cos\nu, \sin\nu)$, where $\nu \in E_\nu$. E_ν is the domain of the migration dip angle.

2.5-D BORN MODELING EQUATION

We reduce the 3-D Born modeling formula to 2.5-D by restricting the wave propagation to a vertical plane, the (x_1, x_3)-plane, so $x_2 = 0$. This is done by setting the out-of-plane slowness p_2 to zero, reducing the kinematic ray equations to 2-D (Ettrich *et al.*, 2001). The rays still travel in a 3-D medium so the ray amplitudes exhibit 3-D geometrical spreading. We proceed as in Bleistein (1986) to use the method of stationary phase to approximate the integral in the out-of-plane variable x_2. The 1-D stationary phase formula approximates the integral

$$I(\omega) = \int f(\sigma)e^{i\omega T(\sigma)}d\sigma \approx \sqrt{\frac{2\pi}{|\omega||\partial_\sigma^2 T(\sigma_0)|}}f(\sigma_0)e^{i\omega T(\sigma_0)+i(\pi/4)\mathrm{sgn}(\omega)\mathrm{sgn}(\partial_\sigma^2 \mathrm{T}(\sigma_0))}, \tag{18}$$

for sufficiently large $|\omega|$ and $\partial_\sigma^2 T(\sigma_0) \neq 0$, where σ_0 is the stationary point, such that $\partial_\sigma T(\sigma)|_{\sigma=\sigma_0} = 0$ (Bleistein, 1984).

We apply the stationary phase formula to the Born integral in equation (12) and integrate with respect to x_2. The stationary point is at

$$\partial_{x_2} T(\mathbf{x}^r, \mathbf{x}, \mathbf{x}^s) = p_2^s(\mathbf{x}) + p_2^r(\mathbf{x}) = 0, \tag{19}$$

which gives $p_2^s = p_2^r = 0$ for $x_2 = 0$. From equation (19) and (10) it follows that

$$\partial_{x_2}^2 T(\mathbf{x}^r, \mathbf{x}, \mathbf{x}^s)\big|_{x_2=0,p_2=0} = (\partial_{x_2} p_2^s + \partial_{x_2} p_2^r)|_{x_2=0,p_2=0} = \frac{1}{Q_2^\perp(\mathbf{x}, \mathbf{x}^s)} + \frac{1}{Q_2^\perp(\mathbf{x}, \mathbf{x}^r)}, \tag{20}$$

where $Q_2^\perp(\mathbf{x}, \mathbf{x}^s)$ and $Q_2^{r\perp}(\mathbf{x}, \mathbf{x}^r)$ are the out-of-plane relative geometrical spreading factors for the rays from the source $\mathbf{x}^s$ and receiver $\mathbf{x}^r$, respectively, to the imaging point $\mathbf{x}$. Furthermore, $\mathrm{sgn}(\partial_{\mathrm{x}_2}^2 \mathrm{T}(\mathbf{x}^r, \mathbf{x}, \mathbf{x}^s)\big|_{\mathrm{x}_2=0}) = 1$, since the out-of-plane relative geometrical spreading is positive from (20). Inside the Born integral there will be a division by

$$\Big[Q_2^\perp(\mathbf{x}^r, \mathbf{x})Q_2^\perp(\mathbf{x}, \mathbf{x}^s)\partial_{x_2}^2 T(\mathbf{x}^r, \mathbf{x}, \mathbf{x}^s)\Big]^{1/2} = \Big[Q_2^\perp(\mathbf{x}^r, \mathbf{x}) + Q_2^\perp(\mathbf{x}, \mathbf{x}^s)\Big]^{1/2} = \mathcal{L}_R^\perp(\mathbf{x}^r, \mathbf{x}, \mathbf{x}^s), \tag{21}$$

which is the out-of-plane relative geometrical spreading factor given by (Ettrich *et al.*, 2001)

$$\mathcal{L}_R^\perp(\mathbf{x}^r, \mathbf{x}, \mathbf{x}^s) = \left[\int_{ray} \frac{V_2}{p_2} d\tau\right]^{1/2}, \tag{22}$$

where V_2 is the x_2-component of the group velocity and τ denotes traveltime.

Defining an in-plane amplitude factor

$$A^{||}(\mathbf{x}, \mathbf{x}^s) = \frac{e^{-i(\pi/2)\kappa(\mathbf{x},\mathbf{x}^s)\mathrm{sgn}\,\omega}}{4\pi[\rho(\mathbf{x})v^s(\mathbf{x})\rho(\mathbf{x}^s)v(\mathbf{x}^s)]^{1/2}|Q_2^{||}(\mathbf{x}, \mathbf{x}^s)|^{1/2}}, \tag{23}$$

we obtain the 2.5-D Born integral for point scattering

$$U_{mn}(\mathbf{x}^r, \omega, \mathbf{x}^s) = \sqrt{2\pi}|\omega|^{3/2}e^{i(\pi/4)\mathrm{sgn}\,\omega} \cdot \int_{\mathcal{D}(x_2=0)} h_m(\mathbf{x}^r)\frac{\rho^{(0)}(\mathbf{x})A^{||}(\mathbf{x}^r, \mathbf{x})A^{||}(\mathbf{x}, \mathbf{x}^s)}{\mathcal{L}_R^\perp(\mathbf{x}^r, \mathbf{x}, \mathbf{x}^s)} \cdot S(\mathbf{x})e^{i\omega T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)}h_n(\mathbf{x}^s)dx_1 dx_3. \tag{24}$$

The 2.5-D Born integral corresponds to the response of line scatterers from a point source to a point receiver. This is accounted for by an out-of-plane geometrical spreading computed along the ray and in-plane geometrical spreading from the source and the receivers to the scattering line, respectively. In addition, the integral has been multiplied by a factor $\sqrt{2\pi i/\omega}$ (a half-integration in the time-domain), an additional effect of the line scatterer. All kinematic aspects of the raytracing is restricted to in-plane.

LINE-SCATTERING INVERSION/MIGRATION

In order to prepare for the inversion and to simplify the equations, it is convenient to consider the data contracted with the polarization vectors at the source and receiver and corrected for amplitude and traveltime corresponding to the GRA from the source and receiver to the image point $\mathbf{y}$. Including additional normalizing factors for the migration dip, frequency and phase, this gives the preconditioned data

$$U(\mathbf{x}^r, \omega, \mathbf{x}^s; \mathbf{y}) = \left(\frac{1}{2\pi}\right)^{5/2} \frac{e^{-i(\pi/4)\mathrm{sgn}\,\omega}}{|\omega|^{1/2}} h_m(\mathbf{x}^r; \mathbf{y})U_{mn}(\mathbf{x}^r, \omega, \mathbf{x}^s)h_n(\mathbf{x}^s; \mathbf{y}) \cdot \frac{\mathcal{L}_R^\perp(\mathbf{x}^r, \mathbf{y}, \mathbf{x}^s)}{\rho^{(0)}(\mathbf{y})A^{||}(\mathbf{x}^r, \mathbf{y})A^{||}(\mathbf{y}, \mathbf{x}^s)} e^{-i\omega T(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)}|\mathbf{p}^m(\mathbf{y})|^2. \tag{25}$$

The preconditioning corrects the data and removes the effects of the two-way travel in the medium, from $\mathbf{x}^s$ to the image point $\mathbf{y}$ and to $\mathbf{x}^r$. This is intuitively what the inversion for the medium parameters should do, i.e. removing all effects of the wave propagation apart from the change in the wavefield due to the medium perturbation. Using the inversion formula derived in Appendix A (the inversion is done in detail in Appendix C), an estimate of the perturbation of the medium parameters is given by

$$\hat{\mathbf{c}}^{(1)}(\mathbf{y}) = Re\{\int_0^\infty \int_{E_\nu} \langle\Lambda(\boldsymbol{\nu}^m; \mathbf{y})\rangle^{-1} \cdot \int_{E_\theta} \mathbf{w}(\mathbf{x}^r, \mathbf{y}, \mathbf{x}^s)U(\mathbf{x}^r, \omega, \mathbf{x}^s; \mathbf{y})d\theta d\nu d\omega\}, \tag{26}$$

where the matrix Λ is given by

$$\Lambda(\boldsymbol{\nu}^m;\mathbf{y}) = \int_{E_\theta} \mathbf{w}(\boldsymbol{\nu}^m,\theta;\mathbf{y})\mathbf{w}^T(\boldsymbol{\nu}^m,\theta;\mathbf{y})d\theta, \quad (27)$$

and $\langle\Lambda(\boldsymbol{\nu}^m;\mathbf{y})\rangle^{-1}$ indicates a pseudo-inverse. This factor works as a removal of the effects of the radiation pattern. The study of this matrix and its pseudo-inverse also gives an estimate of the resolution of the parameters for which you invert (De Hoop *et al.*, 1999). For the purpose of velocity estimation and estimation of the geological dip, Brandsberg-Dahl *et al.* (1999) introduced the incomplete inverse GRT as an integration over migration dip angle ν, resulting in an estimate of the perturbations in model parameters which are functions of scattering angle:

$$\hat{\mathbf{c}}^{(1)}(\theta;\mathbf{y}) = Re\left\{\int_0^\infty \int_{E_\nu} \frac{\mathbf{w}(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)}{|\mathbf{w}(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)|^2} U(\mathbf{x}^r,\omega,\mathbf{x}^s;\mathbf{y})d\nu d\omega\right\}. \quad (28)$$

2.5-D-true-amplitude migration may be defined as a process which produces reflectivity as a function of scattering angle. These angle gathers may be analyzed directly, used as input to angle stacks or as input to an inversion algorithm. A simple migration algorithm gives an estimate of the scattering coefficient (Sollid & Ursin, 2001)

$$\hat{S}(\mathbf{y};\theta) = Re\left\{\int_0^\infty \int_{E_\nu} U(\mathbf{x}^r,\omega,\mathbf{x}^s;\mathbf{y})d\nu d\omega\right\}. \quad (29)$$

For an isotropic background medium, the weights $\mathbf{w}(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)$ do not depend on the migration dip and thus

$$\hat{\mathbf{c}}^{(1)}(\theta;\mathbf{y}) = \frac{\mathbf{w}(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)}{|\mathbf{w}(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)|^2}\hat{S}(\theta;\mathbf{y}). \quad (30)$$

THE 2.5-D BORN-HELMHOLTZ MODELING FORMULA

A geological interface defines a surface across which there exists a jump in the medium parameters. This interface will then be the support of the medium perturbation $\mathbf{c}^{(1)}$. Let the function $\phi(\mathbf{x}) = 0$ describe such an interface. The medium perturbation can then be expressed by

$$\mathbf{c}^{(1)}(\mathbf{x}) = \mathbf{c}^{(1)}(\mathbf{x})|_{\phi(\mathbf{x})=0}H(\phi(\mathbf{x})), \quad (31)$$

where $H(\phi(\mathbf{x}))$ is a unit step function at the interface:

$$H(\phi(\mathbf{x})) = \begin{cases} 1 & \mathbf{x} \text{ is below the interface} \\ 0 & \mathbf{x} \text{ is above the interface.} \end{cases} \quad (32)$$

The gradient of $\mathbf{c}^{(1)}(\mathbf{x})$ is now

$$\nabla_\mathbf{x}\mathbf{c}^{(1)}(\mathbf{x}) = \mathbf{c}^{(1)}(\mathbf{x})|_{\phi(\mathbf{x})=0}\delta(\phi(\mathbf{x}))|\nabla_\mathbf{x}\phi(\mathbf{x})|\boldsymbol{\nu}^\phi, \quad (33)$$

where $\delta(\phi(\mathbf{x}))$ is a spatial delta function at the reflecting surface. The surface is cylindrical under assumptions of 2.5-D, i.e. no variations in the out-of-plane direction. $\boldsymbol{\nu}^\phi$ is the unit normal to the interface pointing downwards and correspond to the local geological dip

$$\boldsymbol{\nu}^\phi = |\nabla_\mathbf{x}\phi|^{-1}\nabla_\mathbf{x}\phi. \quad (34)$$

Applying the divergence theorem to the 2.5-D Born line-scattering integral in equation (24) and neglecting a smoothly varying part, gives (Ursin & Tygel, 1997; De Hoop & Bleistein, 1997)

$$U_{mn}(\mathbf{x}^r,\omega,\mathbf{x}^s) = \sqrt{2\pi}|\omega|^{1/2}e^{i(3\pi/4)\mathrm{sgn}\,\omega} \cdot \int_{\mathcal{D}(x_2=0)} h_m(\mathbf{x}^r)\frac{\rho^{(0)}(\mathbf{x})A^{||}(\mathbf{x}^r,\mathbf{x})A^{||}(\mathbf{x},\mathbf{x}^s)}{\mathcal{L}^\perp(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)} \cdot \mathbf{w}^T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)\frac{\nabla_\mathbf{x}\mathbf{c}^{(1)}(\mathbf{x})\cdot\nabla_\mathbf{x}T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)}{|\nabla_\mathbf{x}T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)|^2} \cdot e^{i\omega T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)}h_n(\mathbf{x}^s)d\mathbf{x}. \quad (35)$$

We insert equation (33) and get the 2.5-D Born-Helmholtz modeling integral (see Figure 1)

$$U_{mn}(\mathbf{x}^r,\omega,\mathbf{x}^s) = \sqrt{2\pi}|\omega|^{1/2}e^{i(3\pi/4)\mathrm{sgn}\,\omega} \cdot \int_{\mathcal{D}(x_2=0)} h_m(\mathbf{x}^r)\frac{\rho^{(0)}(\mathbf{x})A^{||}(\mathbf{x}^r,\mathbf{x})A^{||}(\mathbf{x},\mathbf{x}^s)}{\mathcal{L}^\perp(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)} \cdot \mathbf{w}^T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)\mathbf{c}^{(1)}(\mathbf{x})|_{\phi(\mathbf{x})=0}\delta(\phi(\mathbf{x}))|\nabla_\mathbf{x}\phi(\mathbf{x})| \cdot \frac{\boldsymbol{\nu}^\phi\cdot\boldsymbol{\nu}^m}{|\mathbf{p}^m(\mathbf{x})|}e^{i\omega T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)}h_n(\mathbf{x}^s)d\mathbf{x}. \quad (36)$$

The delta function $\delta(\phi(\mathbf{x}))$ can be viewed as making this integral over $\mathcal{D}(x_2 = 0)$ into an integral over the scattering surface. The factor $|\nabla_\mathbf{x}\phi(\mathbf{x})|$ is a Jacobian between $\mathbf{x}$ and the line coordinate. Note that we now have new factors of ω in front of the integrals. The Born-Helmholtz modeling then is

$$U_{mn}(\mathbf{x}^r,\omega,\mathbf{x}^s) = \sqrt{2\pi}|\omega|^{1/2}e^{i(3\pi/4)\mathrm{sgn}\,\omega} \cdot \int_\Sigma h_m(\mathbf{x}^r)\frac{\rho^{(0)}(\mathbf{x})A^{||}(\mathbf{x}^r,\mathbf{x})A^{||}(\mathbf{x},\mathbf{x}^s)}{\mathcal{L}^\perp(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)} \cdot S(\mathbf{x})\frac{\boldsymbol{\nu}^\phi\cdot\boldsymbol{\nu}^m}{|\mathbf{p}^m(\mathbf{x})|}e^{i\omega T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)}h_n(\mathbf{x}^s)d\sigma, \quad (37)$$

which is a line integral in the (x_1, x_3)-plane along the reflecting cylindrical surface and $\mathbf{c}^{(1)}(\mathbf{x})|_{\phi(\mathbf{x})=0}$ is used in formula (14) for the scattering coefficient.

CYLINDRICAL SURFACE-SCATTERING INVERSION/MIGRATION

In the case of the Born-Helmholtz surface-scattering modeling formula (36) the medium perturbation is restricted to that surface. We solve for the medium perturbation localized by the delta function making up the surface (Stolk & De Hoop, 2002)

$$\mathbf{c}^{(1)}(\mathbf{x})|_{\phi(\mathbf{x})=0}\delta(\phi(\mathbf{x}))|\nabla_\mathbf{x}\phi(\mathbf{x})|, \quad (38)$$

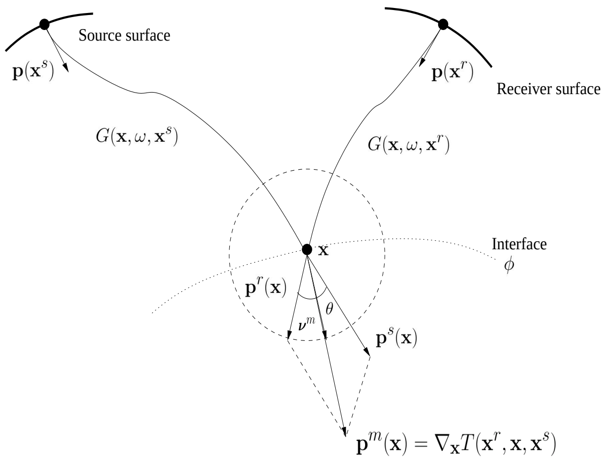

Figure 1. Single scattering with Green's functions connecting the scattering point $\mathbf{x}$ with the source and the receiver, $\mathbf{x}^s$ and $\mathbf{x}^r$, respectively. Scattering point on the interface $\phi(\mathbf{x}) = 0$. The dotted line around the scattering point $\mathbf{x}$ indicates the unit circle on which the migration dip $\boldsymbol{\nu}^m$ is defined. The scattering angle is given by θ.

where $\delta(\phi(\mathbf{x}))|\nabla_{\mathbf{x}}\phi(\mathbf{x})|$ is the so-called *singular function* of the surface. By proceeding in a similar manner as when inverting the Born modeling data $U(\mathbf{x}^r, \omega, \mathbf{x}^s; \mathbf{y})$, we precondition data in equation (36) in the following way

$$U_\Sigma(\mathbf{x}^r, \omega, \mathbf{x}^s; \mathbf{y}) = -i\omega U(\mathbf{x}^r, \omega, \mathbf{x}^s; \mathbf{y}) \frac{|\mathbf{p}^m(\mathbf{y})|}{\boldsymbol{\nu}^\phi(\mathbf{y}) \cdot \boldsymbol{\nu}^m(\mathbf{y})}$$
$$= -\left(\frac{1}{2\pi}\right)^{5/2} h_m(\mathbf{x}^r; \mathbf{y})(i\omega)^{1/2} U^{(1)}_{mn}(\mathbf{x}^r, \omega, \mathbf{x}^s) h_n(\mathbf{x}^s; \mathbf{y})$$
$$\cdot \frac{\mathcal{L}^{\perp}_{R}(\mathbf{x}^r, \mathbf{y}, \mathbf{x}^s)}{\rho^{(0)}(\mathbf{y}) A^{||}(\mathbf{x}^r, \mathbf{y}) A^{||}(\mathbf{y}, \mathbf{x}^s)(\boldsymbol{\nu}^\phi(\mathbf{y}) \cdot \boldsymbol{\nu}^m(\mathbf{y}))}$$
$$\cdot\, e^{-i\omega T(\mathbf{x}^r, \mathbf{y}, \mathbf{x}^s)} |\mathbf{p}^m(\mathbf{y})|^3, \quad (39)$$

where the stationary value $\boldsymbol{\nu}^\phi(\mathbf{y}) \cdot \boldsymbol{\nu}^m(\mathbf{y}) = 1$, i.e. the migration dip and the surface normal coincide, is normally used (De Hoop & Bleistein, 1997). The first line indicates the difference in the preconditioning with the Born modeling data. Inverting this with respect to the perturbation in (38) using the inversion formula in Appendix A, we get

$$\hat{\mathbf{c}}^{(1)}(\mathbf{y})|_{\phi(\mathbf{y})=0} \delta(\phi(\mathbf{y}))|\nabla_{\mathbf{x}}\phi(\mathbf{y})| =$$
$$Re\left\{ \int_0^\infty \int_{E_\nu} \langle \Lambda(\boldsymbol{\nu}^m; \mathbf{y})\rangle^{-1} \right.$$
$$\left. \cdot \int_{E_\theta} \mathbf{w}(\mathbf{x}^r, \mathbf{y}, \mathbf{x}^s) U_\Sigma(\mathbf{x}^r, \omega, \mathbf{x}^s; \mathbf{y}) d\theta d\nu d\omega \right\}, \quad (40)$$

where Λ is defined in equation (27). This is the equivalent result as in equation (26) for the Born inversion but with different preconditioned data. Using the data preconditioned for surface inversion/migration in the incomplete inverse GRT in equation (28) or in the migration formula in equation (29), yields similar angle common-image gathers as for the volume inversion/migration multiplied by the singular function of the reflecting interface, similar to equation (40).

LARGE-CONTRAST MODELING AND MIGRATION

The cylindrical surface-scattering inversion/migration formula derived in the previous section corresponds to using a plane wave reflection coefficient. This is valid for small contrasts in the medium perturbations and precritical reflection angles at the reflecting surface. Using the surface integral formulation of the Born-Helmholtz modeling equation (37), a new linearized reflection coefficient was derived by Schleicher *et al.* (2001). This modeling integral was however not reciprocal. Ursin (2002) extended this to become reciprocal by introducing the horizontal slowness component $\mathbf{p}^h = p^s_h \boldsymbol{\nu}^h$, where $\boldsymbol{\nu}^h$ is the in-plane unit vector orthogonal to $\boldsymbol{\nu}^m$ and p^s_h is the length of $\mathbf{p}^h$. The component is defined from $\mathbf{p}^s(\mathbf{x}) = \mathbf{p}^h(\mathbf{x}) + \boldsymbol{\nu}^m(\mathbf{x})(\mathbf{p}^s \cdot \boldsymbol{\nu}^m)(\mathbf{x})$, where $(\mathbf{p}^s \cdot \boldsymbol{\nu}^m)(\mathbf{x})$ is the projection of $\mathbf{p}^s$ onto $\boldsymbol{\nu}^m$ (see Figure 1). Equivalently we have $\mathbf{p}^r(\mathbf{x}) = -\mathbf{p}^h(\mathbf{x}) + \boldsymbol{\nu}^m(\mathbf{x})(\mathbf{p}^s \cdot \boldsymbol{\nu}^m)(\mathbf{x})$, since the reflection follows Snell's law. Interchanging the source and the receiver and keeping the wave mode in the different parts of the medium the same, results in the same modelling integral. This gives us the condition for reciprocity (Chapman, 1994). We let R be the reciprocal reflection coefficient normalized with respect to the vertical energy flux at the scattering point with the argument of the horizontal slowness. The modelling formula in 2.5-D is then:

$$U^{(1)}_{pq}(\mathbf{x}^r, \omega, \mathbf{x}^s) = \sqrt{2\pi}|\omega|^{1/2} e^{i(3\pi/4)\mathrm{sgn}\,\omega}$$
$$\cdot \int_\Sigma h_p(\mathbf{x}^r) \frac{A^{||}(\mathbf{x}^r, \mathbf{x}) A^{||}(\mathbf{x}, \mathbf{x}^s)}{\mathcal{L}^{\perp}_{R}(\mathbf{x}^r, \mathbf{x}, \mathbf{x}^s)} R(p^s_h \boldsymbol{\nu}^h(\mathbf{x})) 2\rho^{(0)}(\mathbf{x})$$
$$\cdot [V^r(\mathbf{x}) \cos\alpha^r(\mathbf{x}) V^s(\mathbf{x}) \cos\alpha^s(\mathbf{x})]^{1/2}$$
$$\cdot\, e^{i\omega T(\mathbf{x}^r, \mathbf{x}, \mathbf{x}^s)} (\boldsymbol{\nu}^\phi \cdot \boldsymbol{\nu}^m) h_q(\mathbf{x}^s) d\boldsymbol{\sigma}. \quad (41)$$

Here, V^s and V^r are the group velocities associated with the ray connecting the image point with the source and receiver, respectively. α^s and α^r are the angles the group velocity vectors of the rays from the source and receiver make with the surface normal at the reflection point. We precondition the data in equation (41) in the appropriate way. Using the inversion formula in Appendix A, the estimate of reflection coefficient with the restriction to the surface can be written as (note that the preconditioning is here explicit)

$$\hat{R}(p_h^s \boldsymbol{\nu}^h(\mathbf{y}))|_{\phi(\mathbf{y})=0}\delta(\phi(\mathbf{y}))|\nabla_{\mathbf{x}}\phi(\mathbf{y})| = Re\{ \int_0^\infty \int_{E_\nu} - \left(\frac{1}{2\pi}\right)^{5/2} \cdot h_m(\mathbf{x}^r;\mathbf{y})(i\omega)^{1/2}U_{mn}^{(1)}(\mathbf{x}^r,\omega,\mathbf{x}^s)h_n(\mathbf{x}^s;\mathbf{y}) \cdot \frac{\mathcal{L}_R^\perp(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)[V^r(\mathbf{y})\cos\alpha^r(\mathbf{y})V^s(\mathbf{y})\cos\alpha^s(\mathbf{y})]^{-1/2}}{2\rho^{(0)}(\mathbf{y})A^{||}(\mathbf{x}^r,\mathbf{y})A^{||}(\mathbf{y},\mathbf{x}^s)(\boldsymbol{\nu}^\phi(\mathbf{y})\cdot\boldsymbol{\nu}^m(\mathbf{y}))} \cdot e^{-i\omega T(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)}|\mathbf{p}^m(\mathbf{y})|^3 d\nu d\omega\ \}. \quad (42)$$

CONSIDERATIONS WITH BAND-LIMITED SOURCES

The results derived in this paper are results where the source is assumed to be a delta function. In seismic field data the source is some wavelet $s(t)$ which gives a blurring effect in the inversion/migration (De Hoop *et al.*, 1999). The smearing of the inversion/migration result in space can be accounted for in the surface scattering results in this paper (Jaramillo & Bleistein, 1999; Sollid & Ursin, 2001; Ursin, 2002). The singular function

$$\delta(\phi(\mathbf{y}))|\nabla_{\mathbf{x}}\phi(\mathbf{y})|, \quad (43)$$

in equations (40) and (42) is replaced by

$$|\mathbf{p}^\phi(\mathbf{x}^\phi)|s(|\mathbf{p}^\phi(\mathbf{x}^\phi)|\boldsymbol{\nu}^\phi\cdot(\mathbf{y}-\mathbf{x}^\phi)), \quad (44)$$

with $\mathbf{x}^\phi$ defined from $\phi(\mathbf{x}^\phi)=0$. At a specular reflection $|\mathbf{p}^\phi|=|\mathbf{p}^m|$, this is the so-called *stretch factor* (Tygel *et al.*, 1994). Dividing our surface inversion/migration formulas by the stretch factor will thus account for the blurring and yield the correct amplitude behavior.

DISCUSSION AND CONCLUSION

We have derived several results under the restriction of 2.5-D (see introduction) for modeling, inversion and migration. Results are derived for anisotropy with at least one symmetry plane. The medium can be inhomogeneous in-plane, but homogeneous in the out-of-plane direction in order to restrict the wave propagation to the plane of consideration. These restrictions reduce the number of medium parameters one can invert for as compared to the full 3-D case. The stationary phase formula gives the leading-order term with respect to a large parameter, here being ω. From the 3-D Born modeling formula to the 2.5-D requires only a change by a multiplication with a phase factor $\omega^{-1/2}$ as well as spatial factors. We observe the same behavior in the change from 3-D to 2.5-D in the Born-Helmholtz modeling equation (37) and the large-contrast reflection modeling formula (41). All the inversions are done in the same manner. We observe that the inversion formula in Appendix A needs a factor ω in the integral. The equivalent inversion formula in 3-D has ω^2 (Ursin, 2002). This means that the data in the 2.5-D case, which are operated on by these factors in the inversion, will have a factor of $\omega^{-1/2}$ different than the 3-D case. In the time domain these factors become half-integrations. Integration is a smoothing operation, meaning that the data used in 2.5-D inversion are regularized or smoothed by a factor $\omega^{-1/2}$ more than for the 3-D case.

The 3-D case with point source, point or surface scatterer, and point receiver can be reduced to the 2.5-D case, as we have seen, by an approximation on the out-of-plane integral by stationary phase with point source, line or cylindrical surface scatterer and point receiver (equations (24) and (37) respectively). Going from the 2.5-D formulas to the 2-D case, however is not possible in a similar manner. *Ad hoc* methods have been suggested such as multiplying by the appropriate phase factor $\sqrt{2\pi i/\omega}$ and setting the out-of-plane geometrical spreading, $\mathcal{L}_R^\perp$, to 1 (see Stockwell (1995) for reference). Physically, the inability to deduce the 2-D case from the 2.5-D case in any sound mathematical way stems from the fact that the 2-D problem has a line source and line receivers as opposed to point source and point receivers. They do however have the same type of scatterers, i.e. line or cylindrical surface scatterer.

ACKNOWLEDGMENTS

The authors would like to thank the Center of Wave Phenomena for the possibility to stay as guests for the Fall 2001 until Summer 2002. Stig-Kyrre Foss would also like to thank the URE-project at NTNU, Trondheim, Norway for the financial support.

REFERENCES

Aki, K., & Richards, P. G. 1980. *Quantitative seismology. Theory and methods.* Vol. 1. W.H. Freeman, New York.

Beylkin, G. 1984. The inversion problem and applications of the generalized Radon Transform. *Comm. Pure Appl. Math.*, **XXXVII**, 579–599.

Beylkin, G., & Burridge, R. 1990. Linearized inverse scattering problems in acoustics and elasticity. *Wave Motion*, **12**, 15–52.

Bleistein, N. 1984. *Mathematics of wave phenomena.* Academic Press Inc.

Bleistein, N. 1986. Two-and-one-half dimensional in-plane wave propagation. *Geophysical Prospecting*, 686–703.

Brandsberg-Dahl, S., De Hoop, M. V., & Ursin, B. 1999. Velocity analysis in the common scattering-angle/azimuth domain. *Proceedings 69th Ann. Internat. Mtg., Soc. Explor. Geophys.*, 1222–1223.

Brandsberg-Dahl, S., Ursin, B., & De Hoop, M. V. 2000. Seismic velocity analysis in the scattering angle/azimuth domain. *Submitted Geophys. Prosp.*

Brandsberg-Dahl, S., De Hoop, M. V., & Ursin, B. 2001.

Focusing in dip and AVA compensation on scattering-angle/azimuth gathers. *Submitted to Geophysics.*

Burridge, R., De Hoop, M. V., Miller, D., & Spencer, C. 1998. Multiparameter inversion in anisotropic media. *Geophys. J. Int.*, **134**, 757–777.

Červený, V. 1981. Computation of geometrical spreading by dynamic ray tracing. *Pages 61–73 of: SEP-28.* Stanford University.

Červený, V. 2001. *Seismic Ray Theory.* Cambridge University Press.

Chapman, C. H. 1994. Reflection/transmission coefficients reciprocities in anisotropic media. *Geophys. Journal International*, **116**, 498–501.

Chapman, C. H. 1999. *Seismic Ray Theory and Finite Frequency Extensions.* to appear in IASPEI handbook.

De Hoop, M. V., & Bleistein, N. 1997. Generalized Radon transform inversions for reflectivity in anisotropic elastic media. *Inverse Problems*, **13**, 669–690.

De Hoop, M. V., Spencer, C., & Burridge, R. 1999. The resolving power of seismic amplitude data: An anisotropic inversion/migration approach. *Geophysics*, **64**, 852–873.

Ettrich, N., Sollid, A., & Ursin, B. 2001. Out-of-plane geometrical spreading in anisotropic media. *Submitted Geoph. Prosp.*

Geoltrain, S. 1989. *Asymptotic solutions to direct and inverse scattering in anisotropic media.* Ph.D. thesis, Colorado School of Mines.

Goldin, S. V. 1986. *Seismic traveltime inversion.* SEG: Investigations in Geophysics No. 1.

Guillemin, V. 1985. On some results of Gel'fand in integral geometry. *Proceedings of Symposia in Pure Mathematics, American Mathematical Society*, **43**, 149–155.

Jaramillo, H. H., & Bleistein, N. 1999. The link of Kirchhoff migration and demigration to Kirchhoff and Born modeling. *Geophysics*, **64**, 1793–1805.

Klimeš, L. 1997. Phase shift of the Green function due to caustics in anisotropic media. *Expanded Abstracts: 68th Ann. Internat. Mtg. ,Soc. Explor. Geophys.*, 1834–1837.

Schleicher, J., Tygel, M., Ursin, B., & Bleistein, N. 2001. The Kirchhoff-Helmholtz integral for anisotropic elastic media. *Wave Motion*, **34**, 353–364.

Schoenberg, M., & Helbig, K. 1997. Orthoromic media: Modeling elastic wave behavior in a vertically fractured earth. *Geophysics*, 1954–1974.

Sollid, A. 2000. *Imaging of ocean bottom seismic data.* Ph.D. thesis, Norwegian University of Science and Technology, Trondheim.

Sollid, A., & Ursin, B. 2001. *Scattering-angle migration of OBS data in weakly anisotropic media.* Submitted to Geophysics.

Stockwell, Jr., J. W. 1995. 2.5-D wave equations amd high-frequency asymptotics. *Geophysics*, **60**, 556–562.

Stolk, C. C. 2000. Microlocal analysis of a seismic linearized inverse problem. *Wave Motion.* in print.

Stolk, C. C., & De Hoop, M. V. 2002. Modeling and inversion of seismic data in anisotropic elastic media. *Comm. Pure Appl. Math.*, **LV**.

Tsvankin, I. 2001. *Seismic signatures and analysis of reflection data in anisotropic media.* Elsevier.

Tygel, M., Schleicher, J., & Hubral, P. 1994. Pulse distortion in depth migration. *Geophysics*, **59**, 1561–1569.

Ursin, B. 2002. Parameter inversion and migration in anisotropic elastic media. *In process, CWP-rapport 2002.*

Ursin, B., & Tygel, M. 1997. Reciprocal volume and surface scattering integrals for anisotropic elastic media. *Wave Motion*, **26**, 31–42.

APPENDIX A: THE INVERSION FORMULA

The following identity comes from Fourier analysis

$$\frac{1}{(2\pi)^2}\int e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}d\mathbf{k} = \delta(\mathbf{x}-\mathbf{y}), \tag{A1}$$

where $\mathbf{x}, \mathbf{y}, \mathbf{k} \in \mathbb{R}^2$. $\mathbf{k}$ is usually referred to as the wavenumber. We introduce the equivalent of polar coordinates where we let the radius be $k = \sqrt{k_1^2 + k_2^2}$ and the angle ν^m so that

$$\begin{aligned}\mathbf{k} &= k(\cos\nu, \sin\nu) = (k_1, k_2)\\ &= k\boldsymbol{\nu}^m.\end{aligned} \tag{A2}$$

Using this parametrization in the integral (A1) we get

$$\int_0^\infty k \int_0^{2\pi} e^{ik\boldsymbol{\nu}^m\cdot(\mathbf{x}-\mathbf{y})}d\nu dk = 4\pi^2\delta(\mathbf{x}-\mathbf{y}). \tag{A3}$$

In the framework of wave propagation the wavenumber is $\mathbf{k} = \omega|\mathbf{p}^m(\mathbf{y})|$. We substitute this into equation (A3) and take the real part on both sides to get

$$Re\left\{\int_0^\infty \omega d\omega \int_0^{2\pi} |\mathbf{p}^m(\mathbf{y})|^2 d\nu e^{i\omega\nabla_\mathbf{x}T\cdot(\mathbf{x}-\mathbf{y})}\right\} = 4\pi^2\delta(\mathbf{x}-\mathbf{y}). \tag{A4}$$

After a preconditioning of the data in the main text, we use Taylor expansions (Beylkin, 1984) of amplitudes, polarization vectors and phase functions around $\mathbf{x}$ (as done in detail in Appendix C). The first term in the expansion has the form of the inversion formula in equation (A1). The result is the inversion formula

$$Re\left\{\int_0^\infty \omega d\omega \int_{E_\nu} |\mathbf{p}^m(\mathbf{y})|^2 d\nu[1+\mathcal{O}(|\mathbf{x}-\mathbf{y}|)]e^{i\omega[\nabla T\cdot(\mathbf{x}-\mathbf{y})+\mathcal{O}(|\mathbf{x}-\mathbf{y}|^2)]}\right\} = 4\pi^2\delta(\mathbf{x}-\mathbf{y}) + \text{smoother terms.} \tag{A5}$$

Integration of the of the other terms of the Taylor expansion yields smoother amplitude terms by the Bolker conditions (Guillemin, 1985) or the equivalent traveltime injectivity conditions (Stolk, 2000). They are discarded in inversion for the most singular term.

APPENDIX B: 2-D MODELING AND INVERSION

2-D modeling is done with a line source in a 2-D model; then the response will not vary along a receiver line. We derive the 2-D GRA Green's function from the 3-D GRA Green's function by summing the contributions from all point sources along the source line. This gives the response

$$U_{ip} = \int_{-\infty}^{\infty} G_{ip}(\mathbf{x}, \omega, \mathbf{x}^s)dx_2^s, \tag{B1}$$

where $G_{ip}(\mathbf{x}, \omega, \mathbf{x}^s)$ is the 3-D GRA Green's function defined in equation (3). Using the same arguments as in the main text, the stationary point of the integral is found to be at $x_s^s = x_2$, and then

$$\left|\frac{\partial^2 T(\mathbf{x}, \mathbf{x}^s)}{\partial x_2^2}\right|_{x_2=0} = \left|\frac{1}{Q_2^\perp(\mathbf{x}, \mathbf{x}^s)}\right|. \tag{B2}$$

Approximating the integral in equation (B1) by the method of stationary phase then yields the reciprocal 2-D Green's function

$$G_{ip}^{2-D}(\mathbf{x}, \mathbf{x}^s) = |\omega|^{-1/2}e^{i\frac{\pi}{4}\operatorname{sgn}\omega}h_i^s(\mathbf{x})A^{2-D}(\mathbf{x}, \mathbf{x}^s)e^{i\omega T(\mathbf{x},\mathbf{x}^s)}h_p(\mathbf{x}^s), \tag{B3}$$

where the 2-D amplitude function is

$$A^{2-D}(\mathbf{x}, \mathbf{x}^s) = \frac{e^{-i\frac{\pi}{2}\kappa(\mathbf{x},\mathbf{x}^s)\operatorname{sgn}\omega}}{2(2\pi)^{1/2}[\rho(\mathbf{x})v^s(\mathbf{x})\rho(\mathbf{x}^s)v(\mathbf{x}^s)]^{1/2}|Q_2^{||}(\mathbf{x}, \mathbf{x}^s)|^{1/2}}, \tag{B4}$$

and $Q_2^{||}$ is defined in equation (8). The indices i, p take on values $1, 3$ and all vectors also only have components in the x_1 and x_3 direction.

The 2-D Born single scattering integral can be derived from the 2-D equations exactly as in the 3-D case. A line source at $\mathbf{x}^s$ will excite a line scatterer at $\mathbf{x}$, which acts as a secondary line source producing the response as $\mathbf{x}^r$. Taking into account the reciprocity of the GRA Green's function, the 2-D Born integral can be written

$$U_{mn}(\mathbf{x}^r,\omega,\mathbf{x}^s) = i\omega \int_{\mathcal{D}(x_2=0)} h^r_m(\mathbf{x}^r) A^{2-D}(\mathbf{x}^r,\mathbf{x}) A^{2-D}(\mathbf{x},\mathbf{x}^s)\rho^{(0)}(\mathbf{x})S(\mathbf{x})e^{i\omega T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)} h^s_n(\mathbf{x}^s)d\mathbf{x}. \tag{B5}$$

For inversion and migration we precondition the data by

$$U(\mathbf{x}^r,\omega,\mathbf{x}^s;\mathbf{y}) = -i\frac{h_m(\mathbf{x}^r;\mathbf{y})U_{mn}(\mathbf{x}^r,\omega,\mathbf{x}^s)h_n(\mathbf{x}^s;\mathbf{y})}{4\pi^2\rho^{(0)}(\mathbf{y})A^{2-D}(\mathbf{x}^r,\mathbf{y})A^{2-D}(\mathbf{y},\mathbf{x}^s)} e^{-i\omega T(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)}|\mathbf{p}^m(\mathbf{y})|^2, \tag{B6}$$

Using this corrected data gives us the inversion/migration results in equations (26) and (28) in the main text.

The 2-D Born-Helmholtz surface modeling integral is derived equivalently as in the main text and is given by

$$U_{mn}(\mathbf{x}^r,\omega,\mathbf{x}^s) = -\int_{\mathcal{D}(x_2=0)} h_m(\mathbf{x}^r)\rho^{(0)}(\mathbf{x})A^{2-D}(\mathbf{x}^r,\mathbf{x})A^{2-D}(\mathbf{x},\mathbf{x}^s)$$
$$\cdot\, \mathbf{w}^T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)\mathbf{c}^{(1)}(\mathbf{x})|_{\phi(\mathbf{x})=0}\delta(\phi(\mathbf{x}))|\nabla_{\mathbf{x}}\phi(\mathbf{x})| \frac{\boldsymbol{\nu}^\phi\cdot\boldsymbol{\nu}^m}{|\nabla_{\mathbf{x}}T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)|} e^{i\omega T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)}h_n(\mathbf{x}^s)d\mathbf{x}. \tag{B7}$$

Preconditioning the data in the above equation by

$$U_\Sigma(\mathbf{x}^r,\omega,\mathbf{x}^s;\mathbf{y}) = -\omega\frac{h_m(\mathbf{x}^r;\mathbf{y})U_{mn}(\mathbf{x}^r,\omega,\mathbf{x}^s)h_n(\mathbf{x}^s;\mathbf{y})}{4\pi^2\rho^{(0)}(\mathbf{y})A^{2-D}(\mathbf{x}^r,\mathbf{y})A^{2-D}(\mathbf{y},\mathbf{x}^s)} e^{-i\omega T(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)}\frac{|\mathbf{p}^m(\mathbf{y})|^3}{\boldsymbol{\nu}^\phi\cdot\boldsymbol{\nu}^m}, \tag{B8}$$

the inversion result can be found by using inversion formula (40).

APPENDIX C: 2.5-D INVERSION OF THE BORN MODELING EQUATION

By the following inversion/migration we get the most singular part of the medium perturbations $\mathbf{c}^{(1)}$ (De Hoop *et al.*, 1999). This means that we disregard the smoother terms. We perform the inversion/migration by the use of the inversion formula derived in Appendix A

$$Re\left\{\int_0^\infty \omega d\omega \int_{E_\nu} |\mathbf{p}^m(\mathbf{y})|^2 d\nu[1+\mathcal{O}(|\mathbf{x}-\mathbf{y}|)]e^{i\omega[\nabla T\cdot(\mathbf{x}-\mathbf{y})+\mathcal{O}(|\mathbf{x}-\mathbf{y}|^2)]}\right\} = 4\pi^2\delta(\mathbf{x}-\mathbf{y}) + \text{smoother terms}. \tag{C1}$$

where ν^m is the migration dip angle and $\mathbf{y}$ is the imaging point. The imaging point is the spatial point in the subsurface for which we invert for the rock parameters. In order to get our modeling formula into the appropriate form, we precondition our data U_{mn} (24) with factors evaluated at the imaging point and the polarization vectors at the source and receiver given in equation (25)

$$U(\mathbf{x}^r,\omega,\mathbf{x}^s;\mathbf{y}) =$$
$$\left(\frac{1}{2\pi}\right)^{5/2}\frac{e^{-i\frac{\pi}{4}\operatorname{sgn}\omega}}{|\omega|^{1/2}} h_m(\mathbf{x}^r;\mathbf{y})U_{mn}(\mathbf{x}^r,\omega,\mathbf{x}^s)h_n(\mathbf{x}^s;\mathbf{y})\cdot\frac{\mathcal{L}^\perp_R(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)}{\rho^{(0)}(\mathbf{y})A^{||}(\mathbf{x}^r,\mathbf{y})A^{||}(\mathbf{y},\mathbf{x}^s)}e^{-i\omega T(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)}|\mathbf{p}^m(\mathbf{y})|^2. \tag{C2}$$

We perform a Taylor expansion of the resulting amplitude terms, phase function and polarization vectors in the integral around the imaging point $\mathbf{y}$ after this preconditioning

$$\begin{aligned} T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s) - T(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s) &= \nabla T(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)\cdot(\mathbf{x}-\mathbf{y}) + \mathcal{O}(|\mathbf{x}-\mathbf{y}|^2),\\ \frac{A^{||}(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)\mathcal{L}^\perp(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)}{A^{||}(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)\mathcal{L}^\perp(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)} &= 1+\mathcal{O}(|\mathbf{x}-\mathbf{y}|),\\ h_m(\mathbf{x}^r)h_m(\mathbf{x}^r;\mathbf{y}) = h_n(\mathbf{x}^s)h_n(\mathbf{x}^s;\mathbf{y}) &= 1+\mathcal{O}(|\mathbf{x}-\mathbf{y}|), \end{aligned} \tag{C3}$$

where we have used that the polarization vectors are normalized. This means that the preconditioned equation (24) in equation (C2) becomes

$$U(\mathbf{x}^r,\omega,\mathbf{x}^s;\mathbf{y}) = \omega|\mathbf{p}^m(\mathbf{y})|^2\int_{\mathcal{D}(x_2=0)}[1+\mathcal{O}(|\mathbf{x}-\mathbf{y}|)]\mathbf{w}^T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s)\mathbf{c}^{(1)}(\mathbf{x})e^{i\omega[\nabla T\cdot(\mathbf{x}-\mathbf{y})+\mathcal{O}(|\mathbf{x}-\mathbf{y}|^2)]}d\mathbf{x}. \tag{C4}$$

We wish to remove the effects of the radiation pattern in order to invert only for the medium perturbation. Multiplying on both sides with the radiation pattern (16) at the image point, $\mathbf{w}(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)$, and using the Taylor expansion

$$\mathbf{w}(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)\mathbf{w}^T(\mathbf{x}^r,\mathbf{x},\mathbf{x}^s) = \mathbf{w}(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)\mathbf{w}^T(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s) + \mathcal{O}(|\mathbf{x}-\mathbf{y}|), \tag{C5}$$

we can write

$$\mathbf{w}(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)U(\mathbf{x}^r,\omega,\mathbf{x}^s;\mathbf{y}) = \omega|\mathbf{p}^m(\mathbf{y})|^2 \int_{\mathcal{D}(x_2=0)} \mathbf{w}(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)\mathbf{w}^T(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)[1+\mathcal{O}(|\mathbf{x}-\mathbf{y}|)] \cdot \mathbf{c}^{(1)}(\mathbf{x})e^{i\omega[\nabla T\cdot(\mathbf{x}-\mathbf{y})+\mathcal{O}(|\mathbf{x}-\mathbf{y}|^2)]}d\mathbf{x}. \quad \text{(C6)}$$

The radiation pattern is dependent on the scattering angle θ (see equation (16) and Figure 1), so we integrate over θ spanning the domain E_θ

$$\int_{E_\theta} \mathbf{w}(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)U(\mathbf{x}^r,\omega,\mathbf{x}^s;\mathbf{y})d\theta = \omega|\mathbf{p}^m(\mathbf{y})|^2 \int_{\mathcal{D}(x_2=0)} \Lambda(\nu^m;\mathbf{y})[1+\mathcal{O}(|\mathbf{x}-\mathbf{y}|)]\mathbf{c}^{(1)}(\mathbf{x})e^{i\omega[\nabla T\cdot(\mathbf{x}-\mathbf{y})+\mathcal{O}(|\mathbf{x}-\mathbf{y}|^2)]}d\mathbf{x}, \quad \text{(C7)}$$

where the 22×22 matrix Λ is defined in equation (27) in the main text. Introducing the pseudo-inverse $\langle\Lambda(\nu^m;\mathbf{y})\rangle^{-1}$ and multiplying by this factor on both sides we in effect remove the radiation pattern. This yields

$$\langle\Lambda(\boldsymbol{\nu}^m;\mathbf{y})\rangle^{-1}\int_{E_\theta} \mathbf{w}(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)U(\mathbf{x}^r,\omega,\mathbf{x}^s;\mathbf{y})d\theta \approx \omega|\mathbf{p}^m(\mathbf{y})|^2 \int_{\mathcal{D}(x_2=0)} [1+\mathcal{O}(|\mathbf{x}-\mathbf{y}|)]\mathbf{c}^{(1)}(\mathbf{x})e^{i\omega[\nabla T\cdot(\mathbf{x}-\mathbf{y})+\mathcal{O}(|\mathbf{x}-\mathbf{y}|^2)]}d\mathbf{x}. \quad \text{(C8)}$$

Now integrate both sides over the domains of ω and ν as indicated in the inversion formula (C1)

$$\int_0^\infty d\omega \int_{E_\nu} d\nu^m \langle\Lambda(\boldsymbol{\nu}^m;\mathbf{y})\rangle^{-1}\int_{E_\theta} \mathbf{w}(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)U(\mathbf{x}^r,\omega,\mathbf{x}^s;\mathbf{y})d\theta = \int_{\mathcal{D}(x_2=0)} \mathbf{c}^{(1)}(\mathbf{x}) \int_0^\infty \omega d\omega \int_{E_\nu} |\mathbf{p}^m(\mathbf{y})|^2 d\nu[1+\mathcal{O}(|\mathbf{x}-\mathbf{y}|)]e^{i\omega[\nabla T\cdot(\mathbf{x}-\mathbf{y})+\mathcal{O}(|\mathbf{x}-\mathbf{y}|^2)]}d\mathbf{x}. \quad \text{(C9)}$$

By taking the real part here, the last part of the integrals on the right hand side now has the form of inversion formula (C1). Observe that they can be written this way because of the terms we multiplied with in our preconditioning; this serves as a motivation of our choice. We disregard the smoother terms as we are only interested in the most singular term. The resulting delta function collapses the integral over $\mathbf{x}$ and leaves the estimate of $\mathbf{c}^{(1)}$ evaluated at the imaging point

$$\hat{\mathbf{c}}^{(1)}(\mathbf{y}) = Re\left\{\int_0^\infty \int_{E_\nu} \Lambda^{-1}(\boldsymbol{\nu}^m;\mathbf{y}) \int_{E_\theta} \mathbf{w}(\mathbf{x}^r,\mathbf{y},\mathbf{x}^s)U(\mathbf{x}^r,\omega,\mathbf{x}^s;\mathbf{y})d\theta d\nu d\omega\right\}, \quad \text{(C10)}$$

where U is found in equation (C2). This is the inversion result of the 2.5-D Born modeling formula in the main text (26). The inversion of the other modelling equations in the main text are done in a similar manner.