Explicit expressions for map time-migration in weakly anisotropic VTI media

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ABSTRACT
If the velocity of the medium is known and the medium does not incorporate different reflectors that have identical surface seismic measurements that remain identical under small perturbations of the medium, map migration achieves a one-to-one correspondence between the reflections recorded at the surface, and the reflectors in the subsurface, through explicit use of the slopes in the data. Because of this property, it has had many applications in velocity estimation. In this paper, I apply the weak anisotropy approximation to the 3D post-stack and pre-stack map time-migration equations for qP-waves in transversely isotropic homogeneous media with a vertical symmetry axis. In this way the dependence of qP-wave time imaging (in this approximation) on the anellipticity parameter η and the zero-dip NMO velocity $V_{NMO}(0)$ only, is analytically confirmed in the context of map time-migration. The accuracy of the equations is verified with numerical examples. The developed equations allow for the estimation of the anellipticity parameter η in the context of time-migration velocity analysis.

Key words: map time-migration, homogeneous, VTI, weak anisotropy

1 INTRODUCTION

Imaging the subsurface involves measuring seismic waves at the earth-surface and migrating the data down to true subsurface positions. It is known (Guillemin, 1985; ten Kroode *et al.*, 1998; De Hoop & Brandsberg-Dahl, 2000; Stolk & De Hoop, 2002; Douma & De Hoop, 2002) that for arbitrary complex media, the slope information together with the position (space and time) of the reflections are related through a one-to-one mapping to the subsurface position and orientation of the reflectors, provided the medium does not allow different reflectors to have identical surface seismic measurements that persist to be identical under small perturbations of the medium (this is the Bolker condition (Guillemin, 1985)). Most imaging algorithms use the slope information in the data only implicitly, whereas map migration uses this information explicitly.

Map migration is certainly not new. Many authors explicitly use the slope information in data to find the reflection points (Weber, 1955; Graeser *et al.*, 1957; Haas & Viallix, 1976; Musgrave, 1961; Sattlegger, 1964; Cohen, 1998), although they do not always use the term map migration. Also, map migration has had many applications in velocity estimation (Gjoystdal & Ursin, 1981; Gray & Golden, 1983; Maher *et al.*, 1987; Sword, 1987; Iversen & Gjoystdal, 1996; Billette & Lambare, 1998; Iversen *et al.*, 2000).

Douma and de Hoop (2002) derived closed-form expressions for map time-migration in homogeneous VTI media. Alkhalifah & Tsvankin (1995) and Tsvankin (2001) have shown that the time-signatures (e.g. reflection move-out, DMO and post-stack and pre-stack time-migration operators) of qP-waves in VTI media are governed mainly by the anellipticity parameter η and the zero-dip NMO-velocity $V_{NMO}(0)$. Alkhalifah and Tsvankin (1995) numerically confirm this dependence for transversely isotropic media with a vertical symmetry axis. In addition, Tsvankin (1996) shows numerically that the influence of the vertical shear-wave velocity V_{S0} on the kinematics of qP-waves in transversely isotropic media is negligible, while Alkhalifah (1998) shows analytically that all time-related processing depends exactly on η and $V_{NMO}(0)$ only, when $V_{S0} = 0$. The expressions in Douma and de Hoop (2002) for pre-stack map time-migration in homogeneous VTI media, however, depend on the four parameters η, $V_{NMO}(0)$, V_{P0} and V_{S0}. Here, through application of the weak anisotropy approximation (Thomsen, 1986), I derive closed-form expressions for map time-migration in such media, explicit in the anellipticity parameter η and the zero-dip NMO-velocity $V_{NMO}(0)$ only, thus confirming Alkhalifah and Tsvankin (1995). Although the obtained expressions merely confirm our current knowledge about the time-signatures of qP-waves in VTI media, the equations presented are new and can be used to invert for the anellipticity parameter η in the context of time-migration velocity analysis.

The outline of this paper is as follows. First, to establish a connection with earlier work on time-signatures of qP-waves in VTI media, I prove that the expression for zero-offset migrated traveltime of Douma and de Hoop (2002) is equivalent to the expression derived by Cohen (1998) for the vertical two-way traveltime t_m. Then, I proceed to derive the weak-anisotropy approximations for post-stack (or, equivalently, zero-offset) time-migration and demigration respectively, followed by the weak anisotropy approximations for pre-stack (or finite-offset) map migration.

2 POST-STACK MAP MIGRATION

In arbitrary complex media a reflection is kinematically fully determined by the two-way traveltime (t_u), the source and receiver positions ($x_{s,r}, y_{s,r}$), and the horizontal slownesses at the source and receiver ($p^{x,y}_{s,r}$). Similarly, in a time-migration context, the image of a reflector (ignoring the amplitudes in the image) is determined by its horizontal position x_m and y_m, its vertical two-way traveltime t_m, and the wave-vector associated with the imaged reflector (i.e. the dip covector ξ) which can be related to the horizontal slowness components p^x_m and p^y_m in the image. Note that the subscripts u and m denote unmigrated and migrated variables respectively. Douma and de Hoop (2002) show that map migration establishes a one-to-one mapping from the unmigrated quantities, associated with reflections in the data, to the migrated quantities associated with reflectors in the subsurface, and make clear that the one-to-one mapping holds even in the presence of caustics, provided the Bolker condition is satisfied. In addition they derive closed-form expressions for pre-stack map time-migration and demigration in homogeneous transversely isotropic media with a vertical symmetry axis (VTI). Here I apply the weak anisotropy approximation to the post-stack (or zero-offset) expressions for map migration and demigration from (Douma & De Hoop, 2002). Note that all equations from (Douma & De Hoop, 2002) are derived in a right handed reference frame with the y-axis positive in the source-receiver direction, and depth (and thus time) positive downwards.

From Douma and de Hoop (2002) the zero-offset expressions for 3D map migration in homogeneous VTI media are

$$t_m = \frac{V(\theta)t_u}{V_{P0}} \times \left(\sqrt{1 - V^2(\theta)p_u^2} - p_u \left.\frac{dV}{d\theta}\right|_{\theta} \right), \tag{1}$$

$$(x,y)_m = (x,y)_u - \frac{V^2(\theta)p_u^{x,y}t_u}{2} - \frac{V(\theta)p_u^{x,y}t_u}{2} \left.\frac{dV}{d\theta}\right|_{\theta} \sqrt{\frac{1}{V^2(\theta)p_u^2} - 1}, \tag{2}$$

$$p_m^{x,y} = \frac{V(\theta)p_u^{x,y}}{V_{P0}\sqrt{1 - V^2(\theta)p_u^2}}, \tag{3}$$

where the notation (x, y) means x or y, V_{P0} is the vertical P-wave velocity, $V(\theta)$ is the phase velocity at angle θ from vertical, $p_u \equiv \sqrt{(p^x_u)^2 + (p^y_u)^2}$, and $p^{x,y}_m$ are the horizontal slownesses in the seismic image. Eqs. (1) - (3) thus determine the reflector position and orientation from the unmigrated quantities associated with a reflection in the data. Note that the expressions for 2D map migration are found from the 3D expressions by setting $x_m = x_u = 0$ and $p^x_m = p^x_u = 0$.

To establish the connection with earlier work on the time signature of qP-waves in homogeneous VTI media, appendix A shows that eq.(1) is equivalent to the expression derived by Cohen (1998) for the vertical two-way traveltime t_m. Recall that Alkhalifah & Tsvankin (1995) and Tsvankin (2001) have shown that qP-wave time signatures in VTI media are governed mainly by the anellipticity parameter η and the zero-dip NMO-velocity $V_{NMO}(0)$, and largely independent of V_{P0}. I confirm this independence for the 2D expressions for post-stack map time-migration in Figure 1. The values $\eta = 0.0833$, $V_{NMO}(0) = 3.29$ km/s and the horizontal slowness components in this figure are the same values used by Alkhalifah & Tsvankin (1995Figure 17); since the used value of the vertical shear-wave velocity V_{S0} is however omitted in their paper, my Figure 1a closely resembles Alkhalifah and Tsvankin's Figure 17, but is not identical. Figure 1 shows results for $V_{S0} = 0$ (grey) and $V_{P0}/V_{S0} = 1.5$ (black), which in practice can be treated as a lower bound on the ratio of P and S wave velocity. Thus the map time-migration equations are governed mainly by η and $V_{NMO}(0)$ and are largely independent of V_{P0}. In addition, this figure confirms that qP-wave time signatures are completely independent of V_{P0} if V_{S0} is set to zero (Alkhalifah, 1998).

Although the numerical results in Figure 1 suggest that post-stack map time-migration is governed mainly by η and $V_{NMO}(0)$, this dependence is not obvious analytically. In terms of the Thomsen parameters δ and ϵ (Tsvankin, 2001p. 22), the phase velocity $V(\theta)$ is given

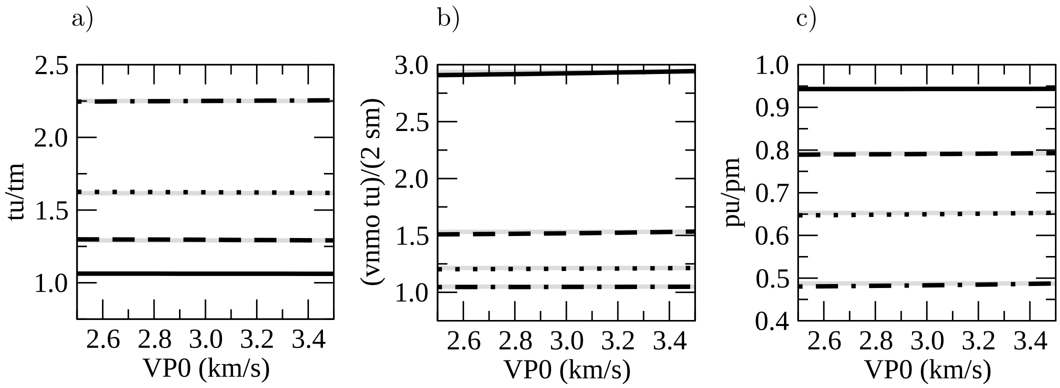

Figure 1. Dependence on V_{P0} of (a) the ratio t_u/t_m, (b) the ratio $V_{NMO}(0)t_u/(2s_m)$ with $s_m \equiv x_u - x_m$, and (c) the ratio p_u/p_m. All ratios are calculated for the values $p_u = 0.10$ s/km (drawn lines), $p_u = 0.18$ s/km (dashed lines), $p_u = 0.22$ s/km (dotted lines) and $p_u = 0.25$ s/km (dashed-dotted lines), with the values $V_{NMO}(0) = 3.29$ km/s, $\eta = 0.0833$, and $V_{P0}/V_{S0} = \frac{3}{2}$ (i.e., $f = \frac{5}{9}$).

by

$$V(\theta) = V_{P0}\left\{1 + \epsilon \sin^2\theta - \frac{f}{2}\right. \quad (4)$$
$$\left.\times\left(1 - \sqrt{\left(1 + \frac{2\epsilon \sin^2\theta}{f}\right)^2 - \frac{2(\epsilon-\delta)\sin^2 2\theta}{f}}\right)\right\}^{\frac{1}{2}} ,$$

with $f \equiv 1 - \frac{V_{S0}^2}{V_{P0}^2}$. This equation can be rewritten in terms of η, $V_{NMO}(0)$ and V_{P0}, using that $\eta = (\epsilon - \delta)/(1+2\delta)$ and $V_{NMO}(0) = V_{P0}\sqrt{1+2\delta}$. Doing this we get

$$V(\theta) = \frac{V_{P0}}{\sqrt{2}}\left\{1 + (\kappa - 1)\sin^2\theta\right. \quad (5)$$
$$\left.+\sqrt{\frac{\kappa\eta}{1+2\eta}\left(\cos 4\theta - 1\right) + \left\{1 + [\kappa - 1]\sin^2\theta\right\}^2}\right\} ,$$

where I defined

$$\kappa(V_{P0}, V_{NMO}(0), \eta) \quad \equiv \quad \frac{V_{NMO}^2(0)}{V_{P0}^2}\left(1 + 2\eta\right) ,$$

and given that the qP-wave phase velocity is largely independent of the vertical S-wave velocity V_{S0}, for simplicity I set $f = 1$. (In practice, however, the influence of V_{S0} is not zero, and it is better to calculate f with a reasonable choice of V_{P0}/V_{S0} ratio, especially when the true V_{P0}/V_{S0} ratio is relatively small, i.e. close to 1.5.) Substituting eq.(5) in the post-stack map time-migration equations (1) - (3), show a functional dependence of the reflector position and orientation on V_{P0}. Thus although the numerical results indicate that the dependence on V_{P0} is very weak, the analytic expressions so far do not indicate this behaviour.

To show analytically that the dependence on V_{P0} is weak, I apply the weak anisotropy approximation to the map time-migration equations in appendix B. The accuracy of these approximations is verified by calculating the approximated values for the 2D migrated quantities shown in Figure 2a. From the top downwards, the figure shows the migrated time, horizontal displacement (i.e. $s_m \equiv x_u - x_m$) and migrated horizontal slowness, normalized by their elliptic counterparts [$t_m/(t_m|_{\eta=0})$, $s_m/(s_m|_{\eta=0})$ and $p_m/(p_m|_{\eta=0})$ respectively]. The grey lines show the approximated values, which are indeed close to their true values (black lines) for $V_{NMO}(0) = 3.29$ km/s, $V_{P0} = 3$ km/s, $f = \frac{5}{9}$ (i.e., $V_{P0}/V_{S0} = \frac{3}{2}$) and $\eta = 0.0833$ (drawn lines), but deviate from the exact values for larger values of η [i.e., $\eta = 0.1666$ (dashed) or $\eta = 0.25$ (dotted)]. Note that overall the weak anisotropy approximation becomes worse for larger angles, but is highly accurate for the position (horizontal and in time) and orientation up to about 60 degree dips for $\eta = 0.0833$. This decrease in accuracy of the weak anisotropy approximations with increasing dip is due to the fact that higher orders of $\sin\theta$ usually go together with higher orders in the Thomsen parameters, which are ignored in the weak anisotropy approximation. Also, ignoring the anellipticity parameter η in post-stack imaging leads to significant errors in the migrated location (at the surface and in time) and orientation of the reflector, since the ratios of the migrated quantities x_m, t_m and p_m to their elliptic equivalents are substantially different from unity, even for moderate dips. The mispositioning in time and orientation of the reflector, as a result of ignoring η, however, is negligible for dips up to about 20 degrees for $\eta = 0.0833$. In contrast, the mispositioning in horizontal position is in this case negligible only to about 10 degrees. Mis-positioning in post-stack imaging as a result of ignoring subsurface anisotropy has been studied by Larner & Cohen (1993) and Alkhalifah & Larner (1994).

3 POST-STACK DEMIGRATION

The post-stack map time-demigration equations in homogeneous VTI media are given by [see (Douma &

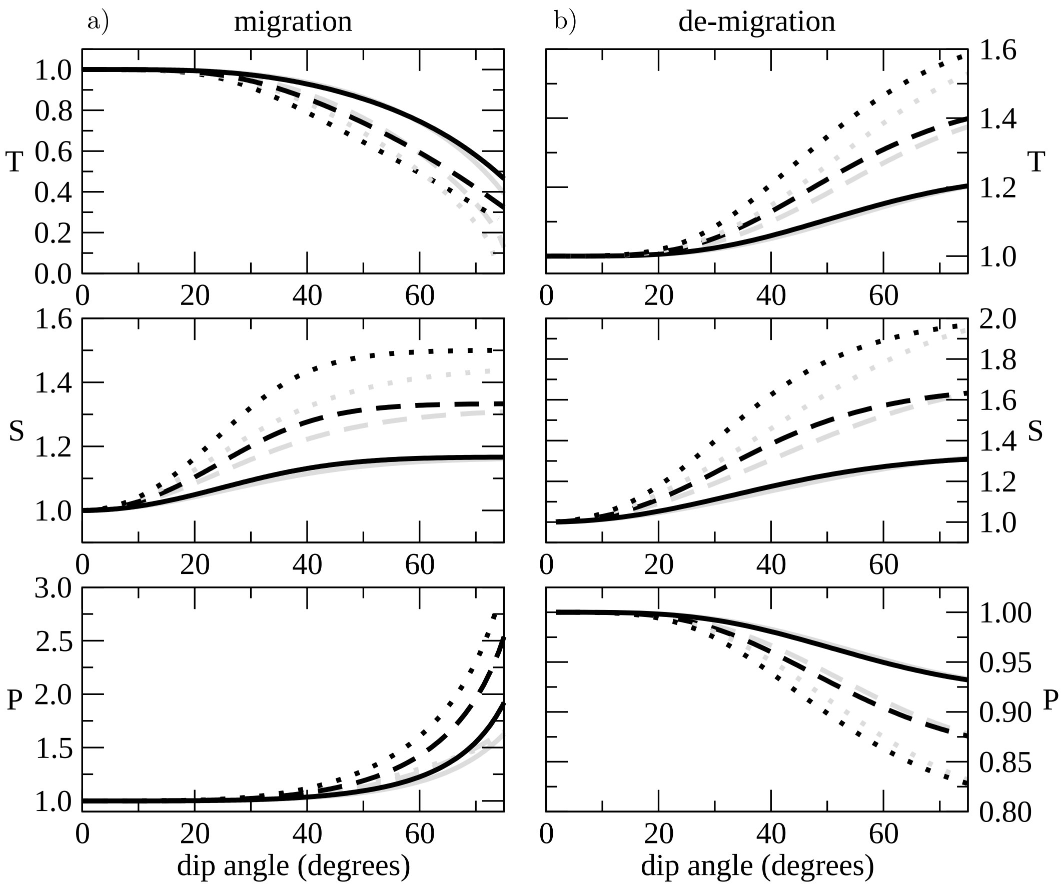

Figure 2. a) Ratios (from the top down) $t_m/(t_m|_{\eta=0})$, $s_m/(s_m|_{\eta=0})$ and $p_m/(p_m|_{\eta=0})$ as a function of reflector dip, using the weak anisotropy approximated equations (grey) and the exact expressions (black). b) Ratios (from the top down) $t_u/(t_u|_{\eta=0})$, $s_u/(s_u|_{\eta=0})$ and $p_u/(p_u|_{\eta=0})$, as a function of reflector dip using the weak anisotropy approximated equations (grey) and the exact expressions (black). All values are calculated with $V_{NMO}(0)$ and f as in Figure 1, $V_{P0} = 3$ km/s, $\eta = 0.0833$ (solid), $\eta = 0.1666$ (dashed), and $\eta = 0.25$ (dotted).

De Hoop, 2002)]

$$t_u = \frac{t_m\sqrt{1+V_{P0}^2 p_m^2}}{\left(\frac{V(\theta)}{V_{P0}} - p_m \left.\frac{dV}{d\theta}\right|_\theta\right)}, \tag{6}$$

$$(x,y)_u = (x,y)_m + \frac{V_{P0}^2 p_m^{x,y} t_m \left(V(\theta) + \frac{1}{V_{P0}p_m} \left.\frac{dV}{d\theta}\right|_\theta\right)}{2\left(V(\theta) - V_{P0}p_m \left.\frac{dV}{d\theta}\right|_\theta\right)}, \tag{7}$$

$$p_u^{x,y} = \frac{V_{P0}p_m^{x,y}}{V(\theta)\sqrt{1+V_{P0}^2 p_m^2}}, \tag{8}$$

with $p_m \equiv \sqrt{(p_m^x)^2 + (p_m^y)^2}$. Then, using eqs.(B1) and (B2) and again consistently linearizing in δ and ϵ, I find

$$t_u \approx t_m \left\{1 + p_m^2 V_{NMO}^2(0) + 6\eta \frac{p_m^4 V_{NMO}^4(0)}{1+p_m^2 V_{NMO}^2(0)}\right\}^{\frac{1}{2}}, \tag{9}$$

$$(x,y)_u \approx (x,y)_m + \frac{V_{NMO}^2(0) p_m^{x,y} t_m}{2} \times \left(1 + 4\eta \frac{p_m^2 V_{NMO}^2(0)}{1+p_m^2 V_{NMO}^2(0)}\right), \tag{10}$$

$$p_u^{x,y} \approx p_m^{x,y} \left\{1 + p_m^2 V_{NMO}^2(0) + 2\eta \frac{p_m^4 V_{NMO}^4(0)}{1+p_m^2 V_{NMO}^2(0)}\right\}^{-\frac{1}{2}}. \tag{11}$$

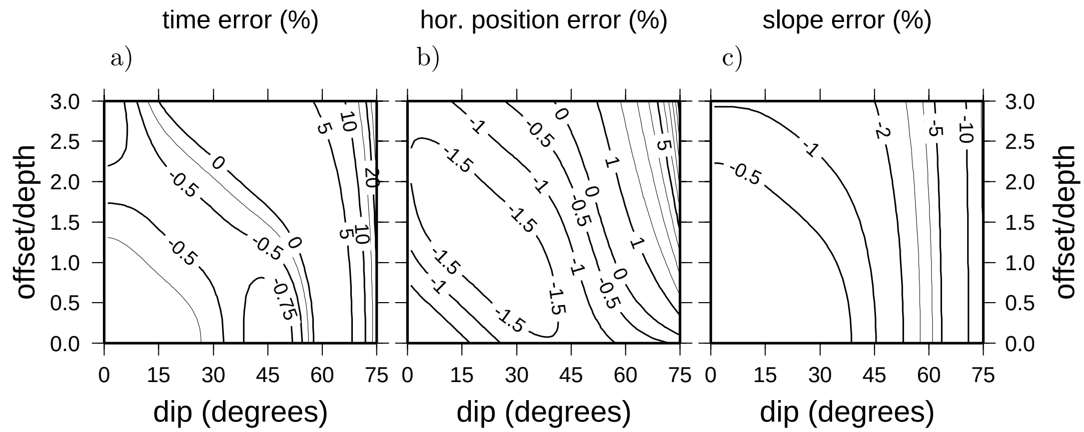

Figure 3. Relative errors of weak anisotropy approximated values with respect to the unapproximated values for (a) migrated time (t_m), (b) horizontal position (s_m) and (c) migrated horizontal slowness (p_m) for 2D pre-stack map migration, as a function of the reflector dip and the offset-to-depth ratio. All values are calculated with $\eta = 0.0833$, $t_m = 2$ s, and $V_{NMO}(0)$, V_{P0} and$f$ as in Figure 1.

For $\eta = 0$ and $V_{NMO}(0) = V_{P0} = v$, these approximations reduce to the isotropic expressions [see Douma & De Hoop (2002)]. To obtain these expressions for the demigration problem, I use the exact expression $\sin\theta = p_m V_{P0}/\sqrt{1 + p_m^2 V_{P0}^2}$.

To assess the accuracy of the approximations, Figure 2b shows the normalized (again by their elliptic equivalents) 2D de-migrated quantities calculated with the exact and approximated equations, for the same values of η, $V_{NMO}(0)$, V_{P0} and f as used in Figure 2a. Again, the approximated values (grey) are in close agreement with the true values (black) for $\eta = 0.833$, but the accuracy of the approximated quantities decreases with increasing dip and η. In the derivation of the weak anisotropy approximations to the demigration equations, it is not necessary to make the additional approximation $\sin\theta \approx p_u V_{P0}$ whenever a $\sin\theta$ term is multiplied with the Thomsen parameters, but we can just use the exact expression $\sin\theta = p_m V_{P0}/\sqrt{1 + p_m^2 V_{P0}^2}$; hence the high accuracy of the unmigrated horizontal slowness even at large dips.

4 PRE-STACK MAP MIGRATION

In appendix C the weak anisotropy approximation for pre-stack (or finite-offset) map time-migration in homogeneous VTI media are derived. Figure 3 shows the accuracy of the approximations as a function of the reflector dip and the offet-to-depth ratio. The values were calculated by first solving the system for 2D pre-stack demigration in VTI media (Douma & De Hoop, 2002) with two non-linear equations with two unknowns for the phase angles at the source and the receiver, viz.

$$\frac{4h}{vt_m} = \left(\frac{\left(r_\theta + \frac{\sqrt{1-r_\theta^2}}{V_r}\,\frac{dV}{d\theta}\big|_r\right)}{\sqrt{1-r_\theta^2} - \frac{r_\theta}{V_r}\,\frac{dV}{d\theta}\big|_r} - \frac{\left(s_\theta + \frac{\sqrt{1-s_\theta^2}}{V_s}\,\frac{dV}{d\theta}\big|_s\right)}{\sqrt{1-s_\theta^2} - \frac{s_\theta}{V_s}\,\frac{dV}{d\theta}\big|_s} \right), \quad (12)$$

$$\nu = -\left(\frac{V_r s_\theta + V_s r_\theta}{V_r\sqrt{1-s_\theta^2} + V_s\sqrt{1-r_\theta^2}} \right), \quad (13)$$

with h the half-offset, $\nu \equiv -\tan\phi$, with ϕ the reflector dip (measured clockwise positive with the horizontal), and $(s,r)_\theta \equiv \sin\theta_{s,r}$ with $\theta_{s,r}$ the phase angles at the source and receiver, respectively. Note that in 2D, $-\pi/2 < \theta_{s,r} < \pi/2$. The angles were found using the Gauss-Newton method (Dennis, 1977). Then, these values were used in the exact migration equations and in their weak anisotropy approximations with $\eta = 0.0833$, $V_{NMO}(0) = 3.29$ km/s, $V_{P0} = 3$ km/s, $f = \frac{5}{9}$ and the migrated time $t_m = 2$ s. The differences (relative to the exact values) between the approximated values and the exact ones are $(t_m^{weak} - t_m^{exact})/t_m^{exact})$, $(s_m^{weak} - s_m^{exact})/s_m^{exact})$, and $(p_m^{weak} - p_m^{exact})/p_m^{exact})$, with $s_m \equiv y_r - y_m$. Figure 3 shows that indeed for all offset-to-depth ratios used, the approximations are highly accurate up to about 60 degrees for $\eta = 0.0833$.

In addition, Figure 4 shows contoured values of the exact equations normalized by their elliptic ($\eta = 0$) counterparts, for the same values of η, $V_{NMO}(0)$, V_{P0}, f and t_m as used in Figure 3. Clearly, ignoring η leads to significant error in the migrated time, lateral location, and orientation of the reflector. The errors are

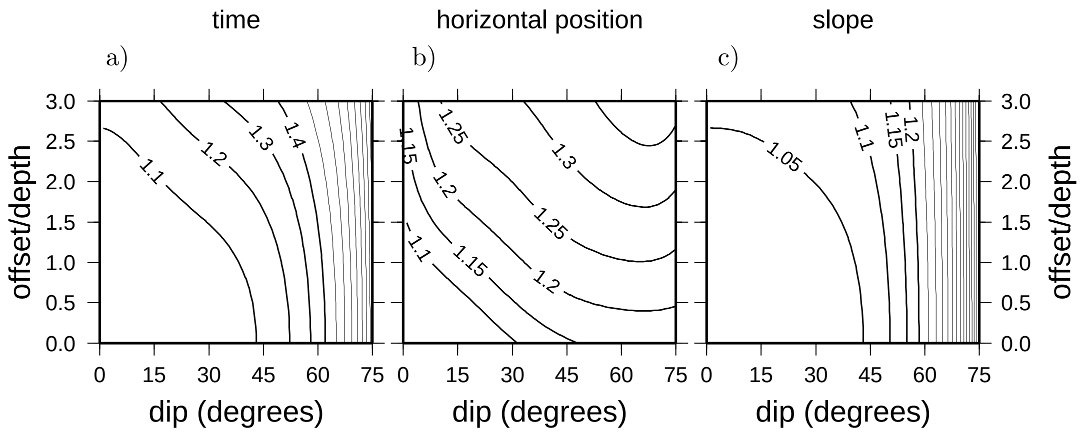

Figure 4. Ratios (a) $t_m/(t_m|_{\eta=0})$, (b) $s_m/(s_m|_{\eta=0})$ and (c) $p_m/(p_m|_{\eta=0})$ for 2D pre-stack map migration, as a function of reflector dip and offset-to-depth ratio. All values used are as in Figure 3.

more substantial for larger offset-to-depth ratios and reflector dips. Note that errors caused by ignoring η for the migrated time and slope show a similar dependence on reflector dip and offset-to-depth ratio, whereas the migrated lateral location shows a maximum error as a function of reflector dip for larger offset-to-depth ratios. Pre-stack migration error as a result of ignoring subsurface anisotropy has been studied by Jaramillo & Larner (1995).

4.1 Pre-stack demigration

While pre-stack map migration in homogeneous VTI media is a straightforward generalization of the post-stack problem, Douma and de Hoop (2002) show that the 3D pre-stack map demigration problem involves the solution of a nonlinear set of four equations with four unknowns, i.e. two azimuth angles and two dips, that cannot be solved in closed-form. Even applying the weak anisotropy approximation to this system does not allow for a solution in closed-form. Therefore, I refrain from treating the pre-stack demigration problem under the weak anisotropy approximation.

5 CONCLUSIONS AND DISCUSSION

Although the post-stack and pre-stack map migration equations for qP-waves in homogeneous VTI media in general depend on $V_{NMO}(0)$, η, V_{P0} and V_{S0}, I show analytically that (at least) under the weak anisotropy approximation the dependence on V_{P0} and V_{S0} disappears, and reconfirm this independence numerically. This is consistent with the result that the time signatures of qP waves in VTI media depend mainly on $V_{NMO}(0)$ and η, showing this in the context of map time-migration. Although this treatment merely confirms this published result, the equations presented are new. The numerical examples presented show that the approximations are accurate (within 5%) up to about 60 degrees for offset-to-depth ratios up to three, at least for η as large as 0.0833 and $V_{NMO}(0) = 3.29$ km/s, and that the errors in the imaged position and orientation caused by ignoring η in homogeneous VTI media can be substantial even for such weak anisotropy.

The equations derived here allow for inversion for the anellipticity parameter η in the context of time-migration velocity analysis. By using data from two (or more) different offsets, we can use the difference between the migrated times, lateral positions or slopes, or all simultaneously, as an objective function to be minimized, by changing η. The optimum value for η then results in the minimum objective function. Such a scheme would be similar to the approach of Iversen *et al.* (2000) who use qP and qSV data to invert for the Thomsen anisotropy parameters ϵ and δ from map migration via ray tracing in heterogeneous anisotropic media. Using the expressions presented, qP data could be used to invert for η instead of ϵ and δ, albeit using straight rays only. In such a scheme, the inverted value for η would be related to a subsurface position, rather than a CMP position as in conventional non-hyperbolic move-out analysis. In most practical situations, however, the lateral variation of η is small, thus rendering such a scheme at first sight obsolete. Nevertheless, it remains to be seen how the sensitivity of the estimation of η from non-hyperbolic move-out analysis compares to that of migration.

Although the equations presented are valid only in VTI media that are homogeneous (i.e., straight rays only), for media with mild lateral and vertical velocity variations, the equations can be used provided the velocity is replaced with the local RMS velocity. Moreover, they can be extended to include vertical heterogeneity through the use of Dix-type averaging (Dix, 1955). Then the value of η obtained from the map time-migration equations presented here, can be treated as an *effective* η, and the interval values of η can be obtained through a layer stripping approach.

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REFERENCES

Alkhalifah, T. 1998. Acoustic approximations for processing in transversely isotropic media. *Geophysics*, **63**(2), 623–631.

Alkhalifah, T., & Larner, K. 1994. Migration errors in transversely isotropic media. *Geophysics*, **59**, 1405–1418.

Alkhalifah, T., & Tsvankin, I. 1995. Velocity analysis for transversely isotropic media. *Geophysics*, **60**, 1550–1566.

Billette, F., & Lambare, G. 1998. Velocity macro-model estimation from seismic reflection data by stereotomography. *Geophysical Journal International*, **135**, 671–690.

Cohen, Jack K. 1998. A convenient expression for the NMO velocity function in terms of ray parameter. *Geophysics*, **63**(1), 275–278.

De Hoop, M. V., & Brandsberg-Dahl, S. 2000. Maslov asymptotic extension of generalized Radon transform inversion in anisotropic elastic media: A least-squares approach. *Inverse Problems*, **16**, 519–562.

Dennis, J.E. Jr. 1977. Nonlinear Least Squares. *Pages 269–312 of:* Jacobs, D. (ed), *State of the Art in Numerical Analysis*. Academic Press.

Dix, C.H. 1955. Seismic velocities from surface measurements. *Geophysics*, **20**, 68–86.

Douma, H., & De Hoop, M. V. 2002. Explicit expressions for map time migration in VTI media and the applicability of map depth migration in the presence of caustics. To be submitted to *Geophysics*.

Gjoystdal, H., & Ursin, B. 1981. Inversion of reflection times in three dimensions. *Geophysics*, **46**(7), 972–983.

Graeser, E., Lode, W., & Pott, G. 1957. Representation of depth-contour maps of arbitrarily curved reflection horizons, including refraction of rays, three-dimensional case. *Geophysical Prospecting*, 135–141.

Gray, W.C., & Golden, J.E. 1983. Velocity determination in a complex earth. *Expanded abstracts of the 53rd Ann. Internat. Mtg. Soc. of Expl. Geophys.*, 577–579.

Guillemin, V. 1985. *Pseudodifferential operators and applications (Notre Dame, Ind., 1984)*. Providence, RI: Amer. Math. Soc. Chap. On some results of Gel'fand in integral geometry, pages 149–155.

Haas, A.G., & Viallix, J.R. 1976. Krigeage applied to geophysics. *Geophysical Prospecting*, **24**, 49–69.

Iversen, E., & Gjoystdal, H. 1996. Event-oriented velocity estimation based on prestack data in time or depth domain. *Geophysical Prospecting*, **44**, 643–686.

Iversen, E., Gjoystdal, H., & Hansen, J.O. 2000. Prestack map migration as an engine for parameter estimation in TI media. *70th Ann. Internat. Mtg. Soc. of Expl. Geophys.*, 1004–1007.

Jaramillo, H., & Larner, K. 1995. Prestack migration error in transversely isotropic media. *CWP Research Report (CWP-185)*.

Larner, K., & Cohen, J. 1993. Migration error in factorized transversely isotropic media with linear velocity variation with depth. *Geophysics*, **58**, 1454–1467.

Maher, S.M., Thorson, J.R., Hadley, D.M., & Swanger, H.J. 1987. Study of comparative interval velocities for map migration. *57th Ann. Internat. Mtg. Soc. of Expl. Geophys.* Session: S3.7.

Musgrave, A.W. 1961. Wave-front charts and three dimensional migrations. *Geophysics*, 738–753.

Sattlegger, J. 1964. Series for three-dimensional migration in reflection seismic interpretation. *Geophysical Prospecting*, **12**, 115–134.

Stolk, C. C., & De Hoop, M. V. 2002. Seismic inverse scattering in the 'wave equation' approach. Submitted to *SIAM J. Appl. Math.*

Sword, C. H. 1987. *Tomographic determination of interval velocities from reflection seismic data: the method of controlled directional reception*. Ph.D. thesis, Stanford Exploration Project, Stanford University.

ten Kroode, A.P.E., Smit, D.-J., & Verdel, A. R. 1998. A microlocal analysis of migration. *Wave Motion*, **28**, 149–172.

Thomsen, L. 1986. Weak elastic anisotropy. *Geophysics*, **51**, 1954–1966.

Tsvankin, I. 1996. P-wave signatures and notation for transversely isotropic media: An overview. *Geophysics*, **61**(2), 467–483.

Tsvankin, I. 2001. *Seismic signatures and analysis of reflection data in anisotropic media*. Elsevier Science Ltd.

Weber, M. 1955. Die bestimmung einer beliebig gekruemmten schichtgrenze aus seismischen reflexionsmessungen. *Geofisica pura e applicata*, **32**, 7–11.

APPENDIX A: COMPARISON WITH COHEN'S (1998) EXPRESSION FOR 2D ZERO-OFFSET MAP MIGRATION

From eqs.(27) and (28) in Cohen (1998) we can deduce, by substituting $z \equiv z_m = V_{p0} t_m/2$ and $\tau \equiv t_u$, that

$$t_m = \frac{t_u}{V_{p0}\left(q - p\frac{dq}{dp}\right)} \,, \tag{A1}$$

where t_m is the migrated two-way traveltime, t_u is the unmigrated two-way traveltime, and p and q are the horizontal and vertical components of the slowness vector. This equation is valid in any vertical symmetry plane of the medium. Although Cohen does not explicitly call this equation an equation for map time-migration, it indeed is, since it makes explicit use of the slope information in the data to find the migrated position in time.

Since the vertical slowness can be written as $q = \sqrt{1/V(\theta)^2 - p^2}$ with $V(\theta)$ the phase velocity, we have

$$\frac{dq}{dp} = -\frac{V(\theta)}{\sqrt{1 - p^2 V(\theta)^2}} \left(\frac{1}{V(\theta)^3} \frac{dV}{dp} + p \right) \,. \tag{A2}$$

To evaluate this expression, we need to find the derivative of the phase-velocity with respect to the horizontal slowness component. Using the chain rule we have

$$\frac{dV}{dp} = \frac{dV}{d\theta}\frac{d\theta}{dp} = \frac{dV}{d\theta}\left(\frac{dp}{d\theta}\right)^{-1} \,. \tag{A3}$$

Since in general we have $p = \sin\theta / V$, we find that

$$\frac{dV}{dp} = \frac{dV}{d\theta} \frac{V(\theta)}{\sqrt{1 - p^2 V(\theta)^2} - p\frac{dV}{d\theta}} \,. \tag{A4}$$

Using this expression in eq.(A2) and substituting the result in eq.(A1) then gives

$$t_m = t_u \frac{V(\theta)}{V_{p0}} \left(\sqrt{1 - p^2 V(\theta)^2} - p\frac{dV}{d\theta} \right) \,, \tag{A5}$$

which is identical to eq.(1) in the main text, when we substitute p_u for p.

APPENDIX B: WEAK-ANISOTROPY APPROXIMATIONS TO THE POST-STACK MAP TIME-MIGRATION EQUATIONS

Under the weak anisotropy approximation (i.e. $(\delta, \epsilon) \ll 1$) we have [e.g. (Tsvankin, 2001, p. 23)]

$$V(\theta) \approx V_{P0}\left(1 + \delta \sin^2\theta + (\epsilon - \delta)\sin^4\theta\right) \,, \tag{B1}$$

and

$$\frac{dV}{d\theta} \approx 2V_{P0} \sin\theta\cos\theta \left[\epsilon - (\epsilon - \delta)\left(1 - 2\sin^2\theta\right)\right] \,. \tag{B2}$$

Using eqs.(B1) and (B2) in (1) and linearizing in δ and ϵ, I find

$$t_m \approx t_u \sqrt{1 - p_u^2 V_{P0}^2 (1 + 2\delta) - 6(\epsilon - \delta) p_u^4 V_{P0}^4 + 4(\epsilon - \delta) p_u^6 V_{P0}^6} \,, \tag{B3}$$

where I used the approximation $\sin\theta \approx p_u V_{P0}$ whenever $\sin\theta$ was multiplied with δ or ϵ, in order to consistently linearize in the Thomsen parameters. Since in the weak anisotropy approximation we have $\eta = (\epsilon - \delta)/(1 + 2\delta) \approx \epsilon - \delta$ and since in general we have $V_{NMO}(0) = V_{P0}\sqrt{1 + 2\delta}$, we can rewrite eq.(B3) to find

$$t_m \approx t_u \sqrt{1 - p_u^2 V_{NMO}^2(0) - 4\eta p_u^4 V_{NMO}^4(0) \left(\frac{3}{2} - p_u^2 V_{NMO}^2(0) \right)} \,. \tag{B4}$$

Similarly, by using eqs.(B1) and (B2) in eqs.(2) and (3) and further linearizing in the Thomsen parameters, I find

$$(x,y)_m \approx (x,y)_u - \frac{V_{NMO}^2(0) p_u^{x,y} t_u}{2} \left(1 + 4\eta p_u^2 V_{NMO}^2(0) \left[1 - \frac{p_u^2 V_{NMO}^2(0)}{2} \right] \right) \,, \tag{B5}$$

$$p_m^{x,y} \approx \frac{p_u^{x,y}}{\sqrt{1 - p_u^2 V_{NMO}^2(0)\left(1 + 2\eta p_u^2 V_{NMO}^2(0)\right)}} \,. \tag{B6}$$

Thus, indeed in the weak anisotropy approximation the dependence of the map time-migration equations on V_{P0} disappears; also for for $\eta = 0$ and $V_{NMO}(0) = V_{P0} = v$ the equations indeed reduce to their isotropic equivalents [see Douma and de Hoop (2002) for the isotropic equations].

APPENDIX C: WEAK-ANISOTROPY APPROXIMATIONS TO THE PRE-STACK MAP TIME-MIGRATION EQUATIONS

For pre-stack (or finite-offset) map migration in homogeneous VTI media, the equations for the reflector location and orientation in the subsurface (Douma & De Hoop, 2002) are

$$
\begin{aligned}
t_m &= \frac{2t_u}{V_{P0}} \left(\frac{1}{V_s\left(\sqrt{1-V_s^2p_s^2} - p_s \left.\frac{dV}{d\theta}\right|_s\right)} + \frac{1}{V_r\left(\sqrt{1-V_r^2p_r^2} - p_r \left.\frac{dV}{d\theta}\right|_r\right)} \right)^{-1} && \text{(C1)} \\
(x,y)_m &= (x,y)_{s,r} - t_u p_{s,r}^{x,y} \left(V_{s,r} + \sqrt{\frac{1}{V_{s,r}^2 p_{s,r}^2} - 1}\, \left.\frac{dV}{d\theta}\right|_{s,r} \right) && \text{(C2)} \\
&\quad \times \frac{\left(\sqrt{1-V_{r,s}^2p_{r,s}^2} - p_{r,s} \left.\frac{dV}{d\theta}\right|_{r,s}\right)}{\frac{1}{V_r}\left(\sqrt{1-V_s^2p_s^2} - p_s \left.\frac{dV}{d\theta}\right|_s\right) + \frac{1}{V_s}\left(\sqrt{1-V_r^2p_r^2} - p_r \left.\frac{dV}{d\theta}\right|_r\right)}\,, \\
p_m^{x,y} &= \frac{1}{V_{P0}} \frac{p_s^{x,y} + p_r^{x,y}}{\sqrt{\frac{1}{V_s^2} - p_s^2} + \sqrt{\frac{1}{V_r^2} - p_r^2}}\,. && \text{(C3)}
\end{aligned}
$$

where $V_{s,r}$ is the phase-velocity at the source or receiver, $p_{s,r}$ is the horizontal slowness at the source or receiver, h is the half-offset, and $p_{s,r} \equiv \sqrt{(p_{s,r}^x)^2 + (p_{s,r}^y)^2}$. Note the order of the subscripts s, r and r, s in eq.(C2).

Comparison of eq.(C1) with eq.(1) reveals that the right hand side of eq.(C1) involves the sum of two fractions with denominators that have the same form as $V_{P0}t_m/t_u$ in eq.(1). Using the weak anisotropy approximation to this form (cf. eq.(B4)) to approximate both fractions in eq.(C1), it follows that the weak anisotropy approximation to t_m for pre-stack map migration is given by

$$
t_m \approx 2t_u \left(\sum_{i=s,r} \frac{1}{\sqrt{1 - p_i^2 V_{NMO}^2(0) - 4\eta p_i^4 V_{NMO}^4(0) \left\{\frac{3}{2} - p_i^2 V_{NMO}^2(0)\right\}}} \right)^{-1}. \qquad \text{(C4)}
$$

Inspection of eqs.(C1) and (C2) reveals that eq.(C2) can be rewritten as

$$
(x,y)_m = (x,y)_{s,r} - \frac{V_{P0} p_{s,r}^{x,y} t_m}{2} \frac{V_{s,r}^2 + \sqrt{\frac{1}{p_{s,r}^2} - V_{s,r}^2}\, \left.\frac{dV}{d\theta}\right|_{s,r}}{V_{s,r}\left(\sqrt{1 - V_{s,r}^2 p_{s,r}^2} - p_{s,r} \left.\frac{dV}{d\theta}\right|_{s,r}\right)}\,. \qquad \text{(C5)}
$$

Since we already have a weak anisotropy approximation for t_m (cf. eq.(C4)), to find the approximated expression for $(x,y)_m$, we need only consider the approximation of the fraction

$$
A \equiv \frac{V_{s,r}^2 + \sqrt{\frac{1}{p_{s,r}^2} - V_{s,r}^2}\, \left.\frac{dV}{d\theta}\right|_{s,r}}{V_{s,r}\left(\sqrt{1 - V_{s,r}^2 p_{s,r}^2} - p_{s,r} \left.\frac{dV}{d\theta}\right|_{s,r}\right)}\,. \qquad \text{(C6)}
$$

The denominator in this fraction is again of the same form as $V_{P0}t_m/t_u$ in eq.(1) with its weak anisotropy approximation given in eq.(B4). We thus have

$$
V_{s,r}\left(\sqrt{1 - V_{s,r}^2 p_{s,r}^2} - p_{s,r} \left.\frac{dV}{d\theta}\right|_{s,r}\right) \approx V_{P0} \sqrt{1 - p_{s,r}^2 V_{NMO}^2(0) - 4\eta p_{s,r}^4 V_{NMO}^4(0) \left\{\frac{3}{2} - p_{s,r}^2 V_{NMO}^2(0)\right\}}\,. \qquad \text{(C7)}
$$

Hence we only need look for a weak anisotropy approximation to the numerator of A. Using eqs.(B1) and (B2) and consistently linearizing in the Thomsen parameters gives

$$V_{s,r}^2 + \sqrt{\frac{1}{p_{s,r}^2} - V_{s,r}^2}\,\frac{dV}{d\theta}\bigg|_{s,r} \approx V_{NMO}^2(0)\left\{1 + 4\eta p_{s,r}^2 V_{NMO}^2(0)\left[1 - \frac{p_{s,r}^2 V_{NMO}^2(0)}{2}\right]\right\}\,. \tag{C8}$$

Then, using this approximation in eq.(C6) together with eq.(C7) we get

$$A \approx \frac{V_{NMO}^2(0)\left\{1 + 4\eta p_{s,r}^2 V_{NMO}^2(0)\left[1 - \frac{p_{s,r}^2 V_{NMO}^2(0)}{2}\right]\right\}}{V_{P0}\sqrt{1 - p_{s,r}^2 V_{NMO}^2(0) - 4\eta p_{s,r}^4 V_{NMO}^4(0)\left\{\frac{3}{2} - p_{s,r}^2 V_{NMO}^2(0)\right\}}}\,. \tag{C9}$$

Finally, using this expression together with eq.(C4) in eq.(C5) then gives

$$\begin{aligned}(x,y)_m \approx (x,y)_{s,r} - p_{s,r}^{x,y} t_u V_{NMO}^2(0)&\left\{1 + 4\eta p_{s,r}^2 V_{NMO}^2(0)\left[1 - \frac{p_{s,r}^2 V_{NMO}^2(0)}{2}\right]\right\}\\ &\times \frac{\sqrt{1 - p_{r,s}^2 V_{NMO}^2(0) - 4\eta p_{r,s}^4 V_{NMO}^4(0)\left\{\frac{3}{2} - p_{r,s}^2 V_{NMO}^2(0)\right\}}}{\sum_{i=s,r}\sqrt{1 - p_i^2 V_{NMO}^2(0) - 4\eta p_i^4 V_{NMO}^4(0)\left\{\frac{3}{2} - p_i^2 V_{NMO}^2(0)\right\}}}\,.\end{aligned} \tag{C10}$$

Note the order of the subscripts s, r and r, s.

Using the weak anisotropy approximation in eq.(C3) and further linearizing in δ and ϵ, I find for the horizontal slowness components in the subsurface

$$p_m^{x,y} \approx \frac{p_s^{x,y} + p_r^{x,y}}{\sum_{i=s,r}\sqrt{1 - p_i^2 V_{NMO}^2(0)\left(1 + 2\eta p_i^2 V_{NMO}^2(0)\right)}}\,. \tag{C11}$$

Note that indeed all weak anisotropy approximations to the migrated quantities for pre-stack migration, reduce to their post-stack equivalent expressions if we set $p_s^{x,y} = p_r^{x,y} = p_u^{x,y}$.