CWP-452

# Quartic moveout coefficient for dipping azimuthally anisotropic layers

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**ABSTRACT**
Interpretation and inversion of azimuthally varying nonhyperbolic reflection moveout requires accounting for both velocity anisotropy and subsurface structure. Here, our previously derived general expression for the quartic moveout coefficient $A_4$ is applied to P-wave reflections in orthorhombic models typical for fractured reservoirs.
The weak-anisotropy approximation for the coefficient $A_4$ in a homogeneous orthorhombic layer above a dipping reflector is controlled by the anellipticity parameters $\eta^{(1)}$, $\eta^{(2)}$ and $\eta^{(3)}$, which are responsible for time processing of P-wave data. The azimuthal variation of the quartic coefficient depends on reflector dip and is quite sensitive to the signs and the relative magnitudes of $\eta^{(1)}$, $\eta^{(2)}$ and $\eta^{(3)}$. The dip-line $A_4$ is proportional to the parameter $\eta^{(2)}$ and rapidly decreases with dip, always going to zero for a dip of 30° where it changes sign. In contrast, the value of $A_4$ on the strike line depends on all three $\eta$ coefficients, but for mild dips it is mostly governed by $\eta^{(1)}$. The contribution of the parameter $\eta^{(3)}$ increases with dip and may lead to a complicated azimuthal signature of the quartic coefficient, with two azimuths of vanishing $A_4$ between the dip and strike directions.
The strong influence of the magnitudes and signs of the anellipticity parameters on the azimuthal pattern of the quartic coefficient suggests that nonhyperbolic moveout recorded in wide-azimuth surveys can help to constrain the anisotropic velocity field. Since for fracture-induced orthorhombic media the parameters $\eta^{(1,2,3)}$ are closely related to the fracture density and infill, the results of azimuthal nonhyperbolic moveout analysis can be also used in fracture characterization.

**Key words:** reflection moveout, azimuthal anisotropy, long speads, wide-azimuth data, P-waves

## 1 INTRODUCTION

Although conventional seismic processing algorithms are largely limited to analysis of hyperbolic moveout on moderate-length spreads, acquisition of long-offset data becomes more and more common. In particular, the technology of ocean-bottom cable is well-suited for recording long-offset reflections for a wide range of source-receiver azimuths. The azimuthal dependence of nonhyperbolic reflection moveout at large offsets may be strongly influenced by elastic anisotropy (Sayers and Ebrom, 1997; Al-Dajani and Tsvankin, 1998) and, therefore, can help in estimating the anisotropic parameters.

In velocity analysis of pure (non-converted) waves, nonhyperbolic moveout is conventionally described using the quartic moveout coefficient $A_4$. Tsvankin and Thomsen (1994) combined the coefficient $A_4$ with the normal-moveout (NMO) velocity $V_{\text{nmo}}$ in a nonhyperbolic moveout equation that proved to be accurate for P-waves and converted PS-waves in anisotropic media (Tsvankin, 2001; Al-Dajani and Tsvankin, 1998). While the formalism for modeling NMO velocity in arbitrarily anisotropic, heterogeneous media has been developed by Grechka and Tsvankin (1998b, 2002) and Grechka, Tsvankin and Cohen (1999), derivation of the quartic moveout coefficient proved to be much more involved because $A_4$ depends on reflection-point dispersal. Analytic expressions for $A_4$ in horizontally layered VTI (transversely isotropic media with a vertical symmetry axis) media and symmetry planes of azimuthally anisotropic media are given in Tsvankin (2001). Al-Dajani and Tsvankin (1998) obtained the azimuthally varying coefficient $A_4$ in layer-cake HTI (transversely isotropic media with a horizontal symmetry axis) media; their results were extended to a horizontal orthorhombic layer by Al-Dajani et al. (1998).

In our previous paper (Pech et al., 2002; hereafter referred to as Paper I), we presented a general expression for the coefficient $A_4$ of pure (non-converted) modes that takes into account reflection-point dispersal on irregular interfaces and is valid for arbitrary anisotropy and heterogeneity. It should be emphasized that this result can be used to model long-spread moveout without time-consuming multi-offset, multi-azimuth ray tracing because all needed quantitites can be computed during the tracing of the zero-offset ray. The equation for $A_4$ was applied in Paper I to study azimuthally varying nonhyperbolic moveout of P-waves in a dipping transversely isotropic (TI) layer with a tilted symmetry axis. For weak anisotropy, the quartic coefficient proved to be proportional to the anellipticity parameter $\eta$ defined as (Alkhalifah and Tsvankin, 1995)

$$\eta \equiv \frac{\epsilon - \delta}{1 + 2\delta}\,, \tag{1}$$

with the azimuthal variation of $A_4$ being highly sensitive to the tilt of the symmetry axis.

Here, we use the expression for the quartic coefficient from Paper I to analyze nonhyperbolic moveout for the more complicated azimuthally anisotropic models with orthorhombic symmetry often used to describe naturally fractured reservoirs (Bakulin et al., 2000). Valuable insight is provided by the weak-anisotropy approximation that represents $A_4$ as a function of three anellipticity coefficients $\eta^{(1)}$, $\eta^{(2)}$, and $\eta^{(3)}$ defined in the symmetry planes of the model. Numerical examples illustrate the high sensitivity of the azimuthally varying quartic coefficient to the signs and relative magnitudes of the parameters $\eta^{(1,2,3)}$.

## 2 NONHYPERBOLIC MOVEOUT AND THE QUARTIC MOVEOUT COEFFICIENT

A detailed analytic description of nonhyperbolic moveout in anisotropic media can be found in Tsvankin (2001). The nonhyperbolic portion of the moveout curve is governed by the quartic moveout coefficient $A_4$ defined by expanding the squared reflection traveltime $t^2$ in a Taylor series in the squared source-receiver offset $X^2$:

$$A_4 = \frac{1}{2}\,\frac{d}{d(X^2)}\left[\frac{d(t^2)}{d(X^2)}\right]\Bigg|_{X=0}\,, \tag{2}$$

Long-spread moveout of P-waves (and, for some models, PS-waves) in both isotropic and anisotropic media can be well-approximated by the following equation suggested by Tsvankin and Thomsen (1994):

$$t^2 = t_0^2 + \frac{X^2}{V_{\rm nmo}^2} + \frac{A_4 X^4}{1 + A X^2}\,, \tag{3}$$

where $t_0$ is the zero-offset time, $V_{\rm nmo}$ is the NMO velocity, and the denominator of the nonhyperbolic term is designed to make the equation convergent at infinitely large offsets. The coefficient $A$ is expressed through $V_{\rm nmo}$, $A_4$, and the horizontal group velocity $V_{\rm hor}$ as

$$A = \frac{A_4}{V_{\rm hor}^{-2} - V_{\rm nmo}^{-2}}\,. \tag{4}$$

NMO velocity for anisotropic, heterogeneous media can be found from the Dix-type averaging equations developed by Grechka, Tsvankin and Cohen (1999) and Grechka and Tsvankin (2002). An analytic expression for the quartic coefficient $A_4$ that has the same level of generality was introduced in Paper I. Since both $V_{\rm nmo}$ and $A_4$ can be computed by tracing only one (zero-offset) ray, equation (3) can serve as a computationally efficient replacement for anisotropic ray tracing in modeling and inversion algorithms.

Paper I shows that the quartic moveout coefficient $A_4$ can be found as a function of the spatial derivatives of the one-way traveltime $\tau$ between the common-midpoint $\mathbf{y}$ and point $\mathbf{x}$ on the reflector (Figure 1):

$$A_4(\mathbf{L}) = \frac{1}{16}\left[\frac{\partial^2\tau}{\partial y_k\,\partial y_l}\,\frac{\partial^2\tau}{\partial y_m\,\partial y_n} + \frac{\tau_0}{3}\,\frac{\partial^4\tau}{\partial y_k\,\partial y_l\,\partial y_m\,\partial y_n} - \tau_0\,\frac{\partial^3\tau}{\partial y_k\,\partial y_l\,\partial x_i}\left(\frac{\partial^2\tau}{\partial x_i\,\partial x_j}\right)^{-1}\frac{\partial^3\tau}{\partial x_j\,\partial y_m\,\partial y_n}\right]L_k L_l L_m L_n\,. \tag{5}$$

Here $\mathbf{L} = [\cos\alpha, \sin\alpha, 0]$ is a unit vector parallel to the CMP line with the azimuth $\alpha$ and $\tau_0$ is the one-way zero-offset traveltime; summation over repeated indices from one to two is implied. All derivatives in equation (5) are evaluated for the zero-offset reflection ray at the CMP location $\mathbf{y}$. Equation (5) is valid for arbitrarily anisotropic, heterogeneous media and reflectors of irregular shape, as long as reflection traveltime can be expanded in a Taylor series near the common midpoint.

Simplifying the spatial derivatives of the traveltime $\tau$ in equation (5) under the assumption of weak anisotropy provides valuable insight into the dependence of the coefficient $A_4$ on the medium parameters. Below, we obtain the weak-anisotropy approximation for the

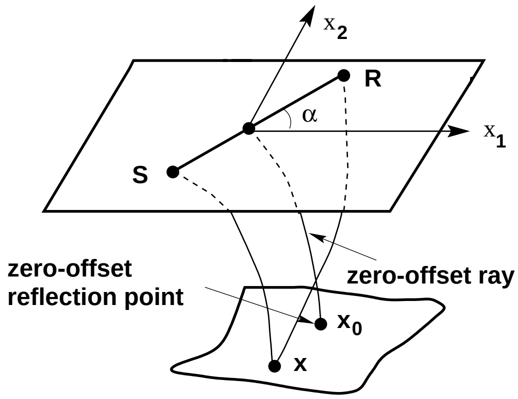


**Figure 1.** Reflection traveltimes from an irregular interface beneath an arbitrarily anisotropic, heterogeneous medium are recorded in a multi-azimuth CMP gather. The quartic moveout coefficient $A_4$ varies with the azimuth $\alpha$ of the CMP line (after Paper I).

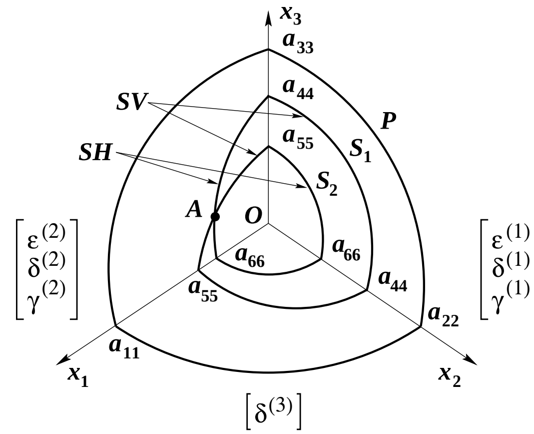


**Figure 2.** Sketch of body-wave phase-velocity surfaces in orthorhombic media (after Grechka, Theophanis and Tsvankin, 1999). The fast shear wave represents an SV (in-plane polarized) mode in the $[x_2,\, x_3]$–plane and an SH mode in the $[x_1,\, x_3]$–plane. $A$ marks a point (conical) singularity where the phase velocities of the two shear waves coincide with each other.

coefficient $A_4$ of P-waves in an orthorhombic layer and analyze its dependence on the anisotropic parameters and reflector dip.

## 3 BRIEF DESCRIPTION OF ORTHORHOMBIC MEDIA

Models with orthorhombic symmetry are often used to describe naturally fractured reservoirs that contain, for example, two orthogonal fracture systems or a single system of penny-shaped cracks embedded in a VTI matrix (e.g., Bakulin et al., 2000; Tsvankin, 2001). Orthorhombic media have three mutually orthogonal planes of mirror symmetry, one of which we assume to be horizontal.

Apart from the wavefront distortions near point shear-wave singularities (such as point $A$ in Figure 2), velocities and traveltimes in the symmetry planes of orthorhombic media can be described by the corresponding VTI equations. Therefore, reflection moveout and other seismic signatures in orthorhombic media are particularly convenient to represent using the notation of Tsvankin (1997, 2001) based on the analogy with Thomsen (1986) parameters for vertical transverse isotropy. The set of Tsvankin's (1997) parameters includes the vertical velocities of the P-wave ($V_{P0}$) and one of the split S-waves ($V_{S0}$; the S-wave is polarized in the $x_1$-direction) and the anisotropic coefficients $\epsilon^{(1,2)}$, $\delta^{(1,2,3)}$, and $\gamma^{(1,2)}$. The parameters $\epsilon^{(1)}$, $\delta^{(1)}$, and $\gamma^{(1)}$ are introduced in the vertical symmetry plane $[x_2,\, x_3]$ using the definitions of Thomsen's coefficients $\epsilon$, $\delta$, and $\gamma$ for VTI media (Figure 2). Similarly, $\epsilon^{(2)}$, $\delta^{(2)}$, and $\gamma^{(2)}$ are defined in the $[x_1,\, x_3]$-plane, and $\delta^{(3)}$ in the horizontal plane $[x_1,\, x_2]$.

An important role in the analysis of the quartic moveout coefficient below is played by the parameters $\eta^{(1)}$, $\eta^{(2)}$, and $\eta^{(3)}$, which quantify deviations from the elliptically anisotropic model in the symmetry planes (Grechka and Tsvankin, 1999):

$$\eta^{(1)} \equiv \frac{\epsilon^{(1)} - \delta^{(1)}}{1 + 2\,\delta^{(1)}} \approx \epsilon^{(1)} - \delta^{(1)}\,, \tag{6}$$

$$\eta^{(2)} \equiv \frac{\epsilon^{(2)} - \delta^{(2)}}{1 + 2\,\delta^{(2)}} \approx \epsilon^{(2)} - \delta^{(2)}\,, \tag{7}$$

$$\eta^{(3)} \equiv \frac{\epsilon^{(1)} - \epsilon^{(2)} - \delta^{(3)}\,(1 + 2\epsilon^{(2)})}{(1 + 2\epsilon^{(2)})\,(1 + 2\delta^{(3)})} \approx \epsilon^{(1)} - \epsilon^{(2)} - \delta^{(3)}. \tag{8}$$

The approximate expressions for $\eta^{(1,2,3)}$ in equations (6)–(8) are obtained by linearizing the exact definitions in the anisotropic parameters.

## 4 P-WAVE QUARTIC COEFFICIENT IN A DIPPING ORTHORHOMBIC LAYER

The model studied here includes a homogeneous orthorhombic layer with a horizontal symmetry plane above a plane dipping reflector (Figure 3). The other two symmetry planes of the layer are vertical and coincide with the coordinate planes $[x_1,\, x_3]$ and $[x_2,\, x_3]$. For simplicity, we assume that the plane $[x_1,\, x_3]$ is also parallel to the dip plane of the reflector, which makes it the only symmetry plane for the model as a whole. Therefore, the zero-offset reflected ray has to be confined to the $[x_1,\, x_3]$-plane (Figure 3).

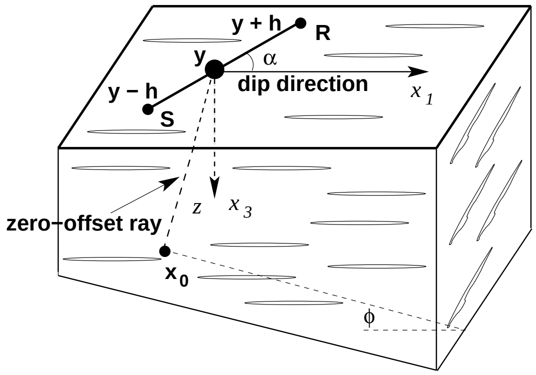


**Figure 3.** Reflected wave is recorded above a homogeneous orthorhombic layer with a dipping lower boundary. The vertical symmetry plane $[x_1,\, x_3]$ coincides with dip plane of the reflector (the dip is denoted by $\phi$).

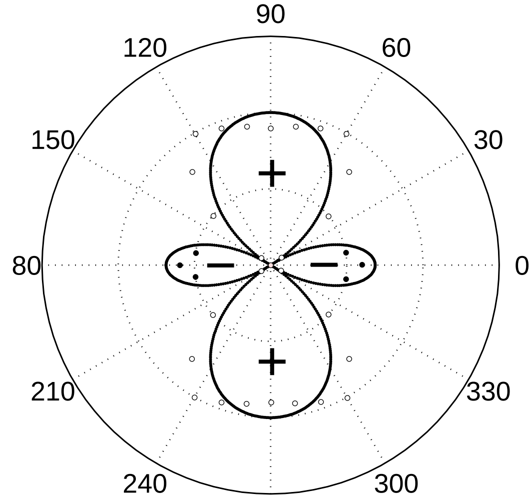


**Figure 4.** Accuracy of the weak-anisotropy approximation for the coefficient $A_4$ in a dipping orthorhombic layer. The circles mark the magnitude of $A_4$ obtained for each azimuth (the numbers on the perimeter) by fitting a quartic polynomial to the ray-traced $t^2(x^2)$–curve on the spreadlength $X_{\max} = 1.5$ km; the reflector depth beneath the CMP is 1 km. The white circles mark positive values of $A_4$, black circles mark negative $A_4$. The solid line is the weak-anisotropy approximation (9); the pluses and minuses indicate the signs of $A_4$ in the corresponding lobes. Both the ray-tracing and weak-anisotropy results are normalized by their respective maximum values of $|A_4|$. Zero azimuth corresponds to the dip plane of the reflector (the dip $\phi = 15°$); the model parameters are $\eta^{(1)} = -0.2$, $\eta^{(2)} = 0.2$, and $\eta^{(3)} = 0.2$.

### 4.1 Weak-anisotropy approximation

The weak-anisotropy approximation for the quartic coefficient $A_4$ of P-waves in this model is derived in Appendix A by linearizing the exact equation (5) in the anisotropic parameters:

$$A_4 = -\frac{1}{2\,t_{P0}^2\,V_{P0}^4}\left\{\eta^{(2)}\cos^2\phi\,\left[2\cos 2\phi\,(1 + \cos 2\alpha\cos 2\phi) + \cos 4\phi - 1\right] + 4\eta^{(1)}\cos^2\phi\,\sin^2\alpha - 2\eta^{(3)}\sin^2\alpha\,(\cos 2\alpha + \cos 2\phi)\right\}, \tag{9}$$

where $t_{P0}$ is the two-way zero-offset traveltime, and $\eta^{(1)}$, $\eta^{(2)}$, and $\eta^{(3)}$ are the anellipticity parameters defined in equations (6)–(8). Equation (9) shows that the azimuthal variation $A_4(\alpha)$ is controlled just by $\eta^{(1,2,3)}$ and reflector dip. This result is not surprising because the parameters $\eta^{(1,2,3)}$, in combination with the NMO velocities from a horizontal reflector in the vertical symmetry planes, fully describe P-wave time-domain signatures for orthorhombic media (Grechka and Tsvankin, 1999).

Note that the quartic coefficient is symmetric not only with respect to the dip direction of the reflector (the dip plane $\alpha = 0°$ is a symmetry plane for the whole model), but also with respect to the reflector strike. Indeed, since $A_4(\alpha) = A_4(-\alpha)$ and $A_4(\alpha) = A_4(\alpha + \pi)$, it follows that $A_4(\alpha) = A_4(\pi - \alpha)$, which means that the quartic coefficient is symmetric with respect to the strike direction $\alpha = \pm 90°$.

Figure 4 confirms that equation (9) is suitable for at least a qualitative description of the quartic moveout coefficient for small and moderate values of the anisotropic parameters. Note that the magnitude of the $\eta$ coefficients in Figure 4 is quite substantial for fracture-induced orthorhombic media (Bakulin et al., 2000). Application of the quartic coefficient in velocity analysis, however, should not be based on the weak-anisotropy approximation, as was shown by Tsvankin and Thomsen (1994) for the simpler VTI model.

### 4.2 Special cases

If the reflector is horizontal ($\phi = 0°$), equation (9) yields the linearized version of the exact solution for $A_4$ in a horizontal orthorhombic layer given by Al-Dajani et al. (1998):

$$A_4\,(\phi = 0°) = -\frac{2}{t_{P0}^2\,V_{P0}^4}\left[\eta^{(2)}\cos^2\alpha - \eta^{(3)}\cos^2\alpha\sin^2\alpha + \eta^{(1)}\sin^2\alpha\right]. \tag{10}$$

Although equation (10) is an approximation valid for small values of the anisotropic coefficients, the term in brackets accurately reproduces the azimuthal variation of the quartic coefficient. For a horizontal layer, $A_4$ in both symmetry-plane directions ($\alpha = 0°$ and $90°$) depends just on the corresponding parameter $\eta$ ($\eta^{(2)}$ in the $[x_1,\, x_3]$–plane and $\eta^{(1)}$ in the $[x_2,\, x_3]$–plane). The coefficient $\eta^{(3)}$ contributes only to the cross-term that reaches its maximum at the azimuth $\alpha = 45°$.

Another special case is that of a dipping VTI layer since transversely isotropic models can be treated as a subset of the more general orthorhombic media. The quartic coefficient in VTI media, which was derived and analyzed in Paper I, can be found from equation (9) by setting $\eta^{(1)} = \eta^{(2)} = \eta$ and $\eta^{(3)} = 0$ (Tsvankin, 1997):

$$A_4^{\rm VTI} = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\cos^4\phi\,\left(1 - 4\sin^2\phi\,\cos^2\alpha\right)\,. \qquad (11)$$

The dip-line ($\alpha = 0°$) and strike-line ($\alpha = 90°$) coefficients $A_4$ for VTI media, which we will need below for comparison with the expressions for orthorhombic media, are given by

$$A_{4,{\rm dip}}^{\rm VTI} = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\cos^4\phi\,\left(1 - 4\sin^2\phi\right)\,, \qquad (12)$$

$$A_{4,{\rm strike}}^{\rm VTI} = -\frac{2\,\eta}{t_{P0}^2\,V_{P0}^4}\cos^4\phi\,. \qquad (13)$$

### 4.3 Dip-line and strike-line expressions

As discussed above, the quartic coefficient in our model is symmetric with respect to the dip and strike directions of the reflector. On the dip line ($\alpha = 0°$), the coefficient $A_4$ from equation (9) takes the form

$$A_{4,{\rm dip}} = -\frac{2\,\eta^{(2)}}{t_{P0}^2\,V_{P0}^4}\cos^4\phi\,\left(1 - 4\,\sin^2\phi\right)\,. \qquad (14)$$

Equation (14) becomes identical to the corresponding VTI equation (12) if the parameter $\eta^{(2)}$ is replaced with $\eta$. Indeed, reflected rays (and their phase-velocity vectors) on the dip line are confined to the symmetry plane $[x_1,\,x_3]$ where the kinematics of wave propagation is the same as that in the VTI model with the vertical velocities $V_{P0}$ and $V_{S0}$ and the anisotropic coefficients $\epsilon = \epsilon^{(2)}$, $\delta = \delta^{(2)}$, $\gamma = \gamma^{(2)}$, and $\eta = \eta^{(2)}$ (Tsvankin, 1997, 2001).

The dip-line quartic coefficient from equation (14) vanishes for the vertical reflector ($\phi = 90°$) and for the dip $\phi = 30°$. If $\eta^{(2)} > 0$, $A_{4,{\rm dip}}$ is negative for mild dips $\phi < 30°$ and becomes positive for $\phi > 30°$, as discussed in Paper I for VTI media (also, see numerical results in Tsvankin, 1995, 2001).

Substitution of $\alpha = 90°$ into equation (9) yields the strike-line quartic coefficient:

$$A_{4,{\rm strike}} = -\frac{2}{t_{P0}^2\,V_{P0}^4}\left[\eta^{(1)}\cos^2\phi - \eta^{(2)}\cos^2\phi\,\sin^2\phi + \eta^{(3)}\sin^2\phi\right]. \qquad (15)$$

It is interesting that the functional dependence of $A_{4,{\rm strike}}$ on the dip $\phi$ is similar to that of the quartic coefficient in a horizontal layer on the azimuth $\alpha$ [equation (10)]. In contrast to the coefficient $A_{4,{\rm dip}}$ from equation (14), $A_{4,{\rm strike}}$ depends on all three parameters $\eta^{(1,2,3)}$ because reflected rays recorded on the strike line $x_2$ deviate from the vertical symmetry plane $[x_2,\,x_3]$. For the same reason, $A_{4,{\rm strike}}$ differs from the strike-line coefficient in VTI media [equation (13)] with $\eta = \eta^{(1)}$ (the parameter $\eta^{(1)}$ is defined in the $[x_2,\,x_3]$-plane). However, equation (15) indicates that the parameter $\eta^{(1)}$ does dominate the quartic coefficient for mild reflector dips $\phi$.

For a vertical interface ($\phi = 90°$), reflected rays travel in the horizontal symmetry plane, and the strike-line quartic coefficient is governed just by the parameter $\eta^{(3)}$:

$$A_{4,{\rm strike}}(\phi = 90°) = -\frac{2\,\eta^{(3)}}{t_{P0}^2\,V_{P0}^4}\,. \qquad (16)$$

Equation (16) coincides with the weak-anisotropy approximation for $A_4$ in a horizontal VTI layer with $\eta = \eta^{(3)}$.

### 4.4 Azimuthal signature

The azimuthal dependence of the quartic coefficient from equation (9) strongly depends on reflector dip as well as the magnitudes and signs of the anellipticity parameters $\eta^{(1,2,3)}$. For mild dips, $A_4(\alpha)$ is largely controlled by the parameters $\eta^{(1)}$ (near the $[x_2,\,x_3]$–plane) and $\eta^{(2)}$ (near the $[x_1,\,x_3]$–plane). As illustrated by Figure 5, if both $\eta^{(1)}$ and $\eta^{(2)}$ are positive and greater by absolute value than $\eta^{(3)}$, $A_4$ typically stays negative for dips smaller than 30°; the same result was obtained in Paper I for a dipping VTI layer [see equation (11)]. Also, as in VTI media, for a dip of 30° $A_4$ goes to zero only in the single (dip) direction (Figure 6). For the $\eta$ parameters from Figures 5 and 6 and dips between 30° and 90°, equation (9) yields two azimuths $\pm\alpha$ for which $A_4 = 0$. If the dip $\phi = 45°$ (Figure 7), the quartic coefficient is positive near the dip plane, vanishes at $\alpha \approx \pm 60°$ and becomes negative close to the strike direction.

The azimuthal variation of the quartic coefficient has a different character if $\eta^{(1)}$ and $\eta^{(2)}$ have opposite signs. In this case, for mild dips $A_4$ changes sign between the dip and strike directions [see equations (14) and (15)], with the azimuthal direction of vanishing $A_4$ dependent on the relative magnitudes of $\eta^{(1)}$ and $\eta^{(2)}$ (Figure 8). Since the quartic coefficient decreases with the dip $\phi$ more rapidly in the dip plane than in the strike direction, the direction where $A_4 = 0$ rotates towards the dip plane as $\phi$ increases. For the example in Figure 8, $A_4$ goes to zero at an angle of about $\pm 35°$ from the dip plane. When the dip reaches 30° (Figure 9), the quartic coefficient goes to zero only in the dip direction, and the azimuthal variation of $A_4$ is similar to that in Figure 6. However, the sign of $A_4$ away from the dip plane in Figure 9 is positive because $\eta^{(1)} < 0$. Finally, for dips larger than 30°, the qurtic coefficient is positive for all azimuths (Figure 10).

To analyze the influence of the parameter $\eta^{(3)}$ on the quartic coefficient, in the next two examples we set $\eta^{(1)}$ and $\eta^{(2)}$ to zero (Figures 11 and 12). Clearly, the

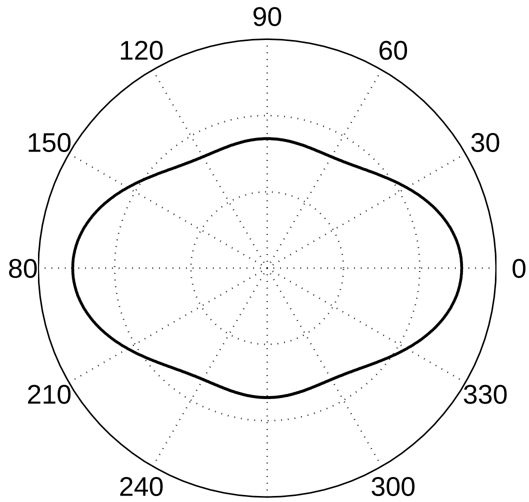


**Figure 5.** Magnitude of the azimuthally-varying quartic moveout coefficient $A_4$ for a dipping orthorhombic layer computed from equation (9). The dip plane of the reflector is at zero azimuth (the azimuth is marked on the perimeter). The anellipticity parameters are $\eta^{(1)} = 0.05$, $\eta^{(2)} = 0.1$, and $\eta^{(3)} = 0.03$; the reflector dip $\phi = 15°$. The other parameters ($t_{P0}$ and $V_{P0}$) change only the scale of the plot (intentionally undefined here). For this model, $A_4 < 0$ for all azimuthal directions.

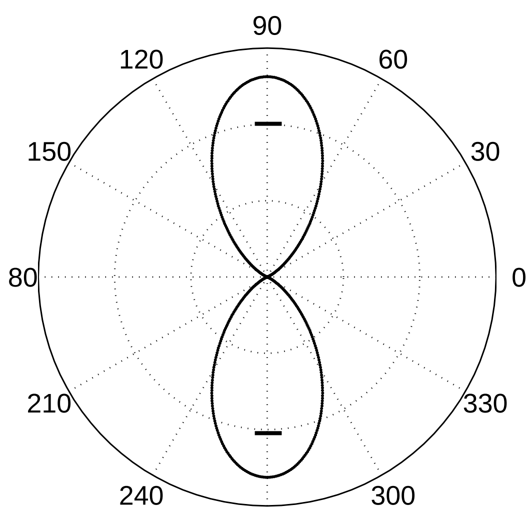


**Figure 6.** Same as Figure 5, but the reflector dip is 30°. The minus signs inside the lobes indicate negative values of $A_4$.

term involving $\eta^{(3)}$ creates a rather complicated azimuthal signature of $A_4$. Since $A_{4,\text{dip}}$ [equation (14)] is proportional to $\eta^{(2)}$, it vanishes for all dips when $\eta^{(2)} = 0$. In addition, the quartic coefficient goes to zero for another azimuth near the strike direction, even for a dip of just 15° (Figure 11). The signature of $A_4$ for

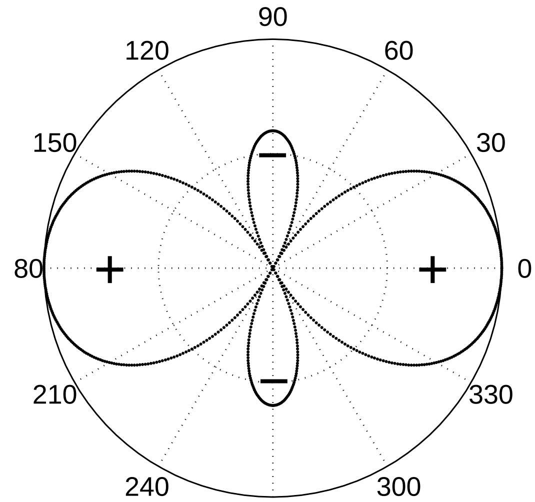


**Figure 7.** Same as Figure 5, but the reflector dip is 45°.

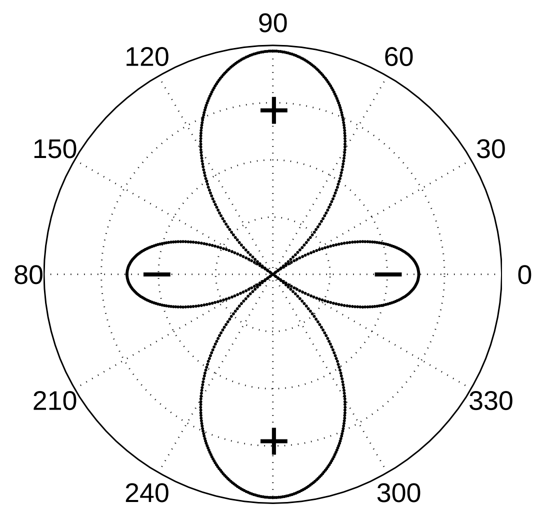


**Figure 8.** Magnitude of the quartic moveout coefficient $A_4$ for an orthorhombic layer computed from equation (9). The anellipticity parameters are $\eta^{(1)} = -0.1$, $\eta^{(2)} = 0.1$, and $\eta^{(3)} = 0.03$; the reflector dip $\phi = 15°$.

a dip of 30° has a similar character, but with different relative magnitudes of the lobes (Figure 12).

For nonzero values of $\eta^{(1)}$ and $\eta^{(2)}$, the contribution of the term proportional to $\eta^{(3)}$ generally increases with reflector dip (Figures 13–15). While the dip-line coefficient $A_4$ depends just on $\eta^{(2)}$, $A_{4,\text{strike}}$ is significantly influenced by $\eta^{(3)}$, in particular for the dips exceeding 30°. If $\eta^{(3)}$ is larger by absolute value than $\eta^{(1)}$ and $\eta^{(2)}$, the quartic coefficient typically goes to zero in at least one off-symmetry direction for a wide range of dips. For example, if the dip $\phi = 30°$, the influence of $\eta^{(3)}$ produces an additional direction of vanishing $A_4$ near the

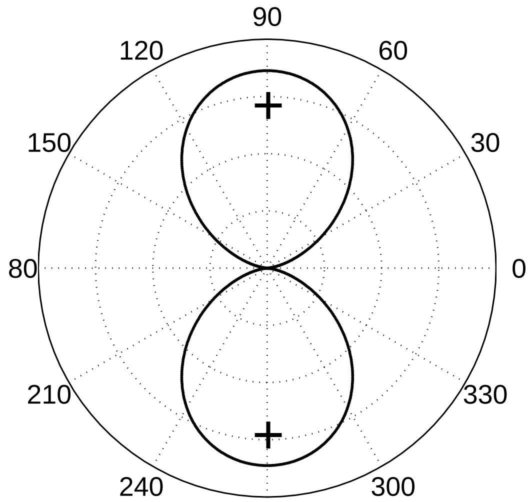


**Figure 9.** Same as Figure 8, but the reflector dip is 30°.

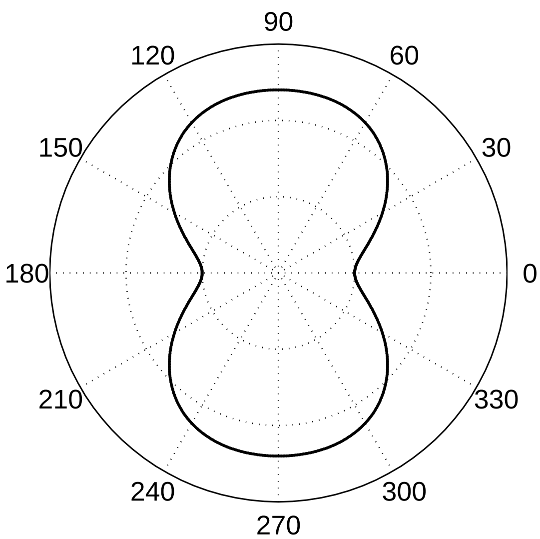


**Figure 10.** Same as Figure 8, but the reflector dip is 45°. The coefficient $A_4$ is positive.

reflector strike, and the azimuthal variation of the quartic coefficient is described by six lobes with alternating signs from one lobe to the next (Figure 14).

## 5 DISCUSSION AND CONCLUSIONS

The general expression for the quartic moveout coefficient $A_4$ derived by Pech et al. (2002; Paper I) was used here to study P-wave nonhyperbolic moveout in a dipping orthorhombic layer. Similar to the NMO ellipse, the coefficient $A_4$ can be computed by tracing a single (zero-offset) ray and then used in the nonhyperbolic moveout equation of Tsvankin and Thomsen (1994) to model re-

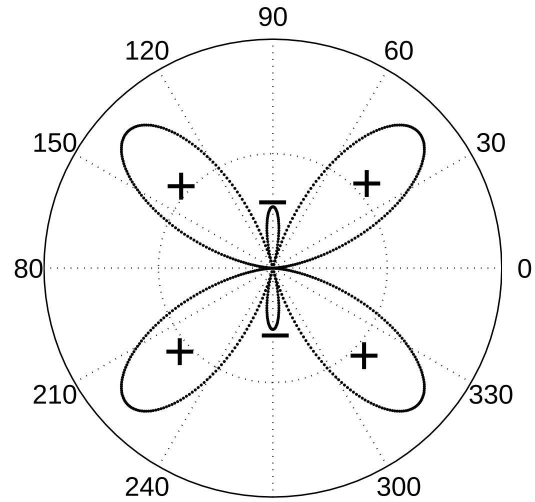


**Figure 11.** Magnitude of the quartic moveout coefficient $A_4$ for an orthorhombic layer computed from equation (9). The anellipticity parameters are $\eta^{(1)} = \eta^{(2)} = 0$ and $\eta^{(3)} = 0.1$; the reflector dip $\phi = 15°$.

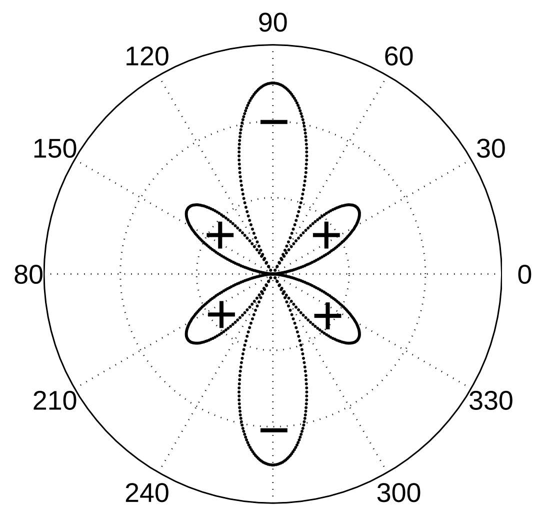


**Figure 12.** Same as Figure 11, but the reflector dip is 30°.

flection traveltimes without time-consuming ray tracing for each source-receiver pair.

The main emphasis of this paper, however, is on the analysis of the azimuthal variation of the P-wave quartic moveout coefficient as a function of the anisotropic parameters and reflector dip. The model consisted of a homogeneous orthorhombic layer with a horizontal symmetry plane and a dipping lower boundary (reflector). It was assumed that the dip plane of the reflector coincides with a vertical symmetry plane of the layer and, therefore, represents a symmetry plane for the whole model. Another symmetry direction for reflection moveout in

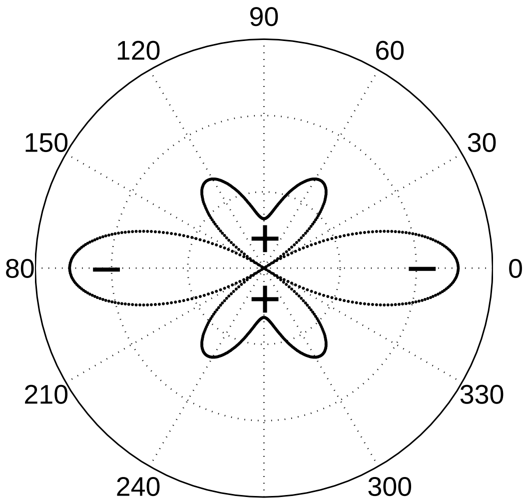


**Figure 13.** Magnitude of the quartic moveout coefficient $A_4$ for an orthorhombic layer computed from equation (9). The anellipticity parameters are $\eta^{(1)} = -0.025$, $\eta^{(2)} = 0.1$, and $\eta^{(3)} = 0.2$; the reflector dip $\phi = 15°$.

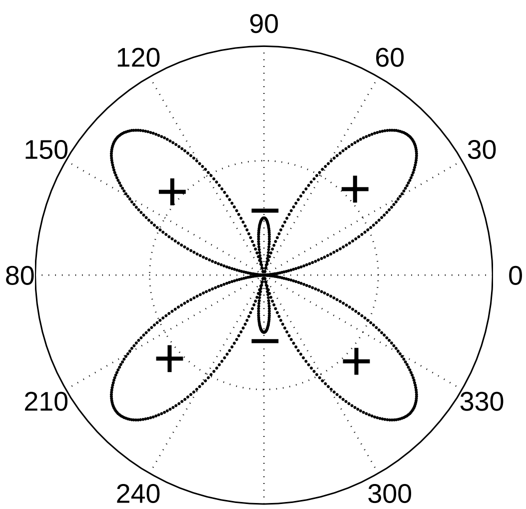


**Figure 14.** Same as Figure 8, but the reflector dip is 30°.

this model is reflector strike, so the azimuthal signature of the quartic coefficient $A_4$ is repeated in each quadrant.

The weak-anisotropy approximation for $A_4$ linearized in Tsvankin's (1997) anisotropic parameters shows that the azimuthal variation of the quartic coefficient is controlled by reflector dip and three anellipticity coefficients $\eta^{(1,2,3)}$. It should be mentioned that the parameters $\eta^{(1,2,3)}$, in combination with the two symmetry-direction NMO velocities, are responsible for all P-wave time-processing steps in orthorhombic media including NMO correction, DMO removal, and time migration.

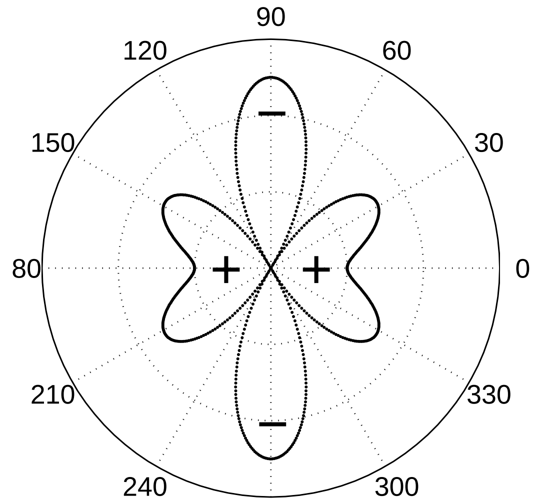


**Figure 15.** Same as Figure 8, but the reflector dip is 45°.

The dip-line quartic coefficient $A_{4,\text{dip}}$ is described by the same equation as in VTI media and depends on a single anisotropic parameter – $\eta^{(2)}$. $A_{4,\text{dip}}$ vanishes for a vertical reflector (dip $\phi = 90°$) and for the dip $\phi = 30°$; the sign of $A_{4,\text{dip}}$ for $\phi < 30°$ is opposite to that of $\eta^{(2)}$. The strike-line $A_4$ depends on all three $\eta$ coefficients but for mild dips is largely governed by $\eta^{(1)}$.

Hence, for dips smaller than 15–20°, the azimuthally varying quartic coefficient mostly depends on the parameters $\eta^{(1)}$ and $\eta^{(2)}$. If $\eta^{(1)}$ and $\eta^{(2)}$ have opposite signs, $A_4$ for mildly dipping reflectors changes sign between the dip and strike directions. The influence of $\eta^{(3)}$ generally increases with dip and may create a rather complicated azimuthal signature of the quartic coefficient, sometimes with two azimuths of vanishing $A_4$ between the dip and strike directions.

The high variability of the azimuthal signature of the quartic coefficient and its sensitivity to the time-processing parameters $\eta^{(1,2,3)}$ can be exploited in the inversion of P-wave data for orthorhombic media. Despite the known instability in estimating the quartic moveout coefficient (Grechka and Tsvankin, 1998a), wide-azimuth long-offset reflection data may be used to determine the sign of $A_4$ and the azimuthal directions of its minimum values. This information can help in constraining the parameters $\eta^{(1,2,3)}$, which are not only needed in velocity analysis, but can also be used in fracture characterization. For example, if the effective orthorhombic anisotropy is caused by two orthogonal systems of penny-shaped cracks embedded in isotropic host rock, all three $\eta$ coefficients vanish for dry (gas-filled) cracks and are positive for cracks filled with fluid (Bakulin et al., 2000).

The exact equation for the quartic moveout coefficient from Paper I can be applied to model nonhyperbolic moveout in more complicated layered orthorhombic media. The analytic results of this work can still provide useful insight into the behavior of $A_4$ in layered media because, at least for mild dips, the effective quartic coefficient at the surface can be approximated by an average of the interval values of $A_4$ computed for the same azimuth (Al-Dajani and Tsvankin, 1998; Al-Dajani et al., 1998).

## 6 ACKNOWLEDGMENTS

We are grateful to members of the A(nisotropy)-Team of the Center for Wave Phenomena (CWP), Colorado School of Mines, for helpful discussions. Partial support for this work was provided by the Chemical Sciences, Geosciences and Biosciences Division, Office of Basic Energy Sciences, U.S. Department of Energy.

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## APPENDIX A: WEAK-ANISOTROPY APPROXIMATION FOR THE QUARTIC MOVEOUT COEFFICIENT IN ORTHORHOMBIC MEDIA

Here we apply the approach discussed in Paper I to obtain a linearized approximation for the P-wave quartic coefficient $A_4$ valid for small values of the anisotropic parameters. The model consists of a homogeneous orthorhombic layer with a horizontal symmetry plane above a plane dipping reflector (Figure 3). It is assumed that the dip plane of the reflector coincides with one of the mutually orthogonal vertical symmetry planes of the overburden.

The one-way traveltime between the common-midpoint (CMP) $\mathbf{y}$ and the plane reflector $z(x_1, x_2)$ is

$$\tau = \frac{\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + z^2(x_1, x_2)}}{V_G}\,, \tag{A1}$$

where $V_G$ is the group velocity. The orientation of the ray that connects the CMP with the reflector can be characterized by the polar angle $a$ and the azimuthal angle $b$:

$$\sin a \equiv \frac{\sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2}}{\sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2 + z^2}}\,, \tag{A2}$$

$$\cos a \equiv \frac{z}{\sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2 + z^2}}\,, \tag{A3}$$

$$\sin b \equiv \frac{(y_2 - x_2)}{\sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2}}\,, \tag{A4}$$

$$\cos b \equiv \frac{(y_1 - x_1)}{\sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2}}\,. \tag{A5}$$

The $P$-wave group velocity $V_G$ linearized in the anisotropic parameters can be determined from the weak-anisotropy approximation for the phase velocity given in Tsvankin (1997b, 2001):

$$V_G = V_{P0}\,\{1 + (\epsilon^{(2)} - \delta^{(2)})(\sin a\,\cos b)^4$$
$$+\,\delta^{(1)}(\sin a\,\sin b)^2 + (\epsilon^{(1)} - \delta^{(1)})(\sin a\,\sin b)^4$$
$$+\cos^2 b\,[\,\delta^{(2)}\sin^2 a - (\delta^{(1)} + \delta^{(2)} - \delta^{(3)}$$
$$-\,2\epsilon^{(2)})\,\sin^4 a\,\sin^2 b\,]\}\,; \qquad \text{(A6)}$$

the angles $a$ and $b$ are defined in equations (A-2)–(A-5).

Next, equation (A1) with the velocity $V_G$ from equation (A6) is substituted into the general equation (5) to evaluate the spatial derivatives of the traveltime, which yields the quartic coefficient $A_4$ as a function of the coordinates of both common midpoint and the zero-offset reflection point. Since the zero-offset ray is confined to the dip plane where all kinematic signatures are described by the corresponding VTI equations, we can relate the horizontal coordinates $x_1^{(0)}$ and $x_2^{(0)}$ of the zero-offset reflection point to the CMP coordinates $y_1$ and $y_2$ by adapting the results of Paper I:

$$y_1 = 2z\tan\phi\,[\,0.5 + \epsilon^{(2)} - (\epsilon^{(2)} - \delta^{(2)})\cos 2\phi\,]$$
$$+\,x_1^{(0)}\,, \qquad \text{(A7)}$$

$$y_2 = x_2^{(0)}\,. \qquad \text{(A8)}$$

Using equations (A7) and (A8) and applying further linearization in the anisotropic parameters, we obtain the following approximation for the $P$-wave quartic moveout coefficient in orthorhombic media:

$$A_4 = -\frac{1}{2\,t_{P0}^2\,V_{P0}^4}\left\{\eta^{(2)}\cos^2\phi\,[\,2\cos 2\phi\,(1\right.$$
$$+\cos 2\alpha\cos 2\phi) + \cos 4\phi - 1]$$
$$+\,4\eta^{(1)}\cos^2\phi\,\sin^2\alpha$$
$$\left.-\,2\eta^{(3)}\sin^2\alpha\,(\cos 2\alpha + \cos 2\phi)\right\}, \qquad \text{(A9)}$$

where $t_{P0}$ is the two-way zero-offset traveltime, and the coefficients $\eta^{(1,2,3)}$ are defined in equations (6)–(8).