

Retrieving the Green’s function in an open system by cross-correlation: A comparison of approaches

Kees Wapenaar¹, Jacob Fokkema¹, and Roel Snieder²

¹*Department of Geotechnology, Delft University of Technology, Delft, The Netherlands*

²*Center for Wave Phenomena, Colorado School of Mines, Golden CO 80401, USA*

ABSTRACT

We compare two approaches for deriving the fact that the Green’s function in an arbitrary inhomogeneous open system can be obtained by cross-correlating recordings of the wave field at two positions. One approach is based on physical arguments, exploiting the principle of time reversal invariance of the acoustic wave equation. The other approach is based on Rayleigh’s reciprocity theorem. Using a unified notation for both approaches, we discuss their similarities and differences.

Key words: seismic interferometry

1 INTRODUCTION

Since the work of Weaver and Lobkis (2001; 2001), many researchers have shown theoretically and experimentally that the cross-correlation of the recordings of a diffuse wave field at two receiver positions yields the Green’s function between these positions. In most cases it is assumed that the diffuse wave field consists of normal modes (with uncorrelated amplitudes) in a closed system. Much less attention has been paid to the theory of Green’s function retrieval in arbitrary inhomogeneous open systems. Nevertheless, the first result stems from 1968, albeit for one-dimensional media, when Claerbout (1968) showed that the seismic reflection response of a horizontally layered earth can be synthesized from the autocorrelation of its transmission response. Recently we generalized this to 3-D arbitrary inhomogeneous media (Wapenaar et al., 2002; Wapenaar, 2003; Wapenaar, 2004). Using reciprocity theorems of the correlation type, we showed in those papers that the cross-correlation of transmission responses observed at the earth’s free surface, due to uncorrelated noise sources in the subsurface, yields the full reflection response (i.e., the ballistic wave and the coda) of the 3-D inhomogeneous subsurface. Weaver and Lobkis (2004) followed a similar approach for a configuration in which the 3-D inhomogeneous medium is surrounded by uncorrelated sources. Derode et al. (2003a; 2003b) derived expressions for Green’s function retrieval in open systems using physical arguments, exploiting the principle of

time reversal invariance of the acoustic wave equation. Their approach can be seen as the ‘physical counterpart’ of our derivations based on reciprocity. In this letter we compare the physical arguments of Derode et al. (2003a; 2003b) with our approach based on Rayleigh’s reciprocity theorem (Wapenaar et al., 2002; Wapenaar, 2003; Wapenaar, 2004). Moreover, we indicate the links with ‘reverse time migration’ and ‘frequency domain migration’, respectively. We use a unified notation, which facilitates the comparison of both approaches.

2 PHYSICAL ARGUMENTS

In this section we summarize the physical arguments of Derode et al. (2003a; 2003b) for deriving expressions for Green’s function retrieval. Consider a lossless arbitrary inhomogeneous acoustic medium in a homogeneous embedding. In this configuration we define two points with coordinate vectors \mathbf{x}_A and \mathbf{x}_B . Our aim is to show that the acoustic response at \mathbf{x}_B due to an impulsive source at \mathbf{x}_A [i.e., the Green’s function $G(\mathbf{x}_B, \mathbf{x}_A, t)$] can be obtained by cross-correlating passive measurements of the wave fields at \mathbf{x}_A and \mathbf{x}_B due to sources on a surface S in the homogeneous embedding. The derivation starts by considering another physical experiment, namely an impulsive source at \mathbf{x}_A and receivers at \mathbf{x} on S . The response at one particular point \mathbf{x} on S is denoted by $G(\mathbf{x}, \mathbf{x}_A, t)$. Imagine that we record this response for

all \mathbf{x} on S , revert the time axis, and feed these time-reverted functions $G(\mathbf{x}, \mathbf{x}_A, -t)$ to sources at all \mathbf{x} on S . Huygens' principle states that the wave field at any point \mathbf{x}' in S due to these sources on S is then given by

$$u(\mathbf{x}', t) \propto \oint_S \underbrace{G(\mathbf{x}', \mathbf{x}, t)}_{\text{'propagator'}} * \underbrace{G(\mathbf{x}, \mathbf{x}_A, -t)}_{\text{'source'}} d^2\mathbf{x}, \quad (1)$$

where $*$ denotes convolution and \propto 'proportional to'. According to this equation, $G(\mathbf{x}', \mathbf{x}, t)$ propagates the source function $G(\mathbf{x}, \mathbf{x}_A, -t)$ from \mathbf{x} to \mathbf{x}' and the result is integrated over all sources on S . Due to the invariance of the acoustic wave equation for time-reversal, the wave field $u(\mathbf{x}', t)$ focusses for $\mathbf{x}' = \mathbf{x}_A$ at $t = 0$. McMechan (1983) exploited this property in a seismic imaging method which has become known as reverse time migration. Derode et al. (2003a; 2003b) give a new interpretation to equation (1). Since $u(\mathbf{x}', t)$ focusses for $\mathbf{x}' = \mathbf{x}_A$ at $t = 0$, the wave field $u(\mathbf{x}', t)$ for arbitrary \mathbf{x}' and t can be seen as the response of a virtual source at \mathbf{x}_A and $t = 0$. This virtual source response, however, consists of a causal and an anti-causal part, according to

$$u(\mathbf{x}', t) = G(\mathbf{x}', \mathbf{x}_A, t) + G(\mathbf{x}', \mathbf{x}_A, -t). \quad (2)$$

This is explained as follows: the wave field generated by the anti-causal sources on S first propagates to all \mathbf{x}' where it gives an anti-causal contribution, next it focusses in \mathbf{x}_A at $t = 0$ and finally it propagates again to all \mathbf{x}' giving the causal contribution. The propagation paths from \mathbf{x}' to \mathbf{x}_A are the same as those from \mathbf{x}_A to \mathbf{x}' , but are travelled in opposite direction. Combining equations (1) and (2), applying source-receiver reciprocity to $G(\mathbf{x}, \mathbf{x}_A, -t)$ in equation (1) and setting $\mathbf{x}' = \mathbf{x}_B$ yields

$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \propto \oint_S G(\mathbf{x}_B, \mathbf{x}, t) * G(\mathbf{x}_A, \mathbf{x}, -t) d^2\mathbf{x}. \quad (3)$$

The right-hand side of equation (3) can be interpreted as the integral of cross-correlations of observations of wave fields at \mathbf{x}_B and \mathbf{x}_A , respectively, due to impulsive sources at \mathbf{x} on S ; the integration takes place along the source coordinate \mathbf{x} . The left-hand side is interpreted as the superposition of the response at \mathbf{x}_B due to an impulsive source at \mathbf{x}_A and its time-reversed version. Since the Green's function $G(\mathbf{x}_B, \mathbf{x}_A, t)$ is causal, it can be obtained from the left-hand side of equation (3) by taking the causal part. The reconstructed Green's function contains the ballistic wave as well as the coda due to multiple scattering in the inhomogeneous medium.

3 RAYLEIGH'S RECIPROCITY THEOREM

In this section we summarize our derivation based on Rayleigh's reciprocity theorem (Wapenaar et al., 2002;

Wapenaar, 2003; Wapenaar, 2004). This reciprocity theorem relates two independent acoustic states in one and the same domain (De Hoop, 1988; Fokkema and van den Berg, 1993). Consider an acoustic wave field, characterized by the acoustic pressure $p(\mathbf{x}, t)$ and the particle velocity $v_i(\mathbf{x}, t)$. We define the temporal Fourier transform of a space- and time-dependent quantity $p(\mathbf{x}, t)$ as $\hat{p}(\mathbf{x}, \omega) = \int \exp(-j\omega t) p(\mathbf{x}, t) dt$, where j is the imaginary unit and ω the angular frequency. In the space-frequency domain the acoustic pressure and particle velocity in a lossless arbitrary inhomogeneous acoustic medium obey the equation of motion $j\omega\rho\hat{v}_i + \partial_i\hat{p} = 0$ and the stress-strain relation $j\omega\kappa\hat{p} + \partial_i\hat{v}_i = \hat{q}$, where ∂_i is the partial derivative in the x_i -direction (Einstein's summation convention applies for repeated lowercase subscripts), $\rho(\mathbf{x})$ the mass density of the medium, $\kappa(\mathbf{x})$ its compressibility and $\hat{q}(\mathbf{x}, \omega)$ a source distribution in terms of volume injection rate density. We consider the 'interaction quantity' $\partial_i\{\hat{p}_A\hat{v}_{i,B} - \hat{v}_{i,A}\hat{p}_B\}$, where subscripts A and B are used to distinguish two independent states. Rayleigh's reciprocity theorem is obtained by substituting the equation of motion and the stress-strain relation for states A and B into the interaction quantity, integrating the result over a spatial domain V enclosed by S with outward pointing normal vector $\mathbf{n} = (n_1, n_2, n_3)$ and applying the theorem of Gauss. This gives

$$\int_V \{\hat{p}_A\hat{q}_B - \hat{q}_A\hat{p}_B\} d^3\mathbf{x} = \oint_S \{\hat{p}_A\hat{v}_{i,B} - \hat{v}_{i,A}\hat{p}_B\} n_i d^2\mathbf{x}. \quad (4)$$

Since the medium is lossless, we can apply the principle of time-reversal invariance (Bojarski, 1983). In the frequency domain time-reversal is replaced by complex conjugation. Hence, when \hat{p} and \hat{v}_i are a solution of the equation of motion and the stress-strain relation with source distribution \hat{q} , then \hat{p}^* and $-\hat{v}_i^*$ obey the same equations with source distribution $-\hat{q}^*$ (the asterisk denotes complex conjugation). Making these substitutions for state A we obtain

$$\int_V \{\hat{p}_A^*\hat{q}_B + \hat{q}_A^*\hat{p}_B\} d^3\mathbf{x} = \oint_S \{\hat{p}_A^*\hat{v}_{i,B} + \hat{v}_{i,A}^*\hat{p}_B\} n_i d^2\mathbf{x}. \quad (5)$$

Next we choose impulsive point sources in both states, according to $\hat{q}_A(\mathbf{x}, \omega) = \delta(\mathbf{x} - \mathbf{x}_A)$ and $\hat{q}_B(\mathbf{x}, \omega) = \delta(\mathbf{x} - \mathbf{x}_B)$, with \mathbf{x}_A and \mathbf{x}_B both in V . The wave field in state A can thus be expressed in terms of a Green's function, according to

$$\hat{p}_A(\mathbf{x}, \omega) = \hat{G}(\mathbf{x}, \mathbf{x}_A, \omega), \quad (6)$$

$$\hat{v}_{i,A}(\mathbf{x}, \omega) = -(j\omega\rho(\mathbf{x}))^{-1}\partial_i\hat{G}(\mathbf{x}, \mathbf{x}_A, \omega), \quad (7)$$

where $\hat{G}(\mathbf{x}, \mathbf{x}_A, \omega)$ obeys the wave equation

$$\rho\partial_i(\rho^{-1}\partial_i\hat{G}) + (\omega^2/c^2)\hat{G} = -j\omega\rho\delta(\mathbf{x} - \mathbf{x}_A), \quad (8)$$

with propagation velocity $c(\mathbf{x}) = \{\kappa(\mathbf{x})\rho(\mathbf{x})\}^{-\frac{1}{2}}$; similar expressions hold for the wave field in state B . Substituting these expressions into equation (5) and using

source-receiver reciprocity of the Green's functions gives

$$2\Re\{\hat{G}(\mathbf{x}_B, \mathbf{x}_A, \omega)\} = \oint_S \frac{-1}{j\omega\rho(\mathbf{x})} \left(\partial_i \hat{G}(\mathbf{x}_B, \mathbf{x}, \omega) \hat{G}^*(\mathbf{x}_A, \mathbf{x}, \omega) - \hat{G}(\mathbf{x}_B, \mathbf{x}, \omega) \partial_i \hat{G}^*(\mathbf{x}_A, \mathbf{x}, \omega) \right) n_i d^2\mathbf{x}, \quad (9)$$

where \Re denotes the real part. Note that the left-hand side is the Fourier transform of $G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t)$; the products $\partial_i \hat{G} \hat{G}^*$ etc. in the right-hand side correspond to cross-correlations in the time domain. Expressions like the right-hand side of this equation have been used by numerous researchers (including the authors) for seismic migration in the frequency domain. Esmersoy and Oristaglio (1988) explained the link with the reverse time migration method, mentioned in the previous section. What is new (compared with migration) is that equation (9) is formulated in such a way that it gives an exact representation of the Green's function $\hat{G}(\mathbf{x}_B, \mathbf{x}_A, \omega)$ in terms of cross-correlations of observed wave fields at \mathbf{x}_B and \mathbf{x}_A . Note that, unlike in the previous section, we have not assumed that the medium outside surface S is homogeneous. The terms \hat{G} and $\partial_i \hat{G}$ under the integral represent responses of monopole and dipole sources at \mathbf{x} on S ; the combination of the two correlation products under the integral ensures that waves propagating outward from the sources on S do not interact with those propagating inward and vice versa. When a part of S is a free surface on which the acoustic pressure vanishes, then the surface integral in equation (5) and hence in equation (9) need only be evaluated over the remaining part of S . Other modifications of equation (9), including the elastodynamic generalization, are discussed in (Wapenaar et al., 2002; Wapenaar, 2003; Wapenaar, 2004). Van Manen and Robertsson (2005) propose an efficient modelling scheme, based on an expression similar to equation (9).

Next we show with subsequent approximations how equation (9) simplifies to equation (3). First we assume that the medium outside S is homogeneous, with constant propagation velocity c and mass density ρ . In the high frequency regime, the derivatives of the Green's functions can be approximated by multiplying each constituent (direct wave, scattered wave etc.) by $-j\frac{\omega}{c} \cos \alpha$, where α is the angle between the pertinent ray and the normal on S . The main contributions to the integral in equation (9) come from stationary points on S (Snieder, 2004; Schuster et al., 2004; Wapenaar et al., 2004). At those points the ray angles for both Green's functions are identical (see also the example in the next section). This implies that the contributions of the two terms under the integral in equation (9) are approximately equal (but opposite in sign), hence

$$2\Re\{\hat{G}(\mathbf{x}_B, \mathbf{x}_A, \omega)\} \approx \frac{-2}{j\omega\rho} \oint_S \partial_i \hat{G}(\mathbf{x}_B, \mathbf{x}, \omega) \hat{G}^*(\mathbf{x}_A, \mathbf{x}, \omega) n_i d^2\mathbf{x}. \quad (10)$$

If we assume that S is a sphere with large enough radius then all rays are normal to S (i.e., $\alpha \approx 0$), hence

$$2\Re\{\hat{G}(\mathbf{x}_B, \mathbf{x}_A, \omega)\} \approx \frac{2}{\rho c} \oint_S \hat{G}(\mathbf{x}_B, \mathbf{x}, \omega) \hat{G}^*(\mathbf{x}_A, \mathbf{x}, \omega) d^2\mathbf{x}. \quad (11)$$

Transforming both sides of this equation back to the time domain yields equation (3) (i.e., the result of Derode et al. (2003a; 2003b)), with proportionality factor $2/\rho c$. Finally we consider a variant of our derivation. The Green's function introduced in equation (8) is the response of an impulsive point source of volume injection rate. Let us define a new Green's function \hat{G}_0 obeying the same wave equation, but with the source in the right-hand side replaced by $-\rho\delta(\mathbf{x} - \mathbf{x}_A)$, hence $\hat{G}_0 = \frac{1}{j\omega} \hat{G}$. Following the same derivation as above, we obtain instead of equation (11)

$$2j\Im\{\hat{G}_0(\mathbf{x}_B, \mathbf{x}_A, \omega)\} \approx \frac{-2j\omega}{\rho c} \oint_S \hat{G}_0(\mathbf{x}_B, \mathbf{x}, \omega) \hat{G}_0^*(\mathbf{x}_A, \mathbf{x}, \omega) d^2\mathbf{x}, \quad (12)$$

where \Im denotes the imaginary part. This expression resembles the results of Weaver and Lobkis (Weaver and Lobkis, 2004) and Snieder (Snieder, 2004), who retrieve the two-sided Green's function from the time-derivative of cross-correlations.

Note that for the derivation of each of the expressions (3) and (9) – (12), we assumed that impulsive point sources were placed on the surface S . This is the approach taken e.g. by Bakulin and Calvert (2004) in their experiment on virtual source imaging. Our derivation also holds for uncorrelated noise sources on S whose source-time function satisfies $s(\mathbf{x}, t) * s(\mathbf{x}', -t) = \delta(\mathbf{x} - \mathbf{x}')C(t)$, with $C(t)$ the autocorrelation of the noise. When the noise is distributed over the surface, the cross-correlation of the observations at \mathbf{x}_A and \mathbf{x}_B leads to a double surface integral. The delta function reduces this to the single surface integral in the theory presented here (Wapenaar et al., 2002; Wapenaar, 2003; Derode, et al., 2003b; Wapenaar, 2004; Weaver and Lobkis, 2004; Snieder, 2004). A further discussion is beyond the scope of this letter.

4 NUMERICAL EXAMPLE

We illustrate equation (10) with a simple example. We consider a 2-D configuration with a single diffractor at $(x_1, x_3) = (0, 600)\text{m}$ in a homogeneous medium with propagation velocity $c = 2000$ m/s, see Figure 1, in which C denotes the diffractor. Further, we define $\mathbf{x}_A = (-500, 100)\text{m}$ and $\mathbf{x}_B = (500, 100)\text{m}$, denoted by A and B in Figure 1. The surface S is a circle with its center at the origin and a radius of 800 m. The solid arrows in Figure 1 denote the Green's function $G(\mathbf{x}_B, \mathbf{x}_A, t)$. We model the Green's functions in equation (10) with the Born approximation, which means

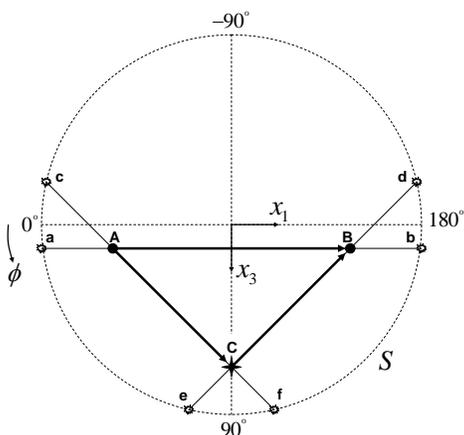


Figure 1. Single diffractor (C) in a homogeneous model. The receivers are at A and B. The integration is carried out along the sources on the surface S . The main contributions come from the stationary points a-d. The contributions from stationary points e and f cancel.

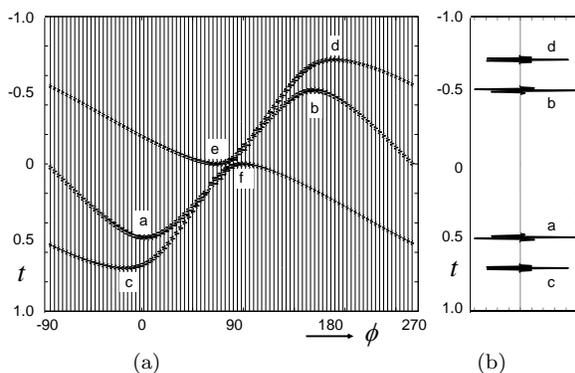


Figure 2. (a) Time domain representation of the integrand of equation (10). (b) The sum of all traces in (a).

that we consider direct waves and first order scattering only. To be consistent with the Born approximation, in the cross-correlations we also consider only the zeroth and first order terms. Figure 2a shows the time-domain representation of the integrand of equation (10) (convolved with a wavelet with a central frequency of 50 Hz). Each trace corresponds to a fixed source position \mathbf{x} on S ; the source position in polar coordinates is $(\phi, r = 800)$. The sum of all these traces (multiplied by $r d\phi$) is shown in Figure 2b. This result accurately matches the directly modelled wave field $G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t)$ (convolved with a wavelet), see Figure 3. The events labelled ‘a’ and ‘c’ in Figure 2 are the direct and scattered arrivals; the events ‘b’ and ‘d’ are the corresponding anti-causal arrivals. This figure clearly shows that the main contribution to these events come from Fres-

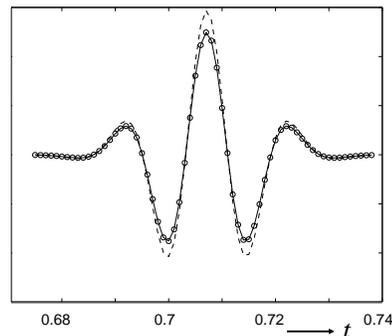


Figure 3. Zoomed-in version of event c in Figure 2b. The solid line is the directly modelled wave field. The circles represent the integration result of equation (10) (i.e., the sum of the traces in Figure 2a). The dashed line represents the integration result of equation (11).

nel zones around the stationary points of the integrand (Snieder, 2004; Schuster et al., 2004; Wapenaar et al., 2004). The sources at these stationary points are marked in Figure 1 with the same labels. We discuss event ‘c’ in more detail. The path ‘cCB’ in Figure 1 represents the scattered wave in $G(\mathbf{x}_B, \mathbf{x}, t)$, for \mathbf{x} at the stationary point ‘c’. The path ‘cA’ represents the direct wave in $G(\mathbf{x}_A, \mathbf{x}, t)$. By correlating these two waves, the traveltime along the path ‘cA’ is subtracted from that along the path ‘cCB’, leaving the traveltime along the path ‘ACB’, which corresponds to the traveltime of the scattered wave in $G(\mathbf{x}_B, \mathbf{x}_A, t)$. This correlation result is indicated by ‘c’ in Figure 2a and the integral over the Fresnel zone around this point is event ‘c’ in Figure 2b. The other events in Figure 2b can be explained in a similar way. Finally, note that there are two more stationary points, indicated by ‘e’ and ‘f’ in Figures 1 and 2a, of which the contributions cancel each other.

A similar numerical evaluation of equation (11) yields the result represented by the dashed curve in Figure 3. We observe that the traveltime of the scattered wave is accurately captured by this equation, but the amplitude is slightly overestimated. By increasing the radius of S to 10 000 m we obtained a result with equation (11) that again accurately matches the directly modelled wave field (not shown).

5 CONCLUSIONS

In the literature several derivations have been proposed for Green’s function retrieval from cross-correlations. We have shown that the derivation by Derode et al. (Derode, et al., 2003a; Derode, et al., 2003b), which is based on physical arguments, leads essentially to

the same result as our derivation based on Rayleigh's reciprocity theorem (Wapenaar et al., 2002; Wapenaar, 2003; Wapenaar, 2004). Moreover, using another definition of the Green's function in Rayleigh's reciprocity theorem, we obtained a representation in terms of the time-derivative of cross-correlations, similar as in Weaver and Lobkis (Weaver and Lobkis, 2004) and Snieder (Snieder, 2004).

REFERENCES

- A. Bakulin and R. Calvert. Virtual source: new method for imaging and 4D below complex overburden. In *Soc. Expl. Geophys., Expanded Abstracts*, pages 2477–2480, 2004.
- N. N. Bojarski. Generalized reaction principles and reciprocity theorems for the wave equations, and the relationship between the time-advanced and time-retarded fields. *J. Acoust. Soc. Am.*, 74:281–285, 1983.
- J. F. Claerbout. Synthesis of a layered medium from its acoustic transmission response. *Geophysics*, 33:264–269, 1968.
- A. T. de Hoop. Time-domain reciprocity theorems for acoustic wave fields in fluids with relaxation. *J. Acoust. Soc. Am.*, 84:1877–1882, 1988.
- A. Derode, E. Larose, M. Campillo, and M. Fink. How to estimate the Green's function of a heterogeneous medium between two passive sensors? Application to acoustic waves. *Applied Physics Letters*, 83(15):3054–3056, 2003.
- A. Derode, E. Larose, M. Tanter, J. de Rosny, A. Tourin, M. Campillo, and M. Fink. Recovering the green's function from field-field correlations in an open scattering medium (L). *J. Acoust. Soc. Am.*, 113(6):2973–2976, 2003.
- C. Esmersoy and M. Oristaglio. Reverse-time wave-field extrapolation, imaging, and inversion. *Geophysics*, 53:920–931, 1988.
- J. T. Fokkema and P. M. van den Berg. *Seismic applications of acoustic reciprocity*. Elsevier, Amsterdam, 1993.
- O. I. Lobkis and R. L. Weaver. On the emergence of the Green's function in the correlations of a diffuse field. *J. Acoust. Soc. Am.*, 110:3011–3017, 2001.
- G. A. McMechan. Migration by extrapolation of time-dependent boundary values. *Geoph. Prosp.*, 31:413–420, 1983.
- G. T. Schuster, J. Yu, J. Sheng, and J. Rickett. Interferometric/daylight seismic imaging. *Geoph. J. Int.*, 157:838–852, 2004.
- R. Snieder. Extracting the Green's function from the correlation of coda waves: A derivation based on stationary phase. *Phys. Rev. E.*, 69:046610–1–046610–8, 2004.
- D. J. van Manen, J. O. A. Robertsson, and A. Curtis. Making waves by time reversal. *Phys. Rev. Lett.*, (submitted), 2005.
- K. Wapenaar. Synthesis of an inhomogeneous medium from its acoustic transmission response. *Geophysics*, 68:1756–1759, 2003.
- K. Wapenaar. Retrieving the elastodynamic Green's function of an arbitrary inhomogeneous medium by cross correlation. *Phys. Rev. Lett.*, 93:254301–1–254301–4, 2004.
- K. Wapenaar, D. Draganov, J. Thorbecke, and J. Fokkema. Theory of acoustic daylight imaging revisited. In *Soc. Expl. Geophys., Expanded Abstracts*, pages 2269–2272, 2002.
- K. Wapenaar, D. Draganov, J. van der Neut, and J. Thorbecke. Seismic interferometry: a comparison of approaches. In *Soc. Expl. Geophys., Expanded Abstracts*, pages 1981–1984, 2004.
- R. L. Weaver and O. I. Lobkis. Ultrasonics without a source: Thermal fluctuation correlations at MHz frequencies. *Phys. Rev. Lett.*, 87:134301–1–134301–4, 2001.
- R. L. Weaver and O. I. Lobkis. Diffuse fields in open systems and the emergence of the Green's function (L). *JASA*, 116(5):2731–2734, 2004.

