Elastic wavefield tomography with probabilistic petrophysical model constraints

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ABSTRACT

Seismic data interpretation is a crucial step for understanding subsurface geology. Without a reasonably accurate subsurface model, image interpretation is erroneous, which can be costly. Seismic exploration and characterization often use elastic wavefield tomography to derive models of subsurface properties. Unconstrained multiparameter inversion may lead to model component updates that do not represent real lithology for independently derived parameters. Incorporating petrophysical information, such as that provided by well logs, during inversion constrains the inverted models and avoids implausible models. To facilitate constrained inversion, we develop a method for elastic wavefield tomography that explicitly imposes petrophysical restrictions in order to update models to have feasible lithology, i.e., have model components that are consistent with the underlying petrophysics. We enforce a feasible region with a model probability density function calculated without user-defined parameters. Inside this feasible region, the inverted models do not need to obey a specific trend, i.e., we do not link the parameters with explicit and potentially inaccurate petrophys-
We demonstrate through elastic models that incorporating probabilistic petrophysical constraints into the inversion objective function leads to reliable models that are superior to models obtained either without constraints or with approximate analytic constraints.
INTRODUCTION

One of the greatest challenges in seismic exploration consists of estimating accurate subsurface elastic models. Seismic tomography is becoming a standard technique for subsurface imaging at various scales, and aims to build high-resolution models of the physical parameters underlying wave propagation. The main approaches to seismic tomography use either traveltimes (Woodward, 1992; Schuster and Quintus-Bosz, 1993; Taillandier et al., 2009) or wavefields (Tarantola, 1984; Mora, 1989; Biondi, 2006; Duan and Sava, 2016; Diaz and Sava, 2017). Wavefield tomography has advantages over traveltime tomography because it recovers parameters with higher resolution (Tarantola, 1986; Pratt, 1999; Operto et al., 2013) by exploiting both the kinematics and dynamics of the observed waveforms.

Full waveform inversion (FWI) is among the most promising techniques in the wavefield tomography category as it delivers highly accurate subsurface models from seismic data (Tarantola, 1984; Pratt, 1999; Sirgue et al., 2004; Virieux and Operto, 2009). FWI operates by iteratively updating model parameters based on the mismatch between observed and simulated data. This optimization exploits seismic wavefield modeling (the forward problem) and solves the associated inverse problem by minimizing an objective function. The gradient of the objective function estimates model perturbations that progressively decrease the data residual. Conventional FWI implementations use the adjoint state method (Plessix, 2006) to compute the objective function gradient, since this method is among the most efficient and requires the least amount of storage.

Early implementations of FWI used the acoustic wave equation (Tarantola, 1984; Pratt, 1990; Pratt et al., 1996). However, acoustic FWI does not take into account elastic effects and the presence of S waves in the data. Several parameters are necessary for describing elastic models (e.g., density $\rho$ and Lamé parameters $\lambda$ and $\mu$, or density and P- and S-wave velocities). Therefore, elastic FWI
(EFWI) better describes the subsurface properties (Tarantola, 1988; Pratt, 1990; Plessix, 2006).

Despite its popularity, FWI is hampered by several challenges undermining its practical application. FWI is a highly non-linear and ill-posed problem, so its convergence is subjected to local minima (Symes, 2008; Virieux and Operto, 2009). Therefore, one needs to include prior information, i.e., model regularization, into inversion to find a plausible solution (Tikhonov and Arsenin, 1977; Tarantola, 2005). Also, the wave extrapolator can become unstable when model updates fluctuate, thus impending further updates.

Elastic FWI suffers from a number of additional practical difficulties. First, using multiple parameters increases the degrees of freedom in the model space, thereby increasing the non-linearity of the inversion (Operto et al., 2013). In addition, as model parameters are updated simultaneously but independently, different model components can be physically contradictory to one-another, creating combinations that are lithologically implausible or impossible. This situation may create inaccurate and unstable forward solutions and lead to poor convergence (Baumstein, 2013). Moreover, similar radiation patterns among various elastic parameters create crosstalk, i.e. different models are viewed similarly in data (Operto et al., 2013; Kamath and Tsvankin, 2016) resulting in ambiguous model inversion. Radiation pattern analysis can partially correct for crosstalk, but is largely ineffective in poorly illuminated areas.

In order to recover models that are self-consistent and properly characterize the subsurface, we need to explicitly impose model constraints during the inversion (Baumstein, 2013; Peters et al., 2015; Duan and Sava, 2016; Manukyan et al., 2018; Zhang et al., 2018; Rocha and Sava, 2018). These constraints can use petrophysical information, such as that contained in well logs or from other sources, as core or basin analysis. Incorporating constraints into the objective function defines model space bounds, forcing the inverted models to be consistent with known petrophysical
Baumstein (2013) and Peters et al. (2015) use the projection onto convex sets (POCS method) to incorporate constraints into acoustic FWI. These constraints impose velocity bounds (box constraints) and specify a minimum smoothness for the model parameters. The updated models must be at the intersection of all convex sets, which leads to more stable simulations and geologically plausible structure. Manukyan et al. (2018) constrain the inversion by assuming that the elastic parameters have structural similarity and thus mitigate parameter trade-off problems and the different spatial resolution of elastic parameters. They show that structurally constrained FWI can deliver higher-quality models when compared to conventional FWI. However, the assumption of structural similarity between different elastic parameters can potentially compromise the final outcome. Zhang et al. (2018) use seismic facies as *a priori* information to constrain the EFWI to derive more reliable and higher-resolution isotropic and anisotropic models. They achieve this goal by adding a model regularization term to the objective function, related to the facies. As the spatial distribution of the facies is unknown, they propose to update the facies at each iteration by using a Bayesian inversion workflow.

Duan and Sava (2016) use a logarithmic penalty function to constrain the elastic wavefield tomography assuming a general linear relationship between the model parameters. Rocha and Sava (2018) also use this type of constraint, but in the context of elastic reflection waveform inversion. Both examples demonstrate that an explicit petrophysical constraint can yield consistent and plausible models even if conventional spatial regularization is not explicitly applied during the inversion. However, complex models do have non-linear petrophysical relationships between the elastic parameters. Moreover, the analytic logarithmic barrier constraint works with user-defined parameters, and their selection may be challenging, especially when different lithofacies are present in the same area. Thus, it is necessary to define more general and automated petrophysical constraints that
provide flexibility and adapt to the specific geologic situation in the imaged area.

We propose elastic wavefield tomography with an objective function that constrains the relationship between elastic parameters using model probability density functions (PDFs). In the following sections, we first discuss general aspects of elastic FWI, emphasizing the mechanisms used to incorporate constraints into the inversion. We demonstrate that PDFs can be used in EFWI without knowing or defining an analytic relationship among the different elastic parameters. Finally we illustrate the features and performance of the proposed method with two synthetic examples.

**ELASTIC WAVEFIELD TOMOGRAPHY**

We consider the isotropic elastic wave equation

\[
\rho \ddot{u} - \lambda \nabla (\nabla \cdot u) - \mu \left[ \nabla \cdot (\nabla u + \nabla u^T) \right] = f,
\]

where \(u(e, x, t)\) is the elastic wavefield, \(f(e, x, t)\) is the source function, \(\lambda(x)\) and \(\mu(x)\) are the Lamé parameters, \(\rho(x)\) is the density and \(e, x\) and \(t\) are, respectively, the experiment index, space coordinates and time. Equation 1 assumes that Lamé parameters vary slowly, such that their spatial gradient can be neglected.

Considering \(u_s\) as the source wavefield, the waveform inversion problem is solved by minimizing an objective function \(\mathcal{J}(u_s, \lambda, \mu)\) consisting of three parts: a term that evaluates the misfit between the simulated and observed data \(\mathcal{J}_D(u_s, \lambda, \mu)\); a model regularization term \(\mathcal{J}_M(\lambda, \mu)\); that enforces spatial correlation of the model parameters, and a term that enforces petrophysical model constraints, \(\mathcal{J}_C(\lambda, \mu)\):
\[ J(u_s, \lambda, \mu) = J_D(u_s, \lambda, \mu) + J_M(\lambda, \mu) + J_C(\lambda, \mu). \] (2)

We can define the data residual

\[ r_D(e, x, t) = W_u(e, x, t)u_s(e, x, t) - d_{obs}(e, x, t), \] (3)

and then the data misfit term

\[ J_D(u_s, \lambda, \mu) = \sum_e \frac{1}{2} \| r_D(e, x, t) \|^2, \] (4)

where \( W_u(e, x, t) \) are weights that restrict the source wavefield \( u_s(e, x, t) \) to the known receiver locations, and \( d_{obs}(e, x, t) \) are the observed data.

To update the model iteratively using a gradient-based method (Tarantola, 1988), one can compute the gradient of \( J_D \) with respect to the model parameters \( \lambda \) and \( \mu \) using the adjoint-state method (Plessix, 2006). This method consists of four steps:

1. Compute the seismic wavefield \( u_s(e, x, t) \) from a source function \( f_s \), such that \( Lu_s - f_s = 0 \), where \( L \) is the linear elastic wave operator derived from equation 1,

2. Compute the adjoint source \( g_s = \partial J/\partial u_s \), which exploits the difference between the observed and simulated data (equation 3),

3. Compute the adjoint wavefield \( a_s(e, x, t) = L^T g_s \), which exploits the adjoint elastic wave operator \( L^T \),

4. Compute the gradient of \( J_D \) with respect to the parameters \( \lambda \) and \( \mu \) by
\[
\begin{bmatrix}
\frac{\partial J_D}{\partial \lambda} \\
\frac{\partial J_D}{\partial \mu}
\end{bmatrix}
= \sum e \begin{bmatrix}
-\nabla (\nabla \cdot \mathbf{u}) \ast \mathbf{a}_s \\
-\nabla (\nabla \mathbf{u} + \nabla \mathbf{u}^T) \ast \mathbf{a}_s
\end{bmatrix},
\]

where the symbol \( \ast \) represents zero-lag time crosscorrelation.

Incorporating a model regularization term into the objective function reduces the inversion ill-posedness, and accelerates convergence towards the global minimum (Asnaashari et al., 2013; Jun et al., 2017; Zhang et al., 2018). Prior information can be estimated from non-seismic data, e.g. well logs, or by incorporating knowledge about the model geometry, e.g., smoothness or consistency with a migrated image. Considering the model residuals:

\[
\begin{align*}
\mathbf{r}_\lambda(x) &= \mathbf{W}_\lambda(x)(\lambda(x) - \bar{\lambda}(x)), \\
\mathbf{r}_\mu(x) &= \mathbf{W}_\mu(x)(\mu(x) - \bar{\mu}(x)),
\end{align*}
\]

the term \( J_M(\lambda, \mu) \) is

\[
J_M(\lambda, \mu) = \frac{1}{2} \| \mathbf{r}_\lambda(x) \|^2 + \frac{1}{2} \| \mathbf{r}_\mu(x) \|^2,
\]

where \( \bar{\lambda}(x) \) and \( \bar{\mu}(x) \) are prior (reference) models. The weighting operators \( \mathbf{W}_\lambda(x) \) and \( \mathbf{W}_\mu(x) \) are related to the model covariance matrices such that \( \mathbf{W}_\lambda = C_{\lambda}^{-\frac{1}{2}} \) and \( \mathbf{W}_\mu = C_{\mu}^{-\frac{1}{2}} \).

The gradient of \( J_M(\lambda, \mu) \) with respect to the model parameters \( \lambda \) and \( \mu \) is
\[
\begin{bmatrix}
\frac{\partial J_M}{\partial \lambda} \\
\frac{\partial J_M}{\partial \mu}
\end{bmatrix} = \begin{bmatrix}
W^T_\lambda W_\lambda (\lambda - \bar{\lambda}) \\
W^T_\mu W_\mu (\mu - \bar{\mu})
\end{bmatrix}
\]

(9)

It is apparent from equations 5 and 9 that the model components \( \lambda \) and \( \mu \) for multiparameter inversion are updated independently and thus can become physically inconsistent with each other, producing inaccurate models, images, and inaccurate geologic interpretation. Explicitly incorporating petrophysical constraints improves the quality of the inversion by restricting the updated models to a feasible region, resulting in more accurate subsurface characterization.

Duan and Sava (2016) propose a logarithmic penalty function \( J_L \) to constrain the inversion to a feasible region by enforcing physical relationships between the updated model parameters:

\[
J_L(\lambda, \mu) = -\eta \sum_x [\log (h_u) + \log (h_l)].
\]

(10)

The scalar parameter \( \eta \) determines the strength of the constraint term in the objective function, and \( h_u \) and \( h_l \) are linear functions that define, respectively, upper and lower boundaries of the feasible region:

\[
h_u = -\lambda + c_u \beta + b_u = 0,
\]

(11)

\[
h_l = \lambda - c_l \beta - b_l = 0.
\]

(12)

The coefficients \( c_u \) and \( c_l \) are the slopes of the boundary lines, and \( b_u \) and \( b_l \) are their intercepts. The distance of a given model parameter in the parameter space \( \{\lambda, \mu\} \) to the boundary lines \( h_u \) and...
$h_i$ determines the value of $J_L$, such that this term dominates the objective function when models are close to either one of the barriers. This constraint forces the inverted models to move away from those boundaries, which act like a barrier that favors model parameters in a given feasible region.

The gradients of $J_L$ with respect to the model parameters $\lambda$ and $\mu$ are

$$
\begin{bmatrix}
\frac{\partial J_L}{\partial \lambda} \\
\frac{\partial J_L}{\partial \mu}
\end{bmatrix} = \eta \begin{bmatrix}
\frac{-1}{\lambda - c_u \mu - b_u} - \frac{1}{\lambda - c_l \mu - b_l} \\
\frac{c_u}{\lambda - c_u \mu - b_u} + \frac{c_l}{\lambda - c_l \mu - b_l}
\end{bmatrix}.
$$

The term $J_L$ assumes that the relationship between the model parameters $\lambda$ and $\mu$ is linear. Therefore, the inversion may not be able to recover complex, nonlinear true physical relationships between the model parameters. Alternatively, if we have access to petrophysical information obtained, for example, from well logs, we can impose a more general kind of petrophysical constraint. In this situation, we can construct a model probability density function (PDF) of the reference well log parameters to characterize the interdependence between the elastic model parameters.

Consider, for example, the crossplot in Figure 1a illustrating a possible petrophysical relationship between the Lamé parameters, $\lambda$ and $\mu$. The linear barrier constraint $J_L$ would not be appropriate for this example, as $\lambda$ and $\mu$ do not share a linear relationship. Instead, we can create a model PDF (Figure 1b) based on the available values of $\lambda$ and $\mu$. For any given model with components $(\lambda_x, \mu_x)$, represented by the red dot in Figure 1b, we would like to evaluate a model space distance to the entire distribution (Figure 2). If this model is close to regions of high probability, we would like the value of the petrophysical constraint $J_P$ to be small. Otherwise, the value of $J_P$ must be large and increasing when model parameters separate more from the distribution. To accomplish this, we construct a weighted sum of the distances between the model with $(\lambda_x, \mu_x)$ components to all the models of the PDF. The weight is proportional to the value of the PDF at a specific model.
cell and inverse proportional to the distance from the analyzed model to that cell.

The model space distance from \((\lambda_x, \mu_x)\) to a cell of the PDF with indices \((i, j)\) is

\[
d_x(i, j) = \sqrt{[\lambda(i) - \lambda_x]^2 + [\mu(j) - \mu_x]^2},
\]

for \(i = 1 \cdots N_\lambda\) and \(j = 1 \cdots N_\mu\) where \(N_\lambda\) and \(N_\mu\) are the numbers of discrete intervals defined for parameters \(\lambda\) and \(\mu\), respectively. Then, the weight from a specific model \((\lambda_x, \mu_x)\) to the cell at coordinates \((i, j)\) with the probability \(P(i, j)\) is

\[
w_x(i, j) = \frac{P(i, j)}{d_x(i, j)},
\]

The weighted distance from the point \((\lambda_x, \mu_x)\) to the entire distribution is

\[
D_x = \sum_{i,j} P(i,j) d_x(i,j),
\]

Therefore, each point in the updated model space is connected to all cells of the model PDF, i.e., we have constructed the distance from this model to the distribution. Since \(D_x\) is small for points that are far from regions of high probability, we can define the petrophysical constraint \(J_P\) as

\[
J_P(\lambda, \mu) = \eta \sum_{x} \frac{1}{D_x},
\]

This definition imposes the condition that all points representing inverted models are as close as possible to the original petrophysical distribution without being drawn especially to any value of the model, but rather close in a statistical sense to the entire distribution. Figure 1c shows the
distribution of the constraint term calculated using equation 17 and the PDF presented in Figure 1b.

The gradient of $J_P$ with respect to the model parameters $\lambda$ and $\mu$ is

$$
\begin{align*}
\begin{bmatrix}
\frac{\partial J_P}{\partial \lambda} \\
\frac{\partial J_P}{\partial \mu}
\end{bmatrix}
= -\eta 
\begin{bmatrix}
\left( \sum_{i,j} P(i,j) [\lambda(i) - \lambda_x] \right) \sum_{i,j} \frac{1}{D_x^2} \\
\left( \sum_{i,j} P(i,j) [\mu(j) - \mu_x] \right) \sum_{i,j} \frac{1}{D_x^2}
\end{bmatrix}
\end{align*}
\tag{18}
$$

The distance to high probabilities in the model parameter space defines the value of $J_P$, such that the gradient of this term dominates in the total gradient of the objective function $J$ if the updated models are far from high probability. Otherwise, this term smoothly forces the models away from regions of low probability.

The probabilistic petrophysical model constraint $J_P$ addresses many issues related to elastic FWI. First, it reduces the non-linearity and ill-posed nature of FWI as it incorporates prior information into the inversion, which reduces the number of model solutions and improves the inversion convergence, preventing instabilities. In addition, it avoids geologically implausible earth models and reduces the artifacts created by the interparameter crosstalk, as it links the elastic parameters during the inversion and restricts them to a feasible region.

**EXAMPLES**

We illustrate our EFWI method with two synthetic examples and compare inversions using only the data misfit term $J_D$, the data misfit with the logarithmic penalty function $J_D + J_L$, and the data misfit with the probabilistic petrophysical constraints $J_D + J_P$. 
The first synthetic example uses a model with negative and positive Gaussian anomalies centered at (1.25, 0.75) and (1.25, 1.75) km, respectively (Figure 3). We simulate 20 vertical displacement sources in a well at \( x = 0.1 \) km and a line of geophones at \( x = 2.4 \) km. Figure 3 shows the true \( \lambda \) and \( \mu \) models along with a crossplot of the models. We choose this simple example to show how the inversion using data misfit with the logarithmic penalty function fails when the model is characterized by two clusters, which represent lithology with different linear relationships between parameters \( \lambda \) and \( \mu \). We initiate the inversion with constant backgrounds and run all inversions for 10 iterations.

Figure 4 shows the recovered \( \lambda \) and \( \mu \) models using the objective function \( J_D \) along with a crossplot in the \( \lambda - \mu \) space. Without imposing petrophysical constraints, the \( \lambda \) and \( \mu \) values deviate from the true model as the similar radiation patterns of different model parameter perturbations create crosstalk among physical properties. This problem is enlarged due to the limited acquisition coverage and different illumination. The inversion does not recover the correct magnitude or size of the \( \lambda \) model. Additionally, the \( \mu \) model has many artifacts due to the crosstalk between the elastic parameters. Although a linear relationship between the parameters characterizes the true model (Figure 3), the crossplot shown in Figure 4 indicates that the parameters \( \lambda \) and \( \mu \) are uncorrelated, which agrees with the fact the data misfit objective function updates them independently, thus leading to unphysical models.

Adding the logarithmic penalty function \( J_L \) to \( J_D \) in the objective function, we obtain the \( \lambda \) and \( \mu \) models shown in Figure 5. The lines in the crossplot correspond to the linear boundaries used to define the logarithmic penalty function \( J_L \). \( J_L \) is not flat in the interval between the upper and lower boundaries and forces the recovered models towards the middle region, as seen in Figure 5. The
point (10.4, 5.2) GPa, which represents the background models, is not mid-way between the two boundaries and, consequently, inversion using $J_L$ changes the background parameters (Figure 5).

Figure 6 shows the recovered $\lambda$ and $\mu$ models using the objective function $J_D + J_P$. For simplicity, we define in this example the PDF as a Gaussian distribution covering the true model. This is not necessary for more complex examples, where the relationship between the parameters is non-linear and derived from actual well logs. The lines in the crossplot correspond to the feasible region used to define the probabilistic constraint $J_P$. Notice that when we include the probabilistic constraint term both anomalies have amplitude and shape closer to the true $\lambda$ and $\mu$ models (Figure 3). Imposing the probabilistic constraint confines the model to the feasible region defined by the PDF, but within this area the models do not follow any predefined trend, which does not happen when we use the logarithmic boundaries. Also, the crossplot shown in Figure 6 has two main linear relationships between the inverted models, what conforms to the true models.

**Marmousi 2 model**

The second synthetic example uses a portion of the Marmousi 2 model (Martin et al., 2002). For this example, we consider that prior petrophysical data from three well logs are available and we use this information to build the petrophysical model constraints. Figure 7 shows the correct $\lambda$ and $\mu$ models, the well locations for our experiment and the crossplot of the model parameters. We simulate a multicomponent ocean bottom seismic survey (OBS) with a line of receiver at the water bottom ($z = 0.17$ km) and 20 evenly spaced pressure sources located at $z = 0.05$ km. Figure 8 shows the crossplot of $\lambda$ and $\mu$ models extracted at the wells located at $x = 1.1$, 2.0 and 3.0 km (Figure 7). We use a 7.5 Hz peak Ricker source wavelet. Figure 9 shows the initial $\lambda$ and $\mu$ models and the corresponding crossplot in the model parameter space.
Figure 10 shows the recovered $\lambda$ and $\mu$ models using the objective function $J_D$. The inverted $\mu$ model has more structural detail than the $\lambda$ model, as it only depends on density and S-velocities, while $\lambda$ depends on density as well as the P- and S-velocities. The wavelength of S waves is shorter than that of P waves, and therefore the inverted $\mu$ model has higher spatial resolution. However, as in this example we do not impose petrophysical constraints, the $\mu$ model has spurious artifacts due to parameter crosstalk. Note that the values of the main structure presented in the initial $\lambda$ model (Figure 9) that conforms to the true $\lambda$ model (Figure 7) are not preserved during the unconstrained inversion (Figure 10). Additionally, the crossplot shown in Figure 10 does not resemble the true model (Figure 7) because the updates are not restricted to a feasible petrophysical region.

Figure 11 shows the recovered $\lambda$ and $\mu$ models using the objective function $J_D + J_L$. We estimate the upper and lower boundaries of the logarithmic penalty function from the well logs as $h_l = \lambda - 1.3\mu - 0.2$ and $h_u = -\lambda + 1.3\mu + 3.15$. We ensure that initial models for the inversion (Figure 9) are within the feasible region. As for the case of inversion using the objective function $J_D$ (Figure 10), the inverted $\mu$ model has spurious artifacts. The main structure presented in the initial $\lambda$ model (Figure 9) is not preserved during this kind of constrained inversion (Figure 11). In addition, the crossplot shown in Figure 11 indicates that the model samples follow the linear trend imposed by the barriers. However this trend does not correspond to the true models (Figure 7).

Figure 12 shows the recovered $\lambda$ and $\mu$ models using the objective function $J_D + J_P$. We estimate the PDF presented in the crossplot as the contour lines from the well log data. The PDF is estimated from the well log data without user-defined parameters. We also ensure that the initial model for inversion (Figure 9) is within the feasible region of this experiment. Note that both $\lambda$ and $\mu$ models in Figure 12 present similar resolution and they are better recovered compared with the cases when we use the objective functions $J_D$ (Figure 10) or $J_D + J_L$ (Figure 11). Additionally, the $\mu$ model artifacts are attenuated when we incorporate the probabilistic petrophysical constraints.
in the inversion. Also, the crossplot built with the inverted models (Figure 12) is closer to the similar crossplot built with the correct model parameters relative to the crossplots obtained for the less robust objective functions $J_D$ (Figure 10) or $J_D + J_L$ (Figure 11).

**CONCLUSIONS**

We incorporate petrophysical information into the elastic full-waveform inversion, using model probability density functions derived from information provided by well logs or from other forms of rock physics analysis. We demonstrate that imposing petrophysical constraints in elastic wavefield tomography improves the quality of the recovered models, by restricting them to a feasible region, and therefore guiding the inversion towards geologically plausible solutions. Two synthetic examples show that the models recovered with the proposed method are closer to the true models, while maintaining robust and realistic petrophysical relationships between the model parameters. This constraint term also helps mitigate the interparameter crosstalk created by similar radiation patterns among the elastic parameters. One can also impose model regularization in addition to petrophysical model constraints to further enhance the geologic realism of models provided by wavefield tomography.

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Marmousi 2 model: Recovered $\lambda$ and $\mu$ models using objective function $J_D$. The inverted $\mu$ model has more structural details than the $\lambda$ model.

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Figure 2: Sketch indicating how each point in the updated model space (red dot) communicates with all cells of the model PDF. Each cell of the model PDF contributes proportionally with its probability value and inverse proportionally to the distance to the analyzed model.
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Figure 3: True $\lambda$ and $\mu$ models and their crossplot in the $\lambda - \mu$ space. The top panels show the sources (dots) located at $x = 0.1$ km and the vertical line corresponds to receivers at $x = 2.4$ km. The negative and positive Gaussian anomalies for the $\lambda$ and $\mu$ models are centered at $(1.25, 0.75)$ km and $(1.25, 1.75)$ km, respectively. The crossplot shows two linear relationships between $\lambda$ and $\mu$: corresponding to each of the Gaussian anomalies.

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Figure 4: Updated $\lambda$ and $\mu$ models using the objective function $J_D$. The Gaussian anomalies are not recovered well and the inverted models present artifacts from inter-parameter crosstalk. The crossplot shows that the parameters do not follow the same trend of the true models (Figure 3).
Figure 5: Updated $\lambda$ and $\mu$ models using the objective function $J_D + J_L$. The lines define the upper and lower boundaries used in the logarithmic penalty function $J_L$. The Gaussian anomalies are not recovered well and the backgrounds are modified during the inversion.

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Figure 6: Updated $\lambda$ and $\mu$ models using the objective function $J_D + J_P$. The contours in the crossplot represent the feasible area provided by the probabilistic petrophysical constraint $J_P$. Both Gaussian anomalies are better recovered for the $\lambda$ and $\mu$ models.

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Figure 7: Marmousi 2 model: True $\lambda$ and $\mu$ models and their crossplot. The vertical lines correspond to the location of the wells, providing information for the petrophysical constraint.

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Figure 8: Crossplot of $\lambda$ and $\mu$ provided by the well logs indicated in Figure 7. The relationship between the model parameters is not linear and does not follow a simple trend.

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Figure 9: Marmousi 2 model: Initial $\lambda$ and $\mu$ models and their crossplot in the $\lambda - \mu$ space.

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Figure 10: Marmousi 2 model: Recovered $\lambda$ and $\mu$ models using objective function $J_D$. The inverted $\mu$ model has more structural details than the $\lambda$ model.

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Figure 11: Marmousi 2 model: Recovered $\lambda$ and $\mu$ models using objective function $J_D + J_L$. The lines in the crossplot define the upper and lower boundaries used in the logarithmic penalty function $J_L$. The inverted model is forced to be in the middle area of the feasible region, which does not reflect correctly the trend shown in Figure 7 for the true model.

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Figure 12: Marmousi 2 model: Recovered $\lambda$ and $\mu$ models using objective function $J_D + J_P$. The contours in the crossplot represent the feasible area for the probabilistic constraint $J_P$. The probabilistic petrophysical constraint leads to better recovered models than unconstrained inversion (Figure 10) and inversion with approximate analytic constraints (Figure 11).