Estimation of microseismic source parameters for 3D orthorhombic media by waveform inversion

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ABSTRACT

Accurate event location and estimation of the source moment tensors are among the main goals of microseismic data processing. An improvement over conventional methods can be achieved by applying anisotropic waveform inversion (WI). Here, we extend our 3D elastic WI algorithm for anisotropic velocity model-building from microseismic borehole data to include estimation of event locations, moment tensors, and origin times. The methodology is designed for microseismic sources embedded in arbitrarily heterogeneous media with orthorhombic or transversely isotropic (TI) symmetry. We illustrate the performance of WI using synthetic data from layered orthorhombic media simulated for typical acquisition geometries. The tests confirm the ability of the algorithm to recover the source parameters provided that the velocity model is known with sufficient accuracy.

1 INTRODUCTION

Processing of microseismic data has emerged as an essential technology in monitoring hydraulic fracturing and characterizing unconventional reservoirs (e.g., Maxwell, 2014, Grechka and Heigl, 2017). Injection of high-pressure fluids and fracturing of tight shale formations induces microseismicity that can provide useful information for optimizing reservoir stimulation. Location of microseismic events, which is one of the main goals of microseismic monitoring, requires an accurate (in most cases, anisotropic) velocity model. It is well known that shales are at least transversely isotropic due to the alignment of clay particles and can exhibit a lower symmetry in the presence of fractures (Tsvankin, 2012).

Another important problem in microseismic studies is estimation of the source mechanisms of the recorded events. The seismic moment tensor $\mathbf{M}$ can provide valuable information about the process of hydraulic stimulation and the induced stress field in the reservoir. In practice, the tensor $\mathbf{M}$ is often obtained by least-squares inversion of the amplitudes of the direct P- and S-waves. For geometries typical in microseismic surveys, the six independent moment-tensor elements can be found from the amplitudes of P-waves recorded in three boreholes or from P- and S-wave amplitudes recorded in two boreholes (Vavryčuk, 2007). Grechka et al. (2016) assume a tensile model of microseismic sources to show that it may be possible to recover the tensor $\mathbf{M}$ even if only one borehole is available.

Transversely isotropic velocity models have become standard in processing and inversion of seismic data (Tsvankin, 2012). Unconventional fractured reservoirs, however, have to be described by orthorhombic or even lower-symmetry models (Tsvankin and Grechka, 2011). One of the most common reasons for orthorhombic anisotropy is a system of parallel vertical fractures embedded in a VTI (TI with a vertical symmetry axis) background (Tsvankin, 1997, 2012), a model believed to be typical for unconventional shale reservoirs. Grechka and Yasevich (2013, 2014) reconstruct a horizontally layered orthorhombic and even the most general, triclinic velocity model along with the microseismic source locations by joint inversion of the traveltimes of the direct P- and S-waves.

Waveform inversion (WI) has proved effective in building high-resolution velocity models from seismic data for imaging purposes (Tarantola, 1984; Pratt, 1999; Virieux and Operto, 2009). The WI objective function typically represents the $L_2$-norm of the difference between the observed and predicted data in the time or frequency domain. Although existing WI methods are largely focused on velocity model-building, waveform inversion can be applied to event location and estimation of the source-mechanisms from earthquake or microseismic data. Kim et al. (2011) obtain gradient expressions for the parameters of earthquake sources using the adjoint-state method (Lions, 1972; Plessix, 2006). They implement iterative nonlinear inversion to estimate the locations and moment tensors for two earthquakes in Southern California using a known isotropic velocity model. Morency and Mellors (2012) adopt the same approach to obtain the source parameters of a geothermal event.

Here, we extend the 3D WI velocity-analysis methodology of Jarillo Michel and Tsvankin (2019) to estimation of the parameters (location, moment tensor, and origin time) of microseismic sources embedded in orthorhombic media. Our algorithm generally follows the framework developed by Jarillo Michel and Tsvankin (2015) for 2D VTI models. First, we describe the inversion methodology that employs the adjoint-
The derivatives of the WI objective function with respect to model parameters is obtained using the adjoint-state method. Then we test the algorithm on synthetic data recorded in several vertical “boreholes” embedded in layered orthorhombic media and evaluate the stability of the inversion in the presence of errors in the velocity model.

2 WAVEFORM-INVERSION METHODOLOGY

The 3D waveform inversion for the source parameters is organized similarly to the 2D WI technique by Jarillo Michel and Tsvankin (2015). However, here we use a lower symmetry (orthorhombic) model and perform wavefield extrapolation with a pseudospectral approach instead of finite-differences to reduce numerical dispersion (Cheng et al., 2016).

The elastic wave equation for a point source embedded in a heterogeneous anisotropic medium can be written as (Aki and Richards, 2002):

\[ \rho \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial}{\partial x_i} \left( c_{ijkl} \frac{\partial u_j}{\partial x_l} \right) = -M_{ij} \frac{\partial (\delta(x - x^s))}{\partial x_j} S(t), \]

(1)

where \( u_i(x, t) \) is the displacement field, \( t \) is the time, \( c_{ijkl} \) is the stiffness tensor \((i, j, k, l = 1, 2, 3)\), \( \rho \) is the density, \( M \) is the source moment tensor, \( x^s \) is the source location, \( S(t) \) is the source signal, and \( \delta(x - x^s) \) is the spatial \( \delta \)-function; summation over repeated indices is implied.

The current implementation of the algorithm is designed to estimate all six independent moment-tensor elements \((M_{11}, M_{12}, M_{13}, M_{22}, M_{23}, \text{and} M_{33})\), the source coordinates \(x_1^s, x_2^s, \text{and} x_3^s\), and the origin time \(t_0\) assuming that the anisotropic velocity model has been estimated. The velocity field is assumed to have orthorhombic or higher symmetry. Note that the VTI model can be treated as a special case of the more general orthorhombic medium (Tsvankin, 1997, 2012).

The gradient of the objective function with respect to the model parameters is obtained using the adjoint-state method. The derivatives of the WI objective function with respect to the source coordinates, origin time, and moment-tensor can be found as (Kim et al., 2011; Jarillo Michel and Tsvankin, 2015):

\[ \frac{\partial F}{\partial x_i} = \int_0^T \frac{\partial [M : \varepsilon^i(x^s, t)]}{\partial x_i} S(T - t) \, dt, \]

(2)

\[ \frac{\partial F}{\partial M_{ij}} = \int_0^T \varepsilon^i_j(x^s, t) S(T - t) \, dt, \]

(3)

\[ \frac{\partial F}{\partial t_0} = \int_0^T M : \varepsilon^i(x^s, t) \frac{\partial S(T - t)}{\partial t} \, dt, \]

(4)

where \( x^s \) is the trial source location, \( T \) is the recording time, and \( \varepsilon^i \) is the adjoint strain tensor. The derivatives in equations 2 – 4 have to be computed only at the trial source position for each microseismic event; note that the gradient equations involve no assumptions about the structure or anisotropic symmetry system of the medium. Here, in contrast to the 2D algorithm of Jarillo Michel and Tsvankin (2015), we carry out gradient calculation for 3D orthorhombic models.

A nondimensionalization approach described by Kim et al. (2011) and Jarillo Michel and Tsvankin (2015) is employed for appropriate scaling of the inversion gradient. This scaling ensures that the derivatives of \( F \) with respect to each model parameter have the same order of magnitude. The moment tensor \( M \) for dislocation-type sources can be represented as (Aki and Richards, 2002; Vavryčuk, 2005):

\[ M_{ij} = \frac{S}{2} \bar{u} c_{ijkl} \nu_k n_l, \]

(5)

where \( S \) is the fault area, \( \bar{u} \) is the time-dependent magnitude of the slip (displacement discontinuity), \( \nu_k \) is the slip direction, and \( n_l \) is the fault normal. If the fault plane coincides with the \([x_1, x_3]\) coordinate plane and the medium is either VTI or orthorhombic, the six independent elements \( M_{ij} \) in equation 5 are given by:

\[ M_{11} = -\frac{S}{2} \bar{u} \sin 2\theta (c_{13} - c_{11}), \]

(6)

\[ M_{13} = \frac{S}{2} \bar{u} \cos 2\theta c_{055}, \]

(7)

\[ M_{33} = -\frac{S}{2} \bar{u} \sin 2\theta (c_{33} - c_{13}), \]

(8)

\[ M_{22} = -\frac{S}{2} \bar{u} \sin 2\theta (c_{23} - c_{12}), \]

(9)

\[ M_{12} = 0, \]

(10)

\[ M_{23} = 0, \]

(11)

where \( c_{11}, c_{13}, c_{33}, c_{055}, c_{23}, \) and \( c_{12} \) are the stiffness coefficients in the two-index Voigt notation and \( \theta \) is the dip angle of the displacement discontinuity measured from the horizontal plane. It is assumed in equations 6 – 11 that the symmetry planes of orthorhombic media coincide with the coordinate planes.

2.1 Synthetic examples

The developed algorithm is tested on microseismic sources embedded in a stack of horizontal orthorhombic layers. All events represent dislocation-type sources with the fault plane that coincides with the \([x_1, x_3]\) coordinate plane.

As an initial test of the WI algorithm, we carry out inversion for the coordinates \( x^s \) and moment tensor \( M \) simultaneously for the orthorhombic model from Table 1 (Figure 1). Multicomponent data are recorded in three vertical boreholes at each grid point. We choose this geometry to confirm the feasibility of estimating both the source coordinates and moment tensor from microseismic data with wide azimuthal coverage. The initial source locations are obtained by increasing all three coordinates by 10 m.

To compute the initial moment tensor, the angle \( \theta \) is increased by \(15^\circ\) (equations 6 - 11). After 20 iterations of waveform inversion, the coordinates \( x^s \) (Figure 2) and tensor \( M \) (Figure 3) are recovered with sufficient accuracy.

In the next test, we use a more realistic acquisition ge-
Waveform inversion in 3D orthorhombic media

<table>
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<th>$V_{S0}$ (m/s)</th>
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<th>$\delta^2$</th>
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Table 1. Interval parameters of a three-layer orthorhombic model.

Figure 1. Microseismic event (blue dot) and receivers (black dots) placed in three boreholes for the model in Table 1: (a) 3D plot; and (b) map view.

Figure 2. Change of the source coordinates (a) $x_s^1$, (b) $x_s^2$, and (c) $x_s^3$ with iterations for the model in Table 1. The actual values are marked by the black lines.

ometry for microseismic surveys, where the data are recorded in just two boreholes (Figure 4). The parameters for this five-layer orthorhombic model (Table 2 and Figure 5) are taken from the inversion results of Grechka and Yaskevich (2014) for Bakken field. The source parameters are perturbed in the same way as in the previous example to obtain the initial model. The accurate inversion results (Figure 6) confirm that the multicomponent wavefield recorded in two vertical boreholes provides sufficient information to recover the event coordinates and all six independent elements $M_{ij}$ of a microseismic event.

It is important to study the performance of the inversion algorithm in the presence of realistic errors in the velocity model. Hence, we repeat the previous test using the same initial model but with the velocities $V_{P0}$ and $V_{S0}$ increased by 5%. Note that the orthorhombic medium is parameterized using Tsvankin’s (1997; 2012) notation, so a change in $V_{P0}$ and $V_{S0}$ produces the corresponding change in the horizontal velocities. The error in the vertical velocities produces some distortions in the inverted parameters but the event coordinates are sufficiently close to the actual values (Figure 7). However, if the velocity errors exceed 7–8%, WI fails to properly refine the source locations $x^s$ and the tensor $M$ (Figure 8). Therefore, although the algorithm can tolerate moderate errors in $V_{P0}$, $V_{S0}$, and other medium parameters, WI-based event lo-
<table>
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<tr>
<th>Layer</th>
<th>Thickness (m)</th>
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<td>0.0</td>
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Table 2. Interval parameters of a five-layer orthorhombic model (after Grechka and Yaskevich, 2014).

Finally, the algorithm is used to estimate the event origin time $t_0$ along with the elements of the moment tensor for the same model (Table 2 and Figure 5). To define the initial model, the angle $\theta$ is increased by $15^\circ$ and the event origin time is increased by 0.004 s. The event coordinates are fixed at their actual values. The inversion successfully resolves both the origin time and the tensor $\mathbf{M}$ (Figure 9). However, simultaneous estimation of the origin time and event coordinates is hampered by parameter trade-offs and requires further study.

3 CONCLUSIONS

We presented a 3D elastic waveform-inversion algorithm designed to estimate the source coordinates, moment-tensor elements, and the origin time of microseismic events. Multicomponent wavefields are supposed to be recorded in at least two...
vertical boreholes embedded in orthorhombic or VTI media. In the current implementation, the anisotropic velocity model is not updated by WI.

The methodology was applied to synthetic data recorded in stratified orthorhombic models. To validate the algorithm, the data were simulated in three boreholes providing wide azimuthal coverage. Then we employed a more realistic geometry for microseismic surveys by inverting data recorded in two boreholes that cross five orthorhombic layers. In both cases, the algorithm recovered the coordinates $x'$ and all six independent moment-tensor elements with sufficient accuracy after less than 20 iterations. The inversion was also shown to be capable of recovering the origin time $t_0$ simultaneously with the source moment tensor.

It is essential to assess the influence of velocity errors on the inversion results. If the vertical velocities $V_{P0}$ and $V_{S0}$ are perturbed by 5%, the estimated source parameters remain sufficiently close to their actual values. However, the increase in the velocity errors to $7-8\%$ causes substantial distortions in the event locations and, especially, in the components of the moment tensor. Ongoing work includes development of a 3D WI methodology designed to update both the source parameters and anisotropic velocity model in iterative fashion.

4 ACKNOWLEDGMENTS

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REFERENCES


Figure 6. Change of the source coordinates (a) $x_1^s$, (b) $x_2^s$, (c) $x_3^s$ and of (d) the moment-tensor element $M_{13}$ with iterations for the model in Table 2.

Figure 7. Change of the source coordinates (a) $x_1^s$ and (b) $x_3^s$ and of (c) the moment-tensor element $M_{13}$ with iterations for the model in Table 2. The vertical velocities $V_{P0}$ and $V_{S0}$ are increased by 5% from the actual values.


Waveform inversion in 3D orthorhombic media

Figure 8. Change of the source coordinates (a) $x_1^s$ and (b) $x_3^s$ and of (c) the moment-tensor element $M_{13}$ with iterations for the model in Table 2. The velocities $V_P^0$ and $V_S^0$ are increased by 7.5% from the actual values.

Figure 9. Change of (a) the origin time $t_0$ and of (b) the moment-tensor element $M_{13}$ with iterations for the model in Table 2. The source coordinates are fixed at their actual values.