Inversion of NMO ellipses for tilted orthorhombic media

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Methodology

• modes: PP, $S_1S_1$, $S_2S_2$ (could use PP+PS = SS)

• input: NMO ellipses, zero-offset times, reflection time slopes

• model: homogeneous layers with plane/curved boundaries

• inversion for: interval medium parameters, interfaces
NMO ellipse

\[ V_{nmo}^{-2} (\alpha) = W_{11} \cos^2 \alpha + 2W_{12} \sin \alpha \cos \alpha + W_{22} \sin^2 \alpha \]

- single layer: \( W_{ij} = f(p_1, p_2, q) \)
- layered model: Dix-type averaging
- tracing only zero-offset ray (correction for curvature)
Workflow

1. Trial model (interval parameters) → $p_1, p_2, \tau_0$
2. Zero-offset rays → Reflectors
3. Effective NMO ellipses $W_{ij}$
4. Misfit in $W_{ij}$ and depths → Update
Objective function

\[ F(m) = \| 1 - \frac{W_{\text{cal}}(m)}{W_{\text{obs}}} \| + \alpha \| 1 - \frac{z_Q(m)}{z_{Q'}(m)} \| \]

\[(Q, Q' = P, S_1, S_2)\]
Orthorhombic medium

Parameters: $V_{P0}$, $V_{S0}$, $\varepsilon^{(1,2)}$, $\delta^{(1,2,3)}$, $\gamma^{(1,2)}$ (Tsvankin, 1997)
Tilted orthorhombic (TOR) medium

Parameters: $V_{P0}, V_{S0}, \varepsilon^{(1,2)}, \delta^{(1,2,3)}, \gamma^{(1,2)}, \beta_1, \beta_2, \beta_3$
Model vector for layered TOR medium

- interval TOR parameters: 
  \( V_{P0}, V_{S0}, \varepsilon^{(1,2)}, \delta^{(1,2,3)}, \gamma^{(1,2)}, \beta_1, \beta_2, \beta_3 \)

- plane dipping interfaces: \( D, \phi, \psi \)

- curved interfaces: \( z \rightarrow \) polynomial function of \( x \) and \( y \)
Data vector

\[ d(Q) = \{ \tau_{0,Q}, \rho_{1,Q}, \rho_{2,Q}, W_{11,Q}, W_{12,Q}, W_{22,Q} \} \]

\[ Q = P, S_1, S_2 \]
Layered model with planar interfaces
Constraints

- reflector coincides with symmetry plane \([x_1, x_2]\): \(\beta_2 = \psi, \beta_3 = \varphi\)
  - \(\delta^{(3)}\) has no influence on NMO ellipses

- vertical velocity:

\[
F(m) = \|1 - \frac{W_{\text{cal}}(m)}{W_{\text{obs}}}\| + \alpha \|1 - \frac{z_Q(m)}{z_Q'(m)}\| + \beta \|1 - \frac{V_{\text{cal \ vert.}, Q}(m)}{V_{\text{obs \ vert.}, Q}}\|
\]
Model with curved interfaces
Reflector reconstruction

• zero-offset reflection point:
  - location
  - reflector normal

• best-fit polynomial $z = f(x, y)$

• depth term in objective function
Challenges

• interface reconstruction

• need to estimate symmetry-plane orientation

• additional model parameters
Advantages

• range of reflector dips and azimuths

• more independent measurements

• additional information from multiple CMPs
TOR layer
Inversion results

**Noise:** 2% for $\tau_0$, 5% for $\rho_{1,2}$ and $V_{nmo}$
Inversion for planar interface

Noise: 2% for $\tau_0$, 5% for $p_{1,2}$ and $V_{nmo}$
Layered TOR model
Layer 1

Noise: 1% for $\tau_0$, 2% for $p_{1,2}$ and $V_{nmo}$

![Graphs showing velocity and angle](image-url)
Layer 2

Noise: 1% for $\tau_0$, 2% for $p_{1,2}$ and $V_{nmo}$

![Graph showing velocity and angle](image)
Layer 3

Noise: 1% for $\tau_0$, 2% for $\rho_{1,2}$ and $V_{nmo}$
Objective function

\[
F(m) = \| 1 - \frac{W^\text{cal}(m)}{W^\text{obs}} \| + \alpha \| 1 - \frac{z_Q(m)}{z_Q'(m)} \| + \beta \| 1 - \frac{V^\text{cal}_{\text{vert.}, Q}(m)}{V^\text{obs}_{\text{vert.}, Q}} \|
\]

\((Q, Q' = P, S_1, S_2)\)
Layer 1 (with constraints)

Noise: 2% for $\tau_0$, 5% for $p_{1,2}$ and $V_{nmo}$

![Graph showing velocity and angle](image)

![Graph showing second and third layers](image)
Layer 2 (with constraints)

Noise: 2% for $\tau_0$, 5% for $p_{1,2}$ and $V_{nmo}$
Layer 3 (with constraints)

Noise: 2% for $\tau_0$, 5% for $p_{1,2}$ and $V_{nmo}$
Summary

• inversion for TOR media with curved interfaces

• constraints on symmetry-plane orientation not critical

• accuracy higher than for plane interfaces

• possible to estimate $\delta^{(3)}$
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