GEOPHYSICS, VOL. 56, NO. 2 (FEBRUARY 1991); P. 182–189, 6 FIGS.

# Born theory of wave-equation dip moveout

Christopher L. Liner*


## ABSTRACT

Wave-equation dip moveout (DMO) addresses the DMO amplitude problem of finding an algorithm which faithfully preserves angular reflectivity while processing data to zero offset.

Only three fundamentally different theoretical approaches to the DMO amplitude problem have been proposed: (1) mathematical decomposition of a prestack migration operator; (2) intuitively accounting for specific amplitude factors; and (3) cascading operators for prestack migration (or inversion) and zero-offset forward modeling.

Pursuing the cascaded operator method, wave-equation DMO for shot profiles has been developed. In this approach, a prestack common-shot inversion operator is combined with a zero-offset modeling operator. Both integral operators are theoretically based on theBorn asymptotic solution to the point-source, scalar wave equation. This total process, termed Born DMO, simultaneously accomplishes geometric spreading corrections, NMO, and DMO in an amplitude-preserving manner. The theory is for constant velocity and density, but variable velocity can be approximately incorporated.

Common-shot Born DMO can be analytically verified by using Kirchhoff scattering data for a horizontal plane. In this analytic test, Born DMO yields the correct zero-offset reflector with amplitude proportional to the angular reflection coefficient. Numerical tests of common-shot Born DMO on synthetic data suggest that angular reflectivity is successfully preserved. In those situations where amplitude preservation is important, Born DMO is an alternative to conventional NMO + DMO processing.


## INTRODUCTION

Many forms of kinematic dip moveout (DMO) have been suggested for common offset (Deregowski and Rocca, 1981; Hale, 1984; Berg, 1985; Notfors and Godfrey, 1987; Bale and Jakubowicz, 1987; Forel and Gardner, 1987; Liner and Bleistein, 1988), and Biondi and Ronen (1987) have extended this work to common-shot DMO. For all their differences, these forms of DMO share a common origin in that they are ultimately based on the dip-corrected NMO equation. Kinematic DMO, by definition, addresses only the issue of traveltimes. Here, I am concerned with the question of DMO amplitude.

DMO amplitude should be of concern because, in practice, data amplitude *is* interpreted after DMO. This interpretation can be prestack amplitude-versus-offset (AVO) analysis or even poststack, postmigration as in bright spot work. If the entire processing stream, including DMO, is not carefully designed with amplitude preservation in mind, then amplitude interpretation becomes suspect.

Every DMO process, kinematic or otherwise, does something to amplitude. Some writers explicitly address this issue, while others, more interested in the imaging aspects, do not. There has been much recent work on DMO amplitude (Jorden, 1987; Beasley and Mobley, 1988; Black and Egan, 1988; Gardner and Forel, 1988; Liner, 1989). Although there may seem to be as many approaches as authors, all of this work actually follows from three fundamentally different approaches to the DMO amplitude problem.

The first approach is *operator splitting*. Beginning with Yilmaz and Claerbout (1979), there have been several careful derivations of DMO from prestack full migration (PSFM). The idea is to manipulate the PSFM operator so that something between NMO and poststack migration is isolated, and identify this as DMO. Many PSFM operators are known, but the one universally used in this approach to

Presented in part at the 59th Annual International Meeting, Society of Exploration Geophysicists. Manuscript received by the Editor August 29, 1989; revised manuscript received August 9, 1990.
*Formerly Golden Geophysical, 1748 Cole Blvd., Ste. 265, Golden, CO 80401; presently Department of Geosciences, University of Tulsa, 600 South College, Tulsa, OK 74104-3189.

DMO is based on the double square root (DSR) equation (see Claerbout, 1984). A particularly elegant exposition is given by Hale (1983). Recently, Black and Egan (1988) revised this method.

DMO amplitudes derived by operator splitting will be as meaningful as those of the original migration operator. Unfortunately, the DSR method of prestack migration is itself not rigorous with respect to amplitude. Specifically, there is no theoretical relationship between the migration result and subsurface reflection coefficients. Also, there is no known method of splitting amplitude-rigorous prestack migration operators such as those discussed by Beylkin (1985) and Bleistein (1987). However, recent work by Jakubowicz and Miller (1989) may lead to progress in this area.

The second approach to the DMO amplitude problem may be termed *intuitive*. This method accounts for specific amplitude factors in a rigorous, but isolated, manner. Deregowski and Rocca (1981), while deriving the DMO impulse response from kinematics, propose an amplitude which is empirically related to the impulse response curvature. They mention that a more rigorous amplitude treatment is possible using ray theory. Later, Deregowski (1985) used ray theory and some a priori conditions, such as operator taper, to find an amplitude term. This analysis presupposes that various processes such as geometric spreading correction have already been performed on the data. A combination of theoretical derivation and a priori assumptions also appears in the amplitude work of Beasley and Mobley (1988). Finally, Gardner and Forel (1988) argue for an amplitude term based on linearized scattering coefficients, operator curvature, spreading, and midpoint sampling.

The third approach to DMO amplitude is *cascaded operators*. Deregowski and Rocca (1981) considered a thought experiment where prestack migration is followed by zero-offset forward modeling. The impulse response for this cascaded process is an operator which maps prestack data to zero-offset: the DMO impulse response. Their analysis was aimed at finding the geometry of the DMO operator. That is, they were concerned only with the kinematics of the cascaded process. However, by using amplitude-rigorous migration and modeling operators, this cascaded approach can take on new meaning. Such a class of migration operators has recently become available (Bleistein, 1987; Beylkin, 1985). To distinguish these amplitude-preserving operators from those of classical migration, they have generally come to be termed "inversion operators." The name derives from their origin in mathematical inverse theory.

Pursuing the cascaded operator approach, an inversion theory wave-equation DMO has been developed (Jorden, 1987; Jorden et al., 1987; Liner, 1989). In this method, a prestack inversion operator is combined with a zero-offset modeling operator. Both integral operators are theoretically based on the Born asymptotic solution to the point-source scalar wave equation. This approach combines the elegance of Deregowski and Rocca's original idea with the wave equation and the amplitude-conscious rigor of mathematical inverse theory.

The initial aim of this report is to create a "first-order" theory of wave-equation DMO as a possible alternative to kinematic DMO. In keeping with this goal, the theory assumes constant density and constant background velocity, but variable background velocity can be approximately incorporated.

## THE WAVE-EQUATION BASIS

The developments below are based on the point-source scalar wave equation:

$$\left[\nabla^2 - \left\{\frac{1+\alpha(\mathbf{x})}{v^2}\right\}\frac{\partial^2}{\partial t^2}\right]P(t, \mathbf{x}) = -\delta(t)\delta(\mathbf{x}), \tag{1}$$

where $P(t, \mathbf{x})$ is the wave field, and $\mathbf{x}$ is a position three-vector. This equation assumes constant density and background velocity. The velocity term is an expansion of the true velocity field $V(\mathbf{x})$ with respect to the constant background velocity $v$,

$$\frac{1}{V^2(\mathbf{x})} \approx \frac{1+\alpha(\mathbf{x})}{v^2}; \qquad |\alpha(\mathbf{x})| \ll 1. \tag{2}$$

The quantity $\alpha(\mathbf{x})$ is termed the velocity perturbation.

The wave equation (1) can be viewed in two fundamentally different ways. If the velocity terms $v$ and $\alpha(\mathbf{x})$ are known, then solving equation (1) for the data $P(t, \mathbf{x})$ will be a forward modeling process. Conversely, if the data and background velocity are known, then solving equation (2) for $\alpha(\mathbf{x})$ is an inverse problem.

## BORN 2.5-D MODELING AND INVERSION

### Zero-offset modeling

The 3-D wave field $P(t, \mathbf{x})$ in equation (1) exists at every point in three-dimensional space. Consider a backscatter experiment where the source and receiver are coincident. Since the background velocity $v$ is constant, the only reflections observed will be due to the perturbation $\alpha(\mathbf{x})$. Within the context of zero-offset modeling $\alpha(\mathbf{x})$ is proportional to the normal-incidence reflection coefficient multiplied by a Heaviside step function. The step function gives the location of the velocity discontinuity and, from $\alpha(\mathbf{x})$, its magnitude is known. This interpretation of alpha holds only for zero-offset modeling, and, for this special case, the perturbation will be termed $\alpha_0(\mathbf{x})$.

If the background $v$ and $\alpha_0(\mathbf{x})$ in equation (1) are known, then an approximate 3-D solution for the data, $P(t, \mathbf{x})$, can be derived from Born theory (Cohen et al., 1986). The interest here is in processing a 2-D line of data, while still allowing for 3-D spreading effects. This is termed 2.5-D theory, and is discussed at length by Bleistein (1986). Modeling and inversion formulas for 2.5-D are derived from their 3-D counterparts by asymptotically eliminating the out-of-plane variables.

Let the recorded line of zero-offset data be represented by $P_0(\omega_0, x_0)$, where $\omega_0$ is the zero-offset frequency and $x_0$ is the zero-offset source-receiver location. The 2.5-D Born modeling formula is

$$P_0(\omega_0, x_0) \sim (-i\omega_0)^{3/2} \int\int dx\, dz\, A_0 e^{i\omega_0\phi_0}\alpha_0(x, z), \tag{3}$$

where $(x, z)$ is a two vector spanning the subsurface. Constant density and velocity are assumed. The amplitude and phase terms are given by

$$A_0 = \frac{1}{16\pi^{3/2}(vr_0)^{3/2}}; \qquad \phi_0 = \frac{2r_0}{v}, \tag{4}$$

where the zero-offset distance is

$$r_0 = \sqrt{(x - x_0)^2 + z^2}, \tag{5}$$

as shown geometrically in Figure 1.

**Prestack inversion**

Starting with the 2.5-D Born forward modeling integral (2), one can derive a 2.5-D inversion formula which will asymptotically recover $\alpha_0(x, z)$ when $P_0(\omega_0, x_0)$ and $v$ are known. This would be a zero-offset inversion formula. The inversion formula needed here, for offset data, is given for 2.5-D and general velocity variation by Bleistein et al. (1987). Let $P_i(\omega_i, x_i)$ denote the recorded offset seismic data where $\omega_i$ is the frequency corresponding to reflection time and $x_i$ is the spatial coordinate parameterizing the input data. In this notation the 2.5-D prestack inversion formula is

$$\alpha_a(x, z) \sim \int\int dx_i \, d\omega_i \, A_i e^{-i\omega_i\phi_i} \frac{P_i(\omega_i, x_i)}{\sqrt{i\omega_i}}, \tag{6}$$

where $\alpha_a(x, z)$ contains the angular reflectivity of the input. Again, constant density and velocity are assumed. The amplitude and phase terms are given by

$$A_i = \frac{2^{3/2}v^{5/2}(r_s r_g)^{3/2}H}{\pi^{1/2}(r_s + r_g)^{1/2}}; \qquad \phi_i = \frac{r_s + r_g}{v}. \tag{7}$$

The input data acquisition geometry (common-shot, common-offset) is specified by the Beylkin determinant (Beylkin, 1985; Bleistein, 1987), $H = H(x, z, x_i)$.

The terms $r_s$ and $r_g$ are defined in terms of generic source and receiver locations $x_s$ and $x_g$ which are in turn functions of the general coordinate $x_i$. From the geometry of Figure 1, it follows that

$$r_s = \sqrt{(x - x_s)^2 + z^2}; \qquad r_g = \sqrt{(x - x_g)^2 + z^2} \tag{8}$$

where $x_s = x_s(x_i)$ and $x_g = x_g(x_i)$.

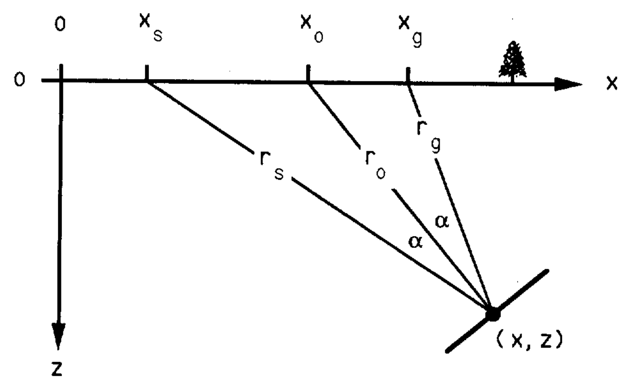


FIG. 1. Geometry of 2.5-D Born DMO.

## 2.5-D BORN DIP MOVEOUT

At this point 2.5-D Born integral operators have been given for zero-offset forward modeling relation (3), and prestack inversion relation (6). Following Jorden (1987), a mapping to zero offset is proposed that is represented by the cascaded process

Born DMO ≡ Zero-offset modeling [Inversion [Data]].

Jorden's (1987) work involved cascading full 3-D operators, which were then analytically reduced within the DMO derivation to 2.5-D. The 3-D operator approach is not pursued here, but is interesting because it opens the possibility of quantifying the out-of-plane component of 3-D DMO (Jorden, 1987).

The cascaded process defining Born DMO will involve substituting $\alpha_a(x, z)$ for $\alpha_0(x, z)$ in equation (3). This means that a process will be designed to pass angular reflection coefficients from the input data through to the zero-offset result. The angular information is then available for further analysis.

Substituting the inversion (6) for $\alpha_0(x, z)$ in equation (3) gives

$$P_0(\omega_0, x_0) \sim (-i\omega_0)^{3/2} \int\int\int\int dx \, dz \, dx_i \, d\omega_i \times A_i A_0 e^{i(\omega_0\phi_0 - \omega_i\phi_i)} \frac{P_i(\omega_i, x_i)}{\sqrt{i\omega_i}}, \tag{9}$$

where all symbols are defined above.

The formula (9) as given is not computationally viable. The subsurface variables $(x, z)$ are not present in either the input data $P_i(\omega_i, x_i)$ or the output $P_0(\omega_0, x_0)$. These were fundamental quantities for the individual modeling and inversion formulas, but for the cascaded operation (9) they are dummy variables. The $x$ and $z$ integrals must be evaluated analytically. This process is outlined in the Appendix for the common-shot input geometry.

Let $P_i(t_i, x_g)$ represent a shot profile which has not been corrected for NMO or geometric spreading. The recorded traveltime is $t_i$ and $x_g$ is the geophone coordinate. The formula for 2.5-D common-shot Born DMO is

$$P_0(t_0, x_0) \sim \int\int dx_g \, dt_i \, A_{\text{shot}} \, \delta(t_i - \phi_i)\partial_{t_i}^{1/2} P_i(t_i, x_g), \tag{10}$$

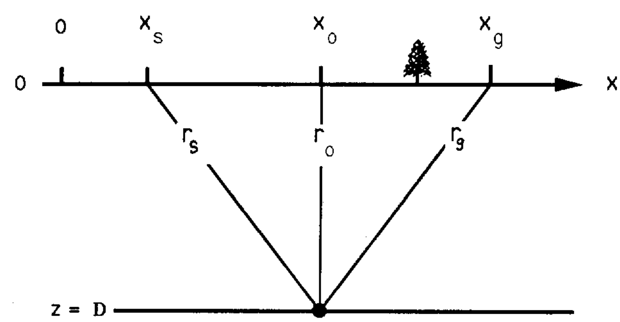


FIG. 2. Model for scattering from a general horizontal plane.

where the fractional derivative operator $\partial_{t_i}^{1/2}$ is defined as frequency-domain multiplication by $\sqrt{i\omega_i}$. The amplitude and phase terms for expression (10) are given by

$$A_{\text{shot}} = \frac{r_0 f^{3/2}}{4(2\pi v r_s)^{1/2} |\beta_g|^{5/2}}, \tag{11}$$

$$\phi_i = \frac{r_s + r_g}{v} = \frac{f}{v}\left\{1 - \frac{r_0^2}{\beta_s \beta_g}\right\}^{1/2}, \tag{12}$$

where $v$ is the constant background velocity and the zero-offset time $t_0$ is

$$t_0 = \frac{2}{v}\{\beta_s \beta_g [1 - (v t_i / f)^2]\}^{1/2}, \tag{13}$$

and where (see Figure 1)

$$r_0 = \frac{v t_0}{2};$$

$$r_s = |\beta_s| v t_i / f;$$

$$r_g = |\beta_g| v t_i / f, \tag{14}$$

$$\beta_s \equiv x_s - x_0;$$

$$\beta_g \equiv x_g - x_0, \tag{15}$$

and the full offset $f$ is

$$f \equiv |\beta_s| + |\beta_g| = |x_s - x_g|. \tag{16}$$

Born DMO simultaneously does spreading corrections, NMO, and DMO. The theoretical development assumes constant density and background velocity. While a constant-velocity DMO might be acceptable, constant-velocity NMO is not. To handle velocity variations approximately, the constant-velocity Born DMO theory can be used with a variable rms velocity field. In this way the velocity field is being treated *consistently* for spreading corrections, NMO, and DMO. By treating the velocity field in an rms fashion, good results are expected until the true raypaths are significantly curved. In general, using the rms velocity field will yield results accurate to leading order in offset squared.

Born DMO is susceptible to the maladies of other $(t, x)$-domain DMO algorithms, including aliasing of the operator at steep dips and operator degeneracy at very near offsets. Some of these issues are discussed in a general setting by Hale (1988).

## ANALYTIC VERIFICATION

Taken literally, the common-shot Born DMO formula (10) implies a weighted integration through the input shot profile to generate each output point. The dominant, and desired, contribution from this integral will come from a stationary point corresponding to the specular reflection geometry of Figure 1. If the range of integration is too narrow (i.e., too

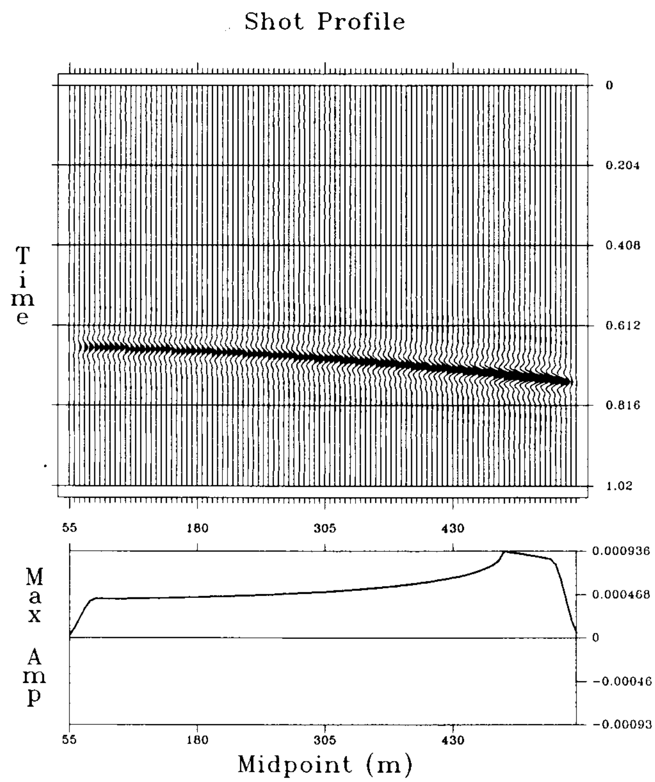


FIG. 3. Ray theory shot profile for a horizontal plane. The amplitude variation with offset is a combination of geometric spreading and angular reflectivity. The model is described in the text.

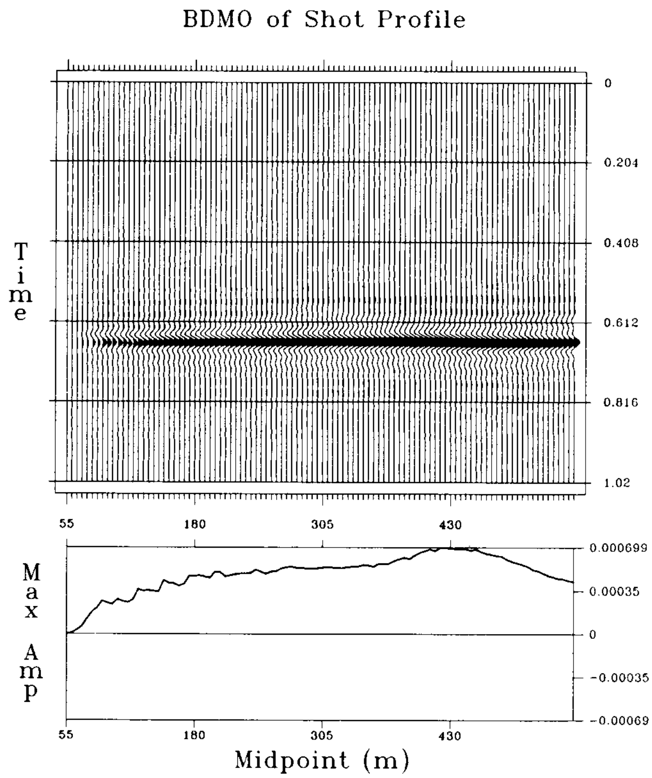


FIG. 4. Common-shot Born DMO of data in Figure 3. amplitude versus offset (AVO) behavior, due to angular reflection coefficient $R(\alpha)$, has been preserved.

few input traces), then unwanted end-point contributions will dominate the output. Output amplitudes will be correct only when well away from end-point effects. If a single spike of amplitude were processed, the output would be nothing but end-point effects. It follows that the impulse response, which is the canonical experiment for kinematic DMO, is inappropriate for evaluating the fidelity of DMO amplitude. The canonical problem for DMO amplitude, as with migration amplitude, is the plane reflector.

Born DMO (10) claims to directly map shot profile data to zero offset; that is, Born DMO accomplishes geometric spreading (GS) correction, NMO, DMO, and inverse GS. Inverse GS introduces the correct zero-offset spreading factor into the output. Furthermore, any angular reflectivity in the shot profile will pass into the zero-offset output. These claims can be verified by applying Born DMO to analytic data for a horizontal plane.

Figure 2 shows geometry for scattering from a general horizontal plane. The source $x_s$ is fixed and there are assumed to be several receivers $x_g$. The depth to the reflector is $D$, and the velocity down to that level is a constant $v$. To leading order, the data received at the geophones will have the form (Bleistein, 1984)

$$P_i(\omega_i, x_g) \sim \frac{R(\alpha)e^{2i\omega_i r/v}}{8\pi r}; \qquad r = \frac{1}{2}\sqrt{x_g^2 + 4D^2}, \tag{17}$$

where $\omega_i$ is the frequency variable for the input reflection time $t_i$. The expression (17) is termed Kirchhoff data for the horizontal plane. The function $R(\alpha)$ is the full nonlinear angular reflection coefficient, and $\alpha$ is the angle of incidence.

Analytically processing relation (17) with the common-shot Born DMO formula (10) yields the desired result

$$P_0(t_0, x_0) \sim \frac{R(\alpha)\delta(t_0 - 2r_0/v)}{8\pi r_0}, \tag{18}$$

where $r_0 = D$. The proof of expression (18) is lengthy and may be found in Liner (1989). From expression (18) it is seen that shot-profile Born DMO applied to horizontal reflector data correctly locates the reflector at the zero-offset location, $t_0 = 2r_0/v$, with the correct zero-offset spreading factor $8\pi r_0$ while preserving the full angular reflection coefficient $R(\alpha)$.

For zero-offset reflection from a horizontal plane, the spreading term $8\pi r_0$ is constant. For a dipping plane or curved surface, it will not be constant and may mask variations in $R(\alpha)$. From expression (18) it follows that input data could be processed directly for $R(\alpha)$ if the amplitude term (11) is multiplied by $8\pi r_0$.

## SYNTHETIC EXAMPLES

It has been shown that the common-shot Born DMO algorithm is theoretically correct. However, computer implementation must be approached carefully if the goal of amplitude-preservation is to be realized in practice. In particular, the effects of operator aliasing and interpolation

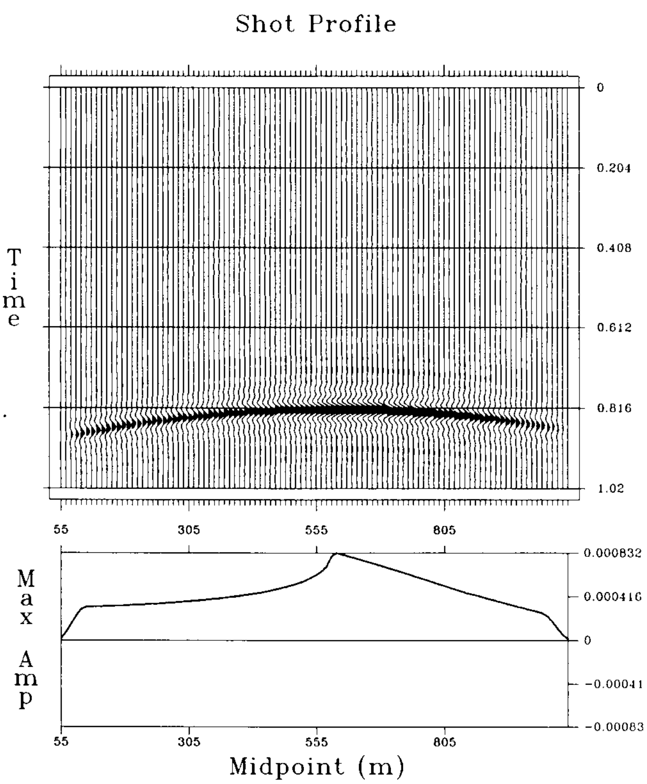


FIG. 5. Ray theory shot profile data for a dipping plane. The model is described in the text.

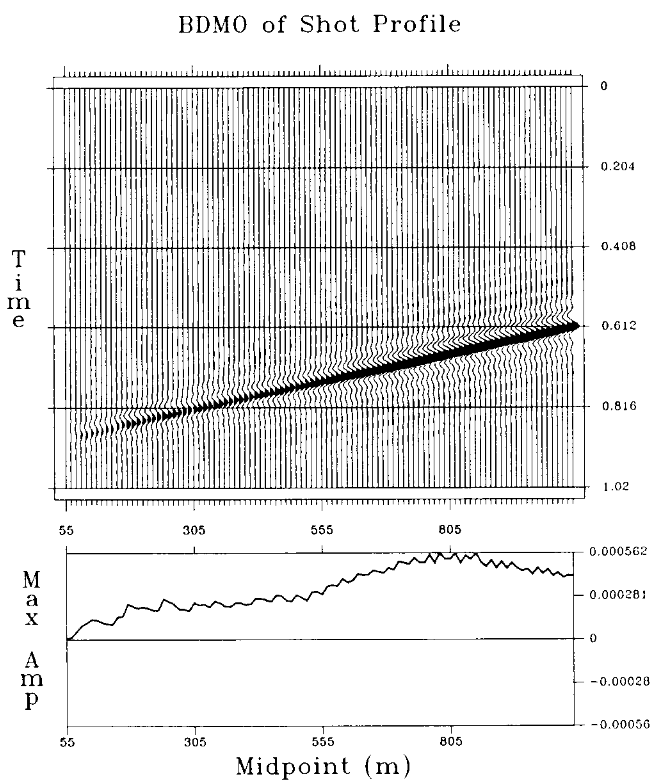


FIG. 6. Common-shot Born DMO of the dipping plane data in Figure 5. The output amplitude contains $R(\alpha)$ and zero-offset spreading.

can be severe. The computer program developed in this study treated operator aliasing by simply truncating the operator when aliasing was imminent. This is a fast, but crude, technique which accounts for much of the "jitter" seen in the synthetics below. For interpolation, an eight-point sinc algorithm was used. To benchmark amplitude preservation of the computer program, a synthetic test was performed (not shown) in which the correct amplitude result was known to be 1.000 for all output traces. Geometrically, the model consisted of a horizontal plane at 1000 m depth. Velocity above the interface was 3000 m/s. The near offset was 100 m, and there were 100 receivers spaced 10 m apart for a far offset of 1100 m. The peak amplitude values, away from end-point effects, were within ±10 percent of the correct value. This result is consistent with the theory, allowing for numerical errors. I assume this order of accuracy is also obtained in the synthetic tests shown below.

The models consisted of the geometry described above. The velocity above the interface is 3000 m/s and velocity below it is 7000 m/s. Forward model data were created using Docherty's (1987) Cshot computer program, a ray-tracing algorithm which incorporates spreading and the geometrical optics reflection coefficient $R(\alpha)$.

The synthetic shot-profile data for the horizontal plane are shown in Figure 3. There are two competing amplitude effects on this data: an increase of $R(\alpha)$ with offset, and a decrease of amplitude with offset due to geometric spreading. Figure 4 is the result of applying Born DMO to the shot-profile data. The output reflector is properly located at the zero-offset time $2r_0/v = .667$ s, and the features of $R(\alpha)$ have been preserved. Since the reflector is horizontal, the zero-offset Born DMO data have a constant spreading factor.

Next consider a single dipping plane. The velocity increases from 3000 m/s to 7000 m/s across the interface. The near offset is 200 m, the receiver interval is 20 m, and there are 100 receivers. At its deepest point, the reflector is 1500 m deep and the dip is 24 degrees. Shot-profile data for the dipping plane are shown in Figure 5. Born DMO of this data to zero-offset gives Figure 6. The amplitude curve indicates variations in $R(\alpha)$ and geometrical spreading.

The tests given here by no means constitute a comprehensive evaluation of Born DMO. They are meant only to show that the process is computationally feasible and may be an alternative to conventional NMO + DMO processing when amplitudes are of concern.

## CONCLUSIONS

A Born theory of dip moveout (Born DMO) has been developed which preserves amplitude while simultaneously performing spreading corrections, NMO, and DMO in a wave-equation sense. The method assumes constant density and background velocity, and is based on the general theory of inversion due to Beylkin (1985) and Bleistein et al. (1987), and follows the work of Jorden (1987).

A formula for common-shot Born DMO was given. The algorithm was analytically applied to Kirchhoff scattering data for a general horizontal plane data. It was shown to preserve the full nonreflection coefficient and introduce the correct zero-offset spreading factor.

Velocity variations can be approximately accounted for while maintaining the speed of the Born DMO algorithm by applying the constant-velocity Born DMO theory, but actually allowing a variable rms velocity field. In general, by using the rms velocity field, the process should be accurate to leading order in offset squared.

The importance of Born DMO to amplitude-versus-offset analysis was illustrated by processing ray theoretical synthetic data for a horizontal and dipping plane. The Born DMO algorithm successfully passed the angular reflection coefficient into the zero-offset output data. For those cases where post-DMO amplitude is an issue, Born DMO gives an alternative to conventional NMO + DMO.

## ACKNOWLEDGMENTS

This work was done while the author was at the Center for Wave Phenomena, Colorado School of Mines. It would not have been possible without the encouragement and insight of Norman Bleistein, Jack Cohen, Shuki Ronen, and Tom Jorden. Intellectual support is acknowledged from Sebastian Geoltrain, and John Stockwell contributed an essential calculation toward the analytic verification of Born DMO. The management of Golden Geophysical is acknowledged for allowing time necessary to complete the writing of this paper.

The author gratefully acknowledges the support of the Consortium Project on Seismic Inverse Methods for Complex Structures at the Center for Wave Phenomena, Colorado School of Mines. Consortium members are: Amerada Hess Corporation; Amoco Production Company; ARCO Oil and Gas Company; Chevron Oil Field Research Company, Conoco, Inc.; GECO; Marathon Oil Company; Mobil Research and Development Corp.; Oryx Energy Company; Phillips Petroleum Company; Shell Development Company; Statoil; Texaco USA; Union Oil Company of California; and Western Geophysical.

The seismic data processing was facilitated by use of the SU (Seismic UNIX) processing line. This software, originated by the Stanford Exploration Project, has been further developed at the Center for Wave Phenomena.

Finally, I acknowledge my wife Janet, whose ability to be supportive of my career at the expense of her own is extraordinary.

## REFERENCES

Bale, R., and Jakubowicz, H., 1987, Post-stack prestack migration: 57th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 714–717.

Beasley, C. J., and Mobley, E., 1988, Amplitude and antialiasing treatment in (*x-t*) DMO: 58th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1113–1116.

Berg, L. A., 1985, Prestack partial migration: Preprint, Meeting of the EAEG, Budapest.

Beylkin, G., 1985, Imaging of discontinuities in the inverse scattering problem by inversion of a causal generalized Radon transform: J. Math. Phys., **26**, 99–108.

Biondi, B., and Ronen, J., 1987, Dip moveout in shot profiles: Geophysics, **52**, 1473–1482.

Black, J. L., and Egan, M. S., 1988, True-amplitude DMO in 3-D: 58th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1109–1113.

Bleistein, N., 1984, Mathematical methods for wave phenomena: Academic Press.

——— 1986, Two-and-one-half dimensional in-plane wave propagation: Geophys. Prosp., **34**, 686–703.

——— 1987, On imaging reflectors in the Earth: Geophysics, **52**, 931–942.
Bleistein N., Cohen, J., and Hagin, F., 1987, Two and one-half dimensional Born inversion with an arbitrary reference: Geophysics, **52**, 26–36.
Claerbout, J. F., 1984, Imaging the earth's interior: Blackwell Scientific Publ.
Cohen, J., Hagin, F., and Bleistein, N., 1986, Three-dimensional Born inversion with an arbitrary reference: Geophysics, **51**, 1552–1558.
Deregowski, S. G., and Rocca, F., 1981, Geometrical optics and wave theory of constant offset sections in layered media: Geophys. Prosp., **29**, 374–406.
Deregowski, S. G., 1985, An integral implementation of dip-moveout: Preprint, Meeting of the EAEG, Budapest.
Docherty, P., 1987, Two-and-one-half dimensional common shot modeling: Center for Wave Phenomena Research rep. no. CWP-050.
Forel, D., and Gardner, G. H. F., 1988, A three-dimensional perspective on two-dimensional dip-moveout: Geophysics, **53**, 604–610.
Gardner, G. H. F., and Forel, D., 1988, Amplitude preservation equations for DMO: 58th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1106–1108.
Hale, I. D., 1983, Dip moveout by Fourier transform: Ph.D. Thesis, Stanford Univ., SEP-36.
——— 1984, Dip moveout by Fourier transform: Geophysics, **49**, 741–757.
——— 1988, DMO Processing: SEG Course Notes, 58th Ann. Internat. Mtg., Soc. Expl. Geophys.
Jakubowicz, H., and Miller, D., 1989, Two-pass 3-D migration and linearized inversion in the $(x, t)$-domain: Geophys. Prosp., **37**, 143–148.
Jorden, T., 1987, Transformation to zero-offset: Center for Wave Phenomena Research rep. no. CWP-052.
Jorden, T., Bleistein, N., and Cohen, J., 1987, A wave equation-based dip moveout: 57th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 718–721.
Liner, C. L., 1988, General theory and comparative anatomy of dip-moveout: Geophysics, **55**, 595–607.
——— 1989, Mapping reflection seismic data to zero-offset: Ph.D. thesis, Colorado School of Mines, CWP-081.
Liner, C. L., and Bleistein, N., 1988, Comparative anatomy of common offset dip-moveout: 58th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1101–1105.
Notfors, C. D., and Godfrey, R. J., 1987, Dip moveout in the frequency-wavenumber domain: Geophysics, **52**, 1718–1721.

## APPENDIX

### OUTLINE DERIVATION OF COMMON-SHOT BORN DMO

Begin with the cascaded Born DMO equation (9)

$$P_0(\omega_0, x_0) \sim (-i\omega_0)^{3/2} \int\!\!\int\!\!\int\!\!\int dx\, dz\, dx_i d\omega_i \times A_1 e^{i(\omega_0\phi_0 - \omega_i\phi_i)} \frac{P_i(\omega_i, x_i)}{\sqrt{i\omega_i}}, \tag{A-1}$$

where the amplitude term is

$$A_1 = A_i A_0 = \frac{v(r_s r_g)^{3/2} H}{2^{1/2}(2\pi)^2 r_0^{3/2}(r_s + r_g)^{1/2}}. \tag{A-2}$$

In what follows the goal is to analytically eliminate $(x, z)$. The major events of the derivation follow Jorden (1987), but there are significant differences in detail.

The first step is to inverse Fourier transform expression (A-1) with respect to $\omega_0$, which occurs on the right-hand side only as $e^{i\omega_0\phi_0}$ and $(-i\omega_0)^{3/2}$. The result is

$$P_0(t_0, x_0) \sim \partial_{t_0}^{3/2} \int\!\!\int\!\!\int\!\!\int dx\, dz\, dx_i\, d\omega_i \times A_1 \delta(t_0 - \phi_0) e^{-i\omega_i\phi_i} \frac{P_i(\omega_i, x_i)}{\sqrt{i\omega_i}}, \tag{A-3}$$

where the correspondence $(-i\omega_0)^{3/2} \rightarrow \partial_{t_0}^{3/2}$ has been used, and offsetting factors of $2\pi$ have been canceled.

The delta function argument is a function of $z$. The zero of the argument defines a critical point $z_c$ given by

$$z_c \equiv \sqrt{r_0^2 - (x - x_0)^2}; \qquad r_0 = \frac{vt_0}{2}, \tag{A-4}$$

where the second equality follows directly from $t_0 = \phi_0$. From a standard $\delta$-function property (Bleistein, 1984, p. 48), it follows that

$$\delta(t_0 - \phi_0) \equiv \frac{\delta(z - z_c)}{\left|\dfrac{\partial\phi_0}{\partial z}\right|_{z = z_c}} = \frac{vr_0}{2z_c}\delta(z - z_c). \tag{A-5}$$

Using expression (A-5) in expression (A-3), the $z$ integration is done analytically, yielding

$$P_0(t_0, x_0) \sim \partial_{t_0}^{3/2} \int\!\!\int\!\!\int dx\, dx_i\, d\omega_i\, A_2 e^{-i\omega_i\phi_i} \frac{P_i(\omega_i, x_i)}{\sqrt{i\omega_i}}, \tag{A-6}$$

where the amplitude term is now given by

$$A_2 = A_1 \frac{vr_0}{2z_c} = \frac{v^2(r_s r_g)^{3/2} H}{2^{3/2}(2\pi)^2 r_0^{1/2}(r_s + r_g)^{1/2} z_c}. \tag{A-7}$$

Isolating the $x$-dependence, write relation (A-6) as

$$P_0(t_0, x_0) \sim \partial_{t_0}^{3/2} \int\!\!\int dx_i\, d\omega_i \frac{P_i(\omega_i, x_i)}{\sqrt{i\omega_i}} \int dx\, A_2 e^{-i\omega_i\phi_i}. \tag{A-8}$$

The $x$ integral is evaluated asymptotically by the method of stationary phase. The stationary phase condition $\partial\phi_i/\partial x = 0$ defines a stationary point $x_c$ given by

$$x_c \equiv x_0 + \frac{r_0^2(\beta_s + \beta_g)}{2\beta_s\beta_g}, \tag{A-9}$$

where

$$\beta_s \equiv x_s - x_0; \qquad \beta_g \equiv x_g - x_0. \tag{A-10}$$

This condition (A-9) is interperted as defining the specular raypaths shown in Figure 1. The critical point $x_c$ will not be defined if $\beta_s = 0$ or $\beta_g = 0$. We conclude that $x_0$ must lie between the source and receiver (i.e., $|x_s| < |x_0| < |x_g|$).

Applying the stationary phase formula (Bleistein, 1984) gives the asymptotic result

$$\int dx\, A_2 e^{-i\omega_i \phi_i} \sim \left[\frac{2\pi v r_s^3 r_g}{\beta_s^2 (r_s + r_g)}\right]^{1/2} \frac{e^{-i\omega_i \phi_i}}{\sqrt{i\omega_i}}. \quad \text{(A-11)}$$

Using the stationary phase evaluation (A-11), write expression (A-8) as

$$P_0(t_0, x_0) \sim \partial_{t_0}^{3/2} \int\int dx_i\, d\omega_i\, A_3 e^{-i\omega_i \phi_i} \frac{P_i(\omega_i, x_i)}{i\omega_i}, \quad \text{(A-12)}$$

where the amplitude term is given by

$$A_3 = A_2 \left[\frac{2\pi v r_s^3 r_g}{\beta_s^2 (r_s + r_g)}\right]^{1/2} = \frac{v^{5/2} r_s^3 r_g^2 H}{(4\pi)^{3/2} r_0^{1/2} (r_s + r_g)|\beta_s| z_c}. \quad \text{(A-13)}$$

The action of the fractional derivative operator $\partial_{t_0}^{3/2}$ will be evaluated analytically. Isolating the $t_0$-dependence of the integrand, write expression (A-12) as

$$P_0(t_0, x_0) \sim \int\int dx_i\, d\omega_i \frac{P_i(\omega_i, x_i)}{i\omega_i} \partial_{t_0}^{3/2} [A_3 e^{-i\omega_i \phi_i}]. \quad \text{(A-14)}$$

Consistent with previous "leading order" approximations, I use the asymptotic relationship

$$\partial_{t_0}^{3/2} [A_3 e^{-i\omega_i \phi_i}] \sim A_3 (-i\omega_i)^{3/2} \left[\frac{\partial \phi_i}{\partial t_0}\right]^{3/2} e^{-i\omega_i \phi_i}. \quad \text{(A-15)}$$

From earlier definitions it follows that

$$\frac{\partial \phi_i}{\partial t_0} = \frac{r_0 f}{2 r_g |\beta_s|}. \quad \text{(A-16)}$$

Using this result, expression (A-14) becomes

$$P_0(t_0, x_0) \sim \int\int dx_i\, d\omega_i\, A_4 e^{-i\omega_i \phi_i} \sqrt{i\omega_i}\, P_i(\omega_i, x_i), \quad \text{(A-17)}$$

where the amplitude is given by

$$A_4 = A_3 \left[\frac{r_0 f}{2 r_g |\beta_s|}\right]^{3/2} = \frac{v^{5/2} r_0 f^{3/2} r_s^3 r_g^{1/2} H}{8(2\pi)^{3/2} (r_s + r_g)|\beta_s|^{5/2} z_c}. \quad \text{(A-18)}$$

Recognizing an exact Fourier transform, the $\omega_i$ integral in expression (A-17) can be performed to give a $(t - x)$-domain formula

$$P_0(t_0, x_0) \sim \int\int dx_i\, dt_i\, A_5 \delta(t_i - \phi_i) \partial_{t_i}^{1/2} P_i(t_i, x_i), \quad \text{(A-19)}$$

where the amplitude term is now

$$A_5 = 2\pi A_4 = \frac{v^{5/2} r_0 f^{3/2} r_s^3 r_g^{1/2} H}{8(2\pi)^{1/2} (r_s + r_g)|\beta_s|^{5/2} z_c}, \quad \text{(A-20)}$$

which is the final form of the general 2.5-D Born DMO.

To this point, only a single source and receiver have been discussed, and the collective geometry of the input data (common-shot or common-offset) has not been specified. The theory depends on the input data geometry only through the Beylkin inversion determinant $H$. The Beylkin determinant for common-shot inversion is given by Jorden (1987). However, that determinant contains a term which depends on the angle of incidence. For Born DMO, this would pass through and show up as a pseudo-AVO effect. The appropriate determinant for Born DMO is

$$H_{\text{shot}} = \frac{2 z_c (r_s + r_g)}{v^3 r_g^3 r_s}. \quad \text{(A-21)}$$

Combining equations (A-20) and (A-21), the common-shot Born DMO amplitude term is found to be

$$A_{\text{shot}} = A_5 (H_{\text{shot}}) = \frac{r_0 f^{3/2}}{4(2\pi v r_s)^{1/2} |\beta_g|^{5/2}}, \quad \text{(A-22)}$$

where the relationship $|\beta_s| r_g = |\beta_g| r_s$ has been used. Gathering up this amplitude term with expression (A-19) and identifying the input $x$-coordinate $x_i$ with the geophone coordinate $x_g$ yields the common-shot Born DMO formula as given in equations (10)–(16).