GEOPHYSICS, VOL. 56, NO. 11 (NOVEMBER 1991); P. 1778–1785, 12 FIGS.

3-D depth migration via McClellan transformations

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ABSTRACT

Three-dimensional seismic wavefields may be extrapolated in depth, one frequency at a time, by two-dimensional convolution with a circularly symmetric, frequency- and velocity-dependent filter. This depth extrapolation, performed for each frequency independently, lies at the heart of 3-D finite-difference depth migration. The computational efficiency of 3-D depth migration depends directly on the efficiency of this depth extrapolation.

McClellan transformations provide an efficient method for both designing and implementing two-dimensional digital filters that have a particular form of symmetry, such as the circularly symmetric depth extrapolation filters used in 3-D depth migration. Given the coefficients of one-dimensional, frequency- and velocity-dependent filters used to accomplish 2-D depth migration, McClellan transformations lead to a simple and efficient algorithm for 3-D depth migration.

3-D depth migration via McClellan transformations is simple because the coefficients of two-dimensional depth extrapolation filters are never explicitly computed or stored; only the coefficients of the corresponding one-dimensional filter are required. The algorithm is computationally efficient because the cost of applying the two-dimensional extrapolation filter via McClellan transformations increases only *linearly* with the number of coefficients N in the corresponding one-dimensional filter. This efficiency is not intuitively obvious, because the cost of convolution with a two-dimensional filter is generally proportional to N^2. Computational efficiency is particularly important for 3-D depth migration, for which long extrapolation filters (large N) may be required for accurate imaging of steep reflectors.

INTRODUCTION

The high computational cost of 3-D poststack depth migration has motivated many papers describing different methods for performing this important step in seismic data processing. This paper describes another method with significant advantages in computational simplicity and efficiency.

The development of accurate 3-D poststack depth migration has been hindered by the fact that commonly used finite-difference migration methods cannot be extended easily from 2-D to 3-D. In particular, 2-D depth migration methods that are based on *implicit* depth extrapolation of the seismic wavefield, such as the common "45-degree" finite-difference method (e.g., Claerbout, 1985), are difficult to extend to 3-D processing.

Many authors, including Brown (1983), Yilmaz (1987, p. 404–405), and Kitchenside and Jakubowicz (1987), have noted the error in *splitting* the depth extrapolation process to extrapolate alternately along the inline (x) and crossline (y) directions, independently. Splitting is the most practical way to extend implicit extrapolation methods from 2-D to 3-D, but as these authors have shown, the errors in splitting depend significantly on reflector dip and azimuth. Specifically, for reflector dips greater than about 20 degrees and reflector azimuths near 45 degrees, measured with respect to either the x or y directions, splitting yields unacceptable errors in reflector positioning. Although techniques for reducing the error in splitting have been demonstrated (Ristow, 1980; Graves and Clayton, 1990), these techniques significantly increase both the computational cost and complexity of implicit depth extrapolation methods.

Some migration methods developed more recently, such as reverse-time migration, may be extended from 2-D to 3-D relatively easily, as demonstrated by Chang and McMechan (1989). Reverse-time migration, because it is based on an explicit backward extrapolation in time, requires no splitting. However, several authors (Reshef and Kessler, 1989;

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Blacquière et al., 1989) have noted that 3-D reverse-time migration is relatively expensive, due to the large number of computations and the large amount of computer memory required.

Reshef and Kessler (1989) discuss the practical aspects of extending a generalized phase-shift method for depth migration (Kosloff and Kessler, 1987) from 2-D to 3-D processing. Because this method is based on explicit depth extrapolation via a two-way wave equation, splitting is unnecessary in their 3-D implementation. However, they note two difficulties with this method. First, as described by Kosloff and Kessler (1987), the method requires the attenuation of exponentially growing evanescent energy via a spatially varying filter. Second, the method uses impedance matching (Baysal et al., 1984) to attenuate unwanted reflections, but fails to attenuate these reflections for waves propagating obliquely to a vertical velocity gradient. Both exponentially growing evanescent energy and unwanted reflections are symptoms of depth extrapolation with a two-way wave equation.

As noted by Claerbout (1985, p. 55), the problems associated with a two-way wave equation can be avoided by using a one-way wave equation for depth extrapolation of the seismic wavefield. However, depth migration via the one-way wave equation has traditionally been accomplished via an implicit extrapolation method (as in 45-degree finite-difference migration), which requires splitting in its extension from 2-D to 3-D processing.

Splitting is unnecessary for *explicit* extrapolation methods. Holberg (1988) describes a method for designing explicit extrapolation filters for the one-way wave equation, and Blacquière et al. (1989) have extended this method for use in 3-D depth migration. Specifically, Blacquière et al. compute and tabulate the coefficients of two-dimensional extrapolation filters. Each of these filters corresponds to a particular ratio of frequency to velocity. Depth extrapolation is then performed for each frequency, independently, by convolving a two-dimensional extrapolation filter with the seismic wavefield. Lateral velocity variations are handled by letting the coefficients of the extrapolation filter (extracted from a table) vary as velocity varies in both the x and y directions.

While direct convolution of two-dimensional extrapolation filters with the seismic wavefield is more accurate than splitting, it is also computationally expensive. The cost of direct convolution is proportional to N^2, where N is the number of coefficients in the corresponding one-dimensional extrapolation filter that one might use in performing 2-D depth migration. In contrast, the cost of the splitting method is proportional to N. The relatively high cost of direct convolution with two-dimensional extrapolation filters motivated Kitchenside and Jakubowicz (1987) and Kitchenside (1988) to investigate alternative extrapolation schemes that combine direct convolution, splitting, and Fourier transform methods to achieve computational efficiency.

The 3-D depth migration method described in this paper is similar to that described by Blacquière et al. (1989), but with two significant advantages:

1) The coefficients of *two-dimensional* depth extrapolation filters are never computed or tabulated. Only the coefficients of the corresponding *one-dimensional* extrapolation filters (identical to those used in 2-D depth migration) are required.
2) The cost of depth extrapolation increases only *linearly* with N, the number of coefficients in the one-dimensional extrapolation filters.

The computational simplicity and efficiency of this method stem from the design and implementation of depth extrapolation filters via McClellan transformations.

DEPTH EXTRAPOLATION FOR 2-D AND 3-D MIGRATION

Two-dimensional migration, when performed by recursive extrapolation of the seismic wavefield in depth, requires an extrapolation filter with a Fourier transform that approximates

$$D(k) \equiv \exp\left\{ i \, \frac{\Delta z}{\Delta x} \left[\left(\frac{\omega \Delta x}{v} \right)^2 - k^2 \right]^{1/2} \right\}, \qquad (1)$$

where ω denotes frequency (in radians per unit time), v denotes velocity, and Δz and Δx denote vertical and horizontal spatial sampling intervals, respectively. Wavenumber k (measured in radians per sample in the x direction) is normalized such that any distance quantity is measured in terms of the number of horizontal sampling intervals Δx. With this normalization, two dimensionless constants, $\Delta z/\Delta x$ and $\omega \Delta x/v$, uniquely specify the desired transform $D(k)$.

The desired transform $D(k)$ defined by equation (1) is appropriate for waves traveling one way, either down or up. In depth extrapolation of CMP stacked data, which corresponds to waves propagating both down and up, one may use the "exploding reflectors" concept to process these data as upgoing waves only, replacing velocity v with half-velocity $v/2$ (e.g., Claerbout, 1985).

The *explicit* depth extrapolation filters of interest here consist of N complex coefficients h_n, with the Fourier transform

$$H(k) \equiv \sum_{n=(-N-1)/2}^{(N-1)/2} h_n e^{-ikn} \approx D(k).$$

The symmetry of $D(k)$ with respect to $k = 0$ implies that the complex extrapolation filter coefficients h_n are even; that is, $h_{-n} = h_n$ for all n. Therefore, the number of coefficients N should be odd. The number of *unique* coefficients h_n is $N_h \equiv (N+1)/2$, and the Fourier transform $H(k)$ may be expressed as

$$H(k) = h_0 + 2 \sum_{n=1}^{N_h - 1} h_n \cos(kn). \qquad (2)$$

Methods for computing the N_h coefficients h_n to obtain a satisfactory approximation $H(k)$ to the desired transform $D(k)$ are described by Holberg (1988) and Hale (1991).

In 2-D migration, extrapolation of a spatially sampled seismic wavefield $P(x_i, z_k, \omega)$ from depth z_{k-1} to depth $z_k \equiv z_{k-1} + \Delta z$ may be performed independently for each frequency ω, by spatial convolution with the one-dimensional extrapolation filter h_n:

$$P(x_i, z_k, \omega) = \sum_{n=(-N+1)/2}^{(N-1)/2} h_n P(x_{i-n}, z_{k-1}, \omega).$$

Velocity variations are handled by letting the coefficients of the extrapolation filter h_n vary as the velocity changes with spatial coordinates x_i and z_k (Holberg, 1988).

For 3-D migration, depth extrapolation may be performed by convolution with a circularly symmetric two-dimensional filter $h_{m,n}$;

$$P(x_i, y_j, z_k, \omega) = \sum_{m=(-N+1)/2}^{(N-1)/2} \times \sum_{n=(-N+1)/2}^{(N-1)/2} h_{m,n} P(x_{i-m}, y_{j-n}, z_{k-1}, \omega).$$

The filter $h_{m,n}$ should have a Fourier transform approximating that specified in equation (1), but with wavenumber k replaced by $k = \sqrt{k_x^2 + k_y^2}$, where k_x and k_y denote inline and crossline wavenumbers, respectively, both normalized by the inline sampling interval Δx, which will here be assumed to be equal to the crossline sampling interval Δy. The desired Fourier transform of the two-dimensional extrapolation filter for 3-D migration is then

$$D(k_x, k_y) = \exp\left\{ i \frac{\Delta z}{\Delta x} \left[\left(\frac{\omega \Delta x}{v} \right)^2 - k_x^2 - k_y^2 \right]^{1/2} \right\}. \quad (3)$$

The assumption above that inline and crossline sampling intervals are equal simplifies the application (in the following section) of McClellan transformations to 3-D depth migration. One may simply resample the seismic wavefield to satisfy this assumption. This spatial resampling would be required for each frequency, but the computational cost of resampling would be insignificant compared with that of depth extrapolation for each frequency. Alternatively, one may use McClellan transformations that account for unequal sampling intervals (Mersereau et al., 1976; Nguyen and Swamy, 1986; Reddy and Hazra, 1987). For simplicity, equal spatial sampling intervals are assumed here.

Splitting in 3-D migration is based on the approximation (e.g., Brown, 1983)

$$D(k_x, k_y) \approx \exp\left(i \frac{\Delta z}{\Delta x} \left\{ \left[\left(\frac{\omega \Delta x}{v} \right)^2 - k_x^2 \right]^{1/2} + \left[\left(\frac{\omega \Delta x}{v} \right)^2 - k_y^2 \right]^{1/2} - \frac{\omega \Delta x}{v} \right\} \right). \quad (4)$$

By separating the k_x and k_y terms in the exponent of equation (4), we approximate the two-dimensional convolution implied by equation (3) as a relatively inexpensive cascade: one-dimensional convolutions in the x direction for all y, followed by one-dimensional convolutions in the y direction for all x, followed by a phase shift. Comparing equations (3) and (4), the splitting approximation is best for $vk_y/\omega\Delta x \approx 0$ or $vk_x/\omega\Delta x \approx 0$, corresponding to reflectors dipping in the x or y directions. Splitting yields large errors for $vk_x/\omega\Delta x \approx vk_y/\omega\Delta x \gg 0$, corresponding to steep reflectors dipping at 45 degrees azimuth between the x and y directions.

McCLELLAN TRANSFORMATIONS

Using McClellan transformations, one can avoid the errors in splitting, without computing the coefficients of two-dimensional extrapolation filters. In other words, we need not compute extrapolation filters with a 2-D Fourier transform approximating $D(k_x, k_y)$. As will be shown below, we need only the one-dimensional filter coefficients h_n used in 2-D migration.

McClellan transformations (McClellan, 1973; Mersereau et al., 1976) are based on the following trigonometric identity

$$\cos(n\theta) = 2\cos(\theta)\cos[(n-1)\theta] - \cos[(n-2)\theta],$$

which is a special case of the recursive formula for the Chebychev polynomials $T_n(x)$ defined by $T_n(\cos\theta) \equiv \cos(n\theta)$ (e.g., Abramowitz and Stegun, 1965, p. 776). With this identity, each of the $\cos(kn)$ terms in equation (2) may be computed recursively from $\cos k$:

$$H(k) = h_0 + 2h_1 \cos k + 2h_2(2\cos k \cos k - 1) + \cdots .$$

Recognizing that the filter $H(k)$ may be written entirely in terms of h_n and $\cos k$, McClellan and Chan (1977) showed that convolution with the filter h_n may be accomplished via the recursive Chebychev filter structure illustrated in Figure 1.

Note that the upper part of the structure in Figure 1 is responsible for computing $\cos(kn)$ through recursive application of the filter $G(k) = \cos(k)$. We may view each multiplication by $G(k)$ as a convolution in space by a filter with Fourier transform $G(k)$. The lower part of this structure simply taps into the upper part as necessary to obtain the filter $H(k)$ of equation (2).

For one-dimensional filters, $G(k) = \cos(k)$ in Figure 1 is the Fourier transform of the filter g_j defined by

$$g_j \equiv \begin{cases} 1/2, & \text{if } j = \pm 1; \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

which has only two nonzero coefficients. Each stage in the Chebychev filter structure implies a convolution with this simple filter. This structure is, however, cumbersome for one-dimensional filters. Direct convolution with the coefficients h_n is both simpler and more efficient than the structure illustrated in Figure 1. We would never use the Chebychev filter structure to perform the one-dimensional convolution with h_n for explicit depth extrapolation in 2-D depth migration.

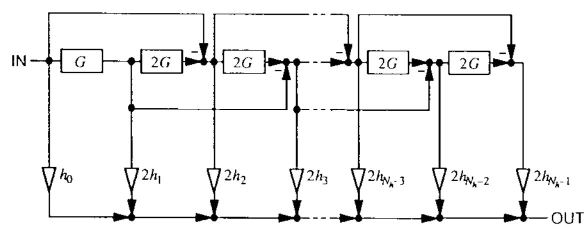

FIG. 1. Chebychev filter structure for convolution with the filter h_n. The upper part of this structure implements the Chebychev recursion for $\cos(kn)$ in terms of $G(k) = \cos k$.

The Chebychev filter structure is most advantageous for two-dimensional filters with quadrantal symmetry, such as circularly symmetric two-dimensional extrapolation filters, for which $k = \sqrt{k_x^2 + k_y^2}$. Such filters may be implemented by the Chebychev filter structure by simply replacing G in Figure 1 with $G(k_x, k_y) = \cos\sqrt{k_x^2 + k_y^2}$.

An exact representation of $G(k_x, k_y)$ in the Chebychev filter structure of Figure 1 would result in a computational cost greater than that of direct two-dimensional convolution. Therefore, McClellan (1973) suggested the approximation

$$G(k_x, k_y) \approx -1 + \tfrac{1}{2}(1 + \cos k_x)(1 + \cos k_y), \quad (6)$$

which is exact for $k_x = 0$, or $k_y = 0$. This approximation is the discrete Fourier transform of the compact two-dimensional filter illustrated in Figure 2. Replacing the coefficients of the one-dimensional filter g_j defined by equation (5) with the two-dimensional filter of Figure 2 *transforms* any one-dimensional filter to a two-dimensional filter with approximate circular symmetry. Note that the Chebychev filter structure is unchanged, that the filter coefficients h_n are the coefficients of the original one-dimensional filter, and that the cost of applying the two-dimensional filter grows *linearly* with the number of filter coefficients N_h.

The coefficients of the McClellan transformation filter illustrated in Figure 2 are independent of the coefficients of the depth extrapolation filter. Therefore, that part of the Chebychev filter structure that performs convolution with the transformation filter (the upper part of Figure 1) can be optimized independently from the part that multiplies the seismic wavefield by the coefficients h_n of the extrapolation filter.

Figure 3 shows contours of constant amplitude and phase (constant k) for any two-dimensional filter designed and implemented via the McClellan transformation of equation (6) and Figure 2. As illustrated in Figure 3, this transformation is best for small k_x and k_y. The transformation is exact for $k_x = 0$ or $k_y = 0$, but exhibits increasing error with increasing k for $k_x \approx k_y$.

Figure 4 shows the coefficients of an improved McClellan transformation filter, which has the Fourier transform

$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$
$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$

FIG. 2. Original McClellan transformation filter—transforms any one-dimensional filter into an approximately circularly symmetric two-dimensional filter, implemented via the Chebychev recursion in Figure 1.

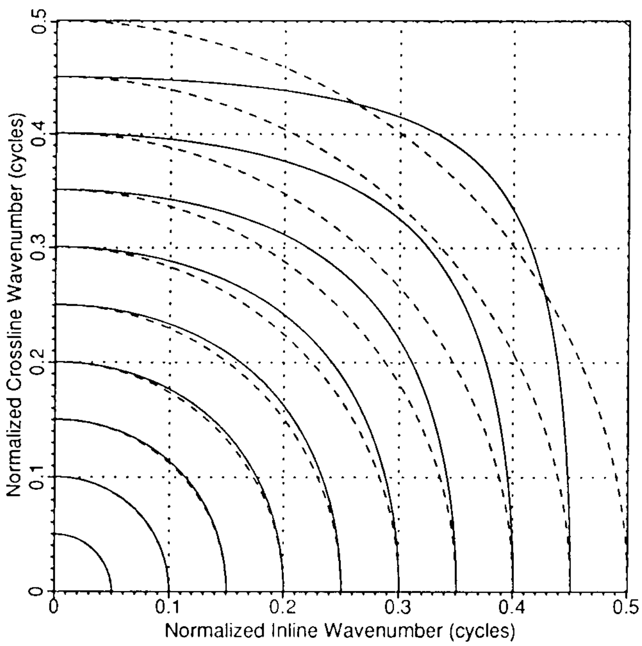

FIG. 3. Contours of constant amplitude and phase (constant k) for two-dimensional filters designed and implemented via the original McClellan transformation of Figure 2. Contours of ideal circularly symmetric filters are plotted with dashed lines. Compare with Figure 5. In this figure, normalized wavenumbers k_x and k_y are measured in cycles per spatial sampling interval. Thus, $k = 0.25$ is one-half the Nyquist wavenumber.

$-\frac{c}{8}$	0	$\frac{c}{4}$	0	$-\frac{c}{8}$
0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	0
$\frac{c}{4}$	$\frac{1}{4}$	$-\frac{1+c}{2}$	$\frac{1}{4}$	$\frac{c}{4}$
0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	0
$-\frac{c}{8}$	0	$\frac{c}{4}$	0	$-\frac{c}{8}$

FIG. 4. Improved McClellan transformation filter for a circularly symmetric two-dimensional filter implemented via the Chebychev recursion in Figure 1. The constant $c \approx 0.0255$ yields a two-dimensional filter that is more nearly circularly symmetric than the original McClellan transformation. Compare this transformation filter with that in Figure 2.

$$G(k_x, k_y) = -1 + \frac{1}{2}(1 + \cos k_x)(1 + \cos k_y) - \frac{c}{2}(1 - \cos 2k_x)(1 - \cos 2k_y). \quad (7)$$

Contours of constant k for this filter are plotted in Figure 5. The constant c in equation (7) was determined by exactly matching a particular k along the diagonal $k_x = k_y$. In this case, the value $k = \pi/3$ was chosen to determine the constant $c \approx 0.0255$. Further improvement is possible, by using larger transformation filters. For a thorough discussion on the design of McClellan transformations, see Mersereau et al. (1976).

The error in McClellan transformations, unlike the error in splitting, depends only on the magnitude of inline and crossline wavenumbers k_x and k_y; this error is independent of frequency ω. For example, Figure 5 suggests that the improved McClellan transformation may be adequate for all wavenumbers less than half-Nyquist. In any case, neither the original nor the improved McClellan transformation exactly yields a circularly symmetric extrapolation filter. In the next section, the accuracy of 3-D migration with the original and improved McClellan transformations is compared with the accuracy obtained by splitting.

3-D MIGRATION IMPULSE RESPONSES

Following closely the work of Blacquière et al. (1989), a 3-D depth migration method was developed; this method is based on a table of extrapolation filters, one filter for each ratio of $\omega \Delta x / v$ tabulated. This table is the same table of *one-dimensional* filters that one might use in 2-D depth migration. Specifically, stable explicit extrapolation filters were computed and tabulated via the modified Taylor-series method described by Hale (1991). In practice, one-dimensional explicit extrapolation filters designed using *any* method, such as that described by Holberg (1988), can be transformed into two-dimensional filters by the McClellan transformations described in the previous section.

Because only the coefficients h_n of the one-dimensional filters must be tabulated, one can afford to sample $\omega \Delta x / v$ in equation (1) finely, thereby avoiding the need to interpolate filter coefficients for a particular $\omega \Delta x / v$. (One may simply choose the coefficients corresponding to the nearest tabulated value of $\omega \Delta x / v$.) The extrapolation filters are then applied via the Chebychev filter structure of Figure 1, which permits the coefficients h_n to change as velocity $v = v(x, y)$ varies laterally with x and y. One can readily verify that doing so is equivalent to letting the coefficients of a two-dimensional extrapolation filter vary with output location, as done by Blacquière et al. (1989).

Because the ability of such a "table-driven" 3-D migration to handle lateral velocity variations has been well demonstrated by Blacquière et al. (1989), the tests described below were performed with constant velocity. These tests serve merely to demonstrate the implementation of the McClellan transformations described in the previous section.

The impulse responses of three different 3-D migration methods are illustrated in Figures 6 through 11). The three methods tested were based on depth extrapolation via:

1) Splitting according to equation (4);
2) The original McClellan transformation illustrated in Figures 2 and 3, and
3) The improved McClellan transformation illustrated in Figures 4 and 5.

In all of these tests, the spatial sampling intervals $\Delta x = \Delta y = \Delta z = 10$ m, the time sampling interval $\Delta t = 10$ ms, and the velocity $v = 2$ km/s. The input trace located at $x = y = 0$ contained a single, zero-phase wavelet centered at $t = 0.46$ s, with frequencies between 0 and 30 Hz.

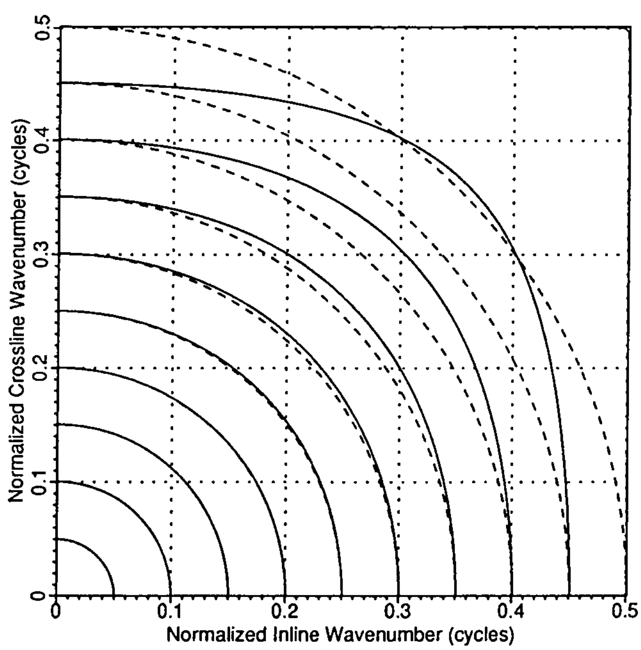

FIG. 5. Contours of constant amplitude and phase (constant k) for two-dimensional filters designed and implemented via the improved McClellan transformation filter of Figure 4. Contours of ideal circularly symmetric filters are plotted with dashed lines. Compare with Figure 3. In this figure, normalized wavenumbers k_x and k_y are measured in cycles per spatial sampling interval. Thus, $k = 0.25$ is one-half the Nyquist wavenumber.

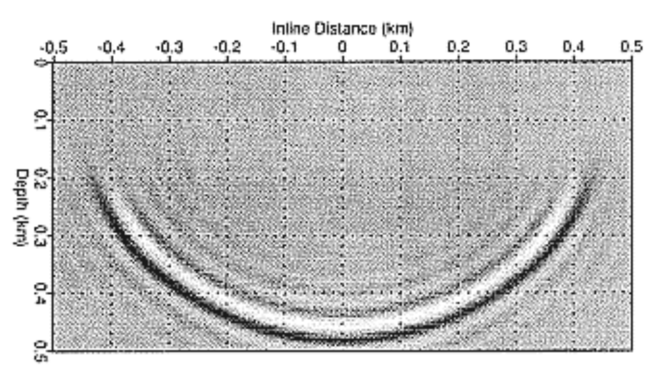

FIG. 6. Impulse response of 3-D migration via splitting, for the crossline coordinate $y = 0$. Compare with Figures 7 and 8.

Figures 6, 7, and 8 illustrate 3-D migration impulse responses for the crossline coordinate $y = 0$. These migration impulse responses are dominated by wavenumber $k_y = 0$, for which splitting and both McClellan transformations are expected to be accurate. Therefore, the impulse responses for $y = 0$ are quite similar.

Figures 9, 10, and 11 illustrate 3-D migration impulse responses for the depth $z = 250$ m. As illustrated in Figures 6, 7, and 8, this depth corresponds to a reflector dip of about 60 degrees. The azimuth-dependent error in splitting is clearly illustrated in Figure 9, which is comparable to Figure 6–27 of Yilmaz (1987, p. 405). The original McClellan transformation yields the nearly circular response seen in Figure 10, with only a slight distortion evident. The improved McClellan transformation yields the apparently circular response exhibited in Figure 11.

Figures 9, 10, and 11 have all been plotted with the same amplitude scaling. The impulse response of 3-D migration via the splitting method (Figure 9) has measurably higher amplitude than either of the impulse responses obtained via McClellan transformation. Although the details are omitted here, a stationary phase estimate of the amplitude of the impulse response for splitting via equation (4) predicts that the amplitude in the inline and crossline directions in Figure 9 is about 40 percent higher than the correct amplitude predicted by equation (3). This difference of 40 percent accounts for the observed difference in amplitude between Figure 9 and Figures 10 and 11. In other words, 3-D migration via the splitting method may yield erroneously high reflector amplitudes, in addition to the more obvious errors in reflector positioning.

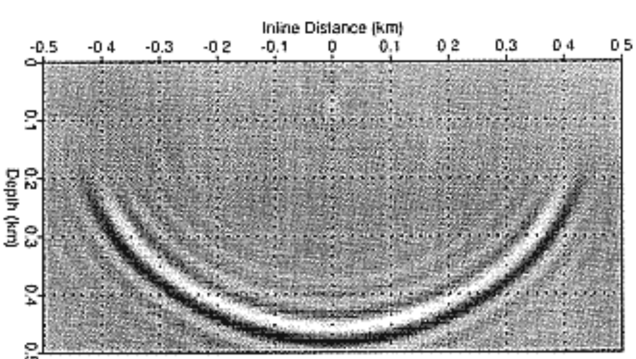

FIG. 7. Impulse response of 3-D migration via the original McClellan transformation, for the crossline coordinate $y = 0$. Compare with Figures 6 and 8.

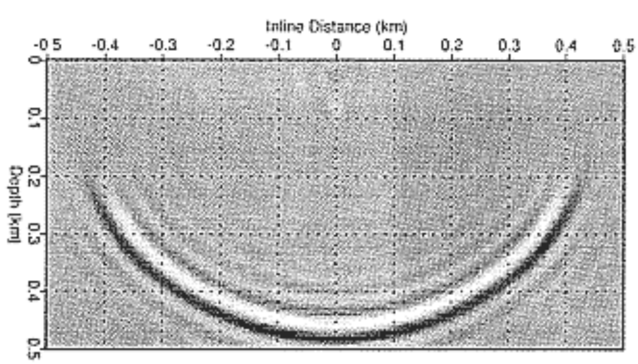

FIG. 8. Impulse response of 3-D migration via the improved McClellan transformation, for the crossline coordinate $y = 0$. Compare with Figures 6 and 7.

COMPUTATIONAL EFFICIENCY

McClellan transformations were first described in GEOPHYSICS by McClellan and Parks (1972) in a paper that showed how transformations could be used to design dip filters. At that time, the goal of the transformation method was

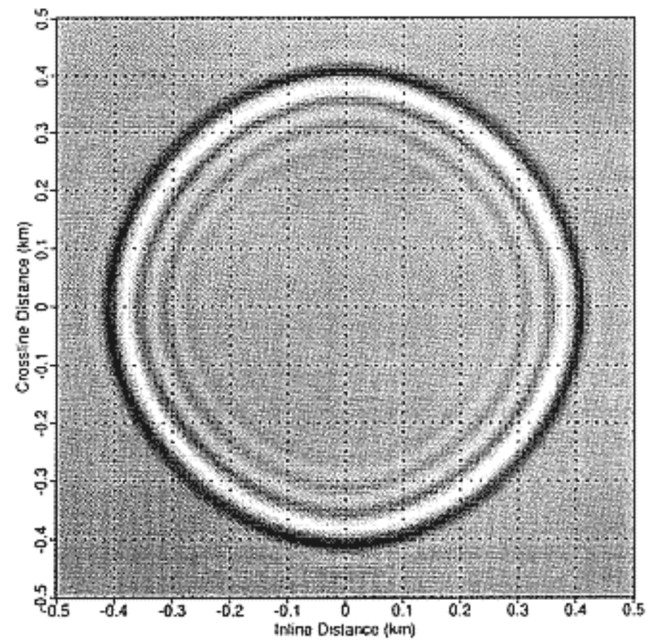

FIG. 9. Impulse response of 3-D migration via splitting, for the depth $z = 250$ m. Compare with Figures 10 and 11.

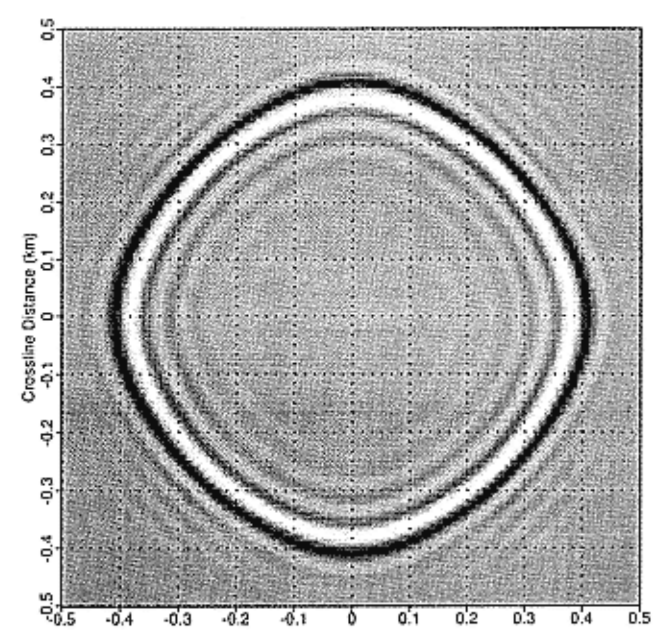

FIG. 10. Impulse response of 3-D migration via the original McClellan transformation, for the depth $z = 250$ m. Compare with Figures 9 and 11.

merely to reduce the computation required to *design* two-dimensional filters. Later, several authors (Mecklenbräuker and Mersereau, 1976; McClellan and Chan, 1977) noted that the transformation method could also significantly reduce the number of arithmetic operations required to *implement* two-dimensional filters, when compared with either direct convolution or fast Fourier transform (FFT) implementations.

A typical application of McClellan transformations in signal processing has been the design and implementation of circularly symmetric low-pass filters (e.g., Hazra and Reddy, 1986). The coefficients of such filters are typically real and constant, so that FFT implementations are a feasible alternative. McClellan transformation of depth extrapolation filters for depth migration is particularly attractive because (1) these filters have complex coefficients and (2) these coefficients may vary spatially. Recalling that a single complex multiplication requires at least six floating point operations (4 multiplications and 2 additions), note that only N_h complex multiplications per output sample are required by the Chebychev filter structure illustrated in Figure 1. Furthermore, unlike an FFT implementation, the Chebychev filter structure permits the filter coefficients h_n to vary as velocity varies with x and y, as required for depth migration.

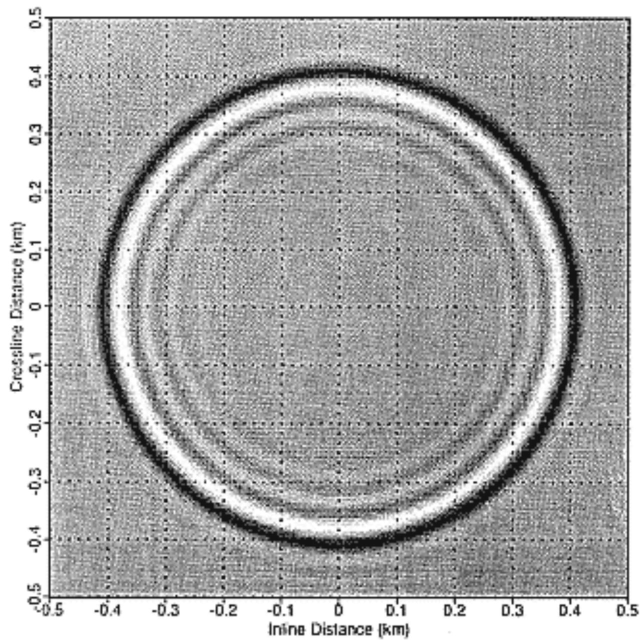

FIG. 11. Impulse response of 3-D migration via the improved McClellan transformation, for the depth z = 250 m. Compare with Figures 9 and 10.

The number of floating point operations (FLOPS) per output sample required by four different depth extrapolation filters are plotted in Figure 12. The four methods shown are splitting, the original McClellan transformation, the improved McClellan transformation, and direct convolution. For each of these methods, the number of (FLOPS) per output sample is plotted for N_h = 10, 20, and 30 coefficients. Although both McClellan transformation methods require more FLOPS than the splitting method to achieve the higher accuracy exhibited in Figures 10 and 11, their cost only grows linearly with the number of filter coefficients N_h. In contrast, the cost of direct convolution grows quadratically as N_h^2. In practice, 3-D depth migration via McClellan transformations will be substantially more efficient than direct convolution.

Care was taken to exploit all possible symmetries in calculating the number of FLOPS plotted in Figure 12. For example, direct convolution with circularly symmetric, two-dimensional extrapolation filters requires only $N_h(N_h + 1)/2$ complex multiplications per output sample. This savings is reflected in Figure 12.

For those readers who wish to reproduce or extrapolate the results for each method plotted in Figure 12, the number of FLOPS was determined to be:

Splitting	$20N_h - 8$
Original McClellan transformation	$32N_h - 26$

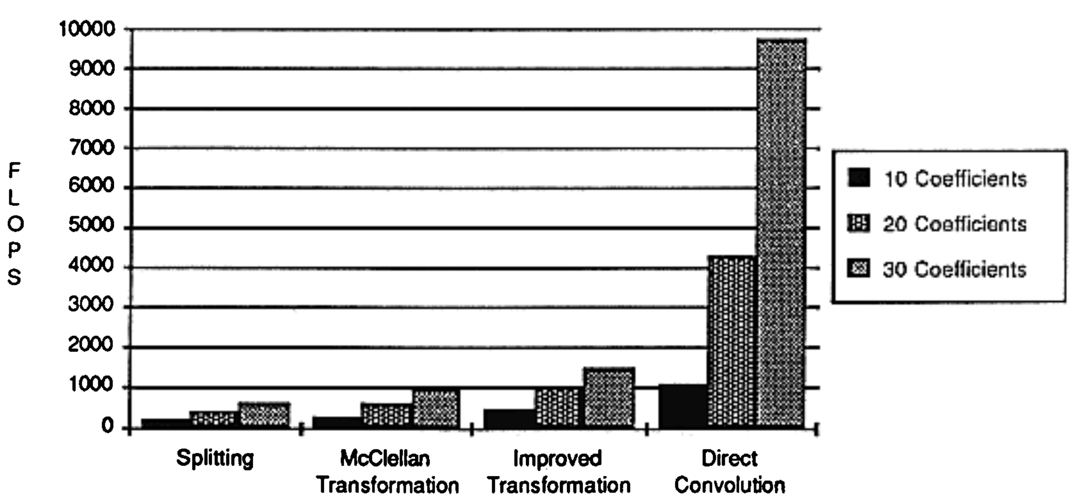

FIG. 12. Number of floating point operations (FLOPS) per output sample required by four different depth extrapolation methods. For each method, the number of FLOPS required is plotted for N_h = 10, 20, and 30 coefficients.

Improved McClellan transformation $52N_h - 46$

Direct convolution $3N_h(N_h + 1) + 2[(2N_h - 1)^2 - 1]$

The use of these formulas to predict computational costs requires the assumption that the cost of a floating point multiplication equals the cost of a floating point addition, which is true for computers typically used today in seismic data processing.

Another important assumption made here is that the Chebychev filter structure can be vectorized (or parallelized) at least as efficiently as direct convolution with a two-dimensional extrapolation filter. Because the coefficients of the small McClellan transformation filter are constants, convolution with this filter may be expressed inline—no short loops that would inhibit vectorization are necessary. Three-dimensional depth migration via McClellan transformations is highly vectorizable.

Those who implement the Chebychev filter structure of Figure 1 should note the similarity of this structure with that of 2nd-order (in time) finite-difference modeling of acoustic waves, in which the wavefield at any time is computed from the wavefield at two previous times. This similarity may be exploited to reduce the software development effort required for efficient implementation of 3-D depth migration via McClellan transformations.

CONCLUSIONS

Given the advantages of performing 2-D depth migration via a table of explicit depth extrapolation filters, McClellan transformations extend these advantages from 2-D to 3-D depth migration. For explicit depth extrapolation filters, McClellan transformations offer significant features:

1) Only the coefficients of one-dimensional extrapolation filters, the same filters used in 2-D depth migration, need be tabulated. This simplifies the design of two-dimensional extrapolation filters for 3-D depth migration, while significantly reducing the amount of computer memory required to hold the tabulated filter coefficients.
2) McClellan transformations are more efficient, requiring fewer floating point operations, than direct convolution with a two-dimensional extrapolation filter. In particular, the cost of 3-D depth migration grows only *linearly* with the length of the tabulated one-dimensional extrapolation filters.
3) Even the simplest McClellan transformation yields significantly greater accuracy than does the splitting method typically used in implicit schemes for 3-D depth migration. The accuracy and computational cost of 3-D depth migration via McClellan transformations depends on the number of coefficients in the transformation filter. More accurate (and, thus, more costly) McClellan transformations may be designed and implemented as necessary.

The computational simplicity and efficiency of McClellan transformations make them an attractive method for implementing 3-D depth migration.

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