Velocity Analysis by Inversion

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ABSTRACT

In conventional inversion methods, imaging structure inside the earth requires reasonable background velocities. In this paper, velocity analysis and structural imaging are done at the same time. The medium is assumed to consist of constant-velocity layers separated by arbitrary, smooth interfaces. The objective of the inversion is to determine layer velocities and locations of the interfaces. The velocity analysis is based on the principle that the images will be distorted when erroneous velocities are used. In particular, the difference between the depths computed by inversions from different experiments can be a measure of the error in velocity. Formulas for sensitivity to velocity error are derived for some special cases. Some computer implementations for both synthetic data and experimental data are done.

INTRODUCTION

The seismic data inversion process may be described as follows: given a scattered field and a background velocity, reflector maps of the earth can be constructed as singular functions of reflectors (Bleistein, 1987). The accuracy and efficiency of this method for a structural image of the earth depends largely on the complexity of the medium and on the quality and complexity of the description of background velocity. We suppose that the medium is made up of constant-velocity layers separated by arbitrary, smooth interfaces. Unfortunately, we have limited information from which to guess these velocities.

The inversion process is, in fact, a form of prestack depth migration. Seismic records, as input, contain information on the traveltime of reflected waves. Using a given velocity model, we map an image from the time domain into the depth domain. Use of incorrect background velocities generally results in a distorted image of the structure. The degree of distortion depends on the magnitude of the velocity error and on the length scale over which signals are propagated with the initial velocity. When a correct velocity is used in prestack migration, the imaged depths at a common location should be the same, regardless of source-receiver offset. Otherwise, if an incorrect background velocity is used, the imaged depths of different source-receiver offsets will differ from one another. Such a deviation, of imaged depths from different offsets, can help us correct the velocity.

Conventional approaches, like the work of Al-Yahya (1989) and MacKay (1992), estimate stacking velocities that are assumed to equal the rms velocities, and then convert the stacking velocities into the interval velocities. Unfortunately, when the velocity has a lateral anomaly, the stacking velocity may be far away from the rms velocity that results in difficulty for velocity analysis (Liu and Bleistein, 1991).

In this paper, we estimate the interval velocities directly. The velocity model is assumed to consist of constant-velocity layers separated by interfaces. The velocity is estimated based on the deviation of imaged depths from different offsets. After the velocity is corrected, the interface (reflector) is determined by picking amplitudes from the output image. To guarantee a successful ray tracing which is used in the inversion code, the modeled interface must be made smooth. This process is then repeated for determining the other velocities and interface shapes for successively deeper layers.

The procedure is as follows:

1. give an initial guess for the velocity model;
2. apply velocity analysis on a fixed output trace to correct velocity at a "nearest" interface (in depth);
3. use the correct velocity to image the interface on the output;
4. smooth the interface both for imaging and continuation of the background velocity below that interface;
5. repeat steps (1) to (4) for the next layer.

MATHEMATICAL PRINCIPLES

We consider the two-dimensional situation. We shall denote by X a 2-D vector, $X = (x, z)$. Let $X_s = X_s(\xi)$ be source positions and $X_r = X_r(\xi)$ be receiver positions located on the datum surface L, where ξ is a position parameter on L. For any point below the surface, $\tau(X_s, X)$ or $\tau(X, X_r)$, respectively, denote traveltimes from X_s to X, or X to X_r.

Suppose we know the total reflection traveltime $T(\xi)$. Any reflection point $X = (x, z)$ must satisfy

$$\tau(X_s, X) + \tau(X, X_r) = T(\xi). \tag{1}$$

For each ξ, the solution of equation (1) is a curve. When ξ varies, we obtain a family of curves.

Theorem 1. For any velocity function, the envelope of a solution family of equation (1) is just the reflector, $z = f(x)$, resulting in traveltime $T(\xi)$.

Proof. The envelope of the family of curves defined by (1) must satisfy that equation and

$$\frac{\partial \tau(X_s, X)}{\partial \xi} + \frac{\partial \tau(X, X_r)}{\partial \xi} = \frac{dT}{d\xi}. \tag{2}$$

Elimination of ξ between these two equations defines the envelope as a function, $z = f(x)$. For example, we could think of solving for $\xi = \xi(x, z)$ in (2) and substituting into (1) to obtain an equation that defines $z = f(x)$ implicitly. Now, differentiation of (2) with respect to x after the substitution for ξ yields the following.

$$\begin{aligned} &\left[\frac{\partial \tau(X_s, X)}{\partial x} + \frac{\partial \tau(X, X_r)}{\partial x}\right] + \left[\frac{\partial \tau(X_s, X)}{\partial z} + \frac{\partial \tau(X, X_r)}{\partial z}\right] \frac{dz}{dx} + \\ &\left[\frac{\partial \tau(X_s, X)}{\partial \xi} + \frac{\partial \tau(X, X_r)}{\partial \xi}\right] \left[\frac{\partial \xi}{\partial x} + \frac{\partial \xi}{\partial z}\frac{dz}{dx}\right] = \frac{dT}{d\xi}\left[\frac{\partial \xi}{\partial x} + \frac{\partial \xi}{\partial z}\frac{dz}{dx}\right]. \end{aligned} \tag{3}$$

However, the envelope condition, (2), allows us to eliminate all terms involving derivatives with respect to ξ, leading to the result that the first line here is equal to zero. That equation can be rewritten as

$$[\nabla_X \, \tau(X_s, X) + \nabla_X \, \tau(X, X_r)] \cdot \frac{dX}{dx} = 0. \tag{4}$$

This is just a statement of the law of reflection. Hence, the curve $z = f(x)$, defined as the envelope of the family of curves in (1), is the reflector. This completes the proof.

Thus, for each ξ, we can determine the position of the reflection point, (x, z), by equations (1) and (2). In general, these equations are difficult to solve. We only consider some special cases here.

Suppose, for example, that the medium velocity is a constant, c, and the datum surface L is the x-axis. Then

$$X_s = (x_s, 0), \qquad X_r = (x_r, 0),$$

$$\tau(X_s, X) = \rho_s / c, \qquad \tau(X, X_r) = \rho_r / c,$$

where

$$\rho_s = \sqrt{(x_s - x)^2 + z^2}, \qquad \rho_r = \sqrt{(x_r - x)^2 + z^2}.$$

For this case, equation (3) and (2) are simplified to

$$\rho_s + \rho_r = cT(\xi), \tag{5}$$

$$\frac{\partial \rho_s}{\partial \xi} + \frac{\partial \rho_r}{\partial \xi} = cT'(\xi). \tag{6}$$

If we fix the horizontal coordinate, x, of the reflection point, then z and ξ can be considered as functions of the velocity c. Differentiating equation (5) with respect to c,

$$\left[\frac{\partial \rho_s}{\partial z} + \frac{\partial \rho_r}{\partial z}\right] \frac{dz}{dc} + \left[\frac{\partial \rho_s}{\partial \xi} + \frac{\partial \rho_r}{\partial \xi}\right] \frac{d\xi}{dc} = c\,T^{'}(\xi) + T(\xi).$$

Now, by using equation (6) to eliminate $\partial\rho_s/\partial\xi$ and $\partial\rho_s/\partial\xi$, and by using

$$\frac{\partial \rho_s}{\partial z} = \frac{z}{\rho_s}, \qquad \frac{\partial \rho_r}{\partial z} = \frac{z}{\rho_r},$$

we have

$$(z/\rho_s + z/\rho_r)\,\frac{dz}{dc} = T(\xi) = (\rho_s + \rho_r)/c.$$

Solving for dz/dc, then

$$\frac{dz}{dc} = \frac{\rho_s \rho_r}{c\,z} > 0. \tag{7}$$

Introduce angles θ and ϕ as in Figure 1. Then,

$$\rho_s = z/\cos(\theta - \phi),$$

$$\rho_r = z/\cos(\theta + \phi),$$

$$\frac{dz}{dc} = \frac{z}{c\cos(\theta - \phi)\,cos(\theta + \phi)}. \tag{8}$$

From (7), it follows that the imaged depth coordinate of reflection for fixed x (i.e, fixed trace location) is erroneous, when an incorrect velocity is used. Moreover, the deviation is positive when c is bigger than the true velocity ($dc > 0$), and negative when c is smaller than the true velocity ($dc < 0$). Let us discuss the relationship between the deviation and the position of the source and receiver. Let c^* denote the true velocity. At $c = c^*$, z is independent of x_s and x_r. With no loss of generality, we suppose that the source is to the left of the receiver, that is, $\theta > 0$.

Common-shot data

From Figure 1,

$$x_s - x = -z\tan(\theta - \phi). \tag{9}$$

If we move from one shot point to another, i.e. x_s varies, but hold x fixed, then differentiating with respect to x_s in equation (9) yields

$$1 = \frac{-z}{\cos^2(\theta - \phi)} \frac{d\theta}{dx_s},$$

or

$$\frac{d\theta}{dx_s} = -\frac{\cos^2(\theta - \phi)}{z}. \tag{10}$$

By differentiating (8) with respect to x_s, using equation (10), and noticing that

$$\frac{\partial z}{\partial x_s} = 0 \text{ at } c = c^*,$$

we find that

$$\frac{\partial^2 z}{\partial x_s\, \partial c} = -\frac{\sin 2\theta}{c\cos^2(\theta + \phi)} < 0, \text{ at } c = c^*. \tag{11}$$

This tells us that the deviation *decreases* as the source moves right ($\Delta x_s > 0$).

Now, we try to obtain the error estimates in velocity analysis, which can be derived from (11). From Figure 1 and the law of sines,

$$\frac{\sin 2\theta}{x_r - x_s} = \frac{\cos(\theta + \phi)}{\rho_s} = \frac{\cos(\theta - \phi)}{\rho_r}. \tag{12}$$

Then (11) becomes

$$\frac{\partial^2 z}{\partial x_s\, \partial c} = \frac{(x_s - x_r)\, \rho_r}{c\, z\, \rho_s}, \text{ at } c = c^*. \tag{13}$$

Hence,

$$\frac{\partial z}{\partial x_s} \approx \left.\frac{\partial z}{\partial x_s}\right|_{c=c^*} + \left.\frac{\partial^2 z}{\partial x_s\, \partial c}\right|_{c=c^*} (c - c^*) = \frac{(x_s - x_r)\, \rho_r (c - c^*)}{c^*\, z\, \rho_s}. \tag{14}$$

Suppose that we have two shots x_{s_1}, x_{s_2} and $x_{s_1} < x_{s_2}$. Then the difference in imaged depths between the two shots can be given by

$$z(x_{s_2}) - z(x_{s_1}) \approx \frac{\partial z(x_0)}{\partial x_s} (x_{s_2} - x_{s_1}) \approx (x_{s_0} - x_{r_0})\, (x_{s_2} - x_{s_1})\, \frac{(c - c^*)}{c^*\, z} \frac{\rho_r}{\rho_s},$$

where $x_{s_0} = (x_{s_2} + x_{s_1})/2$. Thus,

$$\frac{(c - c^*)}{c^*} \approx \frac{(z(x_{s_2}) - z(x_{s_1}))\, z}{(x_{s_0} - x_{r_0})\, (x_{s_2} - x_{s_1})} \frac{\rho_s}{\rho_r} = \frac{\Delta z\, z}{\Delta x_s\, (x_{s_0} - x_{r_0})} \frac{\rho_s}{\rho_r}. \tag{15}$$

Obviously, the quotient ρ_s/ρ_r is greater than 1 for the negative dip angle, and smaller than 1 for the positive dip angle.

Relationship (15) shows us the factors that govern the accuracy of velocity analysis, $(c - c^*)/c^*$, under the assumption that the medium velocity is constant. *The accuracy of velocity analysis is best for large source-to-receiver offset, well-separated shot points, and shallower target. Interestingly, it is better also for reflectors with positive dip(.i.e., receivers located in the down-dip direction relative to the shot point).*

Common-offset data

Let h be half the offset. Then,

$$2\,h = z\,(\tan(\theta-\phi)+\tan(\theta+\phi)).$$

Similar to the deduction of (10), we find that

$$\frac{\partial^2 z}{\partial h\,\partial c} = \frac{2\,\sin 2\theta}{c\,(\cos^2(\theta+\phi)+\cos^2(\theta-\phi))} > 0,\ \text{at}\ c=c^*. \tag{16}$$

Furthermore, from (12), (16) becomes

$$\frac{\partial^2 z}{\partial h\,\partial c} = \frac{2\,h}{c\,z}\frac{2\,\rho_s\rho_r}{\rho_s^2+\rho_r^2},\ \text{at}\ c=c^*, \tag{17}$$

and

$$\frac{\partial z}{\partial h} \approx \frac{2\,h}{c^*\,z}\frac{2\,\rho_s\rho_r}{\rho_s^2+\rho_r^2}\,(c-c^*). \tag{18}$$

Suppose that we have two offsets h_1, h_2 and $h_1 < h_2$. Then the difference in imaged depths for the two offsets is given by the approximation

$$z(h_2)-z(h_1) \approx \frac{\partial z(h_0)}{\partial h}\,(h_2-h_1) \approx 2\,h_0\,(h_2-h_1)\,\frac{(c-c^*)}{c^*\,z}\,\frac{2\,\rho_s\rho_r}{\rho_s^2+\rho_r^2},$$

where $h_0 = (h_2+h_1)/2$. Thus,

$$\frac{(c-c^*)}{c^*} \approx \frac{(z(h_2)-z(h_1))\,z}{2\,h_0\,(h_2-h_1)}\frac{\rho_s^2+\rho_r^2}{2\rho_s\rho_r} = \frac{\Delta z\,z}{2\,\Delta h\,h_0}\frac{\rho_s^2+\rho_r^2}{2\rho_s\rho_r}. \tag{19}$$

The quotient $(\rho_s^2+\rho_r^2)/2\rho_s\rho_r$ is greater than 1 for any given dip angle.

The relationship (19) shows us that *the accuracy of velocity analysis deteriorates with increasing reflector depth and dip, and is best when the two offsets are large and greatly different from one another.*

Note, from (19), we can conclude that any error of velocity and the difference of the offsets results in nonzero deviation Δz. More precisely,

$$\Delta z \approx (c-c^*)(h_2-h_1)\left.\frac{\partial^2 z(h_0)}{\partial h\,\partial c}\right|_{c=c^*}.$$

Therefore, we define the quantity $\partial^2 z/\partial h\partial c$ as *the sensitivity to the velocity error.*

Multiple-layer case

If the medium is made up of more than one layer, the expression for the error estimate should be modified. We consider only a simple model that consists of two horizontal layers in the common-offset situation, as in Figure 2. Differentiating equation (3) with respect to c_2, and using (2), we have

$$\left(\frac{\partial\tau(X_s,X)}{\partial z}+\frac{\partial\tau(X_r,X)}{\partial z}\right)\frac{dz}{dc_2}=-\left(\frac{\partial\tau(X_s,X)}{\partial c_2}+\frac{\partial\tau(X_r,X)}{\partial c_2}\right). \tag{20}$$

For simplicity, we assume that the true value of c_2 is equal to c_1. Then, $\theta_1=\theta_2$, and

$$\tau(X_s,X)=\tau(X_r,X)=z/c_1\cos\theta_1,$$

$$\frac{\partial\tau(X_s,X)}{\partial z}=\frac{\partial\tau(X_r,X)}{\partial z}=\frac{\cos\theta_1}{c_1},$$

$$\frac{\partial\tau(X_s,X)}{\partial c_2}=\frac{\partial\tau(X_r,X)}{\partial c_2}=\frac{-d_2}{c_1^2\cos\theta_1}.$$

From this and (20), we find that

$$\left.\frac{dz}{dc_2}\right|_{c_2=c_1}=\frac{d_2}{c_1\cos^2\theta_1}. \tag{21}$$

Furthermore, the sensitivity to the velocity error is given by

$$\left.\frac{\partial^2 z}{\partial h\,\partial c_2}\right|_{c_2=c_1}=\frac{d_2}{d_1+d_2}\frac{2\tan\theta_1}{c_1}. \tag{22}$$

Compared to (16) (let $\phi=0$), equation (22) shows us that for multiple layers, *the sensitivity to the velocity error in a thin layer is reduced by the ratio of the layer thickness to layer reflector depth.*

Note, for general background velocity, we still can obtain a formula for the depth variation to the background velocity. However, this variation may be negative when the background velocity increases. All discussions are in Appendix A.

SPECIAL TECHNIQUES

Iteration for velocity

If we view the depth difference Δz as a function of velocity c, then the correct velocity c^* is the one for which

$$\Delta z(c)=0.$$

Based on this observation, we use a simple iteration scheme to determine the velocity.

1. Choose initial velocities c_1 and c_2 such that

$$\Delta z(c_1) < 0, \qquad \Delta z(c_2) > 0.$$

2. Compute a new velocity by weighting the initial velocities as follows:

$$c_3 = c_1 \frac{\Delta z(c_2)}{\Delta z(c_2) - \Delta z(c_1)} + c_2 \frac{\Delta z(c_1)}{\Delta z(c_1) - \Delta z(c_2)}.$$

3. If $\Delta z(c_3) = 0$ (or smaller than a given precision), the iteration will stop, with c_3 the desired velocity. If $\Delta z(c_3) > 0$, update c_3 replacing c_2 in step(1); otherwise, if $\Delta z(c_3) < 0$, update c_3 replacing c_1.

Smoothing the interface

Ray tracing in the inversion code is stable when the description of the interface has second-order smoothness. Consequently, smoothing of the interface before ray tracing is desirable.

Let $z = f(x)$ be any continuous function. We solve for a smooth function $g(x)$ that approximates $f(x)$ through the requirement

$$\int (f(x) - g(x))^2 \, dx + \alpha \int \left(\frac{d^2 g}{dx^2} \right)^2 dx = \min, \tag{23}$$

where $\alpha > 0$ is called the *smoothing parameter.* The larger the value of α, the smoother will be $g(x)$.

By calculus of variations, we can change (23) into a differential equation for $g(x)$. For any positive number λ and any smooth function with zero boundary condition, $\eta = \eta(x)$, we define a functional,

$$B(\lambda, \eta) = \int [f(x) - (g(x) + \lambda\eta)]^2 \, dx + \alpha \int \left(\frac{d^2 g}{dx^2} + \lambda \frac{d^2 \eta}{dx^2} \right)^2 dx.$$

Then (23) is equivalent to $\frac{dB}{d\lambda} \mid_{\lambda=0} = 0$, for any η. That is,

$$\int [g(x) - f(x)] \eta(x) \, dx + \alpha \int \frac{d^2 g}{dx^2} \frac{d^2 \eta}{dx^2} \, dx = 0.$$

Using integration by parts, we have

$$\int \left[g(x) - f(x) + \alpha \frac{d^4 g}{dx^4} \right] \eta(x) \, dx = 0.$$

Since η is arbitrary, this equality is satisfied if and only if

$$\alpha \frac{d^4 g}{dx^4} + g(x) = f(x).$$

Taking the Fourier transform gives the solution in the wavenumber domain

$$G(k) = F(k)/(1 + \alpha k^4).$$

This expression shows that high-wavenumber components of $f(x)$ are suppressed in the approximation $g(x)$.

COMPUTER IMPLEMENTATIONS

To demonstrate the effectiveness and the efficiency of our method, we practice some numerical examples by using common-offset experiments. The inversion algorithm is based on the assumption that the medium is two-and-half dimensional (Bleistein, et al., 1987; Hsu, 1991).

Example 1: Modeling data

First, we take synthetic common-offset data from the layered model shown in Figure 3. The input synthetic data were obtained by a common-offset modeling program. The synthetic data are generated for five gathers with common offsets 100 m, 300 m, 500 m, 700 m, and 900 m, with shots and receivers on the horizontal top surface. The first shot is at $x = 100$ m, and the shot point spacing is 20 m. Each offset uses 100 shots and receivers. The sampling interval is 4 ms, and the total reflection time is 2 sec. The inversion output spans the ranges 200 m to 1800 m in x, and 0 to 3000 m in z. Velocity analysis is done through the third layer as in Figure 7. After obtaining the velocity in one layer, we pick an interface from the inversion output with this approximate velocity, then smooth it. The process is repeated recursively through all the layers. The results are as follows:

1. In the first layer, the iterative values of c_1 are 1500, 3000, 2013; the true value is 2000.

2. In the second layer, the iterative values of c_2 are 2013, 3500, 2863, 3022; the true value is 3000.

3. In the third layer, the iterative values of c_3 are 3022, 4500, 4007; the true value is 4000.

Using the model consisting of these approximate velocities and interfaces, we obtain an inversion output which is very close to the inversion output using the exact model consisting of the accurate velocities and interfaces (see Figures 5 and 6).

Now we test the sensitivity to velocity error. The theoretical values are computed from equations (19) and (22). In all layers, take $\Delta c = 500$ m/s, $\Delta h = 400$ m, and $h_0 = 250$ m. In the first layer, the theoretical Δz is 100 m, the measured value is 130 m; in the second layer, the theoretical Δz is 17 m, the measured value is 30 m; in the third layer, the theoretical Δz is 12.5 m, the measured value is 10 m. The least measurable Δz (the sample spacing) is 10 m and this limits the accuracy of velocity analysis. The errors between the theoretical and measured Δz are 1/4 to 1/2 wavelength at the dominant frequency.

Example 2: Marathon data

The input data is from a physical experiment. It was provided to us by Marathon Oil Company. The real medium can be approximated to a two-and-half dimensional model. The data were 296 shots, each shot with 48 receivers. The shot point spacing is 24.38 m, and the receiver interval is 24.38 m. We sorted the data into five gathers of common-offset with offsets 268.2 m, 512.1 m, 755.9 m, 999.7 m, and 1243.6 m. The first shot point is at $x = 0$. For each offset, there are 256 shots and receivers. The sampling interval is 4 ms; the total time is 2 s. The inversion output spans the ranges x from 61 to 6949 m and z from 0 to 3658 m. In this problem, we do not know the "true" velocities. Instead, we have the measured velocities from the experiment. Velocity analysis is done through the fourth layer. The results are as follows:

1. In the first layer, the iterative values of c_1 are 2438, 3962, 3309, 3570; the measured value is 3581.
2. In the second layer, the iterative values of c_2 are 3570, 5182, 4779; the measured value is 4801.
3. In the third layer, the iterative values of c_3 are 4779, 7315, 6681; the measured value is 6831.
4. In the fourth layer, the iterative values of c_4 are 6681, 3962, 4869; the measured value is 4801.
5. In the fifth layer, the iterative values of c_5 are 4869, 9144, 5812, 6234; the measured value is 6066.

After velocity analysis, we obtain a velocity model that consists of constant velocity layers and interfaces; then, we implement inversion for each offset. The post-stack inversion is shown in Figure 9. We can see clearly the steep dip on the third interface, the saw-tooth reflector on the fourth interface, and the flat bottom reflector. In contrast, a poor inversion result (shown in Figure 10 is obtained with the measured velocities provided to us. The structural images are distorted, so we can conclude that this velocity model is incorrect.

CONCLUSIONS

In the previous section, we showed computer implementations of our velocity analysis technique for both synthetic data and experimental data. The numerical results show that our method can obtain high accuracy in the estimate of velocities and the imaging of interfaces. Unfortunately, as a practical inversion method, some questions remain.

Model limitations

The present inversion code requires that the medium be made up of constant-velocity layers separated by smooth interfaces. However, in the actual subsurface, interfaces may touch each other or terminate abruptly, and velocities may vary within a layer. Usually, we separate these interfaces on purpose in order to guarantee a successful inversion and assume velocities are constants, although it may produce a model error. To solve this problem completely, a new inversion code needs to be devised for more general models.

Selection of the output trace

In theory, one output trace with a fixed horizontal coordinate is sufficient to determine the velocity. However, if the error in the model cannot be ignored, such as estimate of velocity may be unstable. A better way is to select more output traces and then to take the average value of these velocities from the different traces.

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REFERENCES

Al-Yahya, Kamal, 1989, Velocity analysis by iterative profile migration: Geophysics, **54**, 718-729.

Bleistein, N., 1987, On the imaging of reflectors in the earth: Geophysics, **52**, 931-942.

Bleistein, N., Cohen, J., and Hagin, F., 1987, Two and one-half dimensional Born inversion with an arbitrary reference: Geophysics, **52**, 26-36.

Hsu, C., 1991, The difference between CXZCO and CXZCS: Center for Wave Phenomena Project Review, CWP-107.

Jeannot, J. P., Faye, J. P., Dennelle, E., 1986, Prestack migration velocities from depth-focusing analysis: 56th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 438-440.

Liu, Z. and Bleistein, N., 1992, Velocity analysis by residual moveout: 62nd Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1034-1037.

MacKay, S., and Abma, R., 1992, Imaging and velocity estimation with depth-focusing analysis: Geophysics, **57**, 1608-1622.

APPENDIX A: APPENDIX A: DEPTH VARIATION TO VELOCITY

Suppose that velocity c depends on parameter λ and variable X, i.e.,

$$c = c(\lambda; X).$$

For a fixed horizontal location x, we differentiate equation (3) with respect to λ; then

$$\left[\frac{\partial \tau_s}{\partial \xi} + \frac{\partial \tau_r}{\partial \xi}\right]\frac{d\xi}{d\lambda} + \left[\frac{\partial \tau_s}{\partial \lambda} + \frac{\partial \tau_r}{\partial \lambda}\right] + \left[\frac{\partial \tau_s}{\partial z} + \frac{\partial \tau_r}{\partial z}\right]\frac{dz}{d\lambda} = T^{'}(\xi)\frac{d\xi}{d\lambda}. \tag{A-1}$$

Using (2), we obtain

$$\left[\frac{\partial \tau_s}{\partial z} + \frac{\partial \tau_r}{\partial z}\right]\frac{dz}{d\lambda} = -\left[\frac{\partial \tau(X_s, X)}{\partial \lambda} + \frac{\partial \tau(X, X_r)}{\partial \lambda}\right].$$

From

$$\frac{\partial \tau(X_s, X)}{\partial z} = \frac{\cos(\theta_s)}{c(\lambda; X)}, \qquad \frac{\partial \tau(X_r, X)}{\partial z} = \frac{\cos(\theta_r)}{c(\lambda; X)},$$

where θ_s or θ_r are angles between raypaths, from the source or the receiver, and the vertical at X. We have that

$$\frac{\cos(\theta_s) + \cos(\theta_s)}{c(\lambda; X)}\frac{dz}{d\lambda} = -\left[\frac{\partial \tau(X_s, X)}{\partial \lambda} + \frac{\partial \tau(X, X_r)}{\partial \lambda}\right].$$

Proposition. If $\partial c(\lambda; X)/\partial \lambda > 0$ and the normal direction of the reflector is upwind, then

$$\frac{dz}{d\lambda} > 0.$$

Proof. By the given condition, $|\theta_s + \theta_r|/2 < 90^0$. Therefore,

$$\cos(\theta_s) + \cos(\theta_r) = 2\cos(\frac{\theta_s + \theta_r}{2})\cos(\frac{\theta_s - \theta_r}{2}) > 0,$$

and

$$\frac{\partial \tau(X_s, X)}{\partial \lambda} < 0, \qquad \frac{\partial \tau(X_r, X)}{\partial \lambda} < 0.$$

From these results it follows that

$$\frac{dz}{d\lambda} > 0.$$

This completes the proof.

Example 1. Velocity $c(z) = c_0 + az$. Here λ represents c_0 or a. When the normal direction is upwind, the imaged depth will increase with increment of c_0 or a. However, When the normal direction is downwind (turning wave), the imaged depth will decrease with increment of c_0 or a.

Example 2. Velocity c is a pieces of constants, $c_1, c_2, ..., c_n$. In this situation, no turning wave exists. Therefore the imaged depth will increase with increment of any c_i.