GEOPHYSICS, VOL. 57, NO. 3 (MARCH 1992); P. 474–477, 5 FIGS.

Short Note

Transformation to zero offset for mode-converted waves

Mohammed Alfaraj* and Ken Larner*

INTRODUCTION

The transformation to zero offset (TZO) of prestack seismic data for a constant-velocity medium is well understood and is readily implemented when dealing with either P-waves or S-waves. TZO is achieved by inserting a dip moveout (DMO) process to correct data for the influence of dip, either before or after normal moveout (NMO) correction (Hale, 1984; Forel and Gardner, 1988). The TZO process transforms prestack seismic data in such a way that common-midpoint (CMP) gathers are closer to being common reflection point gathers after the transformation.

Converted P-SV or SV-P waves are different from ordinary P-P or S-S waves in that the downgoing and reflected upgoing waves travel at different velocities, even in an isotropic, homogeneous medium. This makes the kinematics more complicated than that encountered in the absence of mode conversion. One way of dealing with this complication is to approximate the kinematics of converted waves so that they resemble those of ordinary waves (Sword, 1984). Unfortunately, such an approximation is valid only for small offsets and, consequently, cannot be used to process large-offset seismic data.

To our knowledge, TZO for converted waves has, thus far, been discussed in just two papers. Harrison (1990) and Den Rooijen (1991) have proposed exact solutions for the TZO process for converted waves in a constant-velocity medium. Their methods, which are similar, require prior knowledge of both the downgoing and upgoing velocities. The implementation is carried out in such a fashion that the recorded nonzero-offset time t is directly mapped into its corresponding zero-offset time t_0.

Here, we propose another method to solve the TZO problem for converted waves (either P-SV or SV-P) in a constant-velocity medium. This method does not require prior knowledge of the velocities; rather, it requires only that the *ratio* of the two velocities, downgoing and upgoing, be known.

Our approach is an application to mode-converted waves of the same trick introduced by Forel and Gardner (1988) for TZO processing of ordinary P-waves in a constant velocity medium. In their approach (the Gardner method), a transformation of time and offset that removes the dip dependence of stacking velocity precedes NMO correction and can be implemented exactly without knowledge of the velocity. The stacking velocity can then be determined from velocity analysis performed after that transformation has been performed on the data. The velocity analysis, in turn, yields the dip-independent velocity that best stacks the data.

In this note, we show that the Gardner transformation can be extended and applied to converted waves, if the velocity ratio is known.

THE KINEMATICS OF CONVERTED WAVES

From geometrical considerations, the equation describing the two-way traveltime for a converted wave can be written as

$$t = \frac{\sqrt{(x+h)^2 + z^2}}{v} + \frac{\sqrt{(x-h)^2 + z^2}}{\gamma v}, \qquad (1)$$

where

(x, z) : coordinates of the reflection point,
h : half-offset between source and receiver along the surface $z = 0$,
v : velocity for the path from the source to the reflector,
γv : velocity for the path from the reflector to the receiver ($\gamma < 1$ implies P-SV conversion, and $\gamma > 1$ implies SV-P conversion).

Figure 1 shows the geometry pertaining to equation (1). In that figure, the midpoint is at the origin, s denotes the source location, and g denotes the receiver location. Unlike the situation for ordinary P-waves, for mode-converted waves the angles of incidence and reflection differ (i.e., symmetry is broken), as shown in the figure.

Manuscript received by the Editor February 19, 1991; revised manuscript received August 19, 1991.
*Colorado School of Mines, Dept. of Geophysics, Golden, CO 80401.

Following Sword (1984), z^2 can be obtained from equation (1) as

$$z^2 = \frac{1}{\beta^2}\left\{-\beta\left[(x+h)^2 - \frac{1}{\gamma^2}(x-h)^2\right] + \alpha(vt)^2 - \frac{2vt}{\gamma}\sqrt{(vt)^2 - 4\beta xh}\right\}, \quad (2)$$

where

$$\alpha \equiv 1 + \frac{1}{\gamma^2},$$

$$\beta \equiv 1 - \frac{1}{\gamma^2}.$$

For constant γ, h, v, and t, the graph of z versus x looks like a distorted ellipse. An example of such a pseudo-ellipse is shown in Figure 2.

At an arbitrary subsurface reflection point x, z on the pseudo-ellipse (Figure 3), we construct the circle that is tangent to the pseudo-ellipse and has its center along the line connecting the source and receiver. Let the radius of the circle be R and the center be at the point b, 0 relative to the midpoint. To convert CMP gathers to common-reflection-point gathers, TZO must move the nonzero-offset reflection to a location that is at the distance b from the original midpoint (Hale, 1988). The equation describing the circle is thus

$$(x-b)^2 + z^2 = R^2. \quad (3)$$

To satisfy the tangency requirement, the slope of the pseudo-ellipse and that of the circle must be the same at the point of tangency. Differentiating equations (2) and (3) with respect to x yields, respectively,

$$z\frac{dz}{dx} = \frac{1}{\beta^2}\left\{-\beta\left[(x+h) - \frac{1}{\gamma^2}(x-h)\right] + \frac{vt}{\gamma}\frac{2\beta h}{\sqrt{(vt)^2 - 4\beta xh}}\right\}, \quad (4)$$

and

$$(x-b) + z\frac{dz}{dx} = 0. \quad (5)$$

Now, from equations (2), (3), (4), and (5), the radius R can be obtained as a function of v, t, γ, b, h. Specifically, using equations (2) and (3) to eliminate z^2, using equations (4) and (5) to eliminate $z dz/dx$, and after a fair amount of algebra and simplification, we obtain

$$R^2 = (h^2 - b^2)\left[\frac{(vt)^2}{2h(\alpha h + \beta b)} - 1\right]. \quad (6)$$

Harrison (1990) derived a result that is equivalent to equation (6), differing only in that the downgoing and upgoing velocities are used explicitly. Figure 4 shows how the pseudo-ellipse of Figure 2 is constructed as the envelope of circles whose radii are calculated from equation (6).

Equation (6) will next be used to facilitate the derivation of TZO for converted waves.

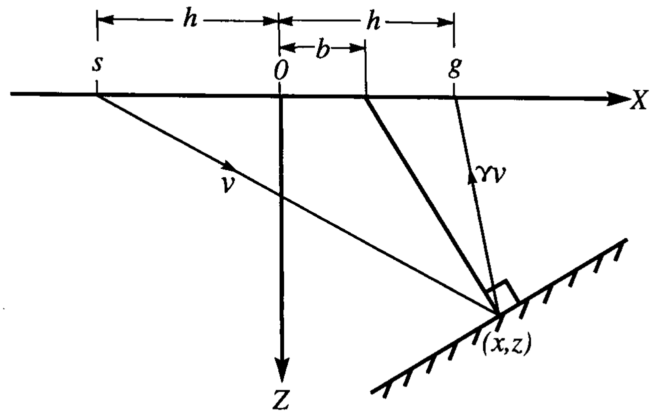

FIG. 1. Depth section depicting a mode-converted, reflection raypath in a homogeneous medium.

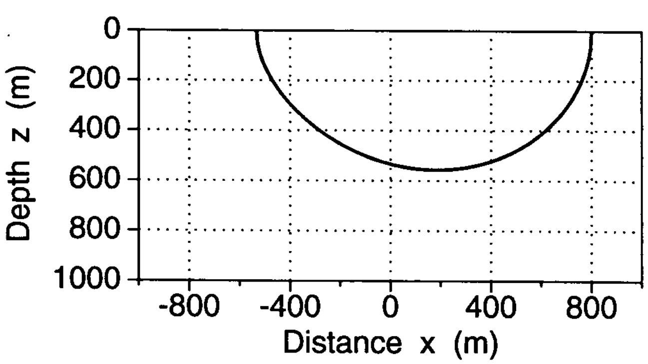

FIG. 2. The locus of z as a function of x looks somewhat like an ellipse, for constant γ, h, v, and t. Here $\gamma = 0.5$, $h = 400$ m, $v = 2000$ m/s, and $t = 1$ s. The midpoint is located at $x = 0$.

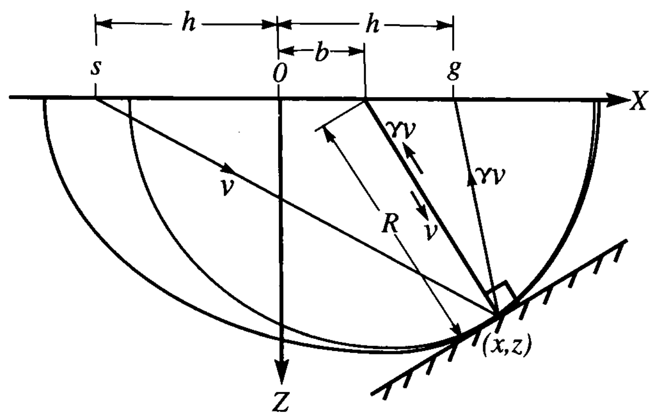

FIG. 3. A circle with radius R and center at $(b, 0)$ is tangent to the pseudoellipse at reflection point (x, z). The displacement b from the midpoint is the same as that in Figure 1.

TZO FOR CONVERTED WAVES

The previous analysis on the kinematics of converted waves was performed in the t, h domain—the physical domain. Following Forel and Gardner (1988), TZO will now be looked at and implemented in a different, nonphysical domain referred to as the t_1, k domain; that is, the data in the t, h domain will be transformed into this t_1, k domain.

Referring to Figure 3, for mode-converted waves (ignoring the fact that mode conversion does not truly occur at normal incidence) the two-way normal-incidence time associated with reflection point (x, z) is given by

$$t_0 = \frac{R}{v} + \frac{R}{\gamma v} = \frac{R}{v}\left(1 + \frac{1}{\gamma}\right),$$

or

$$t_0 = \frac{2R}{v_a}, \tag{7}$$

where the average velocity v_a is given by

$$v_a \equiv \frac{2v}{\sigma}, \tag{8}$$

with

$$\sigma \equiv 1 + \frac{1}{\gamma}.$$

Substitution of equations (7) and (8) for R in equation (6) yields

$$t_0^2 = (h^2 - b^2)\left[\frac{(\sigma t)^2}{2h(\alpha h + \beta b)} - \left(\frac{2}{v_a}\right)^2\right], \tag{9}$$

or

$$t_1^2 = t_0^2 + \left(\frac{2k}{v_a}\right)^2, \tag{10}$$

where

$$k^2 \equiv h^2 - b^2, \tag{11}$$

and

$$t_1 \equiv \frac{\sigma k}{\sqrt{2h(\alpha h + \beta b)}}\, t. \tag{12}$$

The quantity k is the same offset-dependent parameter obtained by Forel and Gardner for ordinary waves. The newly defined time t_1 is a scaled version of the recorded time t. For $\gamma = 1$ (e.g., ordinary P-waves), the scaling factor reduces to k/h, the same factor derived by Forel and Gardner. Note also, that the mapping given by equation (12) is a function of the velocity ratio γ but not of the individual P- and S-waves velocities themselves.

In terms of the transformed offset k and the transformed time t_1, equation (10) is seen to be just a simple hyperbolic time-offset relationship (i.e., standard NMO equation) for converted waves. It maps nonzero-offset data at transformed time t_1 to zero-offset time t_0. Moreover, just as for ordinary waves, by equation (8) the moveout velocity v_a in equation (10) is independent of dip, one of the goals in transforming data to zero offset. Furthermore, as shown in Figure 1, after TZO the zero-offset data and the recorded nonzero-offset data pertain to a *common reflection point*. That is, TZO has removed the problem of reflection-point dispersal for mode-converted data, just as it does for ordinary-wave data. In addition, recall that for ordinary P-wave data, reflection-point dispersal is not an issue when the reflector is horizontal. For mode-converted data, however, it is. TZO, as described here, removes reflection-point dispersal for mode-converted data when the reflector is horizontal, as well as when it has dip.

As an aside, note that for a given offset h, relationship (9) between t_0 and b defines the trajectory of the response of the TZO process to an impulse at the source-receiver midpoint and at time t (i.e., the shape of the TZO operator). Figure 5 is a plot of this function for the same parameters used in Figure 2. Note that equation (9) reduces to an ellipse in t_0, b when $\gamma = 1$ (i.e., $\beta = 0$). For $\gamma \neq 1$, we once again observe the broken symmetry for mode-converted data.

CONCLUSION

We have shown that the problems of dip dependence of stacking velocity as well as reflection-point dispersal can be corrected simultaneously for converted-wave data by simply

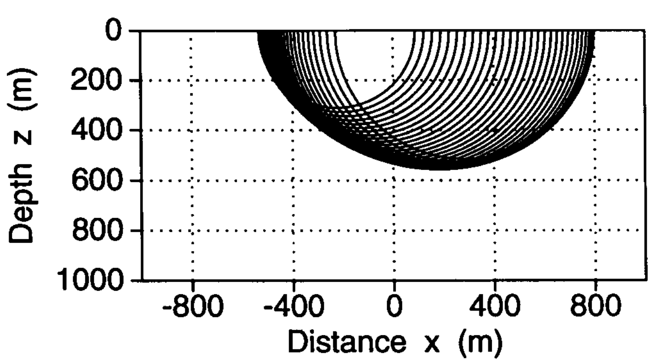

FIG. 4. The curve of Figure 2 is constructed from circles whose radii are given by equation (6).

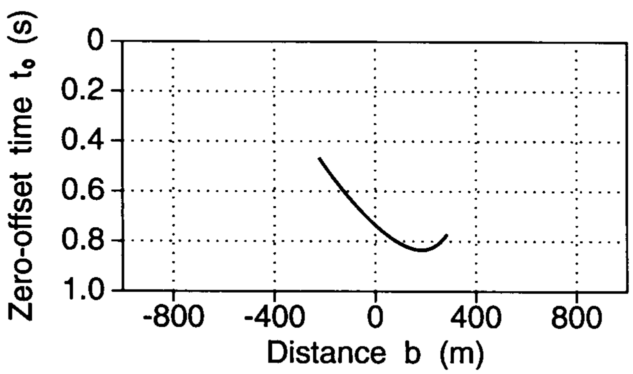

FIG. 5. Shape of the impulse response $t_0(b)$ for the TZO operation for an impulse at time t = 1 s. The parameters here are the same as those in Figure 2.

transforming the data from the t, h domain to the t_1, k domain in accordance with equations (11) and (12), a generalization of the Gardner method that was developed for ordinary P-waves. Furthermore, conventional velocity analysis in the t_1, k domain yields the average velocity v_a as the velocity that best stacks the data (i.e., the stacking velocity). TZO is completed by stacking the individual NMO-corrected t_1, k domain gathers to yield reflections at their true zero-offset reflection times, because t_0 in the t_1, k domain expression is the same as t_0 in the t, h domain. Inverse transformation of the data from the t_1, k domain to the t, h domain, therefore, is not needed. The t_1, k domain is just a vehicle to remove the influence of dip on stacking velocity, to correct for reflection-point dispersal, and in general, to avoid the broken symmetry (from the kinematics of converted waves) arising in the t, h domain. Specifically, its use also removes the reflection-point dispersal that arises for mode-converted data even for horizontal reflectors.

As with Gardner's method, this approach to TZO processing for converted waves holds strictly only in homogeneous, isotropic media. More development is necessary to assess the applicability and value of the method in practice, where, for example, velocity is a function of spatial position as well as of propagation direction. Also, only the kinematics of converted-wave TZO have been considered here. For this TZO process to be of a practical use, issues of amplitude and phase need to be addressed and properly incorporated into the TZO treatment.

ACKNOWLEDGMENTS

We thank Professors Norm Bleistein and Dave Hale for their critical review of the manuscript, the support of the Center for Wave Phenomena, and the reviewers for their valuable suggestions.

REFERENCES

Forel, D., and Gardner, G., 1988, A three-dimensional perspective on two-dimensional dip moveout: Geophysics, **53**, 604–610.

Hale, D., 1984, Dip-moveout by Fourier transform: Geophysics, **49**, 741–757.

——— 1988, Dip moveout processing: Soc. Expl. Geophys. course notes.

Den Rooijen, H., 1991, Stacking of p-sv seismic reflection data using dip moveout: Geophys. Prosp., **39**, 585–598.

Harrison, M., 1990, Dip moveout for converted wave (P-SV) data in a constant velocity medium: 60th Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1370–1373.

Sword, C., 1984, Approximating the kinematics of converted waves: Stanford Explor. Proj. Rep. 41, Stanford University.