Finite-Difference Dip Moveout

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ABSTRACT

From similarities in the kinematics of conventional migration and dip-moveout (DMO), I have developed a finite-difference DMO algorithm applied to normal-moveout-corrected, common-offset seismic data. This algorithm employs a traditional poststack 15-degree finite-difference migration algorithm and a modified velocity function rather than the true migration velocity. Tests with both synthetic and field seismic data show accurate treatment of dips up to 90 degrees when velocity varies with depth.

INTRODUCTION

Many papers have been published on the two most commonly used approaches to dip-moveout (DMO): frequency-wavenumber (f-k) DMO (Jakubowicz, 1990; Hale, 1984) and integral DMO (Hale, 1988). In contrast, only a few papers on finite-difference DMO have been published so far. Since finite-difference algorithms play an important role in conventional migration methods, and DMO is a form of migration, finite-difference algorithms might have application there, as well.

Yilmaz and Claerbout (1980) proposed two finite-difference DMO algorithms from analysis of the double square-root equation. Bolondi, et al. (1982) and Salvador and Savelli (1982) presented another finite-difference DMO algorithm, based on the concept of *offset continuation.* These algorithms have generally low efficiency and low accuracy for handling large-offset seismic sections and reflections from steep reflectors.

Here, I present an approximate finite-difference DMO approach applied to NMO-corrected, common-offset data where velocity is a function of depth. Tests on synthetic and field data show this algorithm to be effective for handling vertical velocity variations, large offsets, and reflections from steep interfaces.

DERIVATION

DMO velocity

In constant-velocity media, conventional poststack migration spreads an impulse at the point $(x = 0, t_0)$ in a zero-offset section to the impulse-response trajectory defined by

$$x^2 + z^2 = (vt_0/2)^2, \tag{1}$$

where x is the CMP coordinate, z is depth, t_0 is zero-offset reflection time, and v is the medium velocity. If we let $\tau \equiv 2z/v$ denote vertical time, or time-equivalent depth, equation (1) becomes

$$x^2 + (v\tau/2)^2 = (vt_0/2)^2,$$

or

$$\frac{x^2}{(vt_0/2)^2} + \frac{\tau^2}{t_0^2} = 1. \tag{2}$$

Equations (1) and (2) show that the impulse response of conventional migration is a circle on the x-z plane, while it is an ellipse on the x-τ plane.

Now, consider the behavior of DMO. In constant-velocity media, the DMO ellipse for an impulse at the fixed point $(x = 0, t_n)$ on an NMO-corrected common-offset section is defined by

$$\frac{x^2}{h^2} + \frac{t_0^2}{t_n^2} = 1, \tag{3}$$

where t_n is the time after NMO correction, and h is half the source-receiver offset (Hale, 1988). If we define a velocity $\tilde{v}$ such that

$$h \equiv \tilde{v} t_n / 2,$$

or

$$\tilde{v}(t_n) \equiv 2h/t_n, \tag{4}$$

and define a depth quantity

$$\tilde{z} \equiv \tilde{v}(t_n) t_0 / 2,$$

then equation (3) becomes

$$x^2 + [\tilde{v}(t_n) t_0/2]^2 = [\tilde{v}(t_n) t_n/2]^2,$$

or

$$x^2 + \tilde{z}^2 = [\tilde{v}(t_n) t_n/2]^2. \tag{5}$$

Equations (3) and (5) indicate that the impulse response of DMO after NMO correction is an ellipse on the x-t_0 plane, while it is a circle on the x-$\tilde{z}$ plane. This behavior of DMO is similar to that of conventional migration.

Comparing equation (3) with equation (2), we find that if the velocity $\tilde{v}(t_n)$ is used instead of the medium velocity v, and τ and t_0 are accordingly replaced by t_0 and t_n in equation (2), then it might appear that any conventional zero-offset migration algorithm can be used to perform DMO processing. With this choice, the impulse response of conventional migration would be just the DMO ellipse defined by equation (3). I call $\tilde{v}$, defined by equation (4), the *DMO velocity*. Furthermore, since both h and t_n are constant for a single impulse at the fixed point $(x = 0, t_n)$, $\tilde{v}$ is a constant. In other words, to get the correct DMO ellipse, a constant DMO velocity $\tilde{v}$ must be used in any conventional migration algorithm. Below we shall see that the situation is unfortunately not quite so simple as this.

Aside from use of a different velocity, the other difference between conventional migration and DMO is that the input to conventional poststack migration is the stacked section, $P(x, t_0)$, and the output is the migrated result, $P(x, \tau)$, while the input to DMO is the NMO-corrected, common-offset section, $P(x, t_n; h)$, and the output is the DMO-corrected, common-offset result, $P(x, t_0; h)$. Here, P denotes seismic data.

Finite-difference DMO for constant-velocity media

In the previous section, we defined in constant-velocity media and showed that conventional migration algorithms could be used to perform DMO. Let us now consider how DMO can be implemented with a traditional poststack, 15-degree finite-difference migration algorithm (Claerbout, 1985). Two reasons for this choice of algorithm are: (1) among all finite-difference migration schemes, this one has the highest computational efficiency, and (2) examples and analysis in later sections show that, for DMO, this seemingly inaccurate algorithm has sufficient accuracy and ability to handle large-offset data where subsurface reflectors are steep. When using a finite-difference algorithm to perform DMO, we must address a complication not yet highlighted: conventional migration depends on the velocity appropriate for the depth point (or its time-equivalent, migrated time τ) from which reflection occurs, not for the unmigrated time t_0. DMO, however, uses the velocity appropriate for the non-DMO-corrected time t_n, not for the DMO-corrected time t_0. According to the derivation of DMO velocity, in the computation of the response of an impulse at the point (x, t_n) $\tilde{v}$ should be a constant from the time t_n to 0, but finite-difference algorithms are recursive processes in the reverse t_n-direction, so that $\tilde{v}$ necessarily increases as t_n decreases.

The impulse responses of finite-difference DMO in Figure 1 show the result of using the velocity defined by equation (4) directly in a poststack migration algorithm. The distance between the projected intersections of the impulse responses with the surface is greater than the offset, $2h$, indicating that the velocity used to obtain this figure is, in effect, too large. For example, to compute the response of the impulse at $t_n = 1.0$ s in this test, a DMO velocity of 2 km/s should be used, but when the computation reaches time $t_n = 0.1$ s recursively in the reverse t_n-direction, finite-difference DMO actually uses a DMO velocity of 20 km/s, a value ten times greater than it should be. To approximately address this problem, I introduce a compensating function $s(t_0)$ into the definition of DMO velocity $\tilde{v}$, defining a new DMO velocity

$$\tilde{\tilde{v}} = 2h/[t_n s(t_0)]. \tag{6}$$

Recognizing that finite-difference DMO operates recursively a great many steps at the minimum processing time, but only one step at the maximum processing time, and the number of recursive steps decreases linearly with t_0, I found empirically that when I choose a $s(t_0)$ function that decreases linearly from about 1.1 at the minimum processing time to 1.0 at the maximum processing time, the impulse responses are squeezed to approximately the correct shapes.

Figure 2 shows impulse responses of finite-difference DMO obtained by using the velocity defined by equation (6), with offset of 2 km in Figure 2a, and 3 km in

Figure 2b. In both cases, the impulse responses project to intersections with the surface that are closer to the shot and receiver points than when $s(t_0)$ is not used. Although dispersion is present in the upper portion of the DMO impulse responses due to use of a 15-degree algorithm, this portion has relatively little influence on the final DMO result because the DMO process only needs the lower portion (Deregowski, 1986; Hale, 1988).

Finite-difference DMO for depth-variable-velocity media

Velocity variation is often ignored in DMO processing, and constant-velocity DMO processing is often used in areas where velocity is known to vary. Many authors, however, have recognized the need to include velocity variation in DMO processing and have attempted to improve the accuracy of DMO approaches when velocity varies with depth. To avoid the high-cost of exact DMO, for example, Hale and Artley (1993) presented an approach that is approximate for $V(z)$ media. Likewise, by introducing Hale's $\gamma(t_n)$ factor in the definition of DMO velocity, we can extend our finite-difference DMO algorithm so that it also can approximately handle velocity variations with depth. Hale (1988) showed that, for depth-variable velocity, the DMO ellipse defined by equation (3) should be modified as

$$\frac{x^2}{\gamma(t_n)h^2} + \frac{t_0^2}{t_n^2} = 1, \tag{7}$$

where

$$\gamma(t_n) = \frac{3v_4^4(t_n)}{2v_2^4(t_n)} - \frac{t_n}{v_2(t_n)}\frac{dv_2}{dt_n} - \frac{1}{2}, \tag{8}$$

$$v_2(t_n) = [\frac{1}{t_n}\int_0^{t_n} v^2(s)ds]^{1/2},$$

and

$$v_4(t_n) = [\frac{1}{t_n}\int_0^{t_n} v^4(s)ds]^{1/4}.$$

If we define

$$\bar{v}(t_n) \equiv 2\sqrt{\gamma}h/t_n \tag{9}$$

and

$$\bar{z} \equiv \bar{v}t_0/2,$$

then we obtain

$$x^2 + \bar{z}^2 = [\bar{v}(t_n)t_n/2]^2 \tag{10}$$

from equation (7). This is also the equation of a circle, now on the $x - \bar{z}$ plane. So, we see from equations (4) and (9) that the constant and depth-variable velocity differ only in the introduction of the factor $\sqrt{\gamma}$ into the definition of DMO velocity. Moreover, since γ is dependent on t_n only, DMO velocity $\bar{v}$, defined by equation (9), is still a function of t_n, just as is $\tilde{v}$ in the constant-velocity case. So, as in the constant-velocity case, we incorporate the function $s(t_0)$ factor into the definition of $V(z)$ DMO velocity, equation (9),

$$\bar{\bar{v}} = 2\sqrt{\gamma}h/[t_n s(t_0)], \tag{11}$$

where, again, $s(t_0)$ decreases linearly from about 1.1 at the minimum processing time to 1.0 at the maximum processing time.

Figure 3 shows the impulse responses of V(z) DMO for offset= 2 km when $\bar{\bar{v}}$ is used. In this test, medium velocity linearly increases from 3 km/s at 0.1 s to 4 km/s at 1.4 s. Comparing Figure 3 with Figure 2a, we see that velocity variation with depth causes the impulse response to narrow slightly. This is consistent with what Hale and Artley (1993) have described.

SYNTHETIC AND FIELD DATA EXAMPLES

Figure 4 shows a geological model containing nine horizontal reflector segments and five dipping reflector segments, with dips ranging from 30 to 90 degrees in 15-degree increments. For the synthetic seismic data, the velocity used increases linearly with depth z, according to $v(z) = 1.5 + 0.8z$ km/s, and the CMP interval is 5 m.

Figure 5a is the zero-offset synthetic section for the model in Figure 4. The goal of applying DMO is to transform offset data into this form. The dashed line in this figure marks the location of the CMP gather shown in Figure 6, and the rectangle shows the region detailed in Figures 5b, 5c, 5d, and 5e. Figure 5b is just a repeat of the portion of the zero-offset section outlined by the rectangle in Figure 5a. Figure 5c is obtained by simply stacking ten NMO-corrected, common-offset sections, whose offsets range from 0.1 to 1.9 km. Note that the dipping reflections, especially the reflection from the 90-degree reflector, have been attenuated, and some noise, due to out-of-phase stacking, appears among the dipping reflections.

The stacked result after NMO and constant-velocity f-k DMO correction (Hale, 1984) is plotted in Figure 5d. Compared with Figure 5c, the stacking quality for dipping reflections in this figure has been significantly improved, but the energy of the dipping reflections (marked by the arrows) is still weaker than in the zero-offset section, Figure 5b. Figure 5e shows the stacked result after NMO and finite-difference $V(z)$ DMO correction. For the γ-factor in the $V(z)$ DMO processing, I used the known rms velocity $v_2(t_n)$and $v_4(t_n)$ from the model. Here, the stacking quality for dipping reflections, especially the reflections from the 70- and the 90-degree reflectors (marked by the arrows), has been further significantly improved and now compares favorably with the ideal result in Figure 5b. The improvement, here, relative to that in Figure 5d is attributable to the use of $V(z)$ DMO, not to the difference between f-k and finite-difference implementation.

Figure 6 shows why the stacked section after $V(z)$ DMO correction is better than the stacked sections after NMO correction. One of the CMP gathers from the model shown in Figure 4 is plotted in Figure 6a. This gather contains six events from the horizontal reflectors, five events from the dipping reflection segments, and several weak diffraction events from the structural corners. The result of NMO correction applied to the CMP gather of Figure 6a is plotted in Figure 6b. Although the events corresponding to the horizontal reflectors in this gather have been well aligned, some problems appear: (1) the reflections from the dipping reflectors have been over-corrected; (2) the barely discernible diffractions at zero-offset times of about 1.65 s, 2 s, and 2.5 s have also been over-corrected; and (3) anomalously, the event at about 0.6 s corresponding to the 30-degree dipping reflector has amplitude increasing with offset. When this NMO-corrected CMP gather is stacked, the poorly aligned events from the dipping reflectors and the diffraction events will be attenuated.

DMO attempts to solve all these problems. Figure 6c shows the result of the finite-difference $V(z)$ DMO correction of Figure 6b. All events that had been over-corrected by NMO in Figure 6b are now aligned close to their zero-offset positions, even for the trace with offset of 3 km. As a result, stacking this CMP gather must enhance all events. Furthermore, although this approach is based on a geometrical argument and it takes no special care in the treatment of amplitude, amplitude values of the event at about 0.6 s from the 30-degree dipping reflector has been altered so that they now decrease with offset, as do those from the other reflectors. In this test, I took the function $s(t_0) = 1.11$ at the minimum processing time. Although I do not show the constant-velocity DMO-corrected CMP gather here, consistent with the results presented in Hale and Artley (1993) the constant-velocity DMO did not align all the over-corrected events as well as does the $V(z)$ DMO.

Figure 7 offers comparisons between common-offset sections before and after DMO. This figure shows the known behavior that DMO moves dipping reflections only a small amount, relative to the full migration distance, in both the horizontal and the vertical directions. Therefore, although a 15-degree approximate algorithm is used, unlike conventional migration, finite-difference DMO generates no visible dispersion or other artifacts even for the reflection corresponding to the 90-degree reflector. Thus, the dip limitation in finite-difference algorithms is much less severe for DMO than for conventional migration. For this reason, we can use a 15-degree algorithm to deal with steep reflections.

Figure 8 shows comparison stack sections across the flank of a salt dome in the Gulf of Mexico. Figure 8a is a conventional stack after NMO correction; Figure 8b is the result of finite-difference $V(z)$ DMO; and Figure 8c shows the result of f-k squeezing $V(z)$ DMO (Hale and Artley, 1993). Both DMO results show the desired enhancement of steep reflections, and their sections are comparable.

CONCLUSION

From the similar kinematics of conventional migration and DMO, I have shown that conventional time-migration algorithms, with an appropriate choice of DMO velocity, can be used to implement DMO processing. Because of its recursive nature, finite-difference implementations require a corrective factor on the velocity to account for the fact that the DMO velocity is a function of input rather than output time. Therefore the finite-difference approach is only approximate. The approach, however, can be readily modified to approximately handle vertical velocity variation.

Examples of applying finite-difference $V(z)$ DMO to both synthetic seismic data for a geological model containing steep reflectors, and field seismic data across a salt dome demonstrate the effectiveness of finite-difference DMO in handling steep reflections for typical ranges of offset and depth-variable velocity.

Because the concept of dip-limited accuracy is much less of a restriction for finite-difference DMO than for conventional migration, a 15-degree algorithm, the most efficient among all finite-difference migration schemes, is used in this paper. Nevertheless, the finite-difference approach is much slower than the f-k squeezing DMO of Hale and Artley (1993). The finite-difference approach, however, may be better suited to treating modest lateral velocity variation.

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FIG. 1. Impulse responses of finite-difference DMO with velocity $\tilde{v}$. Here, the CMP interval is 10 m, medium velocity v =2 km/s, and offset=2 km. The asterisk marks the location of shot, and the wedge marks the location of receiver.

FIG. 2. Impulse responses of finite-difference DMO with velocity $\tilde{\tilde{v}}$. As in Figure 1, the CMP interval is 10 m, and v =2 km/s. (a) Offset=2 km. (b) Offset=3 km. The asterisks mark the location of shots, and the wedges mark the location of receivers.

FIG. 3. Impulse responses of finite-difference DMO for the $V(z)$ case. The CMP interval is 10 m. The asterisks mark the location of shots, and the wedges mark the location of receivers.

FIG. 4. Geological model used to generate synthetic seismic data in Figure 5a.

FIG. 5. (a) Zero-offset synthetic section for the geological model shown in Figure 4. The dashed line marks the location of the CMP gather shown in Figure 6. The rectangle in (a) shows the region detailed in (b), (c), (d) and (e). (b) Detail of zero-offset section taken from (a). (c) Stacked section after NMO correction. (d) Stacked section after NMO and constant-velocity f-k DMO correction. (e) Stacked section after NMO and finite-difference $V(z)$ DMO correction.

FIG. 6. (a) CMP gather for the model showed in Figure 4. (b) After NMO correction. (c) After finite-difference $V(z)$ DMO correction.

FIG. 7. Comparison between the common-offset sections before and after finite-difference $V(z)$ DMO.

FIG. 8. Stacked sections of field data across a salt dome obtained (a) after NMO correction, (b) after finite-difference $V(z)$ DMO correction, and (c) after f-k squeezing $V(z)$ DMO correction. Maximum offset is 3439 m. The two DMO approaches enhance all steep reflections comparably.