Differential-Equation-Based Seismic Data Filtering

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ABSTRACT

Suppressing noise and enhancing useful seismic signal by filtering is one of the important tasks of seismic data processing. Conventional filtering methods are implemented through either the convolution operation or various mathematical transforms. We describe a methodology for studying and implementing filters, which, unlike conventional filtering methods, is based on solving differential equations in the time and space domain. We call this *differential-equation-based filtering* (DEBF). DEBF does not require that seismic data be stationary, so filtering parameters can vary with every time and space point. Examples with 2-D synthetic and field seismic data demonstrate that the DEBF method accomplishes the desired time- and space-varying temporal and moveout filtering at lower cost than conventional frequency-wavenumber-domain filtering. The computational advantage in 3-D would be much greater.

INTRODUCTION

Many different 1-D and 2-D filtering methods have been used in seismic data processing, in both the time and frequency domains. Conventional 2-D moveout filtering methods for suppressing coherent, dipping events, for example, include time-space domain convolution filtering (Treitel et al., 1967) and frequency-wavenumber (f-k) domain filtering (Wiggins, 1966). Although these methods have been applied successfully in seismic data processing, some issues remain outstanding.

- Most commonly used filtering methods have directly or indirectly, explicitly or implicitly, a common basic theoretical assumption: the statistical characteristics of seismic data do not vary with time or space. Seismic data, however, can be quite nonstationary. The amplitudes of seismic traces gradually attenuate, and the dominant frequency of seismic traces gradually reduces with time; spatial variations of subsurface media must cause spatial variations of the characteristics of seismic data; and within common-midpoint (CMP) gathers, the characteristics of seismic traces also vary in both offset and time, among other things because of moveout and angle-dependent reflectivity. As a result, the stationarity assumption of typical filtering methods is not optimal.

- To deal with spatial and temporal variations in data, one typically divides the data into several time- and space-windows, within which the parameters of the filters are kept constant. The final filtered result is a combination of the results obtained from all the time- and space-windows.

The implementation of conventional 2-D time-space domain convolution filtering methods is a process of multitrace convolution. To get well-controlled filtered results, we generally have to use long time and space convolution operators. In such cases, this filtering method generally has lower computational efficiency than does the f-k method. Despite its relative speed, in addition to its problems with handling non-stationary data, the f-k domain filtering has further shortcomings:

- It can produce harmful artifacts related to Gibb's phenomena, spatial aliasing, and side effects.
- It can require a great deal of computer CPU memory.
- It cannot deal efficiently with seismic data whose spatial sampling intervals are variable.

To overcome these shortcomings of conventional filtering, various filtering methods, especially in 2-D, have been developed over the years. These include *tau-p* domain filtering (Noponen and Keeney, 1983), median filtering (Hardage, 1983), depth filtering (McMechan and Sun, 1991), recursive dip filtering (Hale and Claerbout, 1983; Claerbout, 1985) and adaptive filtering. *Tau-p* domain filters can be time-variable, but it is difficult for them to vary in space. While the two parameters of median filtering—length of running window and number of iterations—can be made to vary with space and time, neither offers fine control for the desired suppression of noise. Depth filtering is based on the idea of wavefield downward and upward continuation to remove near-surface noise, such as the direct wave and ground roll. That method, however, is computationally inefficient. The filter parameters in time-space-domain recursive dip filtering of Claerbout (1985, p.122) can vary with both time and space. However, because the algorithm is recursive in the time direction, data must be transposed from trace sequence to time sequence before processing. Although frequency-space-domain recursive dip filtering does not require transposition of the data, filter parameters cannot vary in time, and the system of equations to be solved becomes complex.

The adaptive filtering methods can be divided into two main classes. The first (Anderson and McMechan, 1988) still requires that seismic signal or noise be stationary; the second (Katz and Katz, 1990), which is suitable for variations of seismic data with both time and space, is based on the joint use of several sets of linear basis filters.

With the increasing emphasis on 3-D seismic exploration, a variety of 3-D seismic data processing techniques, such as 3-D velocity analysis, 3-D dip-moveout (DMO) and 3-D migration, have been developed. As far as we know, however, no 3-D filtering methods have been put into practice. Typically, in order to filter 3-D seismic data, one uses existing 2-D filtering methods to filter 2-D sections, first in the inline direction, and then in the crossline direction. When doing so, the 3-D seismic data must be transposed from the inline direction to the crossline direction—a costly step, especially for unstacked data.

Here, we describe a methodology for studying and developing filtering methods based on differential equations. The key steps of this methodology are: (1) design the filters and set up the filtering equations in the frequency domain or in the f-k domain, (2) transform these equations back into the time or time-space domains, as variable-coefficient differential equations, and (3) use a finite-difference algorithm to solve these equations. The approach we use here is similar to that used in some migration methods. These differential-equation-based filtering (DEBF) methods do not require that seismic data be stationary, so their filtering parameters can vary at every temporal and spatial point. That is, the theoretical foundation of these methods is based on nonstationary processes, and thus better fits typical physical processes.

Although our main purpose in this paper is to develop 2-D and 3-D DEBF methods, for simplicity we shall start our discussion with the 1-D case.

ONE-DIMENSIONAL PROBLEM

Principle of 1-D DEBF

Similar to the forms of 1-D Butterworth filters, we define the transfer functions of 1-D filters as follows. For low-frequency-pass filters

$$H_1(x, y, \omega) = \frac{\alpha}{\alpha + \omega^{2n}}, \tag{1}$$

and for high-frequency-pass filters

$$H_2(x, y, \omega) = \frac{\omega^{2n}}{\alpha + \omega^{2n}}, \tag{2}$$

where

$$H_1(x, y, \omega) + H_2(x, y, \omega) = 1, \tag{3}$$

x and y are the spatial coordinates; ω denotes the angular frequency, $\omega = 2\pi f$; f is the frequency; α is a filtering parameter determined by the cut-off frequency; and n is a positive integer that determines the steepness of the boundary between the filter's pass and reject zones. The amplitude curves of $H_1(x, y, \omega)$ are shown in Figure 1, for $n = 1, 2, \cdots, 10$ and a fixed cut-off frequency of 15 Hz. From this figure we can see that the larger the value of n, the steeper the boundary between pass and reject zones. Given relation (3), we need discuss only the low-pass filter $H_1(x, y, \omega)$. Furthermore, any band-pass filter can be formed by cascading a high- and a low-pass filter. Note, in equation (1), that $H_1(x, y, \omega_N) = 1/2$ gives

$$\alpha = {\omega_N}^{2n} = (2\pi f_N)^{2n},$$

where f_N is the cut-off frequency. Suppose we use the 1-D low-pass filter defined by equation (1) to filter seismic data. In the frequency-space domain, we have

$$Q(x,y,\omega) = H_1(x,y,\omega)P(x,y,\omega),$$

or

$$(\alpha + \omega^{2n})Q(x,y,\omega) = \alpha P(x,y,\omega), \tag{4}$$

where $P(x,y,\omega)$ is the Fourier transform of the input seismic data $p(x,y,t)$ with respect to t, and $Q(x,y,\omega)$ is the filtered output in the frequency-space domain. Although we are discussing 1-D filtering here, the seismic data with which we usually deal are generally 2-D or 3-D. That is why we use as arguments (x,y,ω). Inverse Fourier transformation of equation (4) into the time-space domain yields the differential equation

$$[\alpha(x,y,t) + (-1)^n \frac{\partial^{2n}}{\partial t^{2n}}]q(x,y,t) = \alpha(x,y,t)p(x,y,t). \tag{5}$$

Here, we have indicated that the filtering parameter α is dependent on time, as well as space. Strictly this cannot be true. To get equation (5) by inverse Fourier transformation of equation (4), we have assumed that α varies slowly with time. Specifically equation (5) is an acceptable approximation only if the user-specified cut-off frequency varies gradually over the effective length of the filter operator, as is typically true in practice. If α varied more rapidly, α would no longer be simply related to cut-off frequency. Equation (5), with parameter α varying as desired in space and time, is just the 1-D filtering differential equation we desire.

As shown below, we can benefit conceptually and computationally from choosing $n = 1$ when solving equation (5). Setting n to 1, we obtain

$$[\alpha(x,y,t) - \frac{\partial^2}{\partial t^2}]q(x,y,t) = \alpha(x,y,t)p(x,y,t), \tag{6}$$

with

$$\alpha(x,y,t) = 4\pi^2 f_N^2(x,y,t). \tag{7}$$

Through $\alpha(x,y,t)$, the user-specified cut-off frequency f_N in general can vary in both time and space. However, for our simplified 1-D argument in this section, we shall use (t) as short-hand for (x,y,t).

Algorithm for 1-D DEBF

Having derived filtering differential equation (5) or (6) to filter seismic data, the key problem lies in finding a stable and efficient method for solving these equations. Here we use the finite-difference method. Instead of using the ordinary central-difference pattern, we choose (Claerbout, 1985)

$$\frac{\partial^2 q(t)}{\partial t^2} \approx \frac{\delta_{tt}}{\Delta t^2(1+\beta\delta_{tt})}q(t), \tag{8}$$

where Δt is the time sampling interval,

$$\beta = 0.25 - 1/\pi^2,$$

and δ_{tt} is the second-difference operator for a sampled function, $q_n \equiv q(n\Delta t)$, $n = 1, 2, 3, \cdots, L$, and L is the maximum number of sample points in each seismic trace. That is,

$$\delta_{tt} q_n \equiv q_{n+1} - 2q_n + q_{n-1}.$$

Substituting equation (8) into equation (6), for sampled data, gives

$$[\alpha_n - \frac{\delta_{tt}}{\Delta t^2(1 + \beta\delta_{tt})}] q_n = \alpha_n p_n, \tag{9}$$

$$n = 1, 2, 3, \cdots, L,$$

where

$$\begin{aligned} \alpha_n &\equiv \alpha(n\Delta t), \\ q_0 &= q_{L+1} = 0, \\ p_n &\equiv p(n\Delta t), \\ p_0 &= p_{L+1} = 0. \end{aligned}$$

Multiplying the two sides of equation (9) by the denominator, $\Delta t^2(1 + \beta\delta_{tt})$, gives

$$[\theta_n(1 + \beta\delta_{tt}) - \delta_{tt}] q_n = \theta_n(1 + \beta\delta_{tt}) p_n, \tag{10}$$

where we have let

$$\begin{aligned} \theta_n &\equiv \Delta t^2 \alpha_n = 4\pi^2 \Delta t^2 f_{Nn}^2, \\ f_{Nn} &\equiv f_N(n\Delta t). \end{aligned} \tag{11}$$

Equation (10) can be rewritten from its difference-operator form as

$$[\theta_n \mathbf{I} + (1 - \theta_n \beta)\mathbf{T}] \cdot \mathbf{q}_n = \theta_n(\mathbf{I} - \beta\mathbf{T}) \cdot \mathbf{p}_n,$$

$$n = 1, 2, 3, \cdots, L,$$

where

$$\begin{aligned} \mathbf{I} &\equiv [0, 1, 0], \\ \mathbf{T} &\equiv [-1, 2, -1], \\ \mathbf{q}_n &\equiv [q_{n-1}, q_n, q_{n+1}]^T, \\ \mathbf{p}_n &\equiv [p_{n-1}, p_n, p_{n+1}]^T. \end{aligned}$$

We may further express this equation in matrix terms as

$$\mathbf{A} \cdot \mathbf{q} = \mathbf{B}, \tag{12}$$

where

$$\mathbf{B} = \mathbf{C} \cdot \mathbf{p},$$

$$\mathbf{A} = \begin{bmatrix} b_1 & c_1 & 0 & 0 & \cdots & 0 & 0 \\ a_2 & b_2 & c_2 & 0 & \cdots & 0 & 0 \\ 0 & a_3 & b_3 & c_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a_L & b_L \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} e_1 & f_1 & 0 & 0 & \cdots & 0 & 0 \\ d_2 & e_2 & f_2 & 0 & \cdots & 0 & 0 \\ 0 & d_3 & e_3 & f_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & d_L & e_L \end{bmatrix},$$

$$\begin{aligned} \mathbf{q} &= [q_1, q_2, q_3, \cdots, q_L]^T, \\ \mathbf{p} &= [p_1, p_2, p_3, \cdots, p_L]^T, \\ a_n &= c_n = \theta_n\beta - 1, \\ b_n &= \theta_n - 2(\theta_n\beta - 1), \\ d_n &= f_n = \theta_n\beta, \qquad \text{and} \\ e_n &= \theta_n(1 - 2\beta). \end{aligned}$$

Equation (12) is a differential-equation-based, implicit finite-difference implementation of 1-D filtering. Being a diagonally-dominant tridiagonal system of equations, it can be solved relatively efficiently (Claerbout, 1985). The tridiagonal nature of the equation is a result of the choice $n = 1$. As seen in Figure 1, to get sharper cut-off action, we would choose a larger value of n in equation (5). If we did so, however, the resulting system of equations would no longer be tridiagonal, but rather a system of equations with progressively increasing number of nonzero diagonals as n increases. As a result, we would lose the computational advantage of the tridiagonal system. Fortunately, however, we would still have other efficient alternatives, such as the Cholesky decomposition method, whose computational cost grows only linearly with the size L. By solving equation (12) with the user-specified cut-off frequency $f_N(x, y, t)$, differential equation-based 1-D filtering of seismic data can be implemented trace-by-trace.

Examples with synthetic data

To test the effectiveness of the 1-D filtering method presented in this section, we shall study a synthetic data example. The wavelet we used to create synthetic seismic sections is defined by

$$s(t) = \frac{20}{t} e^{-2000t^2} \sin(30\pi t) \cos(62.5\pi t).$$

Frequencies of this wavelet are typical of those in reflection seismic signals.

Figure 2a is a synthetic seismic section consisting of two horizontal reflections with low-frequency (1-10 Hz) "noise" superimposed. Figure 2b shows the amplitude spectra of four traces from Figure 2a. The results of applying time- and space-variable filtering of the data in Figure 2a are shown in Figure 3. The low cut-off frequency in this test is governed by the following parameters: at the second trace, $t200f1, t400f10$ and $t450f40$; and at the 19th trace, $t200f40, t400f10$ and $t450f1$. Here, for example, $t200f1, t400f10$ and $t450f40$ means that before 200 ms, the low cut-off frequency is 1 Hz; at 400 ms, it is 10 Hz; and after 450 ms, it is 40 Hz. The low cut-off frequencies at points between 200 and 450 ms are obtained by linear interpolation. Laterally, these cut-off frequencies are also interpolated linearly between traces along which values are specified. Figure 3a is the filtered section, and Figure 3b is the so-called noise section, the difference between Figure 2a and Figure 3a. Figure 3c shows the amplitude spectra for the 1st, 5th, 10th and 15th traces of Figure 3a. The results in these three figures are just what we expect for such low cut-off filtering.

TWO-DIMENSIONAL PROBLEM

Principle of 2-D DEBF

Having introduced the conception of DEBF with the 1-D case, we now discuss 2-D DEBF. In the f-k domain, we define the transfer functions of the 2-D moveout filters we use as follows. For a high-dip-pass filter

$$H_1(k_x, y, \omega) = \frac{\alpha}{\alpha + \omega^2/ik_x}, \tag{13}$$

and for a low-dip-pass filter

$$H_2(k_x, y, \omega) = \frac{\omega^2/ik_x}{\alpha + \omega^2/ik_x}, \tag{14}$$

where

$$H_1(k_x, y, \omega) + H_2(k_x, y, \omega) = 1. \tag{15}$$

k_x is the wavenumber in the x-direction, and α is a 2-D filtering parameter determined by the user-specified cut-off "dip" or by a combination of the user-specified dominant frequency and cut-off apparent velocity.

Actually, we might have defined other forms for the transfer function, such as $H_1(k_x, y, \omega) = \alpha/(\alpha + \omega^2/k_x^2)$. While such a choice would give a response that is strictly a function of slope k_x/ω, we choose the above forms for simplicity of development, computational efficiency, and stability. Given relation (15), we need only discuss the high-dip-pass filter $H_1(k_x, y, \omega)$. Its amplitude and phase spectra are, respectively,

$$|H_1(k_x, y, \omega)| = \frac{1}{\sqrt{1 + \omega^4/\alpha^2 k_x^2}}, \tag{16}$$

and

$$\theta_1(k_x, y, \omega) = \tan^{-1}(\frac{\omega^2}{\alpha k_x}). \tag{17}$$

Letting $|H_1(k_x, y, \omega)| = \varepsilon$, some chosen amplitude level, then from equation (16)

$$k_x = \pm \frac{\varepsilon}{\sqrt{1-\varepsilon^2}} \cdot \frac{\omega^2}{\alpha}, \tag{18}$$

where ε is a positive constant, less than 1. For different values of ε, equation (18) describes different parabolas that are symmetric with respect to both the ω- and k_x-axis and pass through the origin. That is, the contours of amplitude level of the 2-D filter $H_1(k_x, y, \omega)$ defined by equation (13) are symmetric parabolas. If, further, we let $\varepsilon = 1/\sqrt{2}$, we get a special contour,

$$k_x = \pm \frac{\omega^2}{\alpha}. \tag{19}$$

This is the expression for the half-power contour of $H_1(k_x, y, \omega)$. Now let us see the relationship between $H_1(k_x, y, \omega)$ and the response of an ideal dip filter. In Figure 4a, covering only the range $\omega \geq 0$ and $k_x \geq 0$, the straight line denotes the boundary

$k_x = \omega/V_N$ between the pass and reject zone of an ideal dip filter, while the parabola is the half-power contour of the filter $H_1(k_x, y, \omega)$ defined by equation (19). Here, V_N is the cut-off apparent velocity. Therefore, when using the filter $H_1(k_x, y, \omega)$ as a dip filter, we are, in effect, replacing the straight-line boundary of the ideal filter with the parabolic one. To get desired filtered results, we should make the parabolic boundary of $H_1(k_x, y, \omega)$ as close to the straight line boundary of the ideal dip filter as possible within the range of ω and k_x of interest. This can be done by minimizing the area between the straight and the parabolic line. Over the frequency range of interest (for example, if the range is from 0 to f_{max}), this minimization gives the following expression for the filtering parameter α,

$$\alpha = 2^{4/3}\pi f_N V_N. \tag{20}$$

Here, we take $f_N = f_{max}/2$ as the user-specified dominant frequency. If we use the concept of apparent velocity to characterize 2-D filters, $H_1(k_x, y, \omega)$ is a low-apparent-velocity-pass filter. Figure 4b shows three contours, $\varepsilon = 0.707, 0.6$ and 0.5 of $|H_1(k_x, y, \omega)|$, when α takes the form of equation (20), with $f_N = 35$ Hz and $V_N =$ 3000 m/s. We see that $H_1(k_x, y, \omega)$ is a rough approximation of the ideal dip filter, with a smooth transition in amplitude from the pass to the reject zone.

Using $H_1(k_x, y, \omega)$ defined by equations (13) and (20) to filter seismic data, the filtering equation in the f-k domain can be expressed as

$$Q(k_x, y, \omega) = H_1(k_x, y, \omega) P(k_x, y, \omega),$$

or

$$(i\alpha k_x + \omega^2) Q(k_x, y, \omega) = i\alpha k_x P(k_x, y, \omega), \tag{21}$$

where $P(k_x, y, \omega)$ is the Fourier transformation of the input seismic data $p(x, y, t)$ with respect to x and t; $Q(k_x, y, \omega)$ is the filtered result in the f-k domain. Inverse Fourier transformation of equation (21) into the time-space domain yields

$$[\alpha(x,y,t)\frac{\partial}{\partial x} - \frac{\partial^2}{\partial t^2}]q(x,y,t) = \alpha(x,y,t)\frac{\partial p(x,y,t)}{\partial x}.$$

Similar to the situation for the 1-D filter, the inverse Fourier transformation resulting in this equation is strictly true only when the coefficient α is constant. The differential equation holds approximately as long as α varies slowly in both space and time. If the seismic data with which we deal are only 2-D, we can use the shorthand (x, t) instead of (x, y, t) for the arguments. Thus, the above equation becomes

$$[\alpha(x,t)\frac{\partial}{\partial x} - \frac{\partial^2}{\partial t^2}]q(x,t) = \alpha(x,t)\frac{\partial p(x,t)}{\partial x}, \tag{22}$$

with

$$\alpha(x,t) = 2^{4/3}\pi f_N V_N(x,t). \tag{23}$$

Here, $q(x,t)$ is the filtered output in the time-space domain, and the filtering parameter $\alpha(x,t)$, determined by f_N and $V_N(x,t)$, is a function of both time and space, i.e., it can vary in both temporal and spatial directions. Equation (22) is just the variable-coefficient differential equation we want for 2-D high-dip-pass, or low-apparent-velocity-pass, filtering. To solve this partial differential equation, we use the following conditions:

$$\begin{aligned} &p(x,t)|_{x=-\Delta x} = \eta p(x,t)|_{x=0}, \\ &q(x,t)|_{x=-\Delta x} = \eta q(x,t)|_{x=0}, \\ &q(x,t)|_{t=0} = 0, \quad q(x,t)|_{t=t_{max}} = 0, \end{aligned} \tag{24}$$

where η controls the boundary condition ($0 \leq \eta \leq 1$), Δx is the trace spacing, and $x = 0$ is the location of the first trace in the seismic data.

Algorithm for 2-D DEBF

Using the modified Crank-Nicholson difference pattern to solve equation (22), similar to the 1-D case, we obtain the following difference equation

$$\frac{\alpha_m(n)}{\Delta x}\delta_x q_m(n) - \frac{\delta_{tt}}{2\Delta t^2(1+\beta\delta_{tt})}[q_m(n)+q_{m-1}(n)] = \frac{\alpha_m(n)}{\Delta x}\delta_x p_m(n), \tag{25}$$

$$n = 1,2,3,\cdots,L, \qquad m = 0,1,2,\cdots,M,$$

where

$$\delta_x q_m(n) \equiv q_m(n) - q_{m-1}(n),$$

$$\delta_x p_m(n) \equiv p_m(n) - p_{m-1}(n),$$

and

$$\delta_{tt} q_m(n) \equiv q_m(n+1) - 2q_m(n) + q_m(n-1).$$

Equation (25) can further be written as

$$[\theta_m(n)\mathbf{I} + (1-\theta_m(n)\beta)\mathbf{T}]\cdot\mathbf{q}_m(n) = [\theta_m(n)\mathbf{I} - (1+\theta_m(n)\beta)\mathbf{T}]\cdot\mathbf{q}_{m-1}(n) + \theta_m(n)(\mathbf{I}-\beta\mathbf{T})\cdot[\mathbf{p}_m(n) - \mathbf{p}_{m-1}(n)], \tag{26}$$

$$n = 1,2,3,\cdots,L, \qquad m = 0,1,2,\cdots,M,$$

and the conditions for determining the solution, equation (24), can be written as

$$\begin{aligned} \mathbf{p}_{-1} &= \eta\mathbf{p}_0, \\ \mathbf{q}_{-1} &= \eta\mathbf{q}_0, \\ q_m(0) &= 0, \quad q_m(L+1) = 0, \end{aligned} \tag{27}$$

where Δt is the time sampling interval, L is the maximum length of seismic traces in sample points, M is the maximum number of traces processed,

$$
\begin{aligned}
\mathbf{I} &\equiv [0,1,0], \\
\mathbf{T} &\equiv [-1,2,-1], \\
\alpha_m(n) &\equiv \alpha(m\Delta x, n\Delta t), \\
\beta &= 0.25 - 1/\pi^2, \\
q_m(n) &\equiv q(m\Delta x, n\Delta t), \\
p_m(n) &\equiv p(m\Delta x, n\Delta t), \\
\mathbf{q}_m(n) &\equiv [q_m(n-1), q_m(n), q_m(n+1)]^T, \\
\mathbf{p}_m(n) &\equiv [p_m(n-1), p_m(n), p_m(n+1)]^T, \\
\mathbf{q}_m &\equiv [q_m(1), q_m(2), q_m(3), \ldots, q_m(L)]^T, \\
\mathbf{p}_m &\equiv [p_m(1), p_m(2), p_m(3), \ldots, p_m(L)]^T,
\end{aligned}
$$

and we have let

$$\theta_m(n) \equiv 2\Delta t^2 \alpha_m(n)/\Delta x = 2^{7/3}\pi\Delta t^2 f_N V_{Nm}(n)/\Delta x. \tag{28}$$

If we let

$$
\mathbf{A} = \begin{bmatrix}
b_1 & a_1 & 0 & 0 & \cdots & 0 & 0 \\
a_2 & b_2 & a_2 & 0 & \cdots & 0 & 0 \\
0 & a_3 & b_3 & a_3 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & a_L & b_L
\end{bmatrix},
$$

$$\mathbf{B} = \begin{bmatrix} d_1 & c_1 & 0 & 0 & \cdots & 0 & 0 \\ c_2 & d_2 & c_2 & 0 & \cdots & 0 & 0 \\ 0 & c_3 & d_3 & c_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & c_L & d_L \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} f_1 & e_1 & 0 & 0 & \cdots & 0 & 0 \\ e_2 & f_2 & e_2 & 0 & \cdots & 0 & 0 \\ 0 & e_3 & f_3 & e_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & e_L & f_L \end{bmatrix},$$

$$\begin{aligned} a_n &= \theta_m(n)\beta - 1, \\ b_n &= \theta_m(n) + 2(1 - \theta_m(n)\beta), \\ c_n &= 1 + \theta_m(n)\beta, \\ d_n &= \theta_m(n) - 2(1 + \theta_m(n)\beta), \\ e_n &= \theta_m(n)\beta, \\ f_n &= \theta_m(n)(1 - 2\beta), \end{aligned}$$

equation (26) can be expressed in matrix terms as

$$\begin{gathered} \mathbf{A}\mathbf{q}_m = \mathbf{B}\mathbf{q}_{m-1} + \mathbf{C}(\mathbf{p}_m - \mathbf{p}_{m-1}), \\ m = 0, 1, 2, \cdots, M. \end{gathered} \tag{29}$$

This equation is a differential equation-based implementation of 2-D filtering. As for the 1-D example, equation (29) is a diagonally dominant tridiagonal system of equations, so it too can be solved by using a fast algorithm. Also, from equations (26) and (29), to get the filtered result at a given point, only six adjacent points on two adjacent traces are involved. Thus, to obtain one output trace, only two input traces are used. So, on the one hand, computational operators of this kind of filter are quite short in both temporal and spatial directions and computational efficiency is high; on the other hand, through use of the implicit approach in the time direction and the explicitly recursive approach in the space direction, we, in effect, achieve an extended 2-D impulse response.

Generally, to filter a data set consisting of N traces, each containing N samples, the *f-k* filtering method needs about $(4 \log_2 N + 6)N^2$ floating-point operations, while solution of equation (29), the most costly step in the DEBF method, needs about $24N^2$. Although additional computation for interpolation of filtering parameters in both the time- and the space-direction is needed in the DEBF method, that computation might involve another $2N^2$ operation, so DEBF filtering is faster than the *f-k* method for typical-size data sets.

So far, we have been discussing 2-D high-dip-pass filtering. By relation (15), instead of applying directly $H_2(k_x, y, \omega)$ defined by equation (14), we subtract the filtered result of 2-D high-dip-pass filtering from the original input seismic data to get the result of 2-D low-dip-pass filtering.

Examples with synthetic and field data

We demonstrate the action of the 2-D filtering method first on synthetic and then on field seismic data. Figure 5a contains horizontal events and events with moveout that we shall call *small, moderate*, and *large*. Figure 5b shows a time- and space-variable high-apparent-velocity-pass (or low-dip-pass) filtered result of Figure 5a. In this section any given dipping event has been passed at some time and space points and rejected at the others. In this example, the dominant frequency is 20 Hz, and the cut-off apparent-velocity $V_N(x,t)$ is governed by the following parameters: at 0 km, $t1500V5500$ and $t3510V50$; at 0.625 km, $t1500V5500$ and $t2510V50$, and at 1.25 km, $t1500V5500$ and $t1510V50$. Here, for example, $t1500V5500$ and $t3510V50$ means that before 1500 ms, the cut-off apparent-velocity is 5500 m/s and after 3510 ms, 50 m/s. The values of V_N at all other points are obtained by linear interpolation.

Figure 5c is a detail, corresponding to the box position in Figure 5a, of the high-apparent-velocity-pass filtered results of Figure 5a obtained by using 2-D DEBF, and Figure 5d is that obtained by f-k filtering. Our purpose is to reject the steep events and to pass the horizontal events and those three events with small dips. The cut-off boundary just coincides with the events with moderate dip. Comparing Figure 5c with 5d, we see that f-k filter has a sharper cut-off boundary than does the DEBF used here, but has some artifacts on the side, while DEBF does not.

The second example is a demonstration of noise suppression on a field shot record. Figure 6a contains the raw data, which are contaminated by large-moveout coherent noise in the upper part, and Figure 6b contains the 2-D time- and space-variable DEBF filtered result of Figure 6a showing suppression of the coherent noise. Here, f_N = 30 Hz, and $V_N(x,t)$ is governed by the following parameters: at -1.25 km, $t700V2700$ and $t1000V3500$; at 0.3 km, $t1500V3500$; at 1.3 km, $t800V2200$ and $t1500V3500$; and at 2.3 km, $t1100V1700$ and $t2000V3500$.

The final 2-D filter example is a common-offset field seismic section. The raw section (Figure 7a), especially the right part of this section, has much steeply dipping background noise. In Figure 7b, this noise has been reduced, and the signal-to-noise ratio thereby has been improved.

THREE-DIMENSIONAL PROBLEM

In the previous section, we defined the high-dip-pass filter and discussed its implementation in detail. One way to define the transfer function of a 3-D high-dip-pass filter is

$$H(k_x, k_y, \omega) = \frac{\alpha_x}{\alpha_x + \omega^2/ik_x} \cdot \frac{\alpha_y}{\alpha_y + \omega^2/ik_y} = H_1(k_x, \omega) \cdot H_1(k_y, \omega), \tag{30}$$

where both $H_1(k_x, \omega)$ and $H_1(k_y, \omega)$ are 2-D high-dip-pass filters. $H_1(k_x, \omega)$ is used in the x-direction (say, the inline direction), while $H_1(k_y, \omega)$ is used in the y-direction (say, the crossline direction). Using $H(k_x, k_y, \omega)$ to filter 3-D seismic data, we obtain the filtering equation in the f-k domain

$$\begin{aligned} Q(k_x, k_y, \omega) &= H(k_x, k_y, \omega) \cdot P(k_x, k_y, \omega) \\ &= H_1(k_y, \omega) \cdot H_1(k_x, \omega) \cdot P(k_x, k_y, \omega). \end{aligned} \tag{31}$$

If we let

$$G(k_x, k_y, \omega) = H_1(k_x, \omega) \cdot P(k_x, k_y, \omega) = \frac{\alpha_x}{\alpha_x + \omega^2/ik_x} \cdot P(k_x, k_y, \omega), \tag{32}$$

then equation (31) becomes

$$Q(k_x, k_y, \omega) = H_1(k_y, \omega) \cdot G(k_x, k_y, \omega) = \frac{\alpha_y}{\alpha_y + \omega^2/ik_y} \cdot G(k_x, k_y, \omega). \tag{33}$$

Inverse Fourier transformation of equations (32) and (33) into the time-space domain yields a system of 3-D high-dip-pass filtering differential equations,

$$[\alpha_x(x, y, t)\frac{\partial}{\partial x} - \frac{\partial^2}{\partial t^2}]g(x, y, t) = \alpha_x(x, y, t)\frac{\partial p(x, y, t)}{\partial x}, \tag{34}$$

and

$$[\alpha_y(x,y,t)\frac{\partial}{\partial y} - \frac{\partial^2}{\partial t^2}]q(x,y,t) = \alpha_y(x,y,t)\frac{\partial g(x,y,t)}{\partial y}, \tag{35}$$

where $p(x,y,t)$ is the original input; $g(x,y,t)$ is the intermediate filtered result obtained after equation (34) is used in the x-direction; and $q(x,y,t)$ is the 3-D final filtered result.

Because both equations (34) and (35) are 2-D variable-coefficient differential equations, we can solve them using the algorithm described in the 2-D case. However, in practice, we wish to avoid the costly transposition of 3-D seismic data when implementing the 3-D DEBF method so that the total computational efficiency can be further raised. Fortunately, we can alternately and recursively solve equations (34) and (35) between two adjacent traces in the x-direction and between two adjacent lines in the y-direction. The scheme is shown in Figure 8. We first get the intermediate result, $g(x+1,y+1,t)$, using $g(x,y+1,t), p(x,y+1,t)$ and $p(x+1,y+1,t)$, then get the final result, $q(x+1,y+1,t)$, using $q(x+1,y,t), g(x+1,y,t)$ and $g(x+1,y+1,t)$. As a result, we do not need to transpose 3-D seismic data from the inline direction to the crossline direction and then again back from the crossline direction to the inline direction for subsequent processing. These considerations in 3-D applications suggest further computational advantages (over and above those obtained in 2-D applications) of the DEBF method over *f-k* domain filtering.

As before, implementation of 3-D low-dip-pass DEBF requires only subtraction of the high-dip-pass filtered result from the input data.

CONCLUSION

Unlike conventional filtering techniques, which use convolution operations or mathematical transforms, the DEBF method presented here directly solves variable-coefficient differential equations, and, as a result, the filtering parameters can vary (not too rapidly) at every time and space point. When we use a finite-difference algorithm to solve these differential equations, the filtering processes are transformed into the solution of tridiagonal systems of equations, which have known, stable and efficient solutions. To maintain efficiency, some flexibility in filter characteristics is lost. Specifically, not every desired specification of filter action in *f*-*k* domain can be readily approximated with DEBF method. Nevertheless, we have shown design of practical filter action in 1-D, 2-D, and 3-D.

Because DEBF methods process seismic data trace-by-trace, they require no transposition of seismic data in 2-D and 3-D; they need relatively little computer CPU memory; and (not shown here) they can be adapted to treat seismic data with non-uniform spatial sampling. While non-uniform sampling makes the computation programs more complicated, it should pose no fundamental limitation.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the support of the sponsors of the Consortium Project on Seismic Inversion Methods for Complex Structures at the Center for Wave Phenomena, Colorado School of Mines. This project was also partially supported by the United States Department of Energy, Grant Number DE-FG02-89ER14079. (This support does not constitute an endorsement by DOE of the views expressed in this paper.)

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FIG. 1. Amplitude curves of the low-frequency-pass filter $H_1(\omega)$ for $n = 1, 2, 3, \cdots, 10$, where all the filters have a cut-off frequency of 50 Hz.

FIG. 2. (a) Synthetic data contaminated by 1-10 Hz low-frequency "noise". (b) Amplitude spectra of four of the traces in (a).

FIG. 3. (a) Time- and space-variable filtered result of Figure 2a. The low cut-off frequencies are both time- and space-variable. (b) Time- and space-variable filtered "noise" section. (c) Amplitude spectra of the time- and space-variable filtered section (a) within a 400-ms time window centered on 200 ms.

FIG. 4. (a) Sketch showing the area between the straight-line boundary of an ideal dip filter and the parabolic half-power contour of the 2-D high-dip-pass DEBF filter. (b) Straight-line boundary of an ideal dip filter and three contours of the amplitude spectrum $|H_1(k_x, y, \omega)|$.

FIG. 5. (a) Synthetic seismic section with six horizontal and nine dipping events. (b) Time- and space-variable DEB low-dip-pass filtered section of (a). (c) Detail of DEB low-dip-pass filtered section of (a) to compare with (d). (d) Detail of low-dip-pass filtered section of (a) obtained by using f-k filtering.

FIG. 6. (a) Field shot record with strong, dipping coherent noises in the upper part. (b) Time- and space-variable filtered result of (a). The dipping noise has been reduced, improving the signal-to-noise ratio.

FIG. 7. (a) Common-offset field seismic section with some dipping background noise. (b) DEB filtered section of (a).

FIG. 8. Sketch plan view showing lines of data used in an implementation of the 3-D DEBF method.