A small dip, small offset representation of the DMO operator in a medium with a constant velocity gradient

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ABSTRACT

A small dip, small offset approximation of the dip moveout (DMO) operator is derived in a medium with a constant velocity gradient in the vertical direction. The DMO impulse response is defined by four parameters: the vertical time t_n, the half source-receiver offset h, and two "squeeze" functions $\gamma_x(t_n)$ and $\gamma_y(t_n)$ which are directly related to the central curvature of the DMO operator in the inline and crossline directions.

The expressions obtained show that the DMO operator depends only weakly on the velocity function considered: for a fixed value t_n, the squeeze factors γ_x and γ_y depend solely on the gradient k and are notably independent of the reference velocity V_0. Furthermore, the γ_x factor in the inline direction shows only little sensitivity to variations of the velocity gradient k. However, the γ_y factor in the crossline direction is directly proportional to k.

It is also shown that the approximate representation of the DMO operator remains accurate for large source-receiver offsets. The relative error in curvature in the inline direction is usually less than 10%, even for the largest offsets used in conventional seismic surveys. The curvature of the DMO operator in the crossline direction can be exactly computed for any offset h.

INTRODUCTION

Dip moveout is a simple and cost-effective seismic processing which is usually applied to the data by assuming that the seismic wave velocity is constant. DMO processing in inhomogeneous media is more difficult to implement, because the DMO operator is then a two-dimensional, complex-shaped function of the zero-offset traveltime. In a recent paper (Dietrich and Cohen, 1992, hereafter referred to as Paper I), we derived an *exact* formulation of the DMO operator in a medium characterized by a constant velocity gradient in the vertical direction. Our analytical formulation gives a complete solution of the problem, but does not permit to study (analytically) the overall behaviour and properties of the DMO impulse response in a simple way. In particular, it is often desirable (and instructive) to represent a complex seismic processing operator by a truncated power series to obtain its simplest possible expression. The complexity of the parametric definition of the DMO operator given in Paper I prevents such an analysis.

In this paper, I derive an approximate representation of the DMO operator for a linear velocity-depth function, which is valid for small source-receiver offsets and small dips. The DMO impulse response is basically defined by the curvature at the origin (in two orthogonal directions), and by the vertical time corresponding to a horizontal reflector. The expressions of the curvature are obtained from the normal moveout velocity in the inline and crossline directions.

The central curvature in the inline direction is obtained as a particular case of the general formula derived by Hale (1988) for a vertically inhomogeneous media. The curvature in the crossline direction requires more calculations, but can be derived by extending Hale's approach in the strike direction. The final formulas obtained emphasize the relative insensitivity of the DMO impulse response relative to the velocity profile considered, and confirm the high degree of accuracy of the approximate DMO formula proposed by Hale (1988) in the inline direction, for an arbitrarily complex $V(z)$ medium.

I. GENERAL PROCEDURE

The DMO mapping operator in a depth-dependent medium can be approximately represented by the function

$$t_0 = t_n \left[1 - \frac{x_0^2}{\gamma_x(t_n) h^2} - \frac{y_0^2}{\gamma_y(t_n) h^2} \right] \quad , \tag{1}$$

where t_0 is the two-way traveltime along the normal incidence ray, and (x_0, y_0) are the coordinates of the point of emergence of the ray at the surface of the ground.

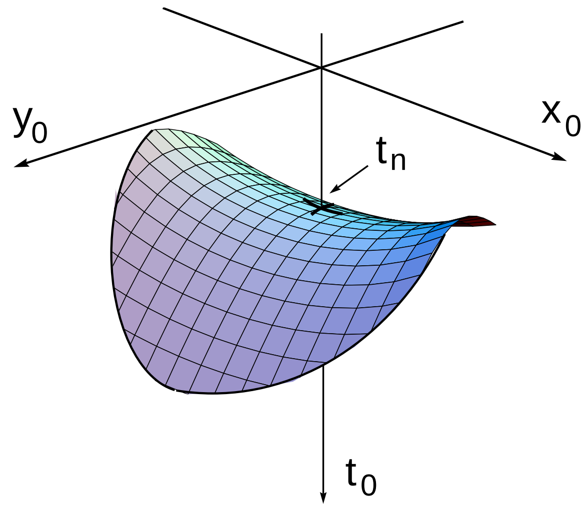

FIG. 1. Small dip, small offset representation of the DMO impulse response in a vertically inhomogeneous medium. The operator is defined by the normal moveout time t_n, and by the central curvature in the inline (x_0) and crossline (y_0) directions.

$\gamma_x(t)$ and $\gamma_y(t)$ are time-variable squeeze functions in the inline and crossline directions respectively, and are related to the curvature of the DMO operator through the relations

$$\left.\frac{\partial^2 t_0}{\partial x_0^2}\right|_{x_0=0} = -\frac{t_n}{\gamma_x(t_n)h^2} \quad \text{and} \quad \left.\frac{\partial^2 t_0}{\partial y_0^2}\right|_{y_0=0} = -\frac{t_n}{\gamma_y(t_n)h^2} \quad . \tag{2}$$

Since γ_x is usually positive, and γ_y, negative, equation (1) generally represents a saddle-shaped operator, as depicted in Figure 1. It can also be noticed that equation (1) differs from the constant velocity DMO ellipse by an additional contribution in the crossline direction, and by the squeeze factors γ_x and γ_y.

The general procedure to derive a small dip, small offset approximation of the DMO operator consists in writing the dip-*dependent* and dip-*independent* NMO equations, and in eliminating the finite-offset time T between these two equations. This procedure has been employed by Hale (1988) in the inline direction, but remains valid in any direction.

The dip-independent NMO equation relating the finite-offset time T to the vertical time t_n is given by

$$T^2 = t_n^2 + \frac{4h^2}{V_2^2(t_n)} \quad , \tag{3}$$

where

$$V_2(t) \equiv \left[\frac{1}{t} \int_0^t v^2(u)du \right]^{1/2} \tag{4}$$

is the root mean square velocity at traveltime t.

The dip-dependent NMO equation relating the finite-offset time T to the zero-offset time T_0 is similarly given by

$$T^2 = T_0^2 + \frac{4h^2}{V_{dip}^2(T_0, \boldsymbol{p})} \quad , \tag{5}$$

where $V_{dip}(T_0, \boldsymbol{p})$ is the moveout velocity at time T_0 and in the direction $\boldsymbol{p}$ traveled by the zero offset ray.

Combining equations (3) and (5), we obtain the expression

$$T_0^2 = t_n^2 + 4h^2 \left[\frac{1}{V_2^2(t_n)} - \frac{1}{V_{dip}^2(T_0, \boldsymbol{p})} \right] \tag{6}$$

which can be written in the form

$$T_0^2 = t_n^2 + \gamma(t_n, \boldsymbol{p}) h^2 \lambda^2(\boldsymbol{p}) \quad , \tag{7}$$

where $\gamma(t_n, \boldsymbol{p})$ is the correction factor at time t_n and in the direction $\boldsymbol{p}$, and λ is the "two-way" ray parameter of the zero offset ray in the direction $\boldsymbol{p}$.

The equivalence between equations (1) and (7) can then be easily established by noting that

$$\lambda_x = \frac{\partial t_0}{\partial x_0} \quad ; \quad \lambda_y = \frac{\partial t_0}{\partial y_0} \tag{8}$$

and

$$T_0^{(x)} = t_0^{(x)} + |\lambda_x x_0| \quad ; \quad T_0^{(y)} = t_0^{(y)} + |\lambda_y y_0| \quad . \tag{9}$$

(T_0 denotes the two-way traveltime along the zero offset ray emerging at the source-receiver midpoint, whereas t_0 represents the two-way traveltime along the normal incidence ray corresponding to a particular reflection point in the subsurface).

Hale (1988) demonstrated that the γ_x factor in the inline direction can be expressed in the form

$$\gamma_x(t_n) = \frac{3}{2}\frac{V_4^4(t_n)}{V_2^4(t_n)} - \frac{t_n}{V_2(t_n)}\frac{\mathrm{d}V_2(t_n)}{\mathrm{d}t_n} - \frac{1}{2} \quad , \tag{10}$$

where

$$V_4(t) \equiv \left[\frac{1}{t} \int_0^t v^4(u) du \right]^{1/4} \quad . \tag{11}$$

The derivation of expression γ_y in the crossline direction can be obtained from equation (7) and requires the knowledge of the moveout velocity V_{dip} in the strike direction.

II. MOVEOUT VELOCITY IN THE STRIKE DIRECTION

According to Levin (1971), the normal moveout velocity for a dipping reflector in a constant velocity medium is given by

$$V_{dip}(\phi,\theta) = V_0[1-\sin^2\phi\cos^2\theta]^{1/2} \quad , \tag{12}$$

where ϕ is the dip angle of the reflector, and θ, the angle between the profile line and the dip line. In the strike direction $\theta=\pi/2$, the moveout velocity is simply equal to the contant velocity V_0 of the medium.

The general expression of the moveout velocity in the crossline direction for a vertically inhomogeneous medium can be obtained from the three-dimensional traveltime equation for dipping layers derived by Diebold (1987). Diebold showed that the traveltime equation along a raypath between a source S and receiver R in a stack of homogeneous layers with interfaces of arbitrary dip and strike can be written as

$$T = \boldsymbol{p}_S\cdot\boldsymbol{X}_S \;+\; \boldsymbol{p}_R\cdot\boldsymbol{X}_R \;+\; \sum_j (q_{S_j}+q_{R_j})z_j \quad , \tag{13}$$

where $\boldsymbol{X}_S$ is the horizontal position vector from the source S to a vertical reference line on which the layer thicknesses z_j are defined; $\boldsymbol{X}_R$ is the horizontal position vector from the reference line to the receiver R; $\boldsymbol{p}_S$ and $\boldsymbol{p}_R$ are the horizontal slowness vectors of the rays departing from the source S, and arriving to the receiver R; and q_{S_j} and q_{R_j} are the vertical slowness components of the source and receiver rays in layer j. The summation is over the layers j traversed by the rays (Figure 2).

The reference line is fixed, but can be chosen anywhere, e.g., through the source, at the receiver, at the reflection point, or elsewhere. The choice of the reference line determines the layer thicknesses z_j and the horizontal vectors $\boldsymbol{X}_S$ and $\boldsymbol{X}_R$, but does not influence the slowness components $\boldsymbol{p}_S$, $\boldsymbol{p}_R$ and q_j.

Diebold's formula is particularly elegant because it is a mere generalization of the well-known traveltime formulas in one- and two-dimensional layered media. Moreover, Richards (1990) showed that the trigonometric proof of equation (13) given in the original paper of Diebold (1987) is hardly needed when the raypath is decomposed in a particular way.

In our problem, it is convenient to put the reference line at the reflection point in the crossline direction $x=0$. With the assumption that the half source-receiver offset h is small, we can write

$$T_0 = 2\;[\;\boldsymbol{p}_0\cdot\boldsymbol{X}_0 \;+\; \tau_0\;] \quad = \quad 2\left[\;p_0^{(y)}y_0 \;+\; \tau_0\;\right] \quad , \tag{14}$$

$$T = 2\;[\;\boldsymbol{p}_S\cdot\boldsymbol{X}_S \;+\; \tau\;] \quad \simeq \quad T_0 \;+\; 2\left[\;p_0^{(y)}\frac{h^2}{y_0} \;+\; \delta\tau\;\right] \quad , \tag{15}$$

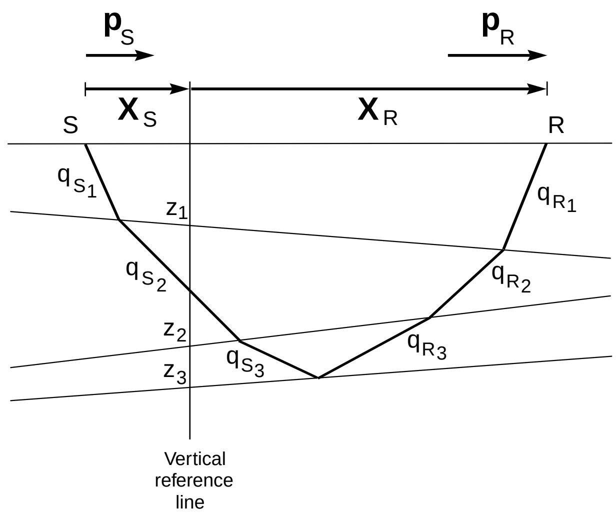

FIG. 2. 2-D schematic of a 3-D raypath from source S to receiver R which is composed of line segments that in general are not co-planar. Dipping interfaces are planar, but are not presumed to share a common strike. The local layer thicknesses z_j are defined by the intersection of the interfaces with the vertical reference line. The q_{S_j}'s and q_{R_j}'s denote the vertical slownesses in each layer, $\mathbf{p}_S$ and $\mathbf{p}_R$ are the horizontal slowness vectors at the source and at the receiver, and $\mathbf{X}_S$ and $\mathbf{X}_R$ represent the horizontal distances traveled along the source and receiver raypaths (after Richards, 1990).

where

$$\tau_0 = \sum_j q_{0_j} z_j \quad ; \quad \tau = \sum_j q_{S_j} z_j \quad \text{and} \quad \delta\tau = -\frac{(p_0^{(y)} h)^2}{2y_0^2} \sum_j \frac{z_j}{q_{0_j}} \quad . \tag{16}$$

We then obtain

$$T^2 = T_0^2 + \frac{4h^2}{V_{dip}^2(T_0, y_0)} + O(h^4) \tag{17}$$

with

$$V_{dip}^2(T_0, y_0) = \frac{2y_0}{p_0^{(y)} T_0}. \tag{18}$$

The above expression for $V_{dip}(T_0, y_0)$ was also given by Witte (1991). When the horizontal distance y_0 traveled along the zero-offset ray is small, it can be shown that

$$V_{dip}(T_0, y_0) \simeq V_2(t_{mig}) \quad , \tag{19}$$

where t_{mig} is the two-way migration (vertical) time corresponding to the reflection at time T_0.

III. THE LINEAR *V(z)* CASE

In a medium characterized by a velocity function $V(z) = V_0 + kz$, the expressions of $V(t)$, $V_2(t)$ and $V_4(t)$ at the two-way traveltime t are respectively given by

$$V(t) = V_0 e^{kt/2} \quad , \quad V_2(t) = V_0 \left[\frac{e^{kt} - 1}{kt} \right]^{1/2} \quad \text{and} \quad V_4(t) = V_0 \left[\frac{e^{2kt} - 1}{2kt} \right]^{1/4} \quad . \tag{20}$$

When these expressions are substituted in equation (10), we find

$$\gamma_x(t_n) = \frac{kt_n \; (e^{kt_n} + 3)}{4 \; (e^{kt_n} - 1)} \quad . \tag{21}$$

In addition, the closed form expression of the traveltime T_0 given in Paper I allows us to write equation (18) in the form

$$V_{dip}^2(T_0, y_0) = V_0 V(t_{mig}) \frac{\sinh kT_0/2}{kT_0/2} \quad , \tag{22}$$

and, after some approximations,

$$V_{dip}^2(T_0, y_0) \simeq V_2^2(t_n) \left[1 - \frac{kt_n \lambda_y^2 V_2^2(t_n)}{16} \right] \quad . \tag{23}$$

The expression of γ_y can then be obtained from equations (6) and (7), and is simply written

$$\gamma_y(t_n) = -\frac{kt_n}{4} \quad . \tag{24}$$

Equations (21) et (24) clearly show that the squeeze functions γ_x and γ_y are independent of the velocity V_0 and depend only on the velocity gradient k. Moreover, it can be demonstrated from equation (21) that

$$\frac{\mathrm{d}\gamma_x}{\gamma_x} \ll \frac{\mathrm{d}k}{k} \quad , \tag{25}$$

which shows that γ_x depends only weakly on k. On the other hand, since γ_y is directly proportional to k, the curvature of the DMO operator in the crossline direction will be affected by errors in the velocity gradient k. (However, it should be recalled that the most energetic contributions of the DMO operator are concentrated along the inline direction – see Paper I).

The behavior of the DMO impulse response predicted by equations (21) and (24) is entirely confirmed by the curves displayed in Figures 3 and 4. Figures 3 and 4 respectively show the *exact* inline and crossline components of the DMO operator (calculated from the equations given in Paper I), for several values of V_0 and k.

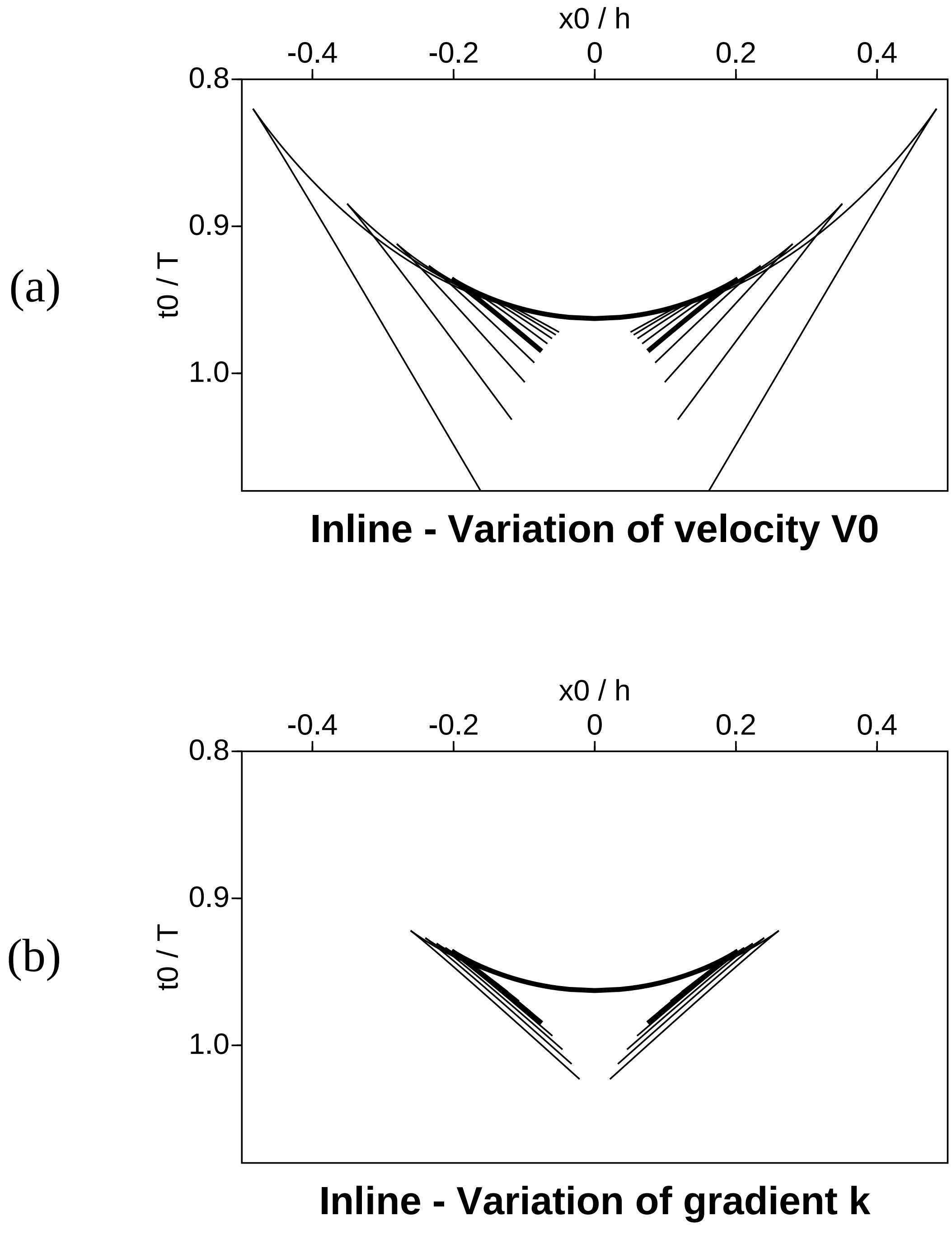

FIG. 3. DMO operators in the inline direction for variations of $\pm 10\%, \pm 20\%, \pm 30\%, \pm 40\%, \pm 50\%$ of velocity V_0 (a), and gradient k (b). The standard values used in this calculation are $V_0 = 1.5$ km/s, $k = 0.3$ /s, $h = 2$ km and $T = 6$ s. The curves have been superposed by using the normal moveout time t_n corresponding to the standard values.

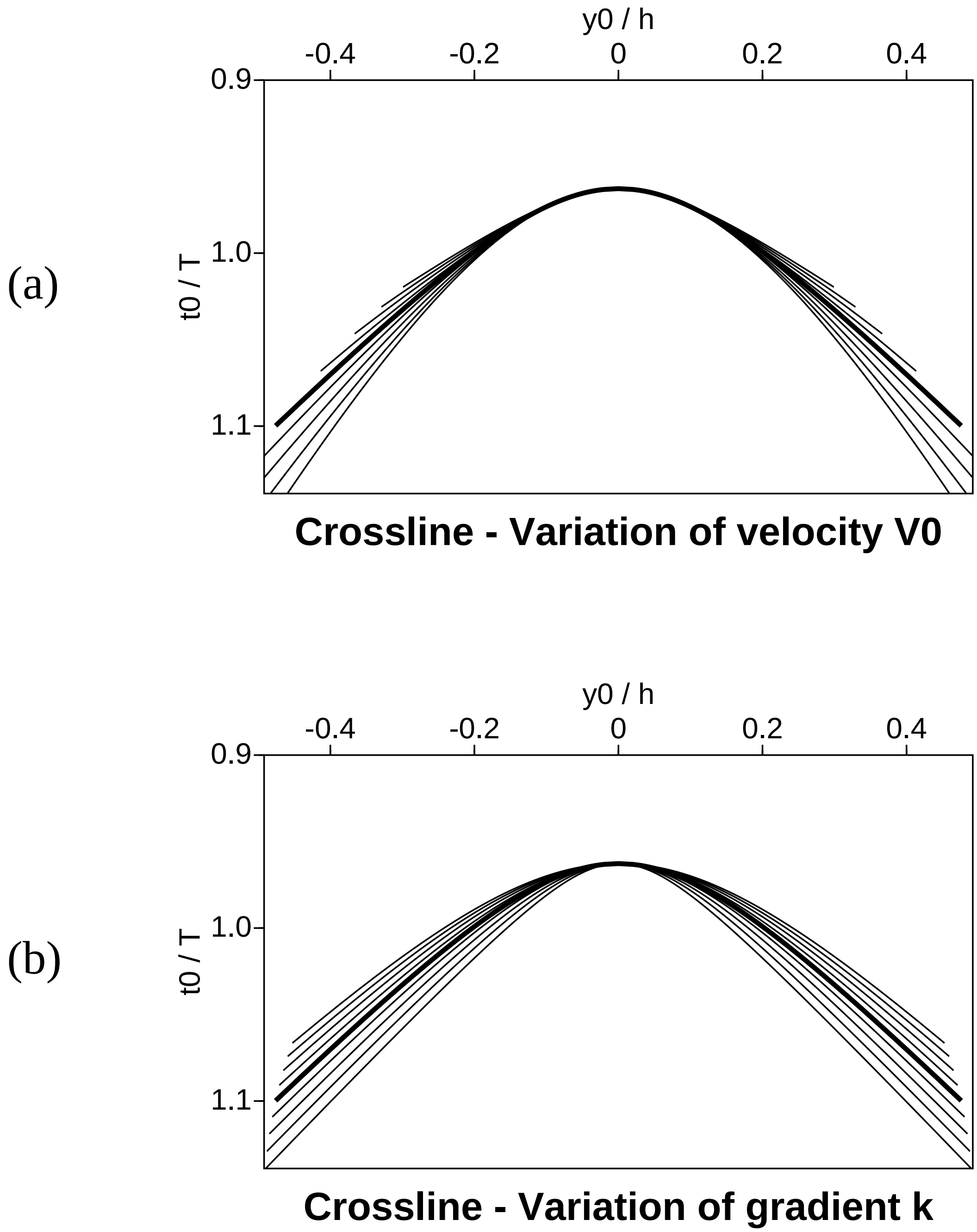

FIG. 4. Same as Figure 3, but in the crossline direction.

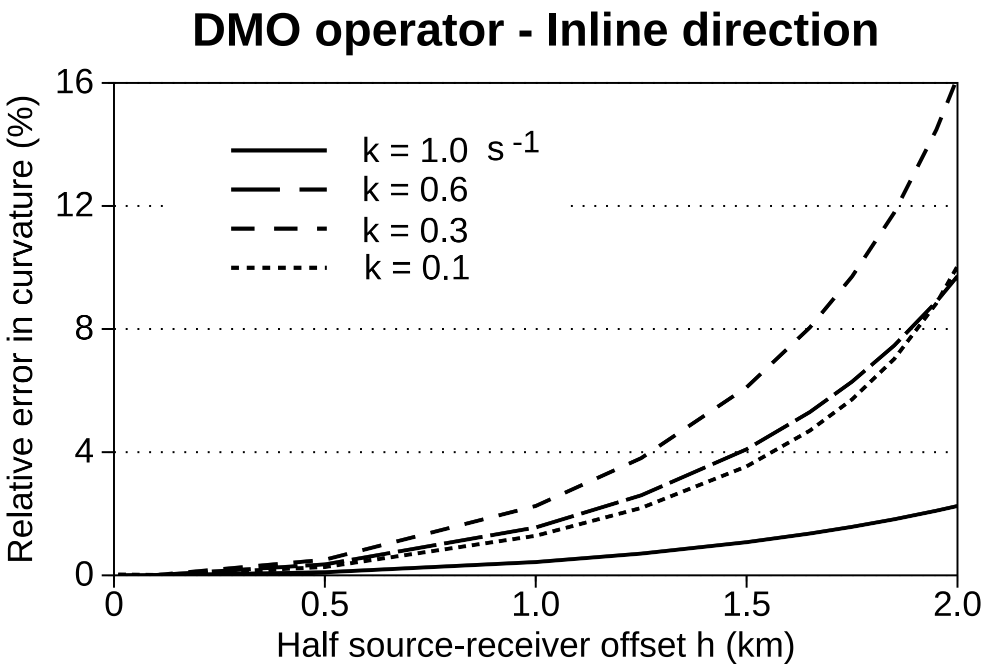

FIG. 5. Accuracy of the DMO approximation in the inline direction. The curves show the relative error in curvature as a function of the half source-receiver offset h, for four different values of the velocity gradient k. The other parameters used in this calculation are $V_0 = 1.5$ km/s and $T = 3$ s .

Besides, it may be noticed from equations (2) and (24) that the curvature in the crossline direction is independent of t_n, and can be exactly computed for any offset h. Figure 5 shows that the curvature computed from equations (2) and (21) in the inline direction remains very accurate when the source-receiver offset is increased.

CONCLUSIONS

An approximate representation of the DMO operator in a medium with a constant velocity gradient has been derived. The approximate operator can be used for any velocity gradient, and remains accurate even for rather large source-receiver offsets. The DMO correction in the inline direction is almost independent of the velocity function considered.

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