2.5D Wave Equations and High-Frequency Asymptotics

John W. Stockwell, Jr.

ABSTRACT

The need for modeling 3-D seismic data in a 2-D setting has motivated investigators to create so-called 2.5-D modeling methods. One such method proposed by Liner (1991) involves the use of an approximate 2.5-D wave operator for constant-density media.

The traveltimes and amplitudes predicted by WKBJ analysis of the Liner 2.5-D wave equation match those predicted by Bleistein's 2.5-D ray-theoretic development for both constant and piecewise constant wavespeed models. However, WKBJ analysis indicates that the Liner 2.5-D variable wavespeed equation will have a maximum amplitude error of $\pm 35\%$ (compared with the $+250\%$ error obtained by using the ordinary 2-D wave equation) in a linear $c(z)$ model where the wavespeed doubles or halves from the beginning to the end of a raypath. The WKBJ series results suggest a modification to the Liner equation that may reduce this error.

Construction of 2.5-D wave equations for variable density (acoustic) problems also follows from the WKBJ analysis.

INTRODUCTION

The recording of seismic data along a line on the Earth's surface is still a widely used technique. The earth model implied by such a dimensionally-constrained recording geometry is a 3-D model with only 2 dimensions of parameter variability, Bleistein (1986). Consequently, there is a need for modeling seismic data in problems with 2-D parameter variability, but yielding waves having full 3-D amplitude behavior. This need has motivated investigators (Deregowski and Brown 1983, Ursin 1978, Hubral 1978, and Newman 1973) to consider 2-D models scaled by an out-of-plane geometric spreading factor to make the amplitudes approximate 3-D amplitudes.

Bleistein (1986) shows how to describe such 2.5-D modeling in a unified manner using the methods of high-frequency asymptotics. In particular, Bleistein develops the ray-theoretic implications of 2.5-D modeling for both constant and variable-density acoustic problems. Bleistein also characterizes the effects of the 2.5-D assumption on the Fourier-like integrals that form the basis of many migration algorithms.

A different approach was taken by Liner (1991), who attempted to create a "2.5-D wave equation" by using the constant wavespeed Green's function as trial solution to the 2-D wave equation. The resulting equation has a form similar to a damped wave equation with a damping coefficient that varies both in space and time. Liner's result differs from a traditional damped wave equation in that it has an additional term containing no derivatives.

The advantage of the Liner approach is that it promises to provide a *wave equation* that could presumably be exploited to permit known wave equation-based techniques (for example, finite-difference modeling) to be applied in 2.5 dimensions. Because numerical implementation of the Liner equation has yielded encouraging results (Bording and Liner 1992 and Bording and Liner 1993), further analysis of this equation is warranted. Therefore, the topic of this paper will largely be a comparison of

Liner's 2.5-D wave equation against the 2.5-D ray theoretic results of Bleistein via a WKBJ series method.

In turn, the WKBJ asymptotic analysis will suggest an approach that removes many of the *ad hoc* aspects of Liner's construction and will even permit the extension of the 2.5-D wave equation concept to variable-density (acoustic) problems.

THE CONCEPT OF A 2.5-D WAVE EQUATION

To approximately describe 3-D wave propagation in a 2-D setting, a reasonable approach might be to construct a damped wave equation

$$\left[\nabla^2 - \gamma(\boldsymbol{x}, t)\frac{\partial}{\partial t} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right] U(\boldsymbol{x}, t) = 0$$

with a spatially and temporally-varying damping coefficient $\gamma(\boldsymbol{x}, t)$ to correct the amplitudes for the effect of out-of-plane geometric spreading. (Here $\boldsymbol{x} \equiv (x, z)$ and ∇^2 is the 2-D Laplacian operator.) Such a "2.5-D wave equation" will probably not be an exact result, but may be best described as an *asymptotic wave operator.* That is, it would be an approximate wave operator having the same asymptotic behavior as the 3-D wave equation for a select collection of models and/or frequency ranges.

The use of approximate wave operators in geophysics has a long tradition. The most notable examples involve the use of the parabolic approximations to the Helmholtz equation (see Claerbout 1970 for exploration geophysics applications; see Tappert 1977 and Hill 1986 for examples of ocean acoustics applications).

THE LINER 2.5-D WAVE OPERATOR

Here is a brief outline of the method that Liner (1991) used to construct a 2.5-D asymptotic wave operator. Liner reasoned that in a constant wavespeed medium, the "2.5-D Green's function" is just the in-plane representation of the 3-D Green's function:

$$G_{2.5D} = \frac{\delta(t - r/c)}{4\pi r},$$

with $r = \sqrt{x^2 + z^2}$ being just the range for the case $y = 0$. The ideal problem to be solved would be to find $\mathcal{L}$ such that

$$\mathcal{L} G_{2.5D} = -\delta(t)\delta(\boldsymbol{x}),$$

where $\boldsymbol{x} \equiv (x, z)$. Then $\mathcal{L}$ would be the "exact 2.5-D operator". Note that the Green's function must also satisfy the homogeneous equation

$$\mathcal{L} G_{2.5D} = 0 \tag{1}$$

for $\boldsymbol{x} \neq \mathbf{0}$ and $t \neq 0$.

Because Liner did not intend to use the operator in the vicinity of the source, he chose to try to construct a nontrivial wave operator that satisfies (1) using the homogeneous 2-D wave equation as starting point. Liner's method was to substitute $G_{2.5D}$ as a trial solution into the homogeneous 2-D wave equation to yield

$$\left[\nabla^2 - \frac{1}{c^2}\frac{d^2}{dt^2}\right] G_{2.5D}(\boldsymbol{x}, t) = -\frac{\delta'(t - r/c)}{4c\pi r^2} - \frac{\delta(t - r/c)}{4\pi r^3}. \tag{2}$$

The two remainder terms on the right result because $G_{2.5D}$ is not a solution to the 2-D wave equation. Liner proposed the following formula

$$\left[\nabla^2 - \frac{1}{c^2}\left\{\frac{d^2}{dt^2} + \frac{c}{r}\frac{d}{dt} + \frac{c^2}{r^2}\right\}\right] U(\boldsymbol{x}, t) = 0, \tag{3}$$

which is just the homogeneous 2-D wave equation with 2 additional terms, to account for the remainder terms in equation (2). Note, while $G_{2.5D}$ is a solution to equation (3), it cannot be assumed to be the *Green's function* of this equation.

Equation (3) is a constant wavespeed equation. To handle variable wavespeed problems, Liner proposed the following formula:

$$\left[\nabla^2 - \frac{1}{c^2}\left\{\frac{d^2}{dt^2} + \frac{1}{t}\frac{d}{dt} + \frac{1}{t^2}\right\}\right] U(\boldsymbol{x}, t) = 0. \tag{4}$$

Liner justified choosing this form by noting that in equation (3) the factors of c/r and c^2/r^2 are, respectively, $1/\tau$, and $1/\tau^2$, where τ is traveltime. Because the signals typically recorded in geophysical investigations have considerably shorter duration than the observed traveltimes, Liner assumed that $t \approx \tau$, where t is absolute time. The method of construction here contains elements that are clearly *ad hoc*, in that the more general forms are being deduced from less general forms—the reverse of the usual mathematical process.

For future reference, equations (3) and (4) will be called, respectively, the "r-equation" and the "t-equation", reflecting the respective r and t dependence of the coefficients of these equations. Note that the t-equation reduces to the r-equation for constant wavespeed media under the assumption of $t \approx \tau$.

Testing the r-equation with WKBJ Asymptotics

If the Liner 2.5-D wave equation is to be considered useful, its solution must match the solutions predicted by other methods of solving the 2.5-D modeling problem. The paper by Bleistein (1986) outlines a ray-theoretic WKBJ asymptotic-based solution to the 2.5-D modeling problem. By deriving the eikonal and transport equations associated with Liner's 2.5-D wave equations, it is possible to make a direct comparison of the high-frequency behavior of these equations with the Bleistein's 2.5-D ray theory. All standard 2-D and 3-D ray theoretic results that follow are taken from Bleistein (1984) section 8.2, while 2.5-D ray theoretic results are taken from Bleistein (1986).

Paralleling the standard development of ray theory, the r-equation is Fourier transformed in time to yield the Helmholtz-like or reduced form

$$\left[\nabla^2 + \frac{\omega^2}{c^2} + \frac{i\omega}{rc} - \frac{1}{r^2c^2}\right] u(\boldsymbol{x}, \omega) = 0. \tag{5}$$

It is sufficient to use

$$u(\boldsymbol{x}, \omega) \sim A(\boldsymbol{x}) \exp[i\omega\tau(\boldsymbol{x})],$$

(just the first term of the traditional WKBJ series) as a trial solution to (5). Here $\tau(\boldsymbol{x})$ is the traveltime and $A(\boldsymbol{x})$ is the amplitude function. Substituting this trial solution into equation (5) yields terms of order $O(\omega^2)$ and $O(\omega)$.

The coefficient of the term of order $O(\omega^2)$ is the same eikonal equation,

$$(\nabla\tau)^2 - \frac{1}{c^2} = 0, \tag{6}$$

that is obtained in 2-D and 2.5-D ray theory.

Similarly, the coefficient of the $O(\omega)$ term is the same (first) transport equation,

$$2\nabla A \cdot \nabla\tau + A\nabla^2\tau + \frac{A}{cr} = 0, \tag{7}$$

that is obtained standard 2-D ray theory, with the exception that it contains an extra term, A/cr. However, Bleistein (1986) does not list a "2.5-D transport equation", meaning that it is necessary to derive such a result to facilitate a comparison between 2.5-D ray theory and Liner's equation.

From standard ray theory

$$\nabla\tau = \frac{\partial \boldsymbol{x}}{\partial\sigma} \qquad \text{and} \qquad \frac{d\tau}{d\sigma} = \frac{1}{c^2}. \tag{8}$$

Here σ is a running parameter along a ray, representing a quantity with units of $[\text{length}]^2/[\text{time}]$ and is related to the traveltime τ through the second expression above. The standard transport equation (either 2-D or 3-D)

$$2\nabla A \cdot \nabla\tau + A\nabla^2\tau = 0, \tag{9}$$

may be rewritten as

$$\frac{dA^2}{d\sigma} = -A^2\nabla^2\tau, \tag{10}$$

by multiplying equation (9) by A and by using the first relation in (8).

Another important result from ray theory is that

$$\nabla^2 A = \frac{d}{d\sigma}\left\{\log[J_{3D}]\right\} \qquad \text{where} \qquad J_{3D} \equiv \frac{\partial(x_1, x_2, x_3)}{\partial(\sigma, \alpha, \beta)}.$$

Here J_{3D} is the 3-D ray Jacobian, and α and β are parameters that label the ray by its initial direction.

For the special case of $x_2 = 0$ and $c(\boldsymbol{x}) = c(x_1, x_3)$—the 2.5-D assumption—$J_{3D}$ becomes σJ_{2D} and equation (10) becomes

$$\nabla^2 A = \frac{d}{d\sigma}\left\{\log[\sigma J_{2D}]\right\} \qquad \text{where} \qquad J_{2D} \equiv \frac{\partial(x_1, x_3)}{\partial(\sigma, \beta)}.$$

Thus,

$$\nabla^2 A = \frac{1}{\sigma} + \frac{d}{d\sigma}\left\{\log[J_{2D}]\right\} = \frac{1}{\sigma} + \nabla^2_{2D} A, \tag{11}$$

where ∇^2_{2D} is the 2-D in-plane Laplacian. This implies that equation (9) becomes

$$2\nabla A \cdot \nabla \tau + A \nabla^2 \tau + \frac{A}{\sigma} = 0 \tag{12}$$

under the 2.5-D assumption. Because the out-of-plane component of the gradient of τ is zero, i.e. $\partial\tau/\partial x_2 = 0$, when (12) is evaluated in the $x_2 = 0$ plane, ∇ and ∇^2 are the 2-D gradient and Laplacian operators, respectively. For constant wavespeed media, $\sigma = c^2 \tau$ and $\tau = r/c$, meaning that this equation reduces to (7) for the case of constant wavespeed.

This shows that the high-frequency behavior of the r-equation, as expressed by leading order WKBJ (ray theoretic) analysis, agrees with Bleistein's 2.5-D ray theory both in traveltime and amplitude.

Progressing wave analysis of the t-equation

The WKBJ analysis of the r-equation in the previous section yields eikonal and transport equations that reduce to forms consistent with the expectations of a 2.5-D theory assuming constant wavespeed. However, the t-equation is of greater interest, because it is intended for use in variable wavespeed problems. This equation is not as easy to analyze with the WKBJ method as the r-equation because the time variability of the coefficients makes forming the temporal Fourier transform more difficult. (This is not impossible, if divisions by it are interpreted as integrations in ω, see Liner and Stockwell, 1993).

There is an alternate asymptotic method called the progressing wave formalism, Lewis (1964), (compare with Červený, Molotkov, and Pšenčík, 1977) that can be used to conduct a WKBJ -like analysis in the time domain.

Here, it is sufficient to use

$$U(\boldsymbol{x}, t) \sim A(\boldsymbol{x}) S\Big(t - \tau(\boldsymbol{x})\Big), \tag{13}$$

as a trial solution of equation (3). Substitution of (13) into (3) yields terms that contain S and S' (the derivative of S), respectively. The S and S' quantities perform the same function as the differing powers of ω do in the standard WKBJ analysis.

The coefficient of the term containing S' is the same 2-D or 2.5-D eikonal equation as equation (6). The coefficient of the term containing S

$$2\nabla A \cdot \nabla \tau + A \nabla^2 \tau + \frac{A}{c^2 t} = 0. \tag{14}$$

is the corresponding (first) transport equation.

Under the assumption that the propagating waveforms have short time histories, τ may be substituted for t in equation (14).

Amplitude comparisons

The eikonal equation associated with the t-equation is the same 2-D eikonal equation that is implied by Bleistein's 2.5-D ray theory, meaning that the t-equation predicts the same traveltimes as Bleistein's 2.5-D ray theory for high-frequency data. Comparing the amplitudes predicted by the t-equation with Bleistein's 2.5-D theory requires that the transport equation (14) be solved for A.

Equation (14) may be rewritten as a differential equation in σ

$$\frac{dA^2}{d\sigma} = -\frac{A^2}{c^2 \tau} - A^2 \frac{d}{d\sigma} \left\{ \log[J_{2D}] \right\},$$

as was equation (11). Here $c^2\tau$ replaces the σ in equation (11). Dividing by A^2 and integrating with respect to σ yields, after some simplification,

$$\log A^2(\sigma) - \log A^2(\sigma_0) = -\int_{\sigma_0}^{\sigma} \frac{d\sigma'}{c^2(\boldsymbol{x}(\tau'))\tau'} - \log[J_{2D}(\sigma)] + \log[J_{2D}(\sigma_0)]. \qquad (15)$$

Making use of equation (8), it is possible to rewrite this equation as

$$A^{\text{Liner}}(\sigma) = \text{const}\ \frac{1}{\sqrt{\tau J_{2D}}}.$$

Here, all parts that are a function of σ_0 have been absorbed into the constant multiplier, and the symbol $A^{\text{Liner}}(\sigma)$ has been substituted for $A(\sigma)$. The corresponding result from Bleistein (1986) is

$$A^{\text{Bleistein}}(\sigma) = \frac{1}{4\pi\sqrt{\sigma J_{2D}}}.$$

For a medium with constant wavespeed c_0, the transport equation associated with the t-equation is the same as that for the standard scalar wave equation, because $\sigma = c_0^2\tau$ in that case. If, however, the wavespeed profile is smoothly varying, beginning as some constant value c_0, then the t-equation result must be the same as Bleistein's result for the constant wavespeed part, meaning that a factor of $1/c_0$ must be present in the constant multiplier.

Therefore, the high-frequency amplitude function for Liner's equation is

$$A^{\text{Liner}}(\sigma) = \frac{1}{4\pi\sqrt{c_0^2\ \tau J_{2D}(\sigma)}},$$

where c_0 is the wavespeed at the beginning of the ray path in a smoothly varying medium.

Because the leading order terms of the WKBJ series are the same for the Liner equation as for the standard wave equation, the same 2.5-D form of the WKBJ Green's function is implied in both cases. This means that, to leading order, the transmission and reflection effects across boundaries will be described by the same asymptotic representations. Thus, as long as the high-frequency condition is honored, the

Liner equation can be expected to describe wave propagation in piecewise constant wavespeed media with the same degree of accuracy as the standard wave equation, provided that $t \approx \tau$.

However, the t-equation will not describe the amplitudes exactly in smoothly varying media, even if $t \approx \tau$ because the out-of-plane spreading factor $1/c_0\sqrt{\tau}$ is not the same as $1/\sqrt{\sigma}$,

AMPLITUDE ERROR ESTIMATES

The maximum and minimum error estimates for a medium with the linear wavespeed profile of the form $c(z) = c_0(1 \pm \eta z)$ are given by the ratio

$$\frac{A^{\text{Bleistein}}}{A^{\text{Liner}}} = \sqrt{\frac{c_0^2 \tau}{|\sigma|}}. \tag{16}$$

Bleistein (1986) gives analytic expressions for σ and τ in the special case of $c(z)$ media

$$\tau = \int_{\xi_3}^{z} \frac{dz'}{c^2(z')\sqrt{1/c^2(z') - \sin^2\beta/c^2(\xi_3)}}$$

and

$$\sigma = \int_{\xi_3}^{z} \frac{dz'}{\sqrt{1/c^2(z') - \sin^2\beta/c^2(\xi_3)}}. \tag{17}$$

Here, ξ_3 and z represent the initial and final x_3-positions on the ray, respectively, with z' being the dummy integration variable. The parameter β is explicitly defined to be the initial ray take-off angle, measured from the vertical. Therefore it is possible to directly compare the Bleistein's 2.5-D amplitudes with the high frequency asymptotic amplitudes of the t-equation for the simple linear profile $c(z) \equiv c_0(1 + \eta z)$.

For the model being considered, the amplitude ratio given by equation (16) is independent of the reference wavespeed c_0, meaning that the maximum depth in the model may be arbitrarily fixed to $z = 1$. Then η will represent the fractional change in wavespeed with depth from its initial value of c_0 at $z = 0$, to its final value at

$z = 1$. For example, $\eta = -0.5$ and $\eta = 1.0$ imply wavespeeds of $c_0/2$ and $2c_0$ at $z = 1$, respectively.

Figure 1 shows a estimate of this amplitude ratio for $-0.5 \leq \eta \leq 1.0$ and $0 \leq \beta \leq 90$ degrees, computed using *Mathematica*™. (The zone that is blank represents, η, β combinations for which no rays exist.) The amplitude ratio shows that the Liner equation overestimates the amplitude when the wavespeed decreases with depth (i.e. when $\eta < 0$) and underestimates the amplitude when the wavespeed increases (i.e. when $\eta > 0$). The maximum size of the error in amplitude is about $\mp 35\%$ in each respective case of halving or doubling the wavespeed from the reference value. From Figure 1 it is apparent that the Liner equation is most acceptable in models where the amplitude change is described by $|\eta| \leq 0.2$, implying amplitude errors $\leq 10\%$.

PROPOSED MODIFICATIONS TO THE LINER EQUATION

The t-equation is useful for modeling constant density media with a range of constant, piecewise constant, and weakly-varying wavespeed profiles. However, insight gained from considering the WKBJ analysis can be used to propose extensions of this equation to more general problems.

The following symbolic form

$$\left[\nabla^2 - \frac{1}{\sigma} \frac{\partial}{\partial t} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] U(\boldsymbol{x}, t) = 0 \tag{18}$$

would have equation (12) as its transport equation. Note that there is no term in (18) that corresponding to the last term in the t-equation, because such a term will cause the higher order transport equations to differ from those implied by Bleistein's 2.5-D ray theory.

However, the practicality of implementing (18) may be questionable because an estimate of σ is needed. Recall, from (8), that $\sigma = \int^\tau c^2(\tau') d\tau'$, meaning that σ is a quantity that can be derived in the most general application only by tracing rays to all points in the medium—not practical for this application. Clearly, an approximation is needed. One approximation is to take $\sigma_n \approx c^2(x,z)n\Delta t$, which is just the $c^2\tau$ approximation (with $t \approx \tau$) that is built into the Liner equation. More sophisticated approximations might be made by assuming a general form of a $c(z)$ profile as an "average medium" and using equation (17) to estimate σ.

Extension to variable density

Bleistein (1986), section 5, develops 2.5-D ray theory for the variable density (acoustic) wave equation

$$\rho(\boldsymbol{x})\nabla \cdot \left(\frac{\nabla U(\boldsymbol{x},t)}{\rho(\boldsymbol{x})}\right) - \frac{1}{c^2(\boldsymbol{x})}\frac{\partial^2 U(\boldsymbol{x},t)}{\partial t^2} = 0. \tag{19}$$

The result of WKBJ analysis of this equation is the same eikonal equation as equation (6) and the (first) transport equation

$$2\nabla\tau \cdot \nabla B + B\rho\nabla\left(\frac{1}{\rho}\right) + B\nabla^2\tau = 0. \tag{20}$$

The amplitude function for 2.5-D models variable density acoustic problem is given by Bleistein (1986) as

$$B = \frac{1}{4\pi}\sqrt{\frac{\rho(\boldsymbol{x}(\sigma,\beta))}{\rho(\boldsymbol{\xi})\ \sigma J_{2D}}}.$$

Here, $\rho(\boldsymbol{\xi})$ is the density at some initial position $\boldsymbol{\xi}$ and σ is the running parameter along the ray.

It is apparent that variable density does not affect the value of the out-of-plane spreading coefficient. It is possible, therefore, to write the transport equation implied by the variable-density 2.5-D theory as

$$2\nabla(B)\cdot\nabla\tau + B\rho\nabla\left(\frac{1}{\rho}\right) + B\nabla^2\tau + \frac{B}{\sigma} = 0.$$

Damped wave operators of the form

$$\rho(\boldsymbol{x})\nabla \cdot \left(\frac{\nabla U(\boldsymbol{x},t)}{\rho(\boldsymbol{x})}\right) - \gamma(\boldsymbol{x},t)\frac{dU(\boldsymbol{x},t)}{dt} - \frac{1}{c^2(\boldsymbol{x})}\frac{d^2U(\boldsymbol{x},t)}{dt^2} = 0$$

will yield eikonal and transport equations compatible with variable density 2.5-D ray theory. It follows that the 2.5-D variable-density wave equation corresponding to Liner's t-equation is

$$\rho(\boldsymbol{x})\nabla\cdot\left(\frac{\nabla U(\boldsymbol{x},t)}{\rho(\boldsymbol{x})}\right)-\frac{1}{c^2}\left\{\frac{d^2}{dt^2}+\frac{1}{t}\frac{d}{dt}+\frac{1}{t^2}\right\}U(\boldsymbol{x},t)=0.$$

Similarly,

$$\rho(\boldsymbol{x})\nabla\cdot\left(\frac{\nabla U(\boldsymbol{x},t)}{\rho(\boldsymbol{x})}\right)-\frac{1}{\sigma}\frac{dU(\boldsymbol{x},t)}{dt}-\frac{1}{c^2}\frac{d^2U(\boldsymbol{x},t)}{dt^2}=0. \tag{21}$$

is the symbolic form that corresponds to equation (18).

CONCLUSIONS

Analysis of the 2.5-D wave equation of Liner (1991) using a WKBJ series method shows that the traveltimes and amplitudes predicted by Liner's equation match those of predicted by Bleistein's 2.5-D ray theory for constant and piecewise-constant wavespeed media.

However, for media with smoothly varying wavespeed profiles the WKBJ analysis reveals that the transport equation associated with the Liner wave equation is not the same as that implied by Bleistein's 2.5-D theory. For a linear $c(z)$ wavespeed profile whose wavespeed halves or doubles from the beginning to the end of a ray path, the amplitude error resulting from this mismatch will be $\mp 35\%$. For models with turning rays, the total amplitude error will tend to be less, because the overestimation of amplitude in one direction will be offset by an underestimation of amplitude as the ray travels in the opposite direction. In either case, the Liner 2.5-D equation will still yield amplitudes that are far more realistic than the 2-D wave equation. For the linear $c(z)$ model discussed above, the corresponding amplitudes for the 2-D wave equation will be 250% too large.

The WKBJ analysis provides a method for creating an alternate asymptotic wave operator that resembles the Liner equation, but that is matched more closely to

Bleistein's 2.5-D ray theory in predicted amplitude behavior. However, there may be little advantage in implementing this alternate scheme because of the difficulty of estimating the parameter σ.

The WKBJ method also may be applied to create 2.5-D wave operators for variable-density (acoustic) problems. Equation (21) is the result of this analysis. Because density variation does not influence the out-of-plane geometric spreading factor, this equation should describe the leading order traveltime and amplitude behavior of waves with the same accuracy as 2.5-D ray theory, provided that density accounts for the majority of the parameter variability of the model.

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FIGURE 1

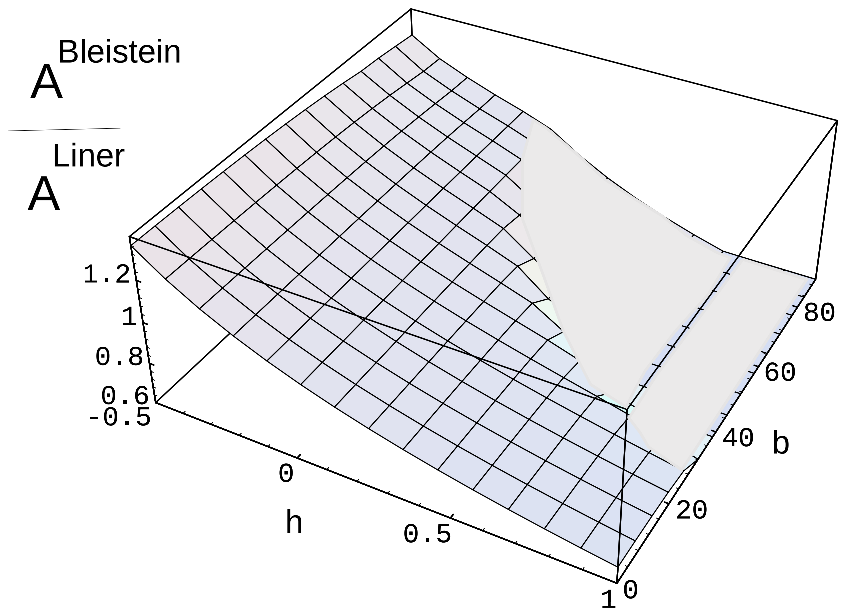

FIG. 1. Ratio of Bleistein's 2.5-D amplitudes vs. WKBJ amplitudes for Liner's 2.5-D wave equation for a linear $c(z)$ model. Here, β is the ray take-off angle and η is the fractional wavespeed change with depth.