Velocity Analysis for Transversely Isotropic Media

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ABSTRACT

The main difficulty in extending seismic processing to anisotropic media is the recovery of anisotropic velocity fields from surface reflection data. We suggest carrying out velocity analysis for transversely isotropic media by inverting the dip-dependence of P-wave moveout velocities. The inversion technique is based on the analytic equation for the normal moveout (NMO) velocity for dipping reflectors in anisotropic media derived by Tsvankin (1993). Recasting this equation as a function of ray parameter provides a mechanism for obtaining the P-wave vertical velocity V_{P0} and Thomsen parameters ϵ and δ using only surface P-wave data. We use the weak-anisotropy approximation for NMO velocity to get analytic insight into the inverse problem and perform the actual inversion by applying the Newton-Raphson method to the exact NMO equation, valid for arbitrary strength of the anisotropy.

Our inversion procedure makes it possible to obtain a family of solutions for V_{P0}, ϵ and δ, all of which have the same NMO velocity as a function of ray parameter. Therefore, it is impossible to recover all three parameters (V_{P0}, ϵ, and δ) from P-wave normal moveout velocity alone, even if a wide range of dips is available. We show that all equivalent models have the same horizontal velocity, long-spread (nonhyperbolic) P-wave moveout for horizontal reflectors, as well as the same time-migration impulse response. In order to describe this family of models, we introduce an effective parameter η that reduces to the difference $\epsilon - \delta$ in the limit of weak anisotropy. Additional steps, such as monitoring the quality of migration image, can be used to ensure the accuracy of the range of solutions. Therefore, inversion of dip-moveout information allows one to perform many important processing steps using only surface P-wave data.

Accurate time-to-depth conversion, however, requires that the anisotropic parameters be resolved individually. If the vertical velocity is known (e.g., from check shots or well logs), the anisotropies ϵ and δ can be found by inverting two P-wave NMO velocities corresponding to a horizontal and a dipping reflector. If no well information is available, the values of ϵ and δ can be obtained by combining our inversion results with shear-wave information, such as the P-SV or SV-SV wave NMO velocities for a horizontal reflector.

Generalization of Tsvankin's single-layer NMO equation for layered anisotropic media with a dipping reflector allows us to extend anisotropic velocity analysis to vertically inhomogeneous media. We show that the influence of a stratified overburden on moveout velocity can be stripped through a Dix-type differentiation procedure.

INTRODUCTION

The importance of anisotropic phenomena in wave propagation is now widely recognized by the exploration community. Errors caused by anisotropy in seismic processing include mis-ties in time-to-depth conversion (Banik, 1984), mispositioning of reflectors in migration (Larner and Cohen, 1993; Alkhalifah and Larner, 1994), distortions of dip-moveout signature (Larner, 1993; Tsvankin, 1993), and AVO response (Wright, 1987, Tsvankin, 1994a).

However, progress in accounting for anisotropy in seismic processing has been slow, mostly due to the difficulty in obtaining anisotropic velocity fields from surface seismic data. For instance, there exist a number of migration algorithms for transversely isotropic media (e.g., VerWest, 1989; Sena and Toksoz, 1993; Alkhalifah, 1994), but their application requires knowledge of the anisotropic velocity model. Clearly, the recovery of several independent elastic coefficients needed to reconstruct the anisotropic velocity function is much more complicated than conventional velocity analysis for isotropic media, especially due to a limited angle coverage of reflection surveys.

Existing work on anisotropic traveltime inversion of reflection data has been done for horizontally homogeneous subsurface models (Byun and Corrigan, 1990; Sena, 1991; Tsvankin and Thomsen, 1995). As shown by Tsvankin and Thomsen (1995), *P*-wave moveout from horizontal reflectors is insufficient to recover the parameters of transversely isotropic media with a vertical symmetry axis (VTI), even if long spreads (twice the reflector depth) are used. The reason for this ambiguity is the trade-off between the vertical velocity and anisotropic coefficients, which cannot be overcome even by using the nonhyperbolic portion of the moveout curve. Tsvankin and Thomsen conclude that the only way to carry out stable inversion of surface reflection data is to combine long-spread *P* and SV moveouts; however, this method encounters many practical difficulties. This means that *P*-wave reflection moveout in horizontally homogeneous media should be supplemented by additional information (e.g., the vertical velocity from check shots or well logs) to make the anisotropic inversion feasible.

The presence of dipping reflectors provides us with the opportunity of extending the angle coverage of the input data without using nonhyperbolic moveout. Here, we develop an inversion technique for transversely isotropic media based on the analytic equation for NMO velocity for dipping reflectors derived by Tsvankin (1993). We recast this equation as a function of ray parameter and use it in inverting dip-dependent *P*-wave NMO velocities for the anisotropic coefficients. Analysis of the stability of the inverse problem by means of the Jacobian matrix is followed by the actual numerical inversion procedure via the Newton-Raphson method. We show that this approach makes it possible to obtain a range of solutions that all have the same NMO velocity for all possible dips, as well as the same nonhyperbolic moveout for horizontal reflectors, and the same time-migration impulse response. Then, we extend our results to vertically inhomogeneous anisotropic media by developing a Dix-type procedure (Dix, 1955) intended to give estimates of the NMO velocity in any individual layer from surface reflection data.

Tsvankin (1993) derived the following equation for the normal moveout (short-spread) velocity for dipping reflectors in a homogeneous anisotropic medium:

$$V_{\rm nmo}(\phi) = \frac{V(\phi)}{\cos\phi} \frac{\sqrt{1 + \frac{1}{V(\phi)} \frac{d^2V}{d\theta^2}}}{1 - \frac{\tan\phi}{V(\phi)} \frac{dV}{d\theta}}, \tag{1}$$

where $V(\phi)$ is the phase velocity as a function of the reflector dip ϕ, and θ is the phase angle measured from vertical.

The NMO equation (1) is a function of phase velocity $V(\theta)$ and its first two derivatives taken at the dip ϕ. Unfortunately, reflection data do not carry any explicit information about the dip; rather, we can expect to recover the ray parameter $p(\phi)$ corresponding to the zero-offset reflection.

$$p(\phi) = \frac{1}{2} \frac{dt_0}{dy} = \frac{\sin\phi}{V(\phi)}, \tag{2}$$

where $t_0(y)$ is the two-way traveltime on the zero-offset (or stacked) section, and y is the midpoint position.

In the numerical analysis of the NMO velocity, the replacement of the angle ϕ by the ray parameter p (horizontal slowness) does not pose any serious problem. Phase velocity and phase angle can be found from the Christoffel equations in a straightforward fashion if horizontal slowness is known, as shown in Appendix A. At the same time, the substitution of p may change the influence of the elastic coefficients on the NMO velocity since the ray parameter itself is dependent on anisotropy.

Equation (1) is valid in symmetry planes of any anisotropic medium and is not restricted to any particular wave type. Here, however, we will use this equation only for *P*-waves in VTI media. In conventional notation, *P*-wave propagation in transversely isotropic models is described by four stiffness coefficients: c_{11}, c_{33}, c_{13}, and c_{44}. The number of independent parameters can be reduced by using the notation suggested by Thomsen (1986); a detailed discussion of Thomsen notation can be found in Tsvankin (1994b). Formally *P*-wave phase and group velocity depend on four Thomsen parameters: *P*- and S-wave vertical velocities V_{P0} and V_{S0} and dimensionless anisotropies ϵ and δ.

$$\epsilon \equiv \frac{c_{11} - c_{33}}{2c_{33}}, \tag{3}$$

$$\delta \equiv \frac{(c_{13} + c_{44})^2 - (c_{33} - c_{44})^2}{2c_{33}(c_{33} - c_{44})}. \tag{4}$$

However, the influence of the shear-wave vertical velocity on P-wave velocities and traveltimes is practically negligible, even for strong anisotropy (Tsvankin and Thomsen, 1994). Tsvankin (1993) shows that the influence of V_{S0} on the P-wave moveout velocity $V_{\rm nmo}$ can be ignored, unless velocity anisotropy is strong and a wide range of V_{S0} is considered. Therefore, in our inversion procedure we will attempt to recover only the parameters V_{P0}, ϵ and δ.

For a horizontal reflector, equation (1) reduces to the well-known formula for NMO velocity given by Thomsen (1986):

$$V_{\rm nmo}(0) = V_{P0}\sqrt{1+2\delta}. \tag{5}$$

The trade-off between the vertical velocity V_{P0} and parameter δ cannot be resolved even if all three zero-dip NMO velocities (for P, SV, and SH waves) are measured (Tsvankin and Thomsen, 1995). On the other hand, if V_{P0} is known (e.g., from check shots or well logs), the zero-dip moveout velocity (5) can be used to obtain δ.

Analytic and numerical analysis performed by Tsvankin (1993) shows that the dip-dependence of P-wave NMO velocities is sensitive to the difference between the anisotropies ϵ and δ. Therefore, if δ has been determined from $V_{\rm nmo}(0)$, we should be able to find ϵ from a single NMO velocity for a dipping reflector. It is also interesting to examine the possibility of recovering all three parameters (V_{p0}, ϵ and δ) from NMO velocities at several distinct dips.

This analysis is performed numerically in the next sections. However, before proceeding with the numerical inversion procedure, we will elucidate the peculiarities of equation (1) by considering the special cases of elliptical and weak anisotropy. Unfortunately, NMO equation (1) is too complex to allow for analytic insight into the contributions of the anisotropic coefficients for general transverse isotropy.

For elliptical anisotropy ($\epsilon = \delta$),

$$V_{\rm nmo}(p) = \frac{V_{\rm nmo}(0)}{\sqrt{1-p^2V_{\rm nmo}^2(0)}}. \tag{6}$$

Equation (6) is a good illustration of the difference between the NMO equations expressed through the dip angle and ray parameter. Tsvankin (1993) showed that if the dip ϕ is used as the argument, the distortion in the NMO velocity due to elliptical anisotropy is proportional to the ratio of the phase velocities $V(\phi)/V(0) = \sqrt{1+2\delta\sin^2\phi}$. Therefore, $V_{\rm nmo}(\phi)$ contains a separate contribution of the parameter δ. However, $V_{\rm nmo}$ expressed through ray parameter (6) is a function just of the zero-dip NMO velocity with no separate dependence on the vertical velocity or on δ. In fact, equation (6) coincides with the NMO formula for isotropic media; the influence of the anisotropy in formula (6) is absorbed by the value of $V_{\rm nmo}(0)$, given by equation (5).

In terms of the inversion procedure discussed here, this result means that for elliptical models the trade-off between V_{PP00} and δ in equation (5) cannot be resolved from the dip-dependence of P-wave NMO velocity (6). Moreover, the reflection moveout for elliptical anisotropy is purely hyperbolic and does not provide any other information except for the short-spread velocity (6). On the other hand, the NMO velocity as a function of p can be easily reconstructed from the NMO velocity for a horizontal reflector; this conclusion has important implications in dip-moveout processing.

In order to understand the behavior of $V_{\rm nmo}(p)$ for non-elliptical models, we use the weak-anisotropy approximation ($\epsilon \ll 1$, $\delta \ll 1$). Equation (1) as a function of ray parameter for weak transverse isotropy is derived in the Appendix.

$$V_{\rm nmo}(p) = \frac{V_{\rm nmo}(0)}{\sqrt{1-y}} \left[1 + (\epsilon - \delta) f(y)\right], \tag{7}$$

where

$$f(y) = \frac{y\,(4y^2 - 9y + 6)}{1-y}, \;\; y = p^2 V_{\rm nmo}^2(0)\,.$$

Note that for elliptical anisotropy the weak-anisotropy approximation (7) reduces to the exact NMO equation (6).

Again, it is interesting to compare equation (7) with the corresponding weak-anisotropy NMO equation as a function of the dip angle given by Tsvankin (1993). Although Tsvankin (1993) emphasized the difference $\epsilon - \delta$ as the most influential parameter in his NMO equation, $V_{\rm nmo}(\phi)$ does contain separate contributions of ϵ and δ. However, when the dip angle is replaced by the ray parameter, the NMO velocity explicitly contains the anisotropies only in the form of the combination $\epsilon - \delta$. Of course, δ is hidden in the value of $V_{\rm nmo}(0)$.

This result has important implications in the inversion procedure. Instead of the three original unknown parameters (V_{P0}, ϵ and δ), in the limit of weak anisotropy, the NMO velocity contains just two combinations of them — $V_{\rm nmo}(0)$ and $\epsilon - \delta$. Therefore, just two distinct dips should provide enough information to recover the two effective parameters and reconstruct the NMO velocity as a function of ray parameter. In the most common case, when the zero-dip NMO velocity has been found by conventional NMO analysis, a single additional dipping reflector makes it possible to recover the difference $\epsilon - \delta$. However, the trade-off between V_{P0}, ϵ and δ cannot be resolved from P-wave NMO velocities, no matter how many reflectors are used. Normal moveout velocities from more than two dipping reflectors just provide redundancy in the estimation of $V_{\rm nmo}(0)$ and $\epsilon - \delta$.

Although these conclusions have been drawn for weak transverse isotropy, the numerical analysis in the following sections leads to similar results for VTI media with arbitrary strength of the anisotropy.

CONDITIONING OF THE PROBLEM

In order to estimate the sensitivity of the NMO velocity to the anisotropic parameters, we evaluate the Jacobian of equation (1) expressed as a function of ray parameter. The Jacobian is obtained by calculating the derivatives of the NMO velocity equation with respect to the model parameters V_{P0}, ϵ and δ. First, we consider the inversion for only one of the three model parameters; second, we consider the case of inverting for two parameters (namely, ϵ and δ) simultaneously; third, we consider the inversion for all three parameters.

Although the NMO velocity equation is nonlinear, its dependence on the anisotropy parameters is smooth enough to use the Jacobian approximation. Figure 1 is a 3-D plot of the values of $V_{\rm nmo}$ as a function of ϵ and δ for a reflector dip of nearly 40 degrees and a vertical velocity V_{P0} of 3.0 km/s. The smoothness of equation (1) over a practical range of ϵ and δ is evident.

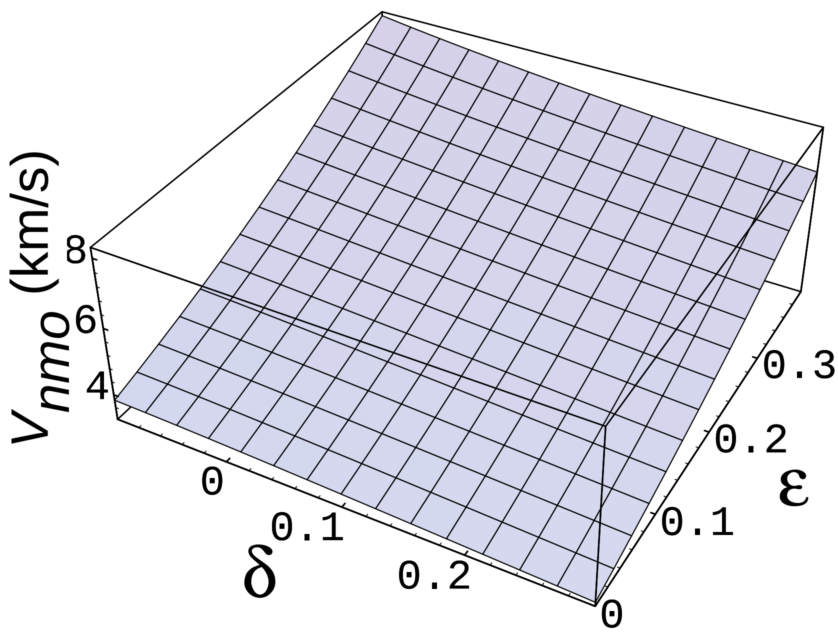

FIG. 1. 3-D plot of the NMO velocity as a function of ϵ and δ for the reflector dip corresponding to $p = 0.2$ s/km; $V_{P0} = 3.0$ km/s.

The derivatives used to form the Jacobian are as follows:

$$d_1(p) = \frac{V_{P0}}{V_{\rm nmo}(p)} \frac{\partial V_{\rm nmo}(p)}{\partial V_{P0}},$$

$$d_2(p) = \frac{1}{V_{\rm nmo}(p)} \frac{\partial V_{\rm nmo}(p)}{\partial \delta},$$

$$d_3(p) = \frac{1}{V_{\rm nmo}(p)} \frac{\partial V_{\rm nmo}(p)}{\partial \epsilon}.$$

The normalization of the derivatives makes them easier to use. For example, $d_3 = 1$ implies that, when solving only for ϵ, a 5 percent error in the measured NMO velocity would cause an error of 0.05 in the calculated value of ϵ.

Figure 2 shows the values of d_1, d_2 and d_3 for a typical VTI medium with $\epsilon = 0.2$, $\delta = 0.1$, and $V_{P0} = 3.0$ km/s. The range of ray parameters used in Figure 2 corresponds to reflector dips from 0 and 60 degrees. As expected (Tsvankin, 1993), $V_{\rm nmo}$ becomes more sensitive to ϵ and V_{P0} with increasing reflector dip, while the dependency on δ surprisingly goes to zero for dips near 27 degrees. Note that in calculating d_2 the other two parameters (V_{P0} and ϵ), rather than $V_{\rm nmo}(0)$, are kept constant.

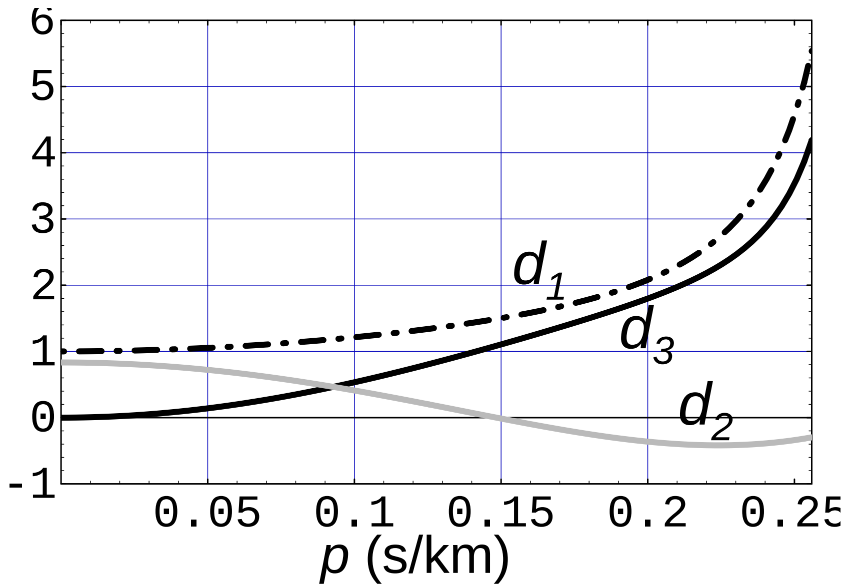

FIG. 2. The derivatives d_1,d_2, and d_3 as functions of ray parameter p. The model parameters are $V_{P0} = 3.0$ km/s, $\epsilon = 0.2$, and $\delta = 0.1$.

If two distinct reflector dips are available, it may be possible to solve for two of the three parameters. The sensitivity of this inversion to errors in the input data (NMO velocities) can be measured from the following Jacobian matrix:

$$J = \begin{pmatrix} d_2(p_1) & d_3(p_1) \\ d_2(p_2) & d_3(p_2) \end{pmatrix} .$$

This Jacobian corresponds to the inversion for ϵ and δ when V_{P0} is known.

The condition number κ for Jacobian matrix J can be computed as follows:

$$\kappa = \sqrt{\frac{\mid \lambda_{max} \mid}{\mid \lambda_{min} \mid}},$$

where λ_{max} and λ_{min} are the maximum and minimum eigenvalues of the matrix

$$A = JJ^T.$$

J^T is the transpose of the matrix J. A large condition number implies an ill-conditioned problem, that is nearly singular, while a low condition number (for example, smaller than 5) usually implies a well-conditioned problem. The absolute errors in the computed anisotropy parameters and the relative error in the computed V_{P0} are close to κ times the relative error in the measured NMO velocity. In most cases, this estimate provides the maximum possible error.

Figure 3 shows the condition number as a function of ray parameter for the inversion of the NMO velocities measured at two dips p_1 and p_2. The dips are ranging from 0 to 60 degrees. The flat (clipped) parts of the 3-D plot correspond to high condition numbers (≥ 6.0). When the dips are close to each other, the problem becomes highly ill-conditioned (i.e., the diagonal line where $p_1 = p_2$). If the difference between the dips is 10 degrees or more, the problem becomes reasonably conditioned, unless both dip angles are large (> 25 degrees); the latter case, however, is hardly typical.

On the whole, the resolution is acceptable if one reflector dip is less than 20 degrees and the other is greater than 20 degrees. In a typical case of horizontal and dipping reflectors, an acceptable resolution in resolving the anisotropies ϵ and δ is achieved (for the model from Figure 3) for dips between 15 and 60 degrees. The best resolution corresponds to dips near 25 degrees (the point along the p_2 axis ($p_1 = 0$ with the minimum condition number in Figure 3).

The inversion for the most practically important case of a horizontal and a dipping reflector needs to be considered in more detail. Figure 4 shows the condition number for the inversion for ϵ and δ (V_{P0} is considered to be known), with reflector dips given by $p_1 = 0.0$ (horizontal reflector) and $p_2 = 0.16$ (near 30-degree dip). The low values of the condition number mean that overall we obtain a reasonably good resolution over a wide range of values of ϵ and δ.

Next, we examine the feasibility of inverting for all three parameters (V_{P0}, ϵ and δ) using NMO velocities for three different dips. The Jacobian matrix for this problem is as follows:

$$J = \begin{pmatrix} d_1(p_1) & d_2(p_1) & d_3(p_1) \\ d_1(p_2) & d_2(p_2) & d_3(p_2) \\ d_1(p_3) & d_2(p_3) & d_3(p_3) \end{pmatrix} . \tag{8}$$

Figure 5 shows the condition number for the Jacobian (8) calculated for reflector dips $p_1 = 0.0$ (horizontal reflector), $p_2 = 0.16$ s/km (near 30-degree dip), and $p_3 = 0.23$ s/km (near 50-degree dip). The huge values of the condition number over the whole range of ϵ indicate that the problem is thoroughly ill-posed. In other words, we could find models with a wide range of V_{P0}, ϵ, and δ that have almost the same NMO velocities for the dips considered here.

Note that for models close to elliptical ($\epsilon = \delta$) the condition number goes to infinity, and the inversion cannot be carried out at all. Above, we obtained this result analytically by showing that the NMO equation for elliptical anisotropy (8) depends

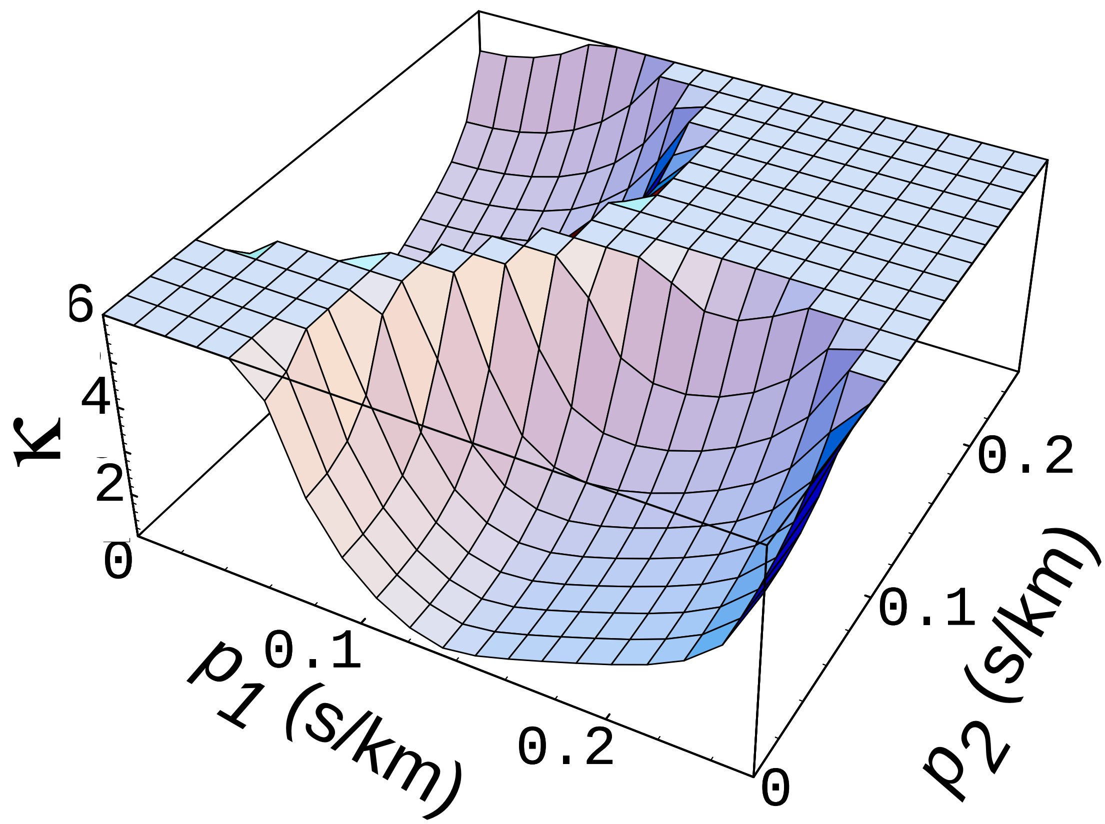

FIG. 3. 3-D plot of the condition number as a function p_1 and p_2 (ray parameters of the two reflectors). The model parameters are the same as in Figure 2.

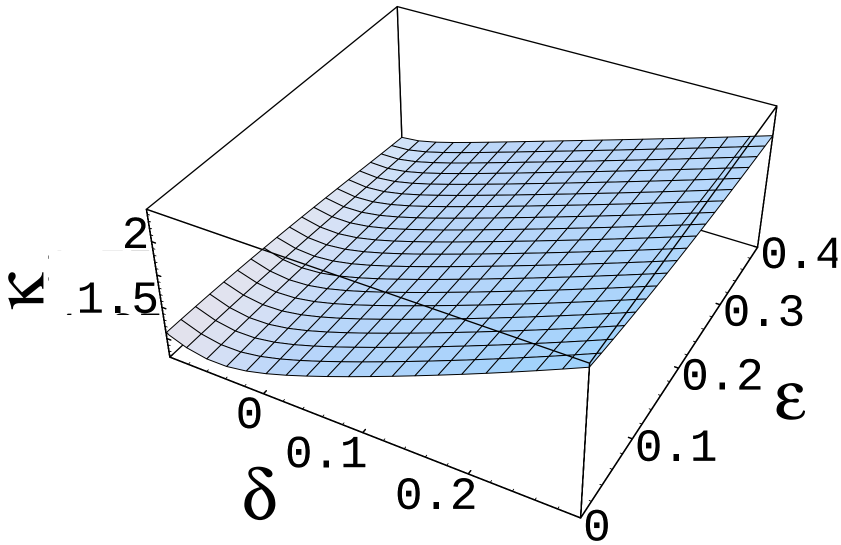

FIG. 4. 3-D plot of the condition number as a function of ϵ and δ. Two reflector dips used in the inversion correspond to ray parameters of $p_1 = 0.0$ s/km (horizontal reflector) and $p_2 = 0.16$ s/km. The vertical velocity $V_{P0} = 3.0$ km/s.

only on the ray parameter and the zero-dip NMO velocity.

Given the uncertainties usually associated with seismic data, we cannot count on resolving all three parameters using this method, even if we had more than three different reflector dips. The ambiguity of the inversion procedure is caused by the trade-off between the anisotropies ϵ and δ in the NMO equation. In the limit of weak anisotropy, this trade-off is demonstrated by the NMO formula (7), which contains only the *difference* $\epsilon - \delta$ and the zero-dip NMO velocity rather than either of the coefficients individually. However, if one of the parameters (V_{P0}, ϵ, δ) is known, the other two can be recovered from NMO velocities for two different dips.

NUMERICAL INVERSION

The above analysis based on the Jacobian matrix is still approximate since the NMO velocity equation is nonlinear. In this section, we perform the actual inversion by means of the Newton-Raphson method and study the range of solutions as well as the sensitivity of the results to errors in the input information. We will concentrate on models with $\epsilon - \delta \geq 0$, which are believed to be most typical for subsurface formations (Tsvankin and Thomsen, 1994).

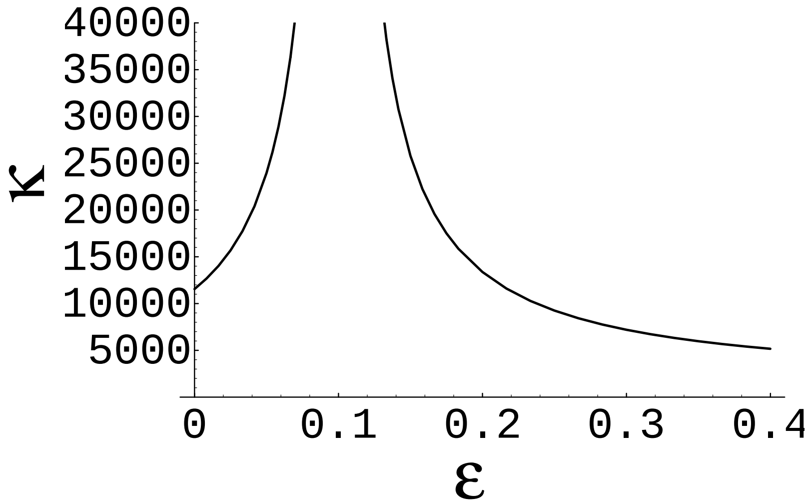

FIG. 5. The condition number as a function of ϵ for the inversion using three reflector dips corresponding to $p_1 = 0.0$ s/km, $p_2 = 0.16$ s/km, and $p_3 = 0.23$ s/km. The vertical velocity $V_{P0} = 3.0$ km/s; δ=0.1.

Inversion using two reflector dips

The input data for the inversion procedure are the P-wave NMO velocities and ray parameters for two different reflector dips; one of the reflectors can be (but is not necessarily) horizontal. The analysis in the previous section indicates that the inversion for all three parameters using three reflector dips is unstable; besides, for our method to be practical we can hardly count on having reliable NMO velocities from more than two distinctly different dips.

In our first example (Figure 6), we consider two reflectors dipping at 20 degrees ($p_1 = 0.11$ s/km), and 50 degrees ($p_2 = 0.23$ s/km) for the same model as in Figure 2. If we know the vertical velocity V_{P0}, the inversion of two NMO velocities should make it possible to recover the anisotropies ϵ and δ. Indeed, as shown in Figure 6, if the actual velocity $V_{P0} = 3.0$ km/s is used in the Newton-Raphson inversion algorithm, we obtain the correct values for both anisotropic parameters.

However, if only surface data are available, the exact vertical velocity may not be known. Therefore, it is interesting to examine the family of solutions corresponding to a range of vertical velocities around the actual value (from 2.6 km/s to 3.5 km/s in Figure 6). For all these solutions, the difference between ϵ and δ is close to the exact value ($\epsilon - \delta = 0.1$). Therefore, in a remarkable agreement with the weak-anisotropy approximation (7), the inversion of the P-wave NMO velocities provides us with a good estimate of the difference $\epsilon - \delta$. The only way to resolve the coefficients indi-

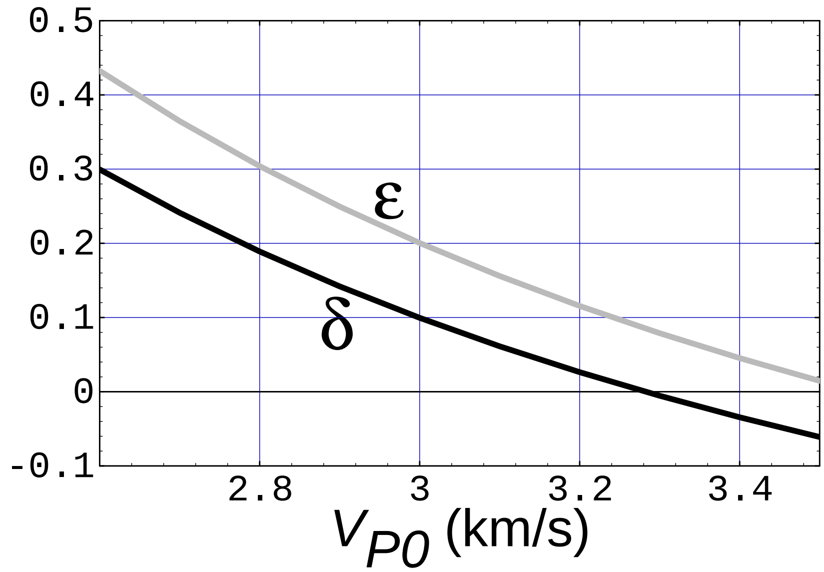

FIG. 6. Parameters ϵ and δ obtained from NMO velocities corresponding to $p_1 = 0.11$ s/km (20 degrees dip for the actual model) and $p_2 = 0.23$ s/km (50 degrees dip). The values of V_{P0} used in the inversion are shown on the horizontal axis. The model parameters are $V_{P0} = 3.0$ km/s, $\epsilon = 0.2$, and $\delta = 0.1$.

vidually is to obtain the vertical velocity V_{P0} using some other source of information (e.g., check shots or well logs).

Another important property of the solutions shown in Figure 6 is that all of them have practically the same NMO velocity as a function of ray parameter for all possible dipping reflectors, not just for the two dips used in the inversion scheme. This is illustrated by Figure 7, which shows that the NMO velocity for any reflector dip is practically the same within the range of solutions in Figure 6.

For instance, the vertical velocity V_{P0} for any of the solutions combined with the corresponding value of δ provides the correct zero-dip NMO velocity (5). Therefore, if we perform the inversion procedure using the NMO velocities for horizontal and dipping reflectors, we end up with the same family of equivalent models as in Figure 6. We conclude that the normal moveout velocities measured at two different dips are sufficient to obtain the NMO velocity for any ray-parameter value.

In essence, we have shown that the exact NMO velocity expressed through ray parameter depends just on the zero-dip NMO velocity and some combination of the anisotropies close to the difference $\epsilon - \delta$. This conclusion is in agreement not only with the weak-anisotropy equation (7), but also with the analysis of the Jacobian matrix in the previous section. In the following, we refer to models obtained by the inversion of P-wave NMO velocity as the "equivalent solutions," or ES.

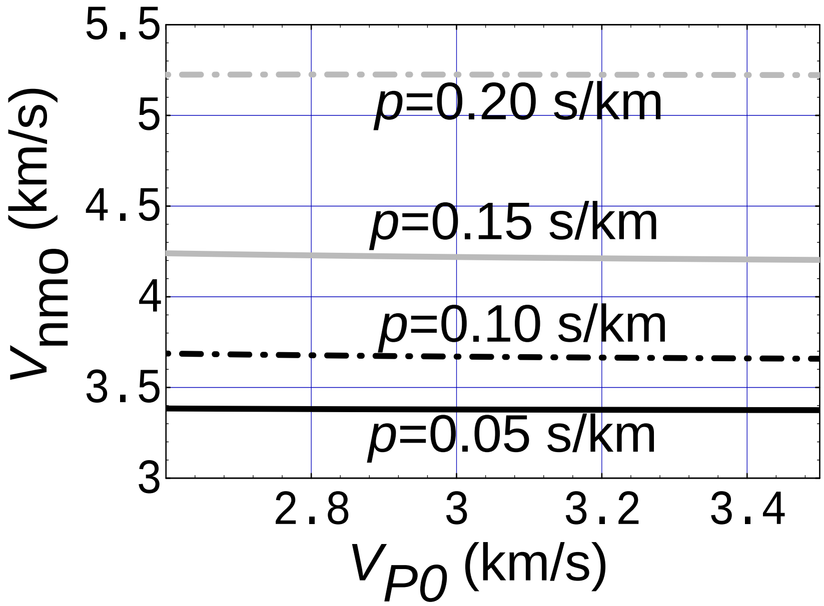

FIG. 7. The NMO velocity for a range of solutions in Figure 6 and four different values of the ray parameter.

Another example, for a medium with stronger anisotropy, is shown in Figure 8. Here, we have considered a typical case of horizontal and dipping reflectors (the dip angle is 40 degrees); however, any pair of dips sufficiently different from each other yields the same family of ES. Here, in contrast with the previous example, the velocity anisotropy is too pronounced for the weak-anisotropy equation (7) to hold, and the inversion does not provide an accurate value of $\epsilon - \delta$, unless we have a good estimate of the vertical velocity. The accuracy of the estimation of $\epsilon - \delta$ is further illustrated by Figure 9, which shows the inversion results for the models with $\epsilon - \delta = 0.1, 0.2$, and 0.3. While the recovery of $\epsilon - \delta$ is unique for elliptical anisotropy ($\epsilon = \delta$, not shown on the plot), it becomes less accurate with increasing $\epsilon - \delta$.

Description of the equivalent solutions

Clearly, the combination of ϵ and δ that describes the family of ES deviates from the difference $\epsilon - \delta$ with increasing anisotropy. An analytic description of this combination for arbitrary strength of the anisotropy is given below.

We have shown that all ES obtained by our inversion technique have the same NMO velocity for all dips, including a horizontal reflector. Therefore,

$$V_{\rm nmo}(0) = V_{P0}\sqrt{1+2\delta} = const \tag{9}$$

within the family of ES. This equation provides a relation between V_{P0} and δ that accurately describes the curves $\delta(V_{P0})$ in Figures 6 and 8.

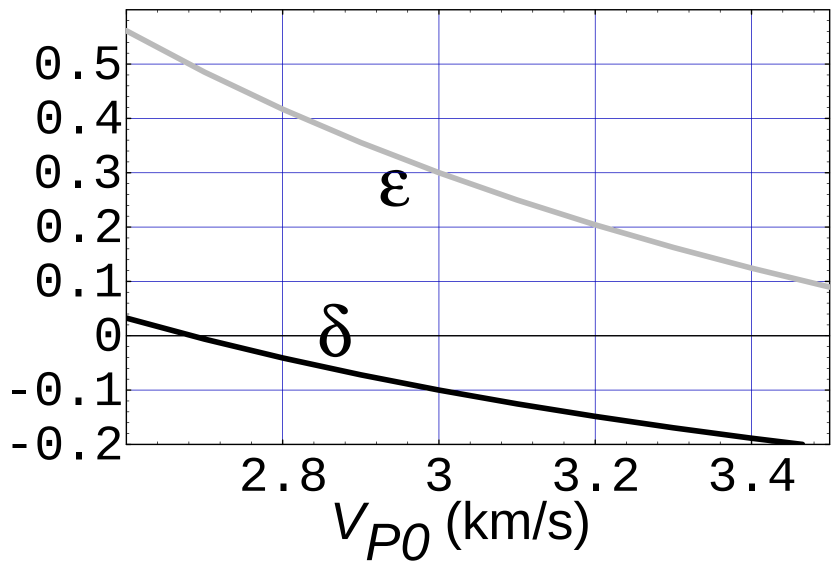

FIG. 8. Inverted values of ϵ and δ as functions of V_{P0} for the model with $V_{P0} = 3.0$ km/s, $\epsilon = 0.3$, and $\delta = -0.1$.

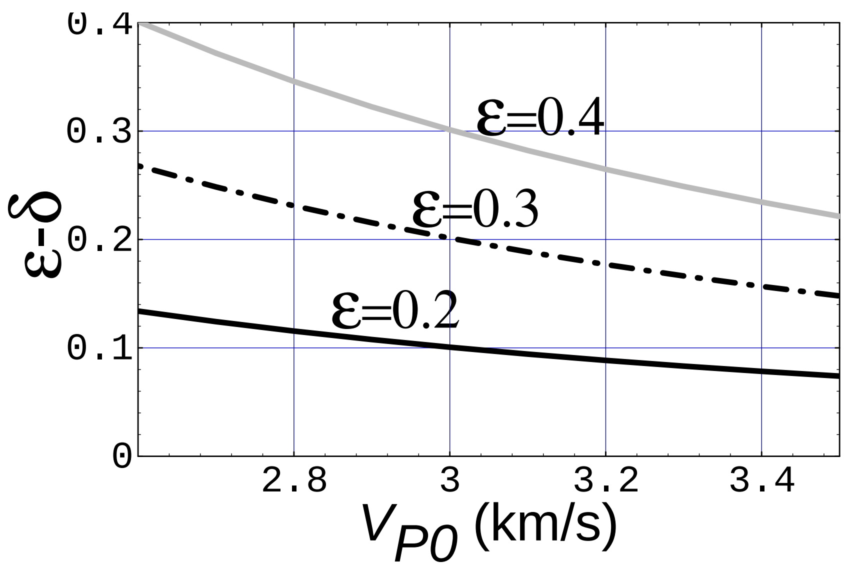

FIG. 9. Inverted value of $\epsilon - \delta$ as a function of V_{P0} for three models with different ϵ; V_{P0}=3.0 km/s, δ=0.1.

However, a single equation is not sufficient to characterize the ES analytically. To obtain another relation between the parameters that would involve ϵ, we examine the behavior of group-velocity curves for the family of ES. Figure 10 shows the group velocity as a function of the group angle for three solutions corresponding to V_{P0}=2.8, 3.0 and 3.2 km/s. The computations were performed for two models with different combinations of ϵ and δ. For both media, all three ES yield the same velocity at an angle of 90 degrees, which coincides with the actual horizontal velocity. This implies that for all ES

$$V_h = V_{P0}\sqrt{1+2\epsilon} = const \tag{10}$$

However, it is more convenient to replace the horizontal velocity by a dimensionless

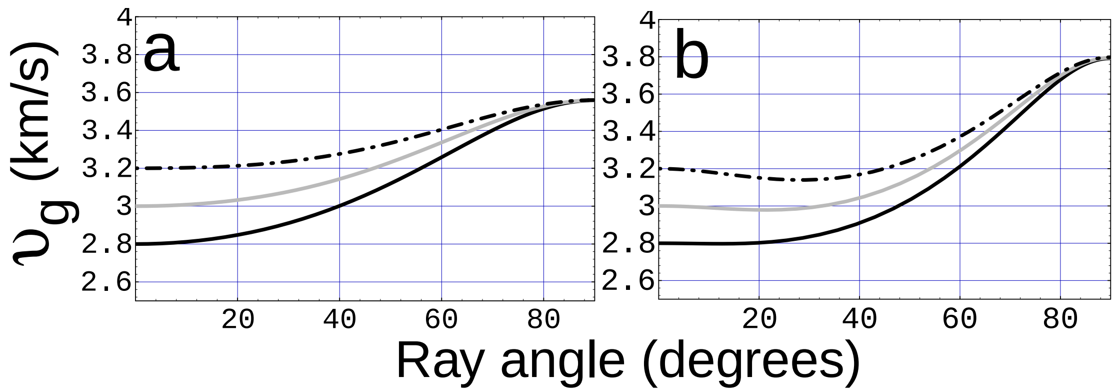

FIG. 10. Group velocity υ_g as a function of the group (ray) angle for solutions from (a) Figure 6 (V_{P0}=3.0 km/s, $\epsilon = 0.2$, $\delta = 0.1$), and (b) Figure 8 (V_{P0}=3.0 km/s, $\epsilon = 0.3$, $\delta = -0.1$). The curves correspond to solutions with V_{P0}=2.8 km/s and the corresponding values of δ and ϵ (black); V_{P0}=3.0 km/s (actual values, gray); and V_{P0}=3.2 km/s (dashed).

parameter, common for all ES, that goes to zero for isotropic media. Combining equations (9) and (10), we choose to define the following anisotropic parameter denoted as η.

$$\eta = 0.5\left(\frac{V_h^2}{V_{\rm nmo}^2} - 1\right) = \frac{\epsilon - \delta}{1+2\delta}\,. \tag{11}$$

Then

$$V_h = V_{\rm nmo}(0)\sqrt{1+2\eta}$$

[compare the form with equations (9) and (10)]. Therefore, our family of ES can be described by two effective parameters: $V_{\rm nmo}(0)$ (or V_h) and η. Only these parameters can be resolved by inverting dip-dependent P-wave NMO velocities. In principle, two distinct dips are sufficient to recover the values of $V_{\rm nmo}(0)$ and η; additional dipping reflectors just provide redundancy in the inversion procedure.

Essentially, by performing numerical inversion we have generalized the weak-anisotropy equation (7) for transversely isotropic media with arbitrary strength of anisotropy. While the weak-anisotropy P-wave NMO equation (expressed through ray parameter) is a function of $V_{\rm nmo}(0)$ and $\epsilon - \delta$, the P-wave NMO velocity for general transverse isotropy is fully characterized by $V_{\rm nmo}(0)$ and η. Clearly, in the limit of weak anisotropy η reduces to the difference $\epsilon - \delta$. Also, note that η is zero not only for isotropy, but also for elliptical anisotropy. In this sense, it is similar to the parameter $\sigma = V_{P0}^2/V_{P0}^2 \, (\epsilon - \delta)$ introduced by Tsvankin and Thomsen (1994) to describe SV-wave moveout.

Equation (11) leads us to another observation. If it is possible to obtain an accurate value for the horizontal velocity V_h (e.g., from head waves traveling along a horizontal reflector or from cross-hole tomography), then the zero-dip velocity $V_{\rm nmo}(0)$ is sufficient to find η and build the P-wave NMO velocity as a function of ray parameter. Dipping reflectors in this case are not needed at all.

The family of ES from our first example in Figure 6 can be represented by $V_{\rm nmo} =$ 3.29 km/s and $\eta = 0.0833$. The example from Figure 8 is characterized by $V_{\rm nmo} = 2.68$ km/s and a much bigger $\eta = 0.5$.

As demonstrated in the following sections, the importance of the family of ES goes well beyond the dip-dependence of P-wave NMO velocities.

Accuracy of the inversion

Next, we study the sensitivity of the inversion procedure to errors in moveout velocities. Figure 11 shows the inverted values of δ and ϵ as a function of errors in the input velocity $V_{\rm nmo}$ (the vertical velocity is fixed at the correct value). For reflectors dipping at 20 and 50 degrees (Figure 11a and b), ϵ is less sensitive to errors and better resolved than is δ. This result might be expected for such reflector dips. If the reflectors are dipping at 0 and 40 degrees (Figure 11c and d), ϵ is still better resolved than δ.

Overall, the condition number κ discussed in the previous section (Figure 3) provides an exaggerated estimate of the errors in the anisotropic parameters. Nevertheless, κ makes it possible to compare the accuracy for different input parameters. For example, if the NMO velocities for 0- and 40-degree dips are used, the condition number ($\kappa = 2.3$, Figure 3) is lower than that for the case of 20- and 50-degree dips ($\kappa = 6.1$). The higher accuracy for the inversion using 0- and 40-degree dips is clearly seen in the lower errors for the parameter δ in Figure 11.

However, it is more instructive to examine the sensitivity of the effective parameter η and the horizontal velocity V_h to errors in the measured values of $V_{\rm nmo}(p)$. In Figure 12, we have introduced errors into the input values of NMO velocities for reflector dips of 0 and 40 degrees. The percentage error in V_h and the absolute error in η are quite small, which indicates that our inversion is reasonably stable. In fact, a 5-percent error in $V_{\rm nmo}$ causes less than a 2.5-percent error in V_h. The percentage

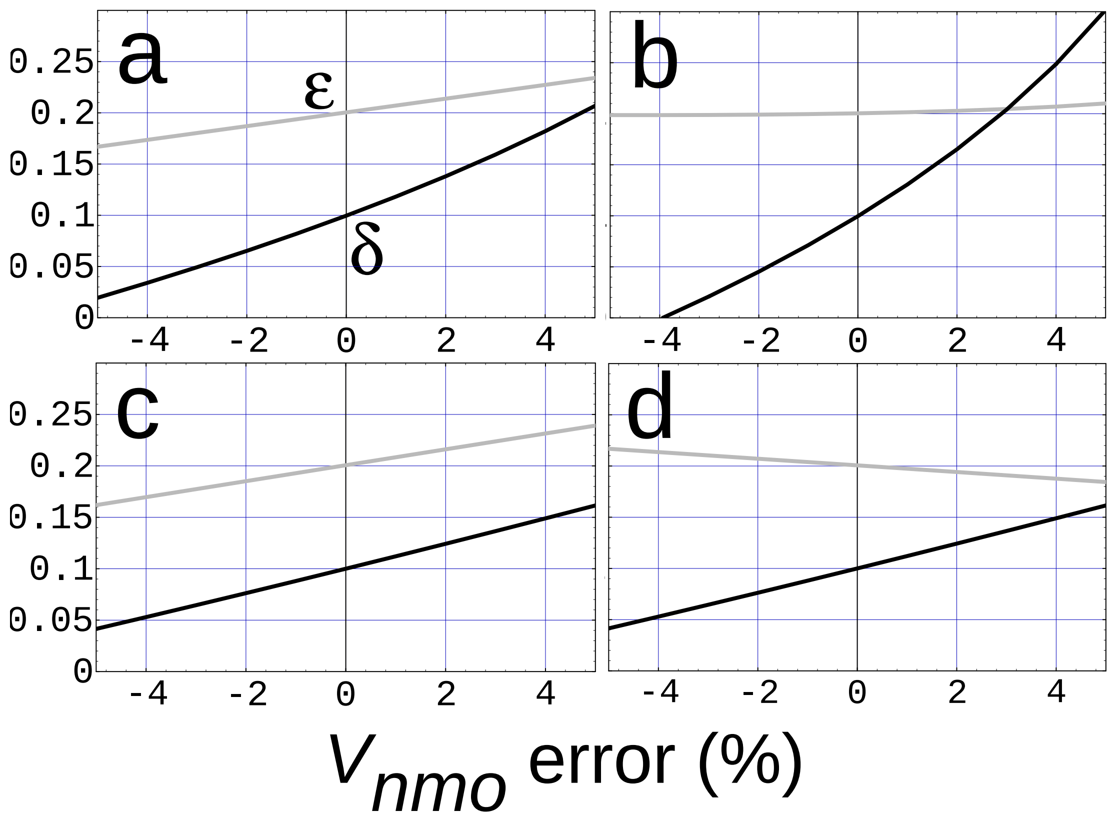

FIG. 11. Inverted values of ϵ and δ as a function of the percent error in the measured NMO velocities. (a) and (b) - NMO velocities at 20- and 50-degree dips are used; (c) and (d) - NMO velocities at 0- and 40-degree dips are used. (a) and (c) - input NMO velocities have the error of the same magnitude and sign; (b) and (d) - input NMO velocities have errors of the same magnitude and opposite sign.

error in η is larger, but this can be expected in the inversion for so small an anisotropic coefficient.

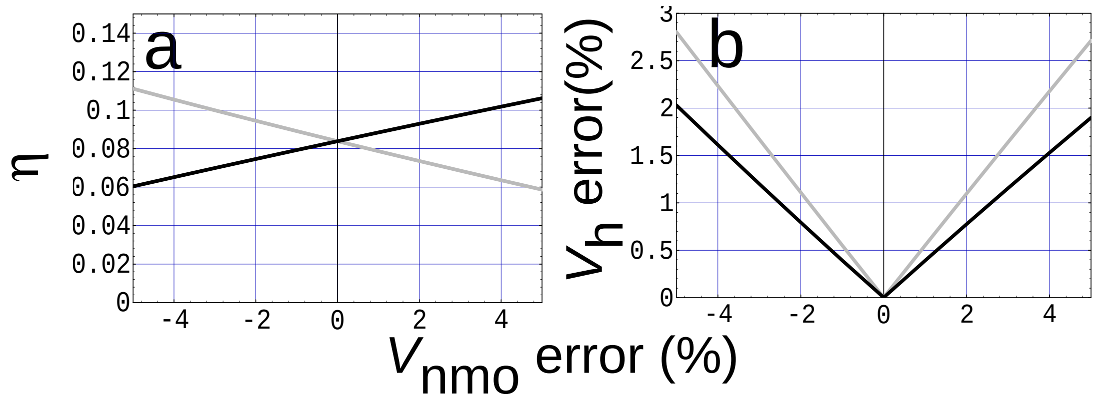

FIG. 12. Dependence of the inverted values of η (a) and V_h (b) on the error in the measured NMO velocities. The model parameters are $V_{P0} = 3.0$ km/s, $\epsilon = 0.2$, and $\delta = 0.1$; reflector dips of 0 and 40 degrees were used. Black lines correspond to errors in the NMO velocity for only the dipping reflector. Gray lines correspond to errors in both V_{nmo} values.

PROPERTIES OF THE FAMILY OF SOLUTIONS

Nonhyperbolic reflection moveout

The inversion of the dip-dependence of P-wave normal moveout velocities enables one to obtain a family of equivalent solutions (ES) described by the zero-dip NMO velocity $V_{\mathrm{nmo}}(0)$ and the effective anisotropic parameter η. In this section, we show that any model with the same $V_{\mathrm{nmo}}(0)$ and η yields the same long-spread (nonhyperbolic) P-wave moveout from a horizontal reflector.

P-wave long-spread moveout in horizontally-layered transversely isotropic media can be well-approximated by the equation (Tsvankin and Thomsen, 1994)

$$t^2(X) = t_{P0}^2 + A_2 X^2 + \frac{A_4 X^4}{1 + A X^2}\,. \tag{12}$$

For a single layer, the coeffficients in formula (12) are

$$A_2 = \frac{1}{V_{P0}^2(1+2\delta)}\,,$$

$$A_4 = -\frac{2(\epsilon-\delta)}{t_{P0}^2 V_{P0}^4}\frac{1+\frac{2\delta}{1-V_{s0}^2/V_{P0}^2}}{(1+2\delta)^4}\,,$$

$$A = \frac{A_4}{\frac{1}{V_h^2} - A_2},$$

where V_h is the horizontal velocity.

Equation (12) remains numerically accurate for long spreads (2 to 3 times, and more, the reflector depth) and pronounced anisotropy. The hyperbolic moveout term, which makes the main contribution to short-spread moveout, depends just on the NMO velocity $V_{\rm nmo}(0)$. The last term in equation (12) describes nonhyperbolic moveout on long spreads.

Substituting the parameters $V_{\rm nmo}(0)$ and η into formula (12) and ignoring the contribution of V_{s0} to the quartic term A_4 (V_{s0} has a negligible influence on P-wave moveout in TI media), we obtain

$$t^2(X) = t_{P0}^2 + \frac{X^2}{V_{\rm nmo}^2(0)} - \frac{2\eta X^4}{V_{\rm nmo}^2(0)[t_{P0}^2 V_{\rm nmo}^2(0) + (1+2\eta)X^2]}. \tag{13}$$

Thus, P-wave long-spread moveout can be adequately described by the vertical traveltime and just the two effective parameters – $V_{\rm nmo}(0)$ and η, with no separate dependence on V_{P0}, ϵ, or δ. The magnitude of nonhyperbolic moveout is strongly dependent on the value of η; if $\eta = 0$, the medium is elliptical and the moveout is purely hyperbolic. Actually, for a given $V_{\rm nmo}(0)$ and t_{P0}, η describes the amount of deviation from hyperbolic moveout. Because our inversion algorithm makes it possible to recover $V_{\rm nmo}(0)$ and η, it therefore provides enough information to build P-wave long-spread moveout curves.

Migration impulse response

Although equation (13) describes moveout for a horizontal reflector, it also can be regarded as the diffraction curve, accurate to a certain dip, on the zero-offset section (post-stack domain). Since time migration is based on collapsing such diffraction curves to their apex, the values of $V_{\rm nmo}(0)$ and η should be sufficient to generate a time-migration impulse response that is accurate up to that certain dip. All lateral position errors described by Larner and Cohen (1993) and Alkhalifah and Larner (1994) for the case of homogeneous media will be eliminated. Poststack depth migration may produce depth errors if the value V_{P0} is inaccurate, but this is a different issue.

Figure 13 shows the time-migration impulse responses (right half only) for different ES from (a) Figures 6 and 8. The curves for all three ES practically coincide with each other, implying that there is no differences between the impulses of the three input models. This confirms that $V_{\rm nmo}(0)$ and η are sufficient to generate an accurate time-migration impulse response for all dips.

This point is illustrated further by Figure 15, which shows anisotropic poststack time migrations (Gazdag's phase-shift migration modified for anisotropic media) of

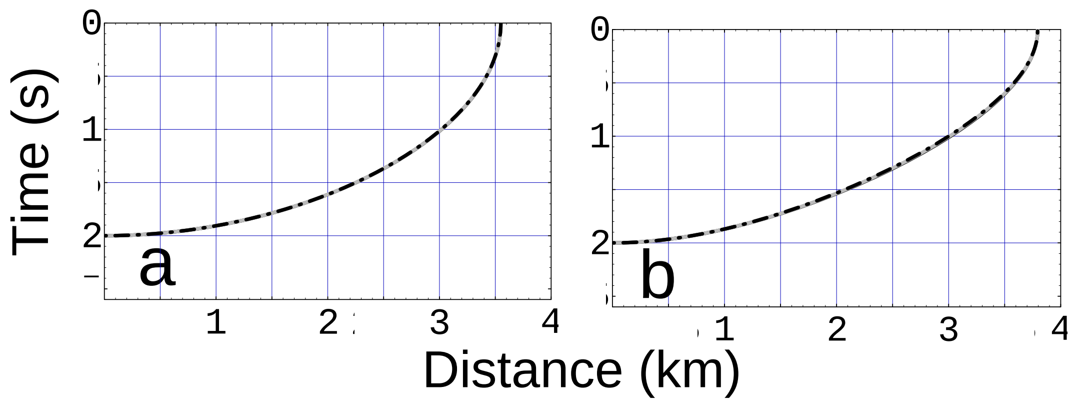

FIG. 13. Anisotropic time-migration impulse response for solutions from (a) Figure 6, and (b) Figure 8. The three curves on each plot correspond to the solutions with V_{P0} = 2.8 km/s (solid black line), 3 km/s (solid gray line), and 3.2 km/s (dashed black line).

the synthetic data generated for the model in Figure 14. The reflectors are embedded in a homogeneous transversely isotropic medium with V_{P0}=3.0 km/s, ϵ=0.2, and δ=0.1 (the same model as in Figure 6). The migrations were performed (a) using the actual model parameters, and (b) using an equivalent solution from Figure 6 with V_{P0}=2.5 km/s, ϵ=0.43, and δ=0.3. Although model (b) is substantially different from the actual one, it has the correct values of $V_{\rm nmo}(0)$ and η and, consequently, produces an accurate image.

Figure 16 shows anisotropic Gaussian beam poststack depth migrations (Alkhalifah, 1994) of the synthetic data generated for the model in Figure 14. The migrations were performed with three velocity models corresponding to three ES from Figure 6; the migration for the actual model is shown in the middle plot (b) of Figure 14. Again, all three models have the same $V_{\rm nmo}(0)$ and η and, therefore, produce equally sharp images. The only difference between the images is a depth shift resulting from differences in the vertical velocities used in the migration. This depth shift is given by the following relation

$$\Delta D = (\frac{V_{P0}}{V_{\rm actual}} - 1)D,$$

where $V_{\rm actual}$ is the true vertical velocity, equal to 3.0 km/s in Figure 16.

Therefore, all ES have the same poststack migration impulse response with a simple depth shift. As we will show in the next section, errors in the effective parameters $V_{\rm nmo}(0)$ and η lead to distortions in migrated images.

Since all ES, characterized by $V_{\rm nmo}(0)$ and η, have the same NMO velocity $V_{\rm nmo}(p)$ in the prestack domain and the same time-migration impulse response in the poststack domain, they should also have the same time-migration impulse in the *prestack* domain. Thus, media with the same $V_{\rm nmo}(0)$ and η yield the same prestack and

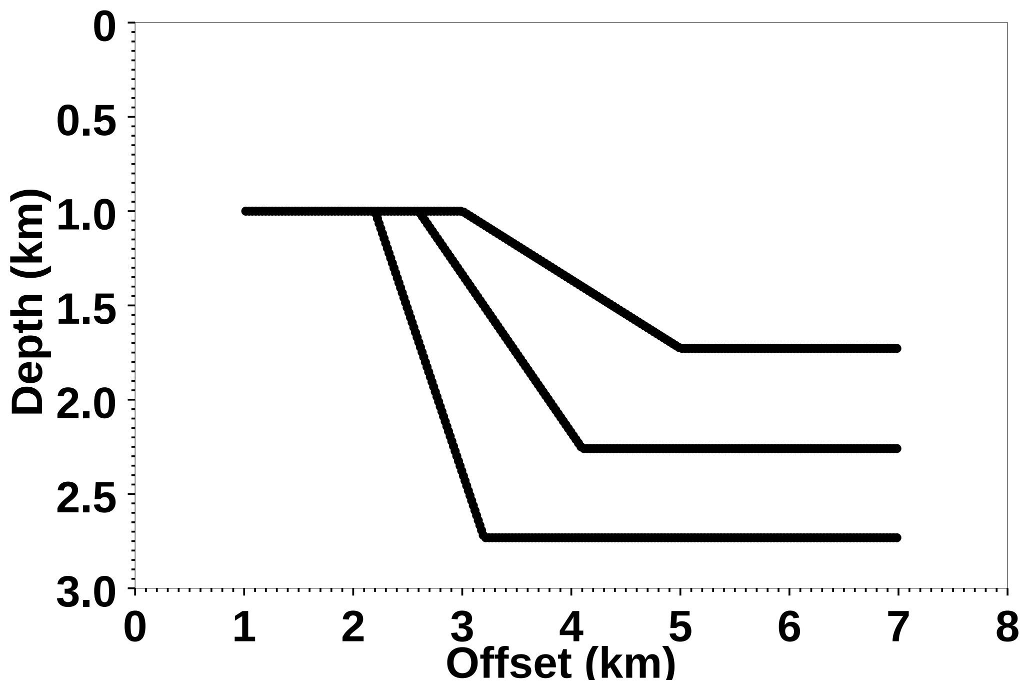

FIG. 14. Model with reflectors dipping at 0,20,40, and 60 degrees.

poststack diffraction curves for surface seismic data. This result might hold even in weakly inhomogeneous media if we consider root-mean-square values.

We conclude that the inversion of P-wave NMO velocities provides enough information to perform all major time-processing steps including DMO, prestack and poststack time migration. However, time-to-depth conversion should be carried out with an accurate value of the vertical velocity that cannot be obtained from NMO velocities alone.

REFINING INVERSION RESULTS USING POSTSTACK MIGRATION

In many cases one can determine the accuracy of the migration algorithm or of the velocity field used in the migration by observing the quality of the migrated image. For example, parabolic shapes, resulting from diffracting edges, imply over-migration, whereas hyperbolic shapes indicate under-migration.

This approach can be used to refine the results of our inversion procedure. Errors in the measured NMO velocity may lead to an inaccurate value of η, which, in turn, may distort the migrated image. In isotropic media, overmigration is usually corrected by reducing the migration velocity. A corresponding correction in transversely isotropic media can be achieved by reducing η.

Figure 17 shows anisotropic poststack time migration of a synthetic data set gen-

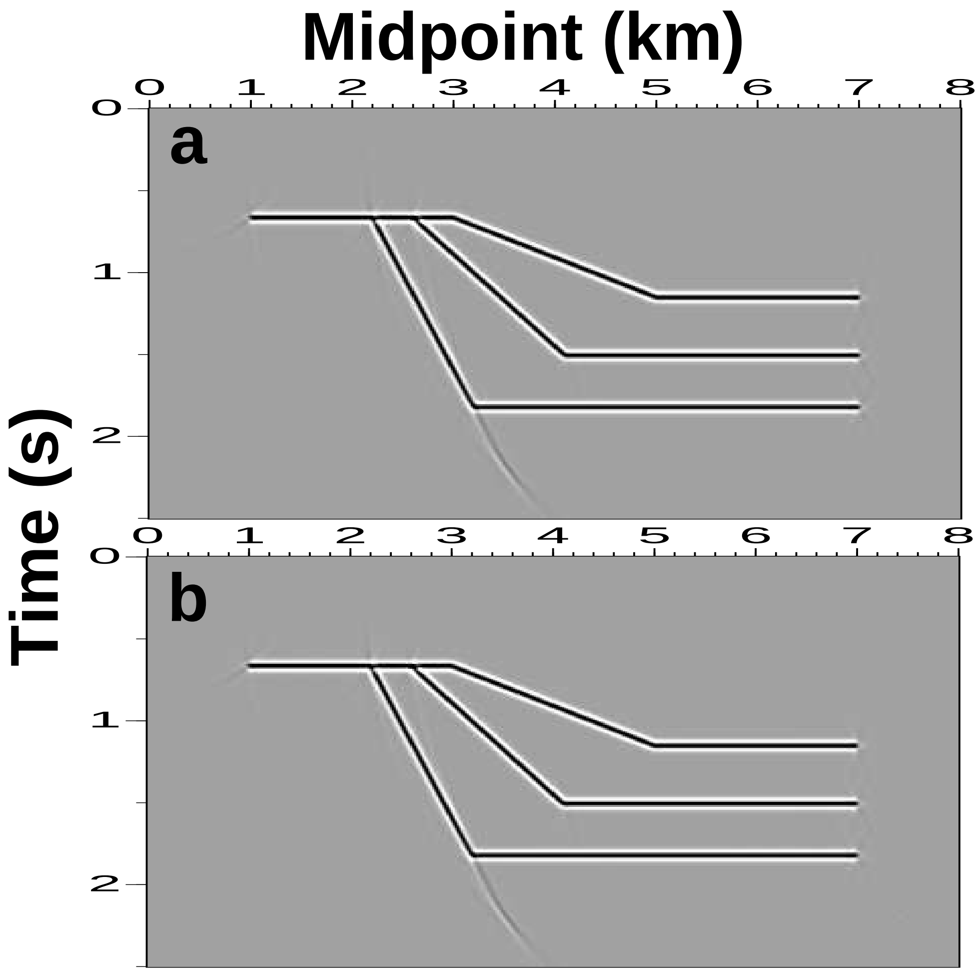

FIG. 15. Anisotropic time migrations of synthetic data generated for the model in Figure 14 using (a) the actual model values of $V_{P0} = 3.0$ km/s, $\epsilon = 0.2$, and $\delta = 0.1$; and (b) an equivalent solution $V_{P0} = 2.6$ km/s, $\epsilon = 0.43$, and $\delta = 0.3$.

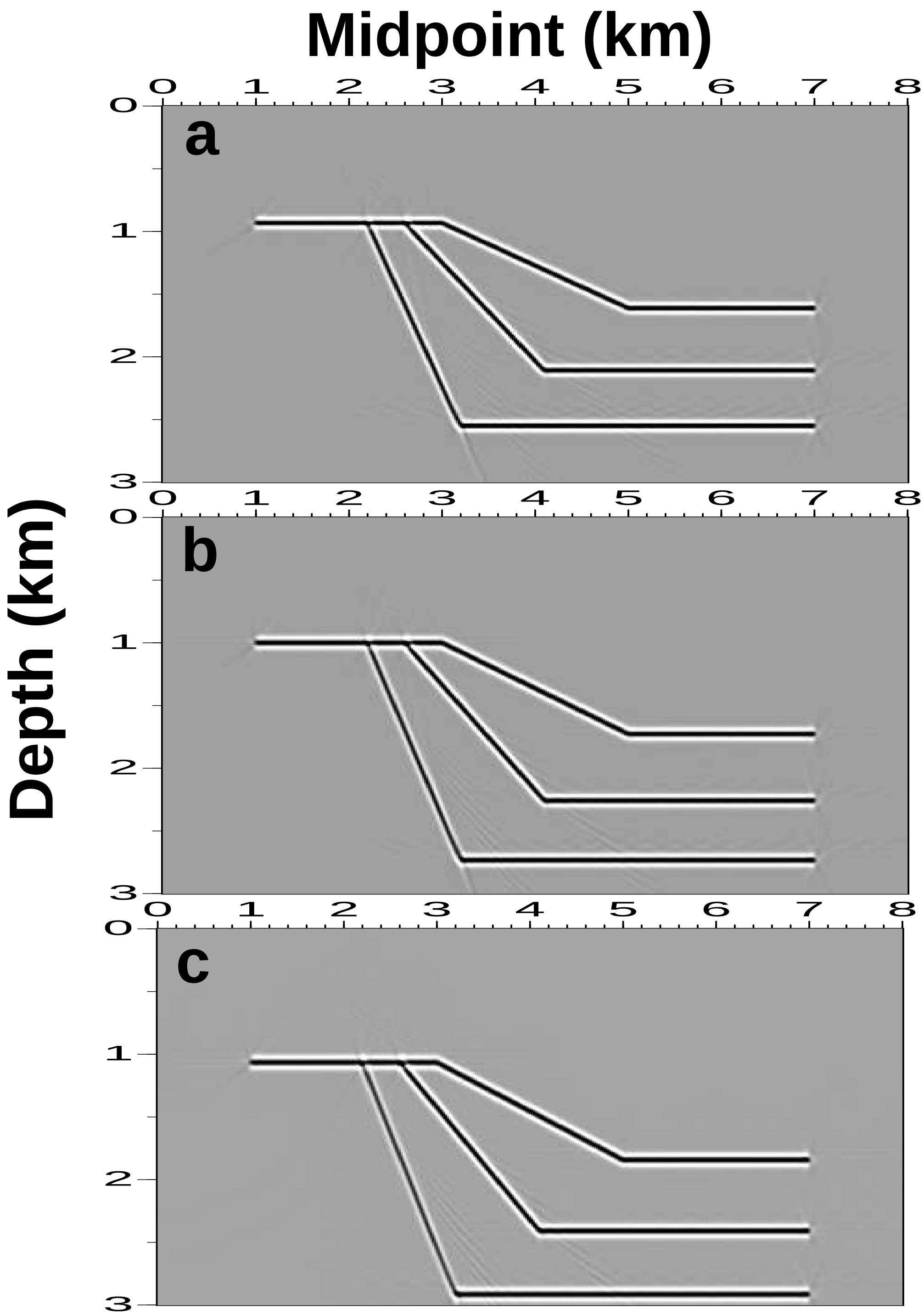

FIG. 16. Anisotropic depth migrations of synthetic data generated for the model in Figure 14 using (a) $V_{P0} = 2.8$ km/s, $\epsilon = 0.3$, and $\delta = 0.19$; (b)$V_{P0} = 3.0$ km/s, $\epsilon = 0.2$, and $\delta = 0.1$ (actual model); (c) $V_{P0} = 3.2$ km/s, $\epsilon = 0.11$, and $\delta = 0.027$.

erated for the model in Figure 14 using inaccurate values of η (the correct value, $V_{\rm nmo}$=3.29 km/s, is used in both cases). The errors, apparent in both cases, show the sensitivity of the migration results to the value of η. Predictably, the distortions are more pronounced for the model with a larger error in η: not only do the reflectors cross, but also the diffracting edges are not imaged well.

Usually we can expect to obtain the zero-dip NMO velocity with a higher accuracy than the parameter η. Therefore, if the image indicates undermigration, (as in Figure 17), we should increase η while keeping $V_{\rm nmo}(0)$ constant; to correct for overmigration, η should be decreased. However, if we have more confidence in the measured value of the NMO velocity for the dipping reflector, a proper choice would be to change both $V_{\rm nmo}(0)$ and η. In fact, then, given error that likely exists in both, the data processor now has two parametrs to adjust.

INVERSION IN A LAYERED MEDIUM

The inversion technique discussed above is designed for a homogeneous medium above the reflector, while realistic subsurface models are, at a minimum, vertically inhomogeneous. Here we generalize the NMO equation of Tsvankin (1993) for layered anisotropic media with a dipping reflector and show that it is possible to recover the NMO velocity in the medium immediately above the reflector via a Dix-type formula.

We consider a layered anisotropic model consisting of a stack of horizontal homogeneous layers above a dipping reflector (Figure 18). It is assumed that the CMP line is perpendicular to the strike of the reflector, and the incidence (sagittal) plane coincides with a plane of symmetry in all layers. Therefore, kinematics of wave propagataion is two-dimensional, i.e., phase and group velocity vectors do not deviate from the incidence plane. The same assumption was made by Tsvankin (1993) in his derivation of the one-layer NMO equation. If the medium is transversely isotropic, the incidence plane should contain the symmetry axis (or it may be the isotropy plane); in an orthorhombic medium, the incidence plane should coincide with one of the three mutually orthogonal symmetry planes.

Since the medium above the reflector is horizontally inhomogeneous, the ray parameter p (horizontal slowness) remains constant between the reflector and the surface. In this case, the short-spread moveout velocity $V_{\rm nmo}$ in CMP geometry is convenient to express as follows (Larner, 1993; Tsvankin, 1993):

$$V_{\rm nmo}^2(p_0) = \lim_{x\to 0} \frac{dx^2}{dt^2} = \frac{1}{t_0} \lim_{p\to p_0} \frac{dx}{dp}\,, \qquad (14)$$

where x is the source-receiver offset, p_0 is the ray parameter of the zero-offset ray ($x=0$), and t_0 is the two-way zero-offset traveltime. Note that the zero-offset ray is not necessarily perpendicular to the reflector in the presence of anisotropy; it is the phase-velocity vector corresponding to the zero-offset ray that should be normal to the reflector.

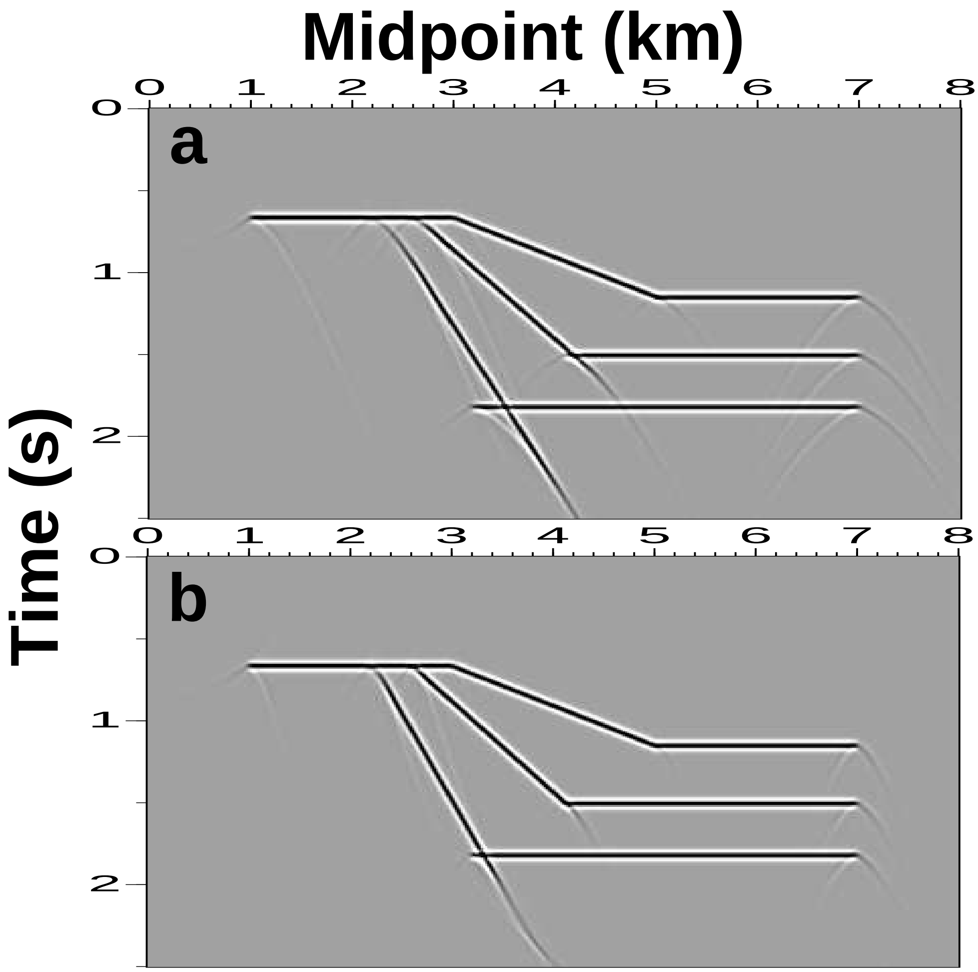

FIG. 17. Anisotropic poststack time migrations of a synthetic data set generated for the model in Figure 14 using distorted values of η: (a) η=0.01, (b) η=0.06; the actual value $\eta = 0.0833$. In both cases $V_{\rm nmo}$=3.29 km/s.

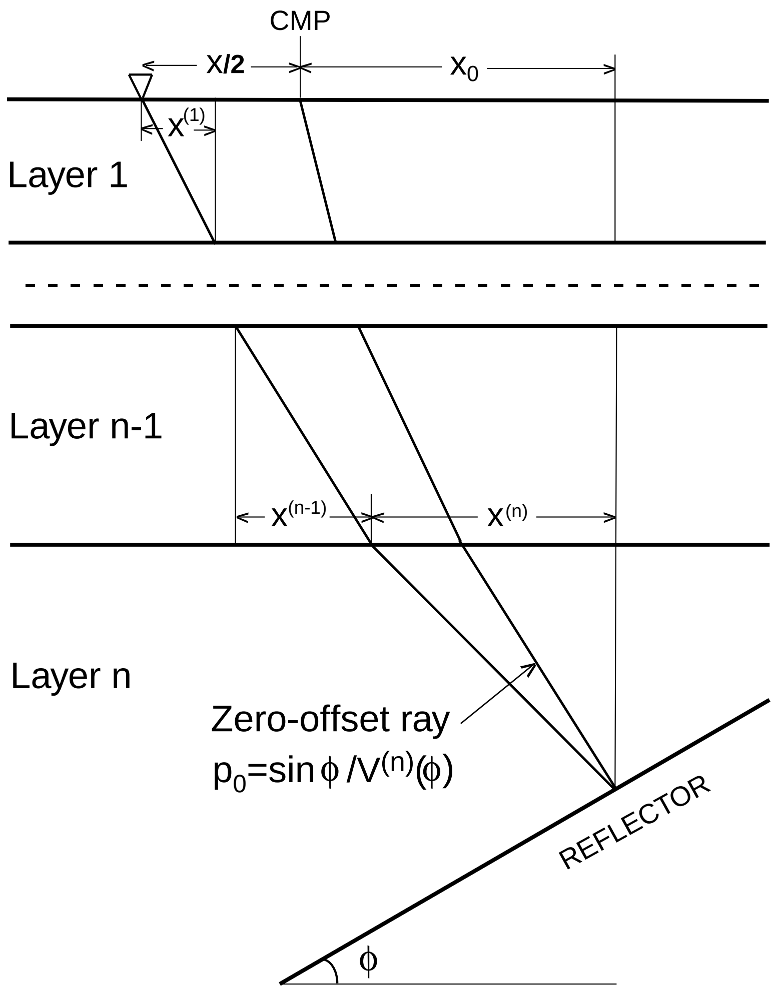

FIG. 18. A stratified anisotropic model that includes a dipping reflector beneath a stack of horizontal homogeneous layers. It is assumed that the incidence (sagittal) plane represents the dip plane of the reflector and a symmetry plane of the medium. $V^{(n)}$ is the phase velocity in the layer immediately above the reflector.

Neglecting the displacement of the reflection point on short spreads used in equation (14) (Tsvankin, 1993), we can represent the offset x as

$$x = 2\,(\sum_{i=1}^{n} x^{(i)} - x_0),$$

where x_i is the horizontal displacement of the ray in layer i, and x_0 is the total horizontal displacement of the zero-offset ray, between the CMP location and the reflection point (Figure 18). Equation (14) now becomes

$$V_{\rm nmo}^2(p_0) = \frac{1}{t_0} \lim_{p \to p_0} \sum_{i=1}^{n} \frac{d(2x^{(i)})}{dp}\,. \qquad (15)$$

From equation (14) it is clear that the summation in equation (15) is performed over the normal-moveout velocities of the individual layers multiplied with the zero-offset times. That is,

$$\lim_{p \to p_0} \frac{d(2x^{(i)})}{dp} = t_0^{(i)}\,[V_{\rm nmo}^{(i)}(p_0)]^2\,, \qquad (16)$$

where $t_0^{(i)}$ is the two-way traveltime along the zero-offset ray in layer i.

Tsvankin (1993) expressed $V_{\rm nmo}^{(i)}$ analytically through the phase angle $\phi^{(i)} = \sin^{-1}[p_0\,V^{(i)}(p_0)]$ corresponding to the zero-offset ray; $V^{(i)}$ is the phase velocity in layer i. In Appendix A and the main text we show how this NMO equation can be rewritten as a function of the ray parameter p_0.

Substituting formula (16) into the equation for the NMO velocity (15) yields

$$V_{\rm nmo}^2(p_0) = \frac{1}{t_0} \sum_{i=1}^{n} t_0^{(i)}\,[V_{\rm nmo}^{(i)}(p_0)]^2\,. \qquad (17)$$

Therefore, the NMO velocity for a dipping reflector under a stratified overburden (Figure 18) is the root-mean-square of the NMO velocities in each layer taken at the ray-parameter value p_0. The traveltimes $t_0^{(i)}$ should be calculated along the zero-offset ray. Equation (17) is quite general in the sense that it does not assume any specific type of anisotropy, although it does require the incidence plane to be a plane of symmetry.

If the reflector is horizontal, equation (17) reduces to the root-mean-square (RMS) of the zero-dip NMO velocities; however, unless the medium is transversely isotropic with a vertical symmetry axis (VTI), the zero-offset ray may deviate from the vertical direction, and t_0 may be different from the vertical time. For the special case of a stack of horizontal VTI layers, formula (17) coincides with the well-known expression discussed by Hake et al. (1984) and Tsvankin and Thomsen (1994).

In order to obtain the NMO velocity in any layer i (including the one immediately above the reflector), we need to apply the Dix formula (Dix, 1955) to the NMO velocities from the top $[V_{\rm nmo}(i-1)]$ and bottom $[V_{\rm nmo}(i)]$ of the layer:

$$[V_{\rm nmo}^{(i)}]^2 = \frac{t_0(i)V_{\rm nmo}^2(i) - t_0(i-1)V_{\rm nmo}^2(i-1)}{t_0(i) - t_0(i-1)}\,, \tag{18}$$

where $t_0(i-1)$ and $t_0(i)$ are the two-way traveltimes to the top and bottom of the layer calculated along the ray with $p = p_0$; all NMO velocities correspond to the ray-parameter value p_0.

The main difference between the NMO equation (18) and the conventional Dix formula is that all NMO velocities and traveltimes in formula (18) should be evaluated at the ray-parameter value corresponding to the dip angle of the reflector $[p_0 = \sin\phi/V^{(n)}(\phi)]$. We are mostly interested in using equation (18) to obtain the normal moveout velocity in the medium immediately above the reflector $[V_{\rm nmo}^{(n)}(p_0)]$ that serves as an input value in the inversion algorithm discussed in the previous sections. Clearly, the recovery of $V_{\rm nmo}^{(n)}(p_0)$ is impossible without obtaining the moveout velocities in the overlying medium for the same value of the ray parameter (p_0). This task is not trivial because conventional NMO velocity analysis for the horizontally-layered overburden provides us only with NMO velocities and traveltimes corresponding to the ray-parameter value $p = 0$.

However, for the special case of isotropic or elliptically anisotropic horizontal layers the normal moveout velocity at any ray-parameter value can be obtained from the zero-dip NMO velocity in a straightforward fashion [equation (6)]:

$$V_{\rm nmo}(p) = \frac{V_{\rm nmo}(0)}{\sqrt{1 - p^2 V_{\rm nmo}^2(0)}}\,. \tag{19}$$

In order to apply equations (17) and (18), it is also necessary to express the traveltime $t_0(p)$ through the zero-offset time and NMO velocity for a horizontal reflector. Below, we derive an equation for $t_0(p)$ valid for isotropy or elliptical anisotropy. For both models, the moveout is purely hyperbolic, and

$$t_0^2(p) = t_0^2(0) + \frac{x^2}{V_{\rm nmo}^2(0)}\,. \tag{20}$$

Substituting $x = t_0 V_0 \tan\psi$ (V_0 is the vertical velocity, ψ is the group angle corresponding to the ray-parameter value p) and expressing ψ through the phase angle θ

$$\tan\psi = \tan\theta\,\frac{V_{\rm nmo}^2(0)}{V_0^2}\,,$$

we find

$$t_0^2(p) = t_0^2(0) \left(1 + \tan^2\theta \, \frac{V_{\rm nmo}^2(0)}{V_0^2}\right) . \tag{21}$$

Using the relation between the phase angle and ray parameter for elliptical anisotropy [equation (A-4)] yields

$$t_0(p) = t_0(0) \sqrt{1 + p^2 \, V_{\rm nmo}^2(p)} \, . \tag{22}$$

Therefore, for isotropy or elliptical anisotropy the two-way traveltime along the ray with any ray-parameter value p can be found just from the vertical traveltime $t_0(0)$ and the NMO velocity $V_{\rm nmo}(p)$ already determined from equation (19).

If the horizontal layers are vertically transversely isotropic (but not elliptically anisotropic), it is necessary to know the value of η in addition to the zero-dip NMO velocity in order to find $V_{\rm nmo}(p)$. As illustrated by Figure 19, the parameters $V_{\rm nmo}(0)$ and η are also sufficient to calculate $t_0(p)$ given the zero-dip time $t_0(0)$.

$$t_0(p) = t_0(0) \, f[\eta, V_{\rm nmo}(0)] \, , \tag{23}$$

where f is independent of the vertical velocity and the individual values of the anisotropies ϵ and δ.

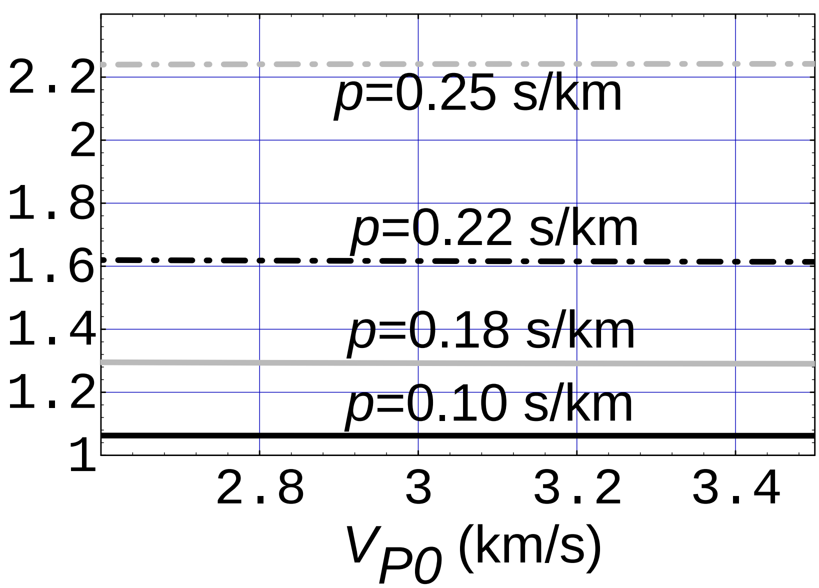

FIG. 19. Curves of the value of f in equation (23), for a range of equivalent solutions and four for different values of the ray parameter p. Here, $V_{\rm nmo}$=3.29 km/s and $\eta = 0.8333$.

Therefore, we propose the following processing sequence designed to strip the influence of the overburden on the NMO velocity: obtain the zero-dip NMO velocities and zero-offset traveltimes for the horizontal layers, use these values to calculate the NMO velocity and traveltime at the ray-parameter value p_0 for the whole horizontally stratified overburden and, finally, calculate the NMO velocity for the medium immediately above the dipping reflector via the Dix-type formula (18). The obtained normal moveout velocity, corresponding to the ray-parameter value p_0, should be combined with the NMO velocity measured for some other dip $p \neq p_0$ (e.g., the zero-dip NMO velocity) for the same layer to perform the single-layer inversion procedure discussed in the previous sections.

The nonhyperbolic moveout equation for a horizontally layered transversely isotropic medium (12) is a function of the quadratic (A_2) and quartic (A_4) moveout coefficients and horizontal velocities (V_h) in each layer, averaged in a complicated fashion. Since A_2, A_4, and V_h in individual layers depend just on $V_{\text{nmo}}(0)$ and η, the total moveout curve is entirely dependent on the values of these two effective parameters averaged over the stack of layers. Likewise, this conclusion holds for time migration in $V(z)$ media.

DISCUSSION AND CONCLUSIONS

We have suggested a method of velocity analysis for transversely isotropic media based on the inversion of the dip-dependence of *P*-wave normal moveout velocities. The algorithm, operating with surface *P*-wave data only, requires NMO velocities and ray parameters to be measured for two different dips; more than two dips provide redundancy that can be used to increase the accuracy of the inversion.

Although this inversion cannot resolve the vertical velocity and anisotropic coefficients individually, it makes it possible to obtain a family of models that have the same moveout velocity for a horizontal reflector $V_{\text{nmo}}(0)$ and the same effective anisotropic parameter $\eta = (\epsilon - \delta)/(1 + 2\delta)$. We have shown that these two parameters are sufficient to obtain NMO velocity as a function of ray parameter, describe long-spread (nonhyperbolic) reflection moveout for a horizontal reflector, and calculate poststack and prestack time-migration impulse responses. This means that the inversion of *P*-wave NMO velocities provides enough information to perform all major time-processing steps including dip moveout, prestack and poststack time migration.

The results of the inversion for η can be refined by inspecting the quality of images generated by poststack migration algorithms. If the image indicates undermigration, we should increase the value of η; to correct for overmigration, η needs to be reduced.

A natural way to include this inversion technique in the processing flow is to apply a Fowler-type dip-moveout method (Fowler, 1989), which transforms CMP data into constant-velocity stacks calculated for a range of stacking velocities. These constant-velocity panels can be conveniently used to pick the values of NMO velocities as well as the corresponding ray parameters required for the inversion procedure. The values of $V_{\rm nmo}(0)$ and η can then be refined by inspecting the output stacked panels, generated by resampling in frequency-wavenumber $(\omega - k)$ domain.

Our analysis suggests an alternative approach to the inverse problem. If it is possible to obtain an accurate value for the horizontal velocity V_h (e.g., from head waves traveling along a horizontal reflector or from cross-hole tomography), then the zero-dip velocity $V_{\rm nmo}(0)$ is sufficient to find η and, therefore, perform the processing steps mentioned above. Dipping reflectors in this case are not needed at all.

Time-to-depth conversion, however, requires accurate values of all parameters. These cannot be found from P-wave NMO velocities alone. Additional information can be obtained from the short-spread moveout velocities of SV or P-SV waves, which provide one more relation between the vertical velocities and the anisotropies ϵ and δ (Tsvankin and Thomsen, 1994). Also, clearly, the vertical velocity can be determined directly if check shots or well logs are available.

The inversion algorithm described here is developed for a homogeneous, transversely isotropic medium above the reflector. To extend the method to vertically inhomogeneous media, we generalized the NMO equation given by Tsvankin (1993) for layered anisotropic media with a dipping reflector. We show that the influence of a stratified isotropic or anisotropic overburden on moveout velocity can be stripped through a Dix-type differentiation procedure. This new NMO formula is valid in symmetry planes of any vertically inhomogeneous anisotropic medium and, therefore, can be used in developing inversion algorithms for more complicated anisotropic models than those considered in this paper.

ACKNOWLEDGMENTS

We are grateful to Ken Larner for helpful discussions and his review of the manuscript. Thanks are due to KACST for financial support of Tariq Alkhalifah's study.

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APPENDIX A: DEPENDENCE OF NMO VELOCITY ON THE RAY PARAMETER

For the purposes of the inversion procedure, we need to recast the NMO velocity as a function of the ray parameter $p(\phi)$ (horizontal slowness) corresponding to the zero-offset reflection. The vertical (m) and horizontal (p) slownesses for P-waves in transversely isotropic media satisfy the following equation (e.g., Larner, 1993)

$$1 = 0.5\{(C_{11}+C_{44})p^2+(C_{33}+C_{44})m^2+\{[(C_{11}-C_{44})p^2-(C_{33}-C_{44})m^2]^2+4(C_{13}+C_{44})^2p^2m^2\}^{\frac{1}{2}}\}.$$

This equation can be solved for m for a known value of the ray parameter p. If both slowness components are obtained, the phase velocity is simply

$$V(p) = \frac{1}{\sqrt{p^2 + m^2(p)}} ,$$

and the phase angle ϕ is given by

$$\phi = \sin^{-1}[V(p)p] .$$

Since phase velocity is a complicated function of the phase angle (or ray parameter) and anisotropic coefficients, it is hardly feasible to find a simple form for $V_{\rm nmo}(p)$ in general transversely isotropic media. Therefore, we will consider the special cases of elliptical and weak anisotropy.

The normal-moveout velocity in elliptically anisotropic media ($\epsilon = \delta$) can be represented as (Tsvankin, 1994)

$$V_{\rm nmo}(\phi) = \frac{V_{\rm nmo}(0)}{\cos\phi} \frac{V_P(\phi)}{V_{P0}} = \frac{V_{\rm nmo}(0)}{pV_{P0}} \tan\phi . \qquad \text{(A-1)}$$

Now we have to obtain the angle ϕ as a function of the ray parameter. The P-wave phase velocity for elliptical anisotropy is given by

$$V_P(\theta) = V_{P0}\sqrt{1 + 2\delta \sin^2\theta} , \qquad \text{(A-2)}$$

θ is the phase angle measured from the symmetry axis. Substituting equation (A-2) into (2), we get

$$p = \frac{\sin\phi}{V_{P0}\sqrt{1 + 2\delta \sin^2\phi}} . \qquad \text{(A-3)}$$

Solving equation (A-3) for the dip angle ϕ yields

$$\sin\phi = \frac{pV_{P0}}{\sqrt{1 - 2\delta\, p^2 V_{P0}^2}} . \qquad \text{(A-4)}$$

Calculating $\tan\phi$ from equation (A-4) and taking into account that $V_{\rm nmo}(0) = V_{P0}\sqrt{1 + 2\delta}$, we get from equation (A-1)

$$V_{\rm nmo}(p) = \frac{V_{\rm nmo}(0)}{\sqrt{1 - p^2 V_{\rm nmo}^2(0)}} . \qquad \text{(A-5)}$$

Therefore, for elliptical anisotropy, P-wave NMO velocity is a function of the ray parameter and zero-dip moveout velocity, with no separate dependence on the coefficient δ.

Now we will carry out a similar derivation for general transverse isotropy ($\epsilon \neq \delta$) using the weak-anisotropy approximation ($\epsilon \ll 1$, $\delta \ll 1$). The weak-anisotropy expression for NMO velocity as a function of the dip angle ϕ was derived by Tsvankin (1994).

$$V_{\rm nmo}(\phi) = \frac{V_P(\phi)}{\cos\phi}\left[1 + \delta + 2(\epsilon - \delta)\sin^2\phi(1 + 2\cos^2\phi)\right]. \tag{A-6}$$

In order to find the dependence of normal moveout velocity on p, we have to obtain the angle ϕ from equation (2). The P-wave phase velocity, linearized in the parameters ϵ and δ, is given by Thomsen (1986).

$$V_P(\theta) = V_{P0}\left(1 + \delta\sin^2\theta\cos^2\theta + \epsilon\sin^4\theta\right). \tag{A-7}$$

The ray parameter (2) then becomes

$$p = \frac{\sin\phi}{V_{P0}\left(1 + \delta\sin^2\theta\cos^2\theta + \epsilon\sin^4\theta\right)}. \tag{A-8}$$

After some algebra, formula (A-8) can be transformed into a quadratic equation for $\sin^2\phi$ with the solution

$$\sin^2\phi = \frac{p^2V_{P0}^2}{1 - 2\delta\, p^2V_{P0}^2}\left[1 + 2(\epsilon - \delta)\, p^4V_{P0}^4\right]. \tag{A-9}$$

Substitution of the angle ϕ from equation (A-9) into (A-6) and further linearization in ϵ and δ leads to the following expression for the NMO velocity:

$$V_{\rm nmo}^2(p) = \frac{V_{\rm nmo}^2(0)}{1 - p^2V_{\rm nmo}^2(0)}\left[1 + 2\,(\epsilon - \delta)\, f(pV_{\rm nmo}(0))\right], \tag{A-10}$$

$$f = \frac{y\,(4y^2 - 9y + 6)}{1 - y}; \quad y \equiv p^2\, V_{\rm nmo}^2(0).$$

In principle, the weak-anisotropy approximation allows us to replace $V_{\rm nmo}(0)$ in the term containing $\epsilon - \delta$ by V_{P0} (it would change only terms quadratic in the anisotropies). However, this means that we can rely only on the terms quadratic in ϵ and δ to distinguish between the true vertical velocity and anisotropies.

Thus, the NMO velocity for weak transverse isotropy expressed through the ray parameter depends on the zero-dip NMO velocity and the difference between ϵ and δ. Comparison between equations (A-10) and (A-5) shows that the weak-anisotropy approximation (A-10) is exact for elliptically anisotropic models ($\epsilon = \delta$).