A Demonstration of Prestack Migration Error in Transversely Isotropic Media

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ABSTRACT

Previous studies have shown that migration errors arise for steep events when poststack migration is done with an algorithm that does not take anisotropy into account. Here, we do a simple demonstration of the imaging errors that arise in prestack migration that ignores anisotropy present in the model. We generated data for an inhomogeneous, transversely isotropic medium with a vertical axis of symmetry (VTI), and analyzed the results of common-offset migration that ignores anisotropy.

Our results show that for the steeper events studied (up to 90 degrees), the lateral position errors observed by Alkhalifah and Larner (1994) for zero-offset migration describe well the errors expected for nonzero-offsets typically encountered in practice; i.e., for the steeper events, migration error is independent of offset. The major problem with ignoring anisotropy in prestack migration is that imaged *depth* is offset-dependent, and the problem becomes worse as reflector dip *decreases*. The offset-dependence of imaged depth, in fact, is most severe for horizontal reflections.

Because group velocity varies with propagation direction in a VTI medium, use of the vertical velocity of the medium as the migration velocity (or even simply as the normal-moveout-correction velocity) will cause nonzero-offset times for horizontal reflectors to be mapped into different imaged depths. The migration velocity that yields the most consistent set of imaged depths of horizontal events for the different offsets is the stacking velocity, obtained from conventional velocity analysis. Unfortunately, those imaged depths, although consistent, differ from the correct depth. In migration of data from isotropic media, depth-focusing analysis seeks to find the migration velocity that images events of all dips at the correct positions, for all offsets. For VTI media, no migration velocity can be found that will image reflections of all dips at the correct positions, independent of offset, when anisotropy is ignored in the migration. More parameters are needed to do a proper migration in VTI media. Alkhalifah and Tsvankin (1994) have found that only one additional parameter is necessary to correct for errors in lateral position.

The inferences, here, are based on the results for tests with just one particular VTI model. We believe, however, that they exemplify generalities we expect to encounter in a planned, more systematic study of prestack migration error stemming

from ignoring anisotropy.

INTRODUCTION

There can be no question of the critical importance of velocity to attaining accuracy in seismic-migration results. For stacked data, erroneous velocity leads to positioning error — lateral error particularly for reflections from dipping interfaces, and depth error particularly for reflections from horizontal and near-horizontal interfaces. Prestack migration with erroneous migration velocity introduces the added complication that the position errors vary with source-to-receiver offset, and consequently lead to dip-dependent wavelet distortion from mis-stacking of out-of-phase signal. These problems, which arise when the wrong velocity is used for migration, are indicative of the problems that similarly arise when the wrong wave-propagation model is used for migration. Specifically, while it is becoming increasingly recognized that the earth's subsurface is anisotropic — sometimes considerably so — most migration performed today, both poststack and prestack, ignores the presence of anisotropy. Use of such isotropic migration algorithms is tantamount, coarsely, to use of the wrong velocity in the migration. In this case, the migration velocity should vary with direction of wave propagation, but the isotropic migration fails to take that variation into consideration.

Larner and Cohen (1993) and Alkhalifah and Larner (1994) have done numerical studies of the positioning errors that arise from ignoring anisotropy, where it is present, in poststack migration. While such errors will depend greatly on the nature of the anisotropy and medium inhomogeneity, those studies of necessity were limited to a particular form of anisotropy and velocity variation — the models had vertical velocity variation (usually constant gradient), and were transversely isotropic (TI), most often with a vertical axis of symmetry. Much could be learned from models with even these restrictions. In particular, Alkhalifah and Larner (1994) found that position errors depend primarily on reflector dip, on vertical migrated time, and on only two of Thomsen's (1986) anisotropy parameters that describe P-waves in TI media — his δ and ϵ. Moreover, Larner and Cohen (1993) found that it is misleading to study migration errors attributable to ignoring anisotropy while at the same time considering only homogeneous media. Vertical velocity variation, which is the norm in the earth's subsurface, tends to ameliorate the most extreme of lateral positioning errors associated with ignoring anisotropy. The depth errors that are caused by use of incorrect velocity in the vertical direction (as happens when velocities are obtained from conventional $T^2 - X^2$ analysis, where the medium is transversely isotropic) could be corrected in a post-migration time-to-depth conversion, if the correct vertical velocity were to become available.

While much was learned in those studies about position errors that arise from poststack migration, the added complications that arise in isotropic prestack migration were not addressed. As an intermediate step, Larner (1993) and Tsvankin (1994) have addressed shortcomings of dip-moveout (DMO) when anisotropy is ignored for VTI media. Larner's numerical studies considered media that are either homogeneous or have vertical variation with constant gradient, and Tsvankin emphasized analytic results for homogeneous media, but also considered inhomogeneous media and media with tilted symmetry axis. Tsvankin's interesting observation is that DMO error depends primarily on one particular combination of Thomsen parameters, the *difference* $\epsilon - \delta$. Both studies found that conventional isotropic DMO that ignores vertical velocity variation can work well for VTI media with vertical velocity variation, often better than isotropic DMO algorithms that attempt to take the velocity variation into account.

It would seem that studies of poststack migration error and DMO error may bracket the problems that can be expected in the migration of unstacked data. Such may not be the case, since we repeatedly find that complications in imaging that arise from ignoring anisotropy defy ready intuition. Lynn et al. (1991), for example, attribute to anisotropy difficulties in prestack imaging of reflections from steep fault planes in data that often respond well to DMO followed by poststack migration. Such might not be surprising when we recognize from Alkhalifah and Larner (1994) that poststack migration error depends primarily on the parameter δ, while DMO error depends primarily on a different parameter — specifically, the difference $\epsilon - \delta$.

Here, we report on the beginning stage of the study of errors that arise in prestack migration when the anisotropy is ignored for VTI media. Our longer term goal is again to do numerical studies of the dependence of the error on Thomsen parameters and on the constant gradient in media with vertical velocity variation. In a preliminary effort, here, we generate data for a simple VTI model, apply prestack isotropic depth migration, and analyze the resulting position errors as a function of reflector dip.

Clearly, the preferred solution to imaging in anisotropic media is to migrate the data with an algorithm that fully takes the anisotropy into account. Such algorithms, for TI media, presently exist. Examples include frequency-wavenumber approaches (Uren, et al., 1990; Gonzalez, et al., 1991), phase-shift migration (Kitchenside, 1991), explicit migration in the ω-x domain (Uzcategui, 1994), Kirchhoff approaches (Ball, 1993; Sena, 1993), and Gaussian beam migration (Alkhalifah, 1994). We can expect more extensions of these approaches to prestack migration in the near future. Once such algorithms are routinely available, however, the overriding remaining issue will be difficulties in estimating the necessary anisotropy parameters well enough for data imaging to benefit sufficiently from use of these algorithms.

THE PLANNED STUDY

We are presently embarking on a systematic numerical study of prestack migration error along the lines of the analysis for poststack migration error done by Larner and Cohen (1993) and Alkhalifah and Larner (1994). In that kinematic approach, two sets of diffraction curves for scattering from a buried point scatterer are computed — one for a specified vertically inhomogeneous, TI medium, and the other for an isotropic medium with velocity approximated, as is typically done in practice, from stacking velocities obtained from conventional, finite-spreadlength $T^2 - X^2$ analysis in regions where reflectors are close to horizontal. In the numerical study, migrated position error for a given reflector dip is computed by comparing the true migrated position with the apex position to which a point on the true diffraction curve (for the TI medium) moves when migration is done based on the erroneous diffraction curve for the isotropic medium (see Larner and Cohen, 1993, for details). In the procedure, each point on the true migration curve can be identified with a particular reflector dip. Therefore, by repeating the procedure for a number of points along the true diffraction curve, we obtain curves of migration error as a function of reflector dip. Then, as in Alkhalifah and Larner (1994), curves are generated for a wide range of TI media, by parametrizing the media in terms of the two Thomsen parameters that largely govern the kinematics dictating migration error, δ and ϵ.

For our planned study of error in prestack migration, we shall follow a similar approach. The main difference is that the diffraction curves for the true TI medium and for the assumed isotropic medium must now be obtained from the summed traveltimes from source to scatterer and from receiver to scatterer, where the sources and receivers are no longer coincident. Since the diffraction curves for the zero-offset problem are already available [two-way travel times can be obtained efficiently from ray tracing based on the ingenious approach of Shearer and Chapman (1988) for factorized anisotropic inhomogeneous (FAI) media with constant velocity gradient], we can readily build the diffraction curves for nonzero source-to-receiver offset from the zero-offset times. Just as migration error was computed from the erroneous apex positions to which points on diffraction curves were migrated in the zero-offset study, our plan is to do the same, but individually for the different constant-offset diffraction curves.

Thus, instead of obtaining just migrated positions (and thus position errors) as a function of dip, for each dip we will now obtain migrated positions as a function of source-to-receiver offset. For the prestack migration problem, reflections now are expected to be imaged at erroneous positions that vary with offset. Such offset-dependence of position error would introduce problems when the data are stacked.

So much for intentions and plans. For this work-in-progress report, we simply demonstrate the issues by constructing a 2-D structural depth model of a particular VTI medium, generate nonzero-offset data for that VTI model, and migrate the data, much as is in practice, with a prestack depth-migration algorithm that does not take anisotropy into account.

THE MODEL

Figure 1 shows the schematic depth section of a structural model that has been much used in studies of imaging within CWP. The model consists of horizontal reflectors with attached segments having dips ranging from 30 to 90 degrees on both sides. This model conveniently allows study of migration action on data from reflectors with a range of depths and dips.

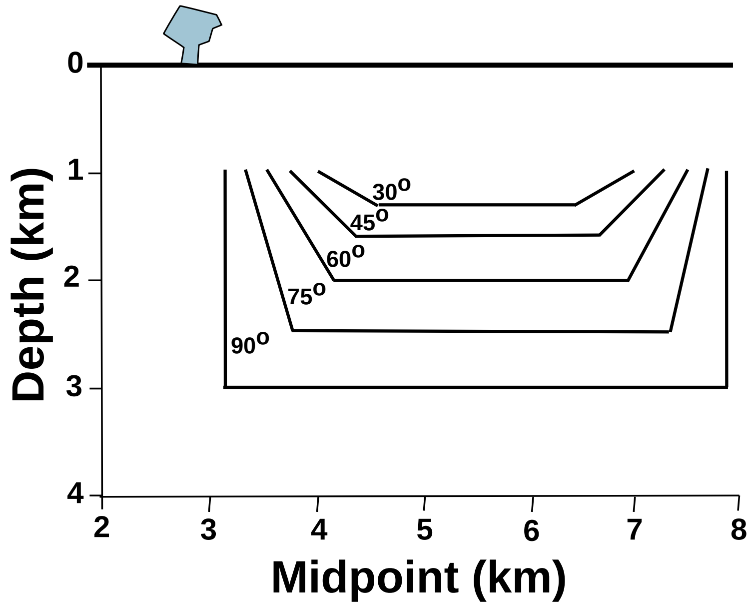


FIG. 1. Schematic depth section showing model consisting of horizontal reflectors with segments having dips ranging from 30 to 90 degrees on both sides.

The particular medium we consider is transversely isotropic with a vertical axis of symmetry (again, VTI), with the Thomsen anisotropy parameters $\epsilon = 0.2, \delta = 0.1$. These values of δ and ϵ are representative of media that are moderately anisotropic in terms of velocity variation with direction. The vertical velocity (i.e., P-wave velocity along the vertical axis-of-symmetry direction) for this model is $v(z) = 2000{+}0.6z$ m/s. Since δ and ϵ are constant throughout, while velocity varies with constant gradient, this medium has the FAI properties that allow us to generate synthetic data using the efficient ray-tracing procedure following the ideas of Shearer and Chapman (1988). The nonzero-offset data were generated by the Kirchhoff algorithm of Alkhalifah (1994) — **SUSYNLVFTI** (**SU** program for generating **SYN**thetic seismograms with **L**inear **V**elocity variation in **FTI** media.)

Figure 2 shows the generated constant-offset sections, for this structural model, for offsets 0 m and 2000 m. For the modeling, the seismic wavelet used was a Ricker wavelet with dominant frequency of 15 Hz, and reflection coefficients are constant throughout (i.e., unrealistically, reflection coefficients are fixed in this program, independent of reflection angle). For the study, we generated synthetic seismic traces across the model for offsets ranging from zero to 4000 m, in 500-m increments.

The reflections from the horizontal portions of the reflectors in Figure 2 occur at later times on the nonzero-offset section than on the zero-offset section because of the familiar action of normal moveout (NMO). Note, however, that the positions of the reflections from the steeper interfaces are much the same on the two sections. This is a manifestation of the well-known characteristic that NMO decreases as reflector dip increases (i.e., stacking velocity increases with reflector dip). This behavior will be a

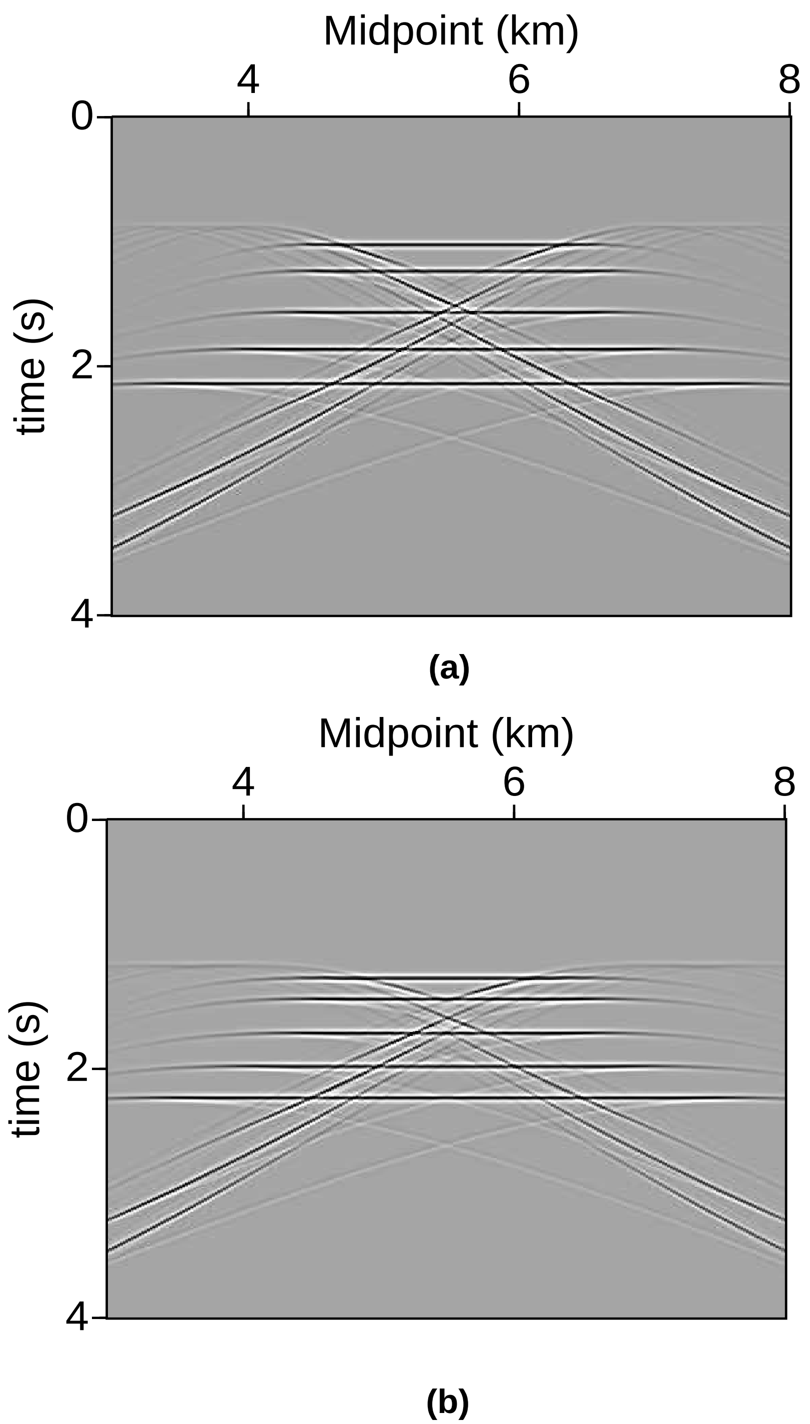


FIG. 2. Synthetic data data corresponding to a VTI medium with $\epsilon = 0.2$ and $\delta = 0.1$, for (a) zero offset and (b) 2000-m offset.

factor in understanding of the migration results, below.

One more point to note in Figure 2. As shown, the reflections from the steepest reflectors are truncated at the left and right boundaries. The truncation was for display only. Actually data were generated for midpoints that range from 0 to 10 km.

IMAGING RESULTS

For the migrations, we used the Kirchhoff depth-migration program (**SUINVVXZ** – **SU** program for doing **INV**ersion for **V**(**X**,**Z**) media) of Liu (1993). As is the case for most migration algorithms in practice, this algorithm is based on the assumption that the medium is isotropic. To demonstrate the quality of imaging of data for which this algorithm was designed, Figure 3 shows a 2000-m offset synthetic section and the migration result, where the medium here *is isotropic* and has the same structure as the model in Figure 1. Also, the velocity function is the same as the vertical velocity function, $v(z) = 2000 + 0.6z$ m/s, used to generate the VTI model data shown in Figure 2.

Clearly, the migration result has the desired accuracy. All reflectors, including the one with 90-degree dip, are positioned well, with constant amplitude, as expected for this model. We conclude that the migration algorithm works as intended, if the medium is isotropic. Such will not be the case when the isotropic algorithm is applied to the data for the VTI medium.

Figures 4 and 5 show migration results in the VTI medium for offsets 500, 1500, 2500, 3500 m. For the migration, the velocity function used was the true vertical velocity function $v(z) = 2000 + 0.6z$ m/s. We can make several observations based on comparisons of the four sections shown in these two figures. Most apparent, images of the dipping interfaces are considerably mispositioned, the lateral positioning error increasing with increasing reflector dip. The mispositioning of the dipping reflectors seen on the zero-offset section, in particular, is just that predicted by the numerical studies of Alkhalifah and Larner (1994). Of interest, the mispositioning of the steepest events is relatively independent of source-to-receiver offset. We shall discuss this further, below. Next, note that while the horizontal reflector is imaged at the correct depth on the zero-offset section, the imaged depths decrease with increasing offset. The isotropic migration algorithm has not taken into account the fact that the propagation velocity for offset raypaths differs from the vertical velocity. Clearly, these data will be distorted (out of focus) when they are stacked to produce the final migrated result, as we shall see below.

Certainly, if the medium were isotropic, some of the phenomena seen here would also be observed if the migration were performed with an erroneous velocity function. Let's see how migrations of these same four offsets would look if our data were migrated with a different velocity function. Figures 6 and 7 again show migration results for offsets 500, 1500, 2500, 3500 m. For the migration, a special choice of migration velocity function $v_m(z)$ was used: $v_m(z) = v(z)\sqrt{1+2\delta}$. This choice of

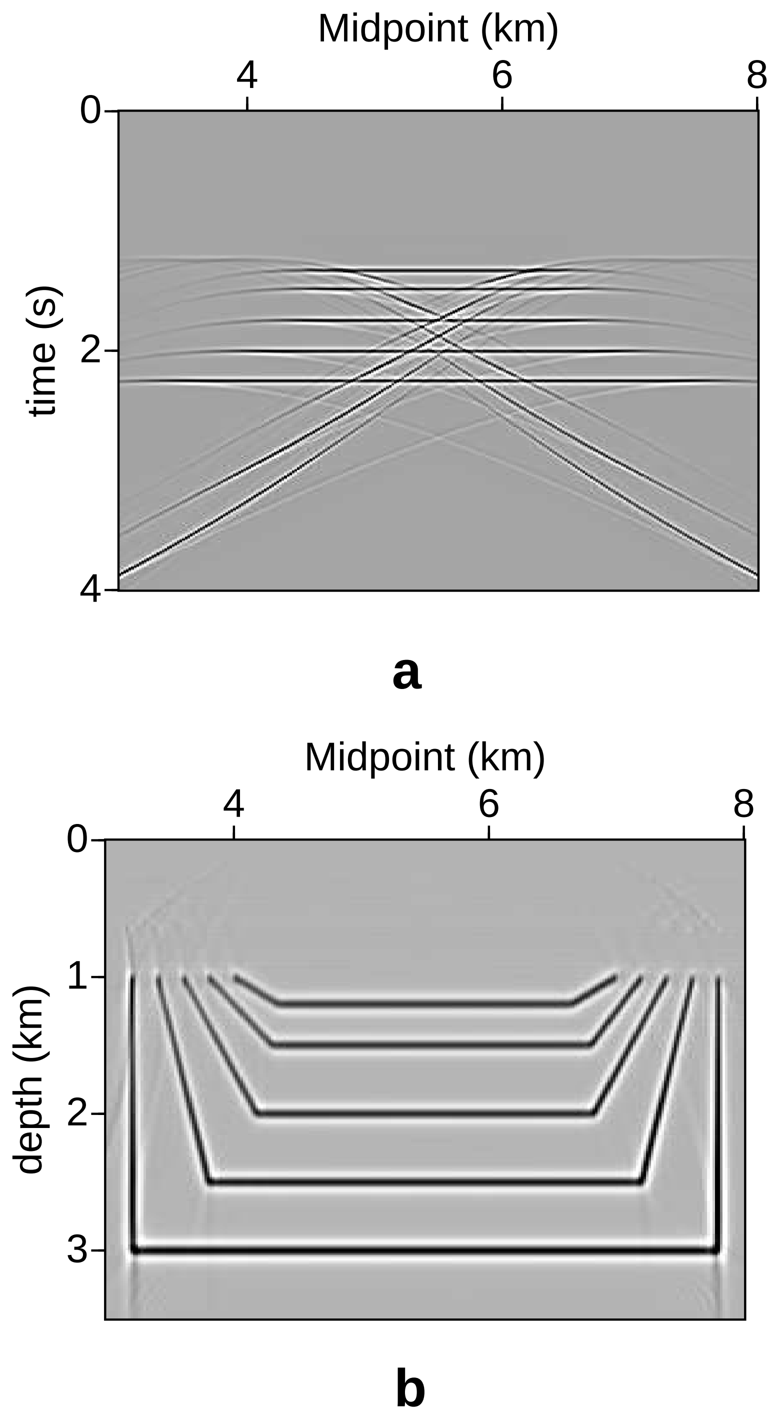
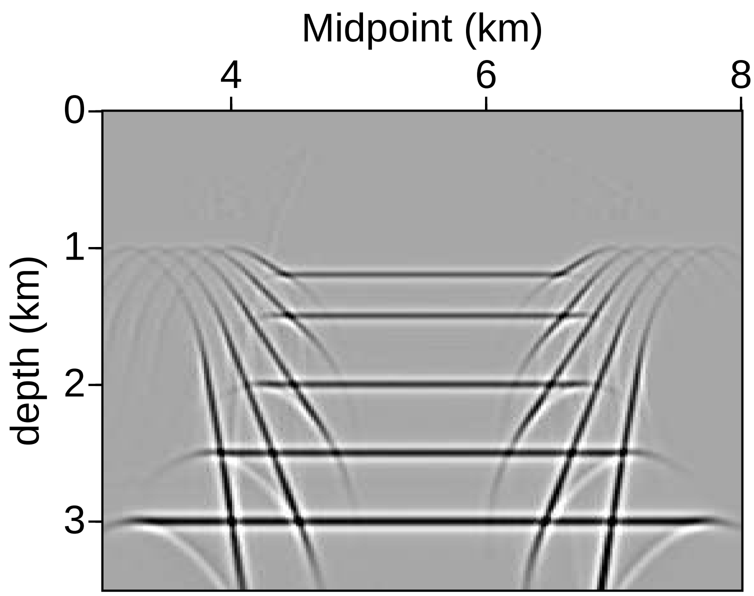


FIG. 3. (a) Synthetic common-offset (2000-m offset) section for an isotropic medium with the reflector geometry shown in Figure 1. (b) Result of isotropic migration.



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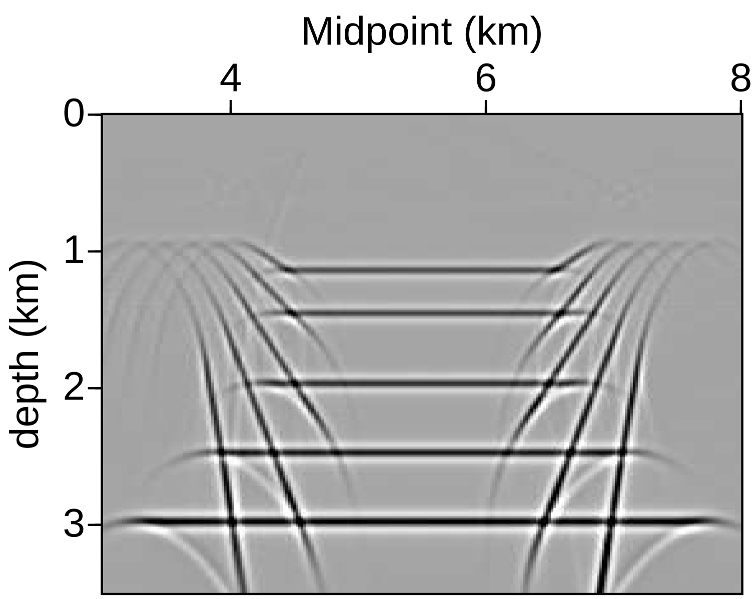

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FIG. 4. Isotropic prestack migration results for the VTI model, for offsets (a) 500 m, and (b) 1500. Migration velocity equals $v(z)$, the true vertical velocity of the medium.

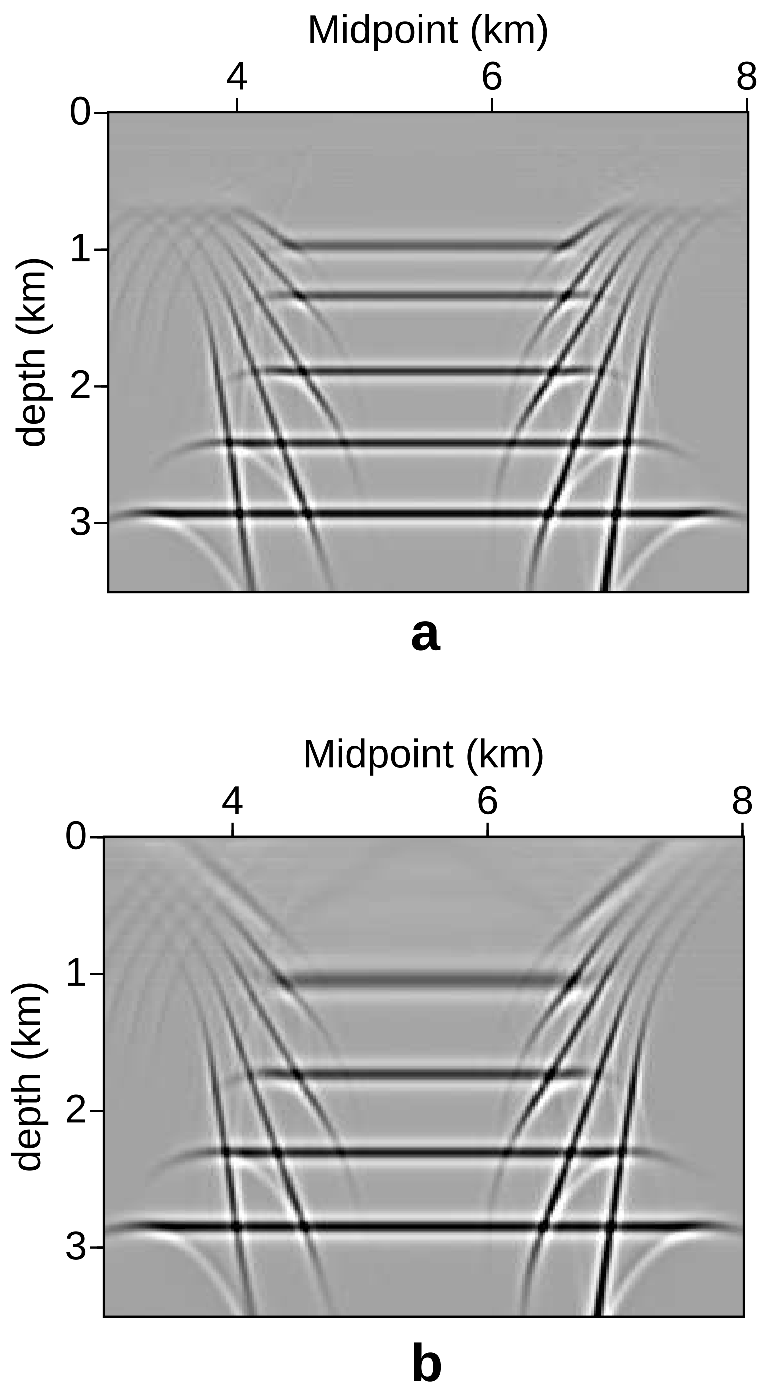


FIG. 5. Isotropic prestack migration results for the VTI model, for offsets (a) 2500 m, and (b) 3500. Migration velocity equals $v(z)$, the true vertical velocity of the medium.

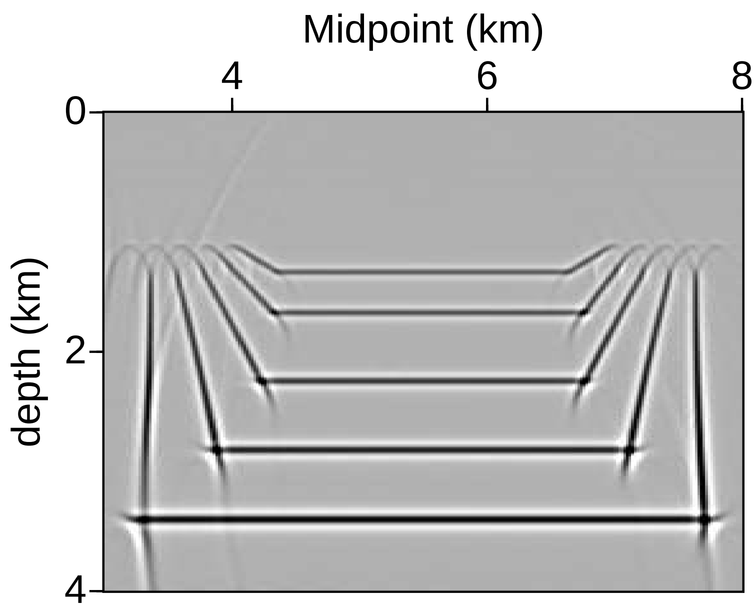
velocity function is founded on the result (Thomsen, 1986) that the zero-offset limit of stacking velocity, for data from horizontal reflectors, differs from the true vertical velocity by the factor $\sqrt{1+2\delta}$. Then, due to spreadlength bias, stacking velocity for finite-spreadlength is larger yet. Thus, our choice makes $v_m(z)$ an approximation to the velocity function that would be obtained from conventional velocity analysis performed using the reflection data from the horizontal reflectors, as might be done in practice.

Migration with a velocity function that differs from the true vertical velocity function can be expected to introduce changes in the imaged depths and in the lateral positions of migrated reflections from dipping reflectors. Clearly, for the combination of zero offset and horizontal reflectors, use of a velocity that differs from the true vertical velocity will image the reflections at the wrong depth. Use of $v_m(z)$, which approximates velocities derived from the stacking velocity, however, has resulted in imaged depths that are rather independent of offset. Thus, a stack of the various offset sections would yield well-focused images of the horizontal reflectors, albeit at the wrong depths.

The imaged reflections from dipping reflectors, again, are mispositioned. As observed by Alkhalifah and Larner (1994) for zero-offset migration, the errors in positioning will differ for the two different choices of velocity function — true vertical velocity and velocity derived from stacking velocity. Moreover, they found that the relative sensitivities to the Thomsen parameters δ and ϵ differ for these two different choices of velocity function. In Figures 6 and 7, as happened when $v(z)$ was used for the migration, the imaged positions of the steep reflectors again are relatively independent of offset. Moreover, for the choice $v_m(z)$, imaged positions of reflections for all dips are consistent from one offset to another. In reviewing Figures 4 and 5, one gets the impression that the bulk of the offset-dependent mispositioning of events from reflectors with intermediate dip is attributable to the offset-dependent imaged depths rather than to offset-dependence of lateral position error.

The imaged positions of the steep events in Figures 6 and 7 are much more accurate than those in the imaged positions of the steep events in Figures 4 and 5. This result is consistent with the findings of Alkhalifah and Larner (1994) for zero-offset migration. It is only fortuitous, however, that the reflectors are imaged as well as here when $v_m(z)$ is used for this VTI medium. In other tests with VTI media characterized by different values of ϵ and δ, the lateral positioning is not so accurate (although the general result that lateral and vertical position error are relatively independent of offset continues to hold), when $v_m(z)$ is used.

Figure 8 compares stacks of the prestack-migrated data obtained using the two velocity functions $v(z)$ and $v_m(z)$ for the migrations. Wide-angle data have been muted, much as is done in practice, but no residual moveout correction has been made to improve the focusing of reflections. These stacked sections support the points mentioned above: (1) the offset-dependence of imaged depths when $v(z)$ is used (but with correctly-imaged depth on the zero-offset section), (2) the relative little offset-



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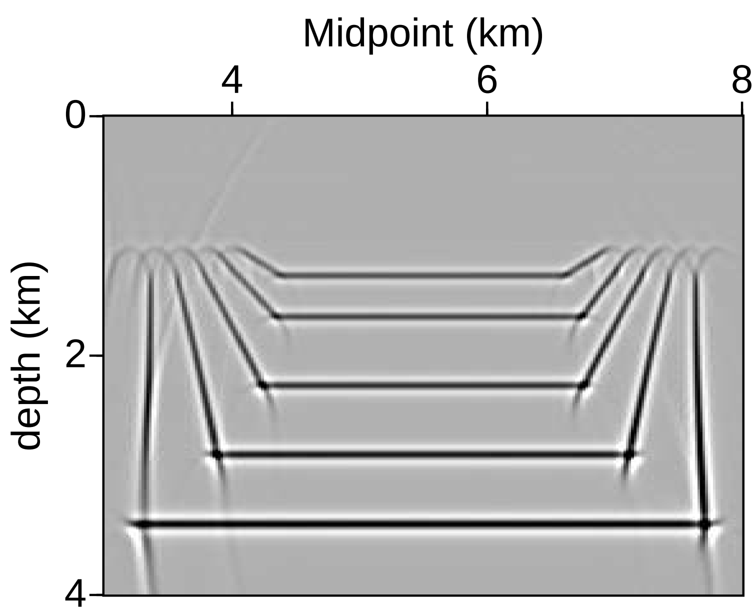

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FIG. 6. Isotropic prestack migration results for the VTI model, for offsets (a) 500 m, and (b) 1500 m. Migration velocity is $v_m(z) = v(z)\sqrt{1+2\delta}$, derived from the NMO velocity (i.e., zero-offset limit of stacking velocity) for the medium.

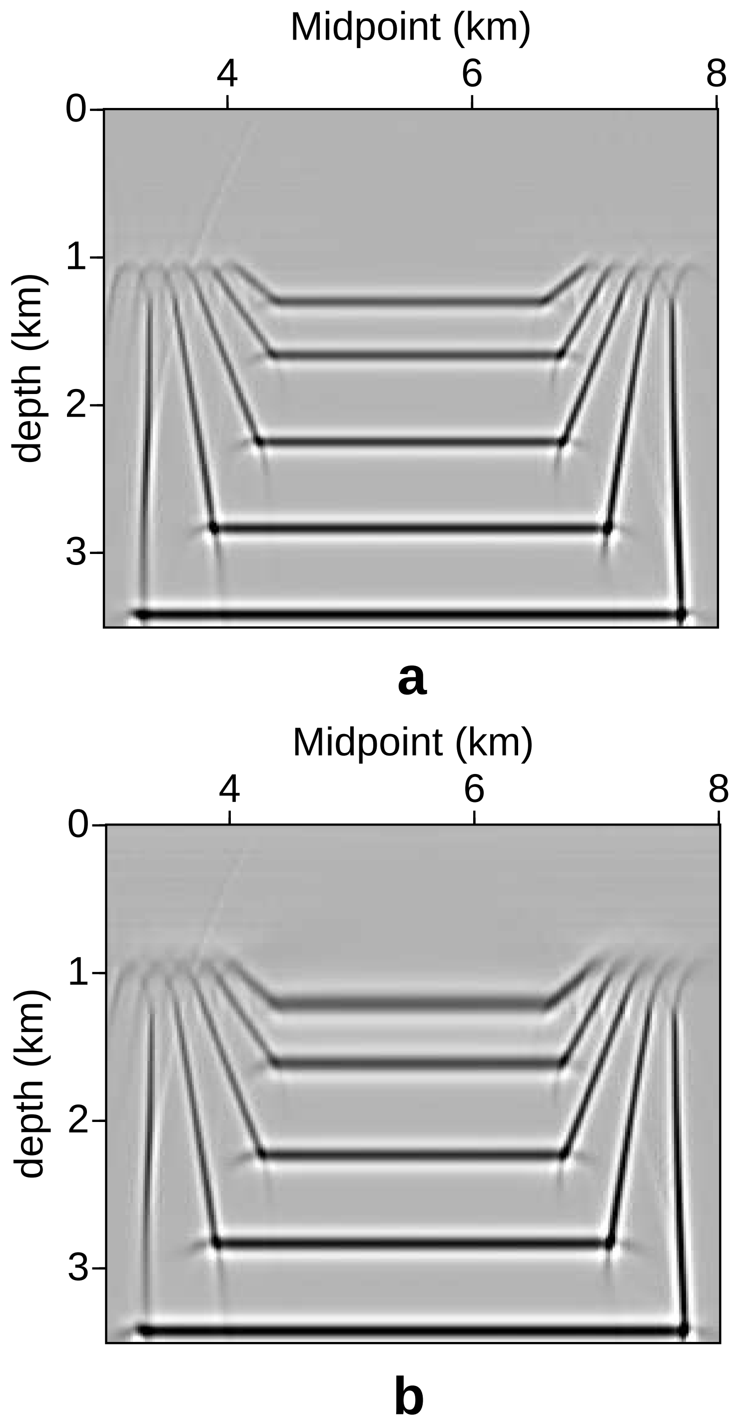

FIG. 7. Isotropic prestack migration results for the VTI model, for offsets (a) 2500 m, and (b) 3500. Migration velocity is $v_m(z) = v(z)\sqrt{1 + 2\delta}$, derived from the NMO velocity (i.e., zero-offset limit of stacking velocity) for the medium.

dependence of imaged depth when $v_m(z)$ is used, (3) the erroneous depths when $v_m(z)$ is used, (4) the relative independence of lateral positioning error with offset of steep events in both cases, and (5) the existence of lateral positioning error in both cases (but worse when $v(z)$ is used). Clearly, migration with $v_m(z)$ has yielded a better-focused section, but with features erroneously positioned in depth. Specifically, as predicted by Thomsen (1986), the imaged depth is too large by the factor $\sqrt{1+2\delta}$, an error of about 10 percent in this case.

DISCUSSION

The above observations for this particular VTI medium hold for data from other media that we have similarly modeled. For media with larger values of the difference $\epsilon - \delta$, the migration error for steep events is larger than that seen for migration velocity $v_m(z)$, here. This is due, in part, to the fact that, for larger $\epsilon - \delta$, the quality of the approximation of stacking velocity by the root-mean-squared (rms) average of $v_m(z)$ is poorer than for the medium studied here.

If an estimate of the value of the parameter δ for a given field data set were to become available, the results of isotropic migration using migration velocity derived from the stacking velocity could be depth-corrected by the factor $\sqrt{1+2\delta}$. Although the events would likely be well stacked, and depths would then be reasonably well corrected, steep events would nevertheless still be mispositioned laterally by the isotropic migration. Only depth migration with an algorithm that takes anisotropy into account and correct values for the appropriate anisotropy parameters, could be expected to yield an accurate and well-focused migration.

Some of the mispositioning seen in Figures 6, 7, 4, and 5 is reminiscent of problems of mispositioning and poor focusing of isotropic data migrated with an incorrect velocity function. Figure 9, for example, compares stacks of isotropic data that have been migrated with the correct velocity function (a) and with a velocity that is 10 percent too low (b). For the data migrated with the correct velocity, the stacked migration result is excellent — features are in the correct positions and are well focused. For the migration with the erroneous velocity, the features are mispositioned and poorly focused.

Methods of depth-focusing analysis (Al-Yahya, 1989; Jeanot, et al., 1986; MacKay and Abma, 1992), which aim to find the migration velocity function that yields the best focusing of the stacked, migrated data have been used to upgrade estimates of migration velocity. The results of our demonstration study, however, suggest that where the algorithm ignores the anisotropy of the medium, depth-focusing analysis will be unsuccessful: The velocity that gives the best focusing on the stacked, migrated data, yields neither the correct depths nor the correct lateral positions of imaged reflections.

The observation that the lateral positioning errors become progressively less dependent on offset as dip increases might be explained as follows. Recall that the

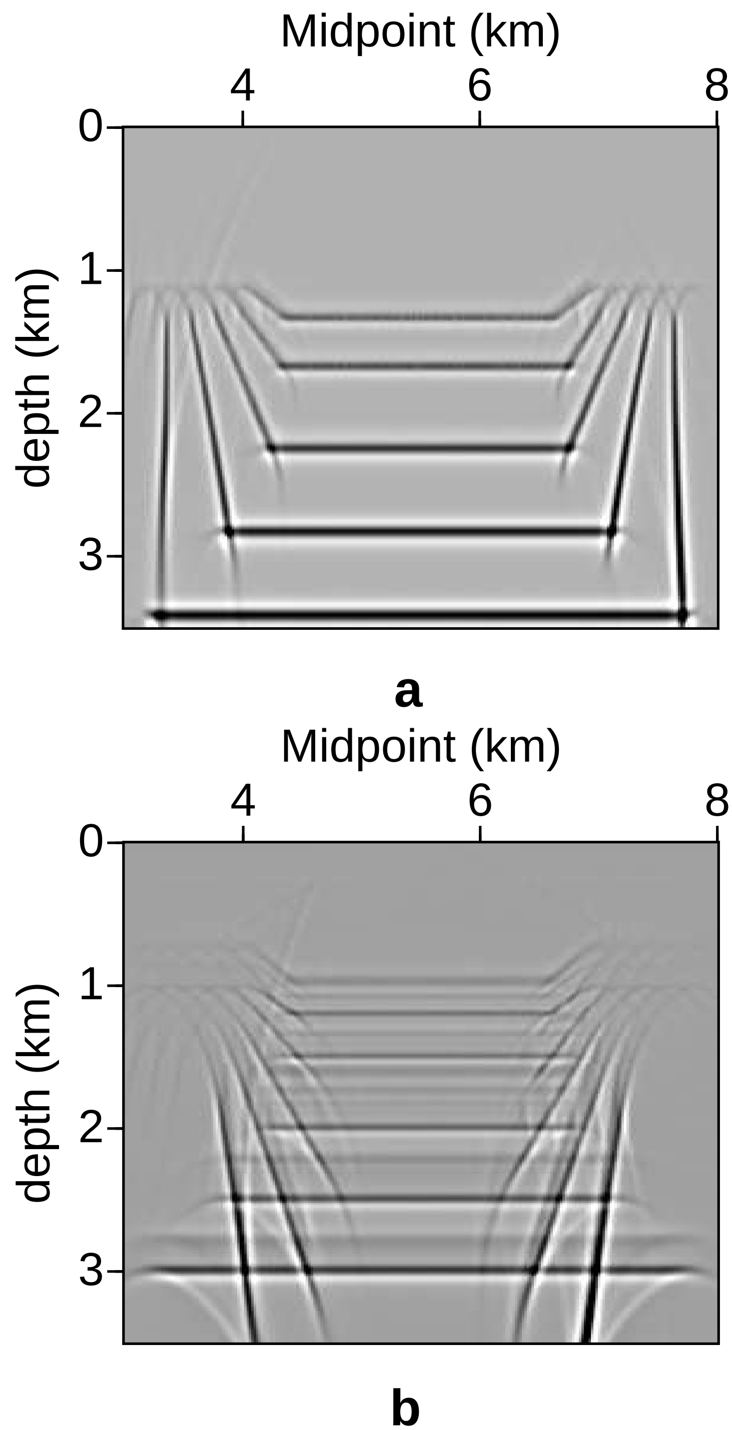

FIG. 8. Stacks of the nine migrated common-offset sections (offsets from zero to 4000 m) for the VTI model. (a). Migration velocity equals $v_m(z) = v(z)\sqrt{(1+2\delta)}$, derived from the NMO velocity. (b). Migration velocity equals $v(z)$, the true vertical velocity function.

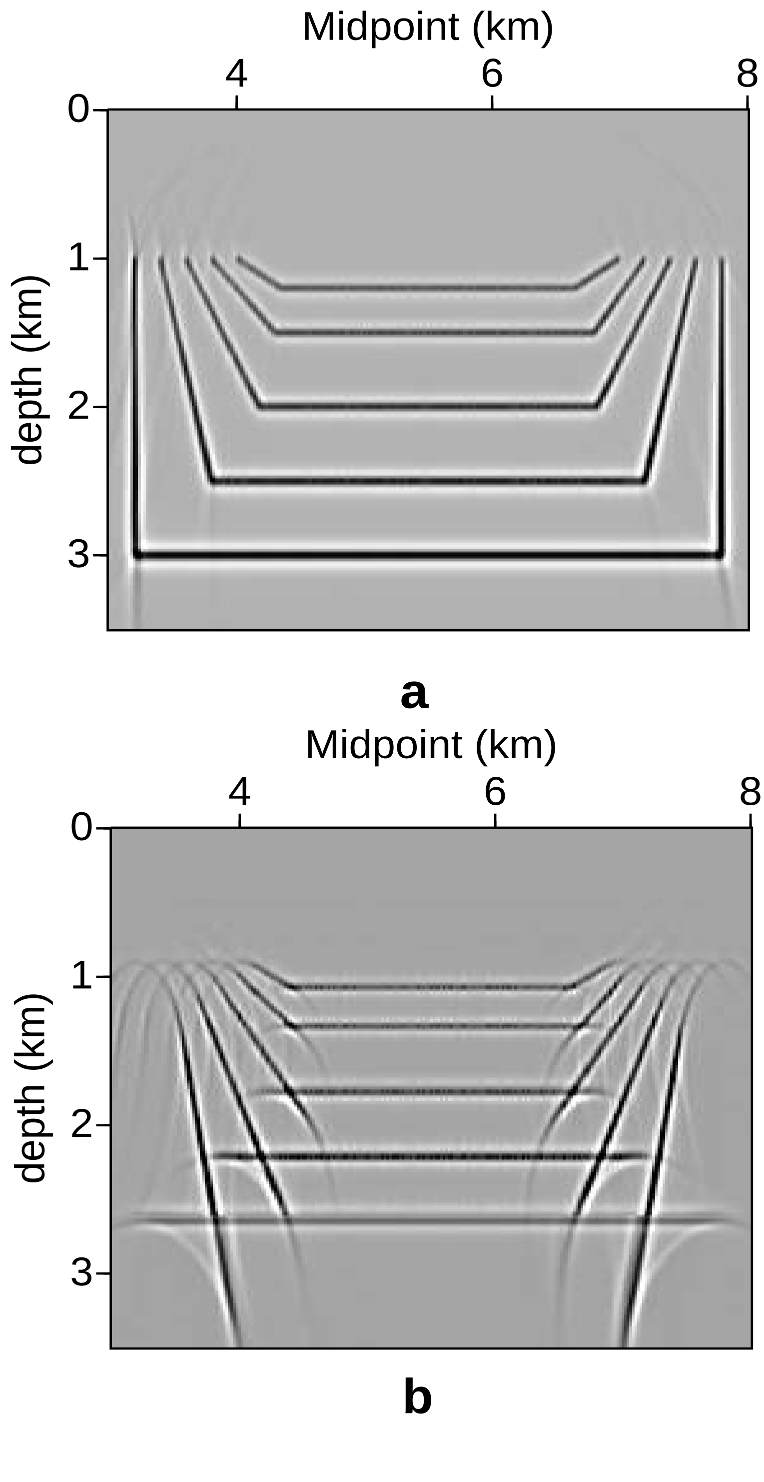

FIG. 9. Stacks of the nine migrated common-offset sections (for offsets from zero to 4000 m) for data from an isotropic medium. (a). The correct velocity is used for the migration. (b). The migration velocity is 10 percent lower than the true velocity.

steeper reflections on the unmigrated data (Figure 2) are in positions that are relatively independent of offset. Again, this reflects the known result for isotropic data that the amount of NMO decreases with increase of reflector dip (i.e., stacking velocity becomes infinite as reflector dip approaches 90 degrees). The same holds even more so for typical anisotropic data (NMO velocity increases more rapidly with reflector dip in VTI media for which $\epsilon - \delta > 0$ than in isotropic media).

Now, consider the schematic section in Figure 10, which shows zero-offset and nonzero-offset reflection raypaths for two situations — a horizontal reflector and a dipping reflector beneath a homogeneous layer. The offsets are identical in the two examples, as are the depths of the reflection points. Consider in the comparison the angles from vertical of the various raypaths in the two figures. For the horizontal reflector, the angles from vertical for the nonzero-offset path differ considerably from the angle (zero in this case) from vertical of the zero-offset path. In contrast, for the dipping reflector, the angles from vertical for the nonzero-offset path are much closer to the angle for the zero-offset path. While the raypaths for inhomogeneous media would no longer be straight, the tendency of nonzero-offset raypaths to approach the raypath for zero offset as reflector dip increases continues to hold. The same is true for TI media. The only difference, not of consequence here, is that for the TI case, the zero-offset angle no longer is the mean of the angles for the two legs of the nonzero-offset path.

This tendency immediately explains the familiar result that stacking velocity approaches infinity as reflector dip increases to 90 degrees. Simply, the raypaths for nonzero offset approach the zero-offset raypath, as do the reflection times. Now, consider the lateral position error, as a function of dip, that arises when anisotropy is ignored. For zero-offset, the error arises because the raypath to steep reflectors is at a large angle from vertical, and waves experience a propagation velocity that differs from that along the vertical path to a horizontal reflector. This accounts for the errors observed by Alkhalifah and Larner (1994) for zero offset.

Finally, consider nonzero offset. As can be inferred from Figure 10, the direction dependence of velocity in the TI medium introduces little offset-dependence of the lateral positioning error for steep reflectors because the direction of travel of waves along the nonzero-offset path is close to that for the waves along the zero-offset path. Moreover, the velocity along the zero-offset path lies between the velocities along the downward and upward legs of the nonzero-offset path, i.e., the velocity along the zero-offset path is close to the mean velocity for the nonzero-offset path, whereas it does not for reflections from the horizontal interface. Therefore, reflections from steep reflectors start in nearly the same positions for all offsets on the unmigrated data, and they get migrated by about the same amount.

For horizontal reflectors, those raypath directions differ considerably for different offsets, and hence so do the velocities experienced. Reflections from horizontal reflectors, however, experience no lateral migration (as long as the medium is laterally homogeneous and the axis of TI symmetry is vertical). Again, however, nonzero-

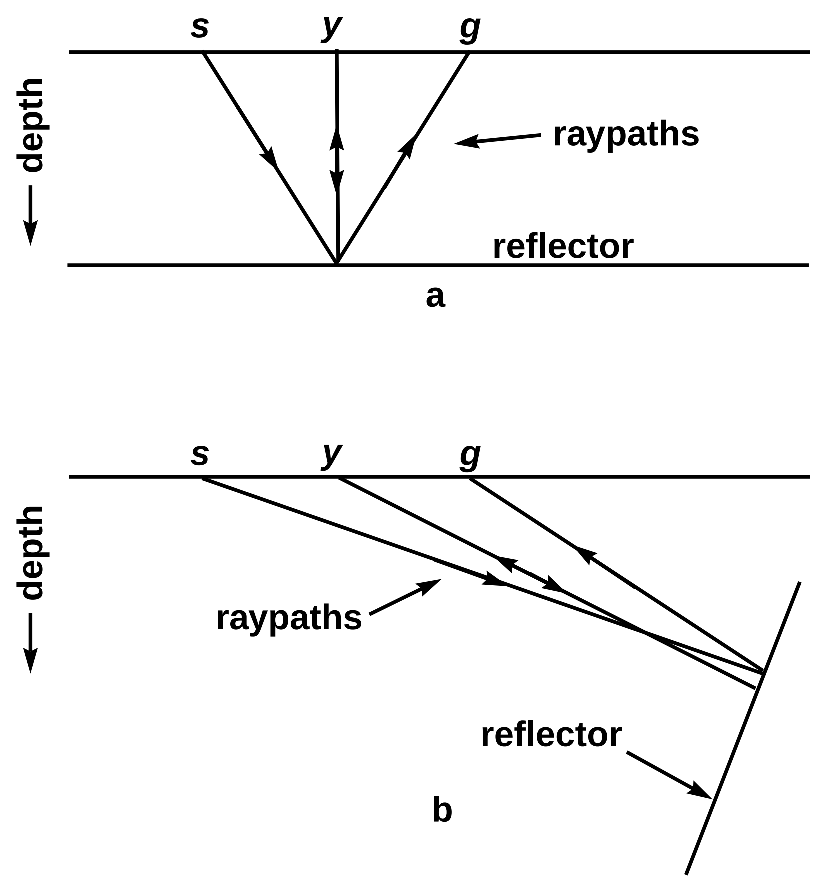

FIG. 10. Schematic depth sections showing reflection raypaths for zero offset and nonzero-offset to a plane reflector beneath a homogeneous layer. (a) Horizontal reflector. (b) Dipping reflector. The source point is denoted by s, midpoint by y, and geophone location by g.

offset reflections from horizontal reflectors do get mispositioned in depth because the propagation velocity differs from that for vertical propagation.

For reflectors with dip beyond 90 degrees, the raypaths for nonzero-offset again depart from that for zero offset. However, for the largest dips typically treated effectively in practice (perhaps about 125 degrees), the departures are still not large.

CONCLUSION

Previous studies have shown that migration errors arise for steep events from anisotropic media when poststack migration is done with an algorithm that does not take anisotropy into account. In their systematic numerical study of lateral positioning errors as a function of dip for a wide range of VTI media, Alkhalifah and Larner (1994) showed that, when migration velocities are derived from stacking velocity for horizontal reflectors, the lateral positioning errors depend primarily on the Thomsen parameter δ. In contrast, when the poststack isotropic migration is done with the true vertical velocity, the errors depend on the other important Thomsen parameter ϵ. In both cases, the errors do not become large until dip exceeds about 50 degrees (for the wide range of anisotropies considered). We plan to extend their methodology to do a similar systematic numerical study of prestack migration error as a function of dip and of the two parameters, δ and ϵ. For now, however, we have simply done a demonstration of the imaging errors for a few VTI models.

Our results show that for the steepest events studied (up to 90 degrees), the lateral position errors observed by Alkhalifah and Larner (1994) for zero-offset migration describe the errors expected for typical maximum offset encountered in practice. However, these results show that the issue with ignoring anisotropy in prestack migration — the offset-dependence of imaged depth — develops for the less steep events, unless the migration velocity is based on the stacking velocity.

While the results, here, pertain to just a few particular VTI models, we believe that the demonstration captures the essence of problems to be expected when doing conventional prestack migration where the subsurface is actually anisotropic. What remains to be seen is how the severity of the problems varies over the range of values of δ and ϵ typically expected in practice. This will be the subject of our planned, more systematic numerical study. Recent analytic studies of Alkhalifah and Tsvankin (1994) suggest that only one new anisotropy parameter [$\eta = (\epsilon - \delta)/(1 + 2\delta)$], in addition to NMO velocity, is necessary to characterize prestack as well as poststack time migration in VTI media. While just these two parameters may suffice to give accurate lateral positioning of data migrated with an anisotropic algorithm, the parameter δ is still required in order to obtain a proper estimate of depth.

As mentioned above, a key step in prestack depth migration today is the depth-focusing analysis (or some counterpart process) aimed at obtaining improved estimates of migration velocity. If, however, the anisotropy of the medium is ignored, depth-focusing analysis will fail to yield a migration velocity function that simultaneously yields a quality stack, with correct lateral and vertical positioning of reflections. Yet, processes such as depth-focusing analysis have been judged necessary in order to get acceptable velocity information to do an accurate prestack depth migration. Accurate prestack depth migration of data from media of the structural complexity of, for example, the Marmousi model (Versteeg and Grau, 1991) requires considerable effort in development of the velocity model. Our results here suggest that, complex as is the Marmousi model, it nevertheless is too simplistic. It should be anisotropic.

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