P-wave Signatures and Parametrization of Transversely Isotropic Media: An Overview

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ABSTRACT

Progress in seismic inversion and processing in anisotropic media depends on our ability to relate different seismic signatures to the anisotropic parameters. While the conventional notation (stiffness coefficients) is suitable for forward modeling, it is inconvenient in developing analytic insight into the influence of anisotropy on wave propagation. Here, I give a consistent description of P-wave signatures in transversely isotropic media using the notation suggested by Thomsen (1986).

Although originally developed for weak transverse isotropy, Thomsen notation simplifies the description of wave phenomena for any strength of the anisotropy. I show that the influence of transverse isotropy on P-wave propagation is practically independent of the vertical S-wave velocity V_{S0}, even in models with strong velocity variations. Therefore, the contribution of transverse isotropy to P-wave kinematic and dynamic signatures is controlled just by two anisotropic parameters, ϵ and δ, with the vertical velocity V_{P0} being no more than a scaling coefficient in homogeneous models. Also, I demonstrate that the weak-anisotropy approximation ($\epsilon \ll 1$, $\delta \ll 1$) can be effectively used to gain analytic insight into a wide range of wave-propagation problems.

This study shows that there is no simple correlation between the strength of velocity anisotropy and the distortions of reflection moveouts and amplitudes. The influence of transverse isotropy on P-wave normal-moveout (NMO) velocity in a horizontally layered medium, small-angle reflection coefficient, and point-source radiation in the symmetry direction is entirely determined by δ. Another group of signatures of interest in reflection seismology, such as the dip-dependence of NMO velocity, magnitude of nonhyperbolic moveout, time-migration response, and the shape of the radiation pattern near vertical, are dependent on both anisotropies (ϵ and δ) and are primarily governed by the difference $\epsilon - \delta$, i.e., by deviations from the elliptically anisotropic model (for elliptical anisotropy, $\epsilon = \delta$). Since P-wave signatures are so sensitive to the value of $\epsilon - \delta$, application of the elliptical-anisotropy approximation in P-wave processing may lead to significant errors.

Most analytic expressions given in the paper can be applied not only to vertical transverse isotropy but also to transversely isotropic media with a tilted symmetry

axis. Moreover, the equation for normal-moveout velocities can be used in symmetry planes of any anisotropic media (e.g., orthorhombic).

INTRODUCTION: CONVENTIONAL AND THOMSEN NOTATION

The influence of anisotropy on seismic processing and interpretation is well documented in the literature (e.g., Banik, 1984; Winterstein, 1986; Sams et al., 1993; Larner, 1993). However, the large number of independent parameters required to describe anisotropic models makes seismic inversion and processing in anisotropic media extremely difficult. Further progress in extending seismic algorithms to anisotropic media requires a better understanding of the relations between seismic signatures and the parameters of the anisotropic velocity field. In spite of significant attention devoted to this problem, there is no consistent description of body-wave velocities and amplitudes, even for the simplest type of anisotropy – transverse isotropy (TI). One of the main reasons for this situation is that the notations used by different authors are incompatible with each other and often are not convenient in describing seismic wave propagation.

Although the matter of notation seems trivial, it is of utmost importance in studying wave propagation in anisotropic media. Historically, wave propagation was described using the elastic (stiffness) coefficients c_{ij}. Since both Hooke's law and the wave equation are expressed through the stiffnesses c_{ij}, these coefficients are convenient to use in all types of forward-modeling algorithms. The problems arise when it is necessary to go beyond specific examples and find the effective parameters that govern seismic wavefields in anisotropic media. As we will see below, the conventional notation is not well-suited for this purpose. Without understanding of the relations between the medium parameters and seismic signatures it is hardly possible to make qualitative estimates of the influence of anisotropy on seismic wavefields and, even more importantly, to develop inversion and processing algorithms for anisotropic media. The main disadvantages of the conventional notation for transverse isotropy can be summarized as follows:

1. The strength of the anisotropy is hidden in the elastic coefficients. The medium is isotropic if $c_{11} = c_{33}$, $c_{44} = c_{66}$ and $c_{13} = c_{33} - 2c_{44}$; clearly, it is difficult to estimate the degree of velocity anisotropy just from inspection of the elastic constants. Also, the condition satisfied by elliptically anisotropic media is complicated.

2. Since the most common anisotropic model is transverse isotropy with a vertical axis of symmetry (VTI), and most reflection data are acquired at small offsets, it would be useful to have a parameter responsible for P-wave velocity near the symmetry axis. However, no such parameter exists in the conventional notation.

3. $P - SV$ propagation is described by four stiffness coefficients: c_{11}, c_{33}, c_{44}, and c_{13}. As I show below, by using Thomsen notation it is possible to cut down on the number of independent parameters needed to describe P-wave signatures. Also, the inversion of P-wave traveltime data for the coefficients c_{ij} is ambiguous because the

trade-off between c_{44} and c_{13} cannot be resolved from P-wave data alone.

4. The expressions for normal-moveout velocities in the conventional notation are complicated. Since seismic processing operates with reflection moveouts, it is important to have easily tractable equations for NMO velocities in anisotropic media.

An improvement over the conventional notation can be achieved by targeting the combinations of elastic constants most suitable for the description of seismic wavefields. This task for transverse isotropy has been fulfilled by Thomsen (1986), who suggested an efficient alternative to the conventional notation.

The idea of Thomsen notation is to separate the influence of the anisotropy from the "isotropic" quantities, i.e., P and S velocities along the symmetry axis (I omit the qualifiers in "quasi-P-wave" and "quasi-SV-wave"). Five independent elastic coefficients needed to describe vertical transverse isotropy (c_{11}, c_{33}, c_{44}, c_{66}, and c_{13}) can be replaced by the vertical velocities V_{P0} and V_{S0} of P- and S-waves respectively and three dimensionless anisotropic parameters ϵ, δ and γ (Thomsen, 1986):

$$V_{P0} \equiv \sqrt{\frac{c_{33}}{\rho}}\,, \tag{1}$$

$$V_{S0} \equiv \sqrt{\frac{c_{44}}{\rho}}\,, \tag{2}$$

$$\epsilon \equiv \frac{c_{11} - c_{33}}{2c_{33}}\,, \tag{3}$$

$$\delta \equiv \frac{(c_{13} + c_{44})^2 - (c_{33} - c_{44})^2}{2c_{33}(c_{33} - c_{44})}\,, \tag{4}$$

$$\gamma \equiv \frac{c_{66} - c_{44}}{2c_{44}}\,, \tag{5}$$

where ρ is the density.

P- and SV-wave signatures depend on four coefficients V_{P0}, V_{S0}, ϵ, and δ, while the SH-wave is fully described by the vertical velocity V_{S0} and parameter γ.

One subtle point in the relation between the conventional notation and Thomsen parameters needs to be mentioned. From equations (1) through (5) it is clear that V_{P0}, V_{S0}, ϵ, δ, and γ are uniquely defined by the stiffness coefficients. The inverse transition from Thomsen parameters to the stiffnesses, however, is unique for only four coefficients (c_{11}, c_{33}, c_{44}, and c_{66}). The fifth coefficient, c_{13}, can be uniquely determined from equation (4) only if the sign of the sum $c_{13} + c_{44}$ is specified. In principle, it is possible for the coefficient c_{13}, as well as for the sum $c_{13} + c_{44}$, to be negative (Helbig and Schoenberg, 1987). Helbig and Schoenberg (1987) also show that while phase velocities are not dependent on the sign of $c_{13} + c_{44}$, P-wave polarizations in media with $c_{13} + c_{44} < 0$ become anomalous. However, since models with negative

$c_{13} + c_{44}$ are extremely rare, for practical purposes of seismic modeling and processing it can be assumed that $c_{13} + c_{44} > 0$.

Some of the advantages of Thomsen notation are immediately obvious. The dimensionless anisotropies ϵ, δ, and γ go to zero for isotropic media and, therefore, conveniently characterize the strength of the anisotropy. The parameter ϵ, close to the fractional difference between the horizontal and vertical P-wave velocities, defines what is often simplistically called the "P-wave anisotropy." Likewise, γ represents the same measure for SH-waves. The parameter δ is responsible for near-vertical P-wave velocity variations; as shown by Thomsen (1986), it is δ rather than ϵ that determines the influence of anisotropy on short-spread reflection data.

Another advantage of Thomsen parameters is the simplicity of the elliptical condition: in elliptically anisotropic media, $\epsilon = \delta$. In this case, the P-wave slowness surface and wavefront are elliptical, while the SV-wave velocity is independent of angle, imlpying a spherical wavefront. For SH-waves, transverse isotropy always means elliptical anisotropy with the degree of the velocity variations determined by the parameter γ. At the same time, for P and SV-waves elliptical anisotropy is a special case of transverse isotropy that is seldom satisfied by subsurface formations (Thomsen, 1986). As we will see below, the sensitivity of P-wave signatures to deviations from elliptical anisotropy makes the difference $\epsilon - \delta$ one of the most important parameters in seismic processing.

Existing laboratory and field data indicate that in most cases the horizontal velocity of P and SH-waves is larger than the vertical velocity, i.e., that ϵ and γ are predominantly positive (Thomsen, 1986). Also, most measurements made at seismic frequencies indicate that $\epsilon > \delta$ (Thomsen, 1986; Tsvankin and Thomsen, 1994a). For instance, $\epsilon > \delta$ for transversely isotropic media due to thin bedding of isotropic layers (Berryman, 1979).

The introduction of the dimensionless anisotropic coefficients allowed Thomsen (1986) to develop the weak-anisotropy approximation ($\epsilon \ll 1$, $\delta \ll 1$, $\gamma \ll 1$) by linearizing seismic velocities, group and polarization angles in ϵ, δ, and γ. The weak-anisotropy approximation is an extremely powerful tool in understanding the behavior of seismic wavefields in anisotropic media. At the same time, it should not be regarded as a substitute for the exact equations in modeling, inversion, and processing algorithms. Below, I use the weak-anisotropy approximation to gain analytic insight into different wave-propagation problems.

The importance of Thomsen parameters, however, goes well beyond the weak-anisotropy approximation. Although originally designed for weakly anisotropic models, the dimensionless anisotropic parameters are convenient to use in TI media with arbitrary strength of velocity anisotropy. This point, which is not well understood in the literature, will be repeatedly stressed as we progress through the paper. One of the main advantages of Thomsen notation, discussed below, is the possibility it offers to describe the influence of transverse isotropy on P-wave propagation by just two parameters: the anisotropies ϵ and δ.

In the following sections, I give a systematic description of body-wave velocities, polarizations, and amplitudes in transversely isotropic media. The present discussion is focused on P-waves, which represent a majority of data being acquired in oil and gas exploration. However, most of the analytic developments below can be easily applied to SV-waves as well. Analytic treatment of SH-waves is straightforward because SH-wave anisotropy is elliptical.

KINEMATIC PROPERTIES

Phase and group velocity

Here, I present two refinements to the phase-velocity equations given by Thomsen (1986). First, I demonstrate that the *exact* phase velocity for P- and SV-waves can be expressed in a relatively simple fashion through Thomsen parameters. Second, I derive the phase-velocity term quadratic in the anisotropies ϵ and δ and use it to estimate the influence of V_{S0} on the P-wave phase velocity.

The formula for the P-wave phase velocity in standard notation can be found, for instance, in White (1983):

$$\begin{aligned} 2\rho V^2(\theta) \;=\;& (c_{11}+c_{44})\sin^2\theta + (c_{33}+c_{44})\cos^2\theta \\ &+ \{[(c_{11}-c_{44})\sin^2\theta - (c_{33}-c_{44})\cos^2\theta]^2 \\ &+ 4(c_{13}+c_{44})^2\sin^2\theta\cos^2\theta\}^{1/2}\,. \end{aligned} \qquad (6)$$

θ is the phase angle measured from the symmetry axis. To obtain the SV-wave velocity, the plus sign in front of the radical should be replaced with a minus.

Dividing both parts of equation (6) by the squared vertical velocity V_{P0}^2 (1) and substituting the anisotropic coefficients ϵ (3) and δ (4) yields, after some algebra, the phase-velocity function expressed through Thomsen parameters:

$$\begin{aligned} \frac{V^2(\theta)}{V_{P0}^2} \;=\;& 1 + \epsilon\sin^2\theta - \frac{f}{2} \\ &+ \frac{f}{2}\sqrt{1 + \frac{4\sin^2\theta}{f}(2\delta\cos^2\theta - \epsilon\cos 2\theta) + \frac{4\epsilon^2\sin^4\theta}{f^2}}\,, \end{aligned} \qquad (7)$$

where

$$f = 1 - V_{S0}^2/V_{P0}^2 \qquad (8)$$

is the only term containing the S-wave vertical velocity.

For elliptical anisotropy ($\epsilon = \delta$), the P-wave phase velocity becomes (exactly)

$$\frac{V^2(\theta)}{V_{P0}^2} = 1 + 2\delta \sin^2\theta \,. \tag{9}$$

Equation (7) can be simplified a little further by separating out under the radical a "non-elliptical" term containing $\epsilon - \delta$:

$$\begin{aligned} \frac{V^2(\theta)}{V_{P0}^2} &= 1 + \epsilon \sin^2\theta - \frac{f}{2} \\ &+ \frac{f}{2}\sqrt{\left(1 + \frac{2\epsilon \sin^2\theta}{f}\right)^2 - \frac{8(\epsilon - \delta)\sin^2\theta \cos^2\theta}{f}} \,. \end{aligned} \tag{10}$$

As before, the SV-wave velocity can be obtained by putting a minus sign before the radical. From comparison of equation (10) with equation (6) it is clear that the exact phase velocity, both for P- and SV-waves, becomes no more complicated if Thomsen parameters are used.

Let us now transform the phase-velocity equation (10) under the assumption of weak anisotropy ($\epsilon \ll 1$, $\delta \ll 1$). Expanding the radical in a Taylor series and dropping terms quadratic in the anisotropies ϵ and δ, we obtain

$$\frac{V^2(\theta)}{V_{P0}^2} = 1 + 2\delta \sin^2\theta \cos^2\theta + 2\epsilon \sin^4\theta \,. \tag{11}$$

Taking the square root and linearizing equation (11) further in ϵ and δ leads to Thomsen's (1986) weak-anisotropy approximation:

$$V(\theta) = V_{P0}\,(1 + \delta \sin^2\theta \cos^2\theta + \epsilon \sin^4\theta) \,. \tag{12}$$

The P-wave phase velocity, linearized in ϵ and δ (12), is independent of the shear-wave vertical velocity V_{S0}. In order to estimate the contribution of V_{S0}, I refine the weak-anisotropy approximation by adding terms that are quadratic in the anisotropies.

Note that for elliptical anisotropy the weak-anisotropy formula for the squared velocity (11) reduces to the exact phase velocity (9). Hence, our refinement to the weak-anisotropy approximation for the squared velocity should contain the difference $\epsilon - \delta$. Therefore, equation (10), which contains an explicitly separated non-elliptical term, is ideally suited for our purposes. By expanding the radical in equation (10) in a Taylor series and retaining the terms quadratic in ϵ and δ, we find

$$\frac{V^2(\theta)}{V_{P0}^2} = 1 + 2\delta \sin^2\theta \cos^2\theta + 2\epsilon \sin^4\theta + \frac{4}{f}(\epsilon - \delta)\,(\epsilon \sin^2\theta + \delta \cos^2\theta)\,\sin^4\theta \cos^2\theta \,. \tag{13}$$

Equation (13) represents the weak-anisotropy approximation for the squared P-wave phase velocity that contains the terms linear and quadratic in the anisotropies ϵ and δ. As expected, the quadratic term vanishes for elliptical anisotropy. However, if we continued the derivation to find a similar expansion for the phase velocity itself, it would contain a quadratic "elliptical" term that does not go to zero for $\epsilon = \delta$.

Now we can estimate the influence of the shear-wave vertical velocity V_{S0} on the P-wave phase velocity $V(\theta)$. The contribution of V_{S0} [or f, see equation (8)] is limited to the quadratic term in equation (13). For a practically important range of V_{S0} corresponding to $1.5 < V_{P0}/V_{S0} < 2.5$, f (and the quadratic term as a whole) changes from 1.19 to 1.8, or by about 50 percent. This is a substantial variation that can lead to tangible phase-velocity changes provided the quadratic term itself is significant. However, the quadratic term vanishes for $\theta = 0°$ and $\theta = 90°$ and remains relatively small at intermediate angles. Of course, this conclusion applies to moderate ϵ and δ, for which equation (13) can be used. For instance, for a model with $\epsilon = 0.5$, $\delta = 0$, formula (13) predicts just a 1.8 percent maximum variation in the P-wave phase velocity corresponding to V_{P0}/V_{S0} ratio changing from 1.5 to 2.5. Moreover, the estimates of the influence of V_{S0} based on equation (13) turn out to be overstated because the contribution of the neglected higher-order terms reduce the influence of V_{S0} even further. For the same model with $\epsilon = 0.5$, $\delta = 0$, the exact phase velocity changes by no more than 0.6 percent for the range of the shear-wave vertical velocity considered above, as compared to 1.8 percent estimated from formula (13).

Figure 1 shows the influence of V_{S0} on the exact P-wave phase velocity for several combinations of ϵ and δ. With increasing difference $\epsilon - \delta$, the curves corresponding to two extreme values of V_{S0} diverge slightly from each other, but the overall contribution of the variations in the S-wave vertical velocity remains practically negligible, even for uncommonly strong velocity anisotropy. Comparison of the two media with $\epsilon = 0.8$ in Figure 1 demonstrates that the magnitude of the influence of V_{S0} depends more on the difference between ϵ and δ than on the individual values of the coefficients: the dependence of the P-wave phase velocity on V_{S0} is less pronounced for the model with larger δ but smaller $\epsilon - \delta$.

Phase velocity is the fundamentally important function in anisotropic wave propagation because it determines (along with its derivatives) the expressions for group and moveout velocities, as well as the group (ray) angles. In general, any kinematic P-wave signature in a homogeneous, transversely isotropic medium can be represented as

$$K_P = K_P^{isot}\left[1 + L(\epsilon, \delta) + Q(\epsilon, \delta, V_{S0})\right], \tag{14}$$

where K_P^{isot} is the isotropic signature ($\epsilon = 0$, $\delta = 0$), L is the term linear in the anisotropies ϵ and δ, and Q denotes the quadratic and other higher-order terms in ϵ and δ, which contain a contribution of V_{S0}. Since the S-wave vertical velocity V_{S0} influences only the second correction term Q, its influence is not significant unless, in some particular case, the anisotropic terms dominate the function K_P (one such

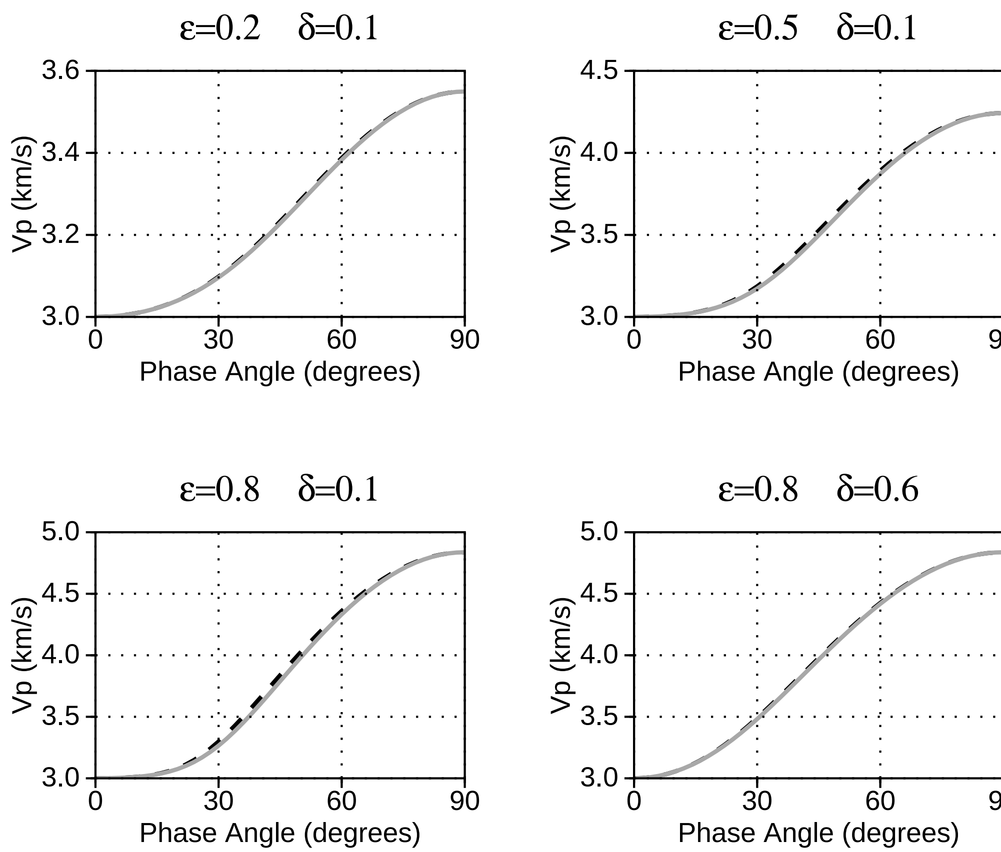

FIG. 1. The influence of V_{S0} on the P-wave phase velocity. The dashed curve corresponds to $V_{P0}/V_{S0} = 1.5$, the gray curve to $V_{P0}/V_{S0} = 2.5$.

case is described below). The weak dependence of the P-wave phase velocity on the S-wave vertical velocity implies that the influence of V_{S0} can be expected to be small in all kinematic problems involving P-waves.

The group velocity is expressed through phase velocity as (Thomsen, 1986)

$$V_{gr} = V \sqrt{1 + \left(\frac{1}{V}\frac{dV}{d\theta}\right)^2}, \tag{15}$$

and the derivative $\frac{dV}{d\theta}$, linearized in ϵ and δ, is given by

$$\frac{dV_P(\theta)}{d\theta} = V_{P0} \sin 2\theta \left(\delta \cos 2\theta + 2\epsilon \sin^2\theta\right), \tag{16}$$

Since the term containing the first derivative of phase velocity is squared, it has only quadratic and higher-order terms in ϵ and δ. Therefore, in the weak-anisotropy approximation, linearized in ϵ and δ, the group velocity as a function of the phase angle coincides with the phase velocity. However, the group velocity should be evaluated at the group angle expressed through the phase angle as (Berryman, 1979)

$$\tan\psi = \frac{\tan\theta + \frac{1}{V}\frac{dV}{d\theta}}{1 - \frac{\tan\theta}{V}\frac{dV}{d\theta}}\,. \tag{17}$$

Dropping the terms quadratic in ϵ and δ from equation (17) yields for P-waves (Thomsen, 1986)

$$\tan\psi = \tan\theta\,[1 + 2\delta + 4(\epsilon - \delta)\sin^2\theta]\,. \tag{18}$$

Due to the presence of terms linear in the anisotropies in equation (18), the difference between the group and phase angles is more pronounced than that between the absolute values of the phase and group velocities. Also note that in the symmetry direction and in the isotropy plane, the phase and group velocities and angles coincide with each other. On the whole, the conclusion about the weak influence of the S-wave vertical velocity drawn for the phase-velocity function, holds for P-wave group velocity as well.

In reflection seismology, we are more interested not in the group velocity itself, but rather in the behavior of reflection moveout, which depends on the group velocity. Normal-moveout velocities and nonhyperbolic moveout for the P-wave in transversely isotropic media are examined in the following sections.

Normal moveout velocity for horizontal reflectors

Anisotropy causes two major distortions of reflection moveouts. First, the normal (short-spread) moveout velocity is not equal to the root-mean-square (RMS) vertical velocity (Banik, 1984; Thomsen, 1986; Tsvankin and Thomsen, 1994a). Second, anisotropy may enhance deviations from hyperbolic moveout; reflection moveout is generally nonhyperbolic even in a single homogeneous anisotropic layer (Hake et al., 1984).

Reflection moveout is usually approximated by the Taylor series expansion near vertical (Taner and Koehler, 1969):

$$t_T^2 = A_0 + A_2 x^2 + A_4 x^4 + \ldots, \tag{19}$$

with the coefficients

$$A_0 = t_0^2, \quad A_2 = \left.\frac{dt^2}{dx^2}\right|_{x=0}, \quad A_4 = \frac{1}{2}\frac{d}{dx^2}\left.\left(\frac{dt^2}{dx^2}\right)\right|_{x=0}, \tag{20}$$

t_0 is the two-way vertical arrival time.

The quantity of most practical importance in exploration is the normal-moveout velocity $V_{\rm nmo}$, which determines the hyperbolic moveout on short spreads.

$$V_{\rm nmo} = 1/A_2 = \left.\frac{dx^2}{dt^2}\right|_{x=0} . \tag{21}$$

In a horizontally layered transversely isotropic model, the normal-moveout velocity is equal to the RMS average of the NMO velocities in the individual layers (Hake et al., 1984):

$$V_{\rm nmo}^2 = \frac{1}{t_0}\sum_{i=1}^{N}[V_{\rm nmo}^{(i)}]^2 \Delta t^{(i)} , \tag{22}$$

where $V_{\rm nmo}^{(i)}$ and $\Delta t^{(i)}$ are the NMO velocity and vertical traveltime in layer i.

The values of normal-moveout velocities in a single layer for different wave types can be expressed through the anisotropic coefficients as follows (Thomsen, 1986):

$$V_{\rm nmo}^2(P) = V_{P0}^2(1+2\delta) , \tag{23}$$

$$V_{\rm nmo}^2(SV) = V_{S0}^2(1+2\sigma) , \tag{24}$$

$$V_{\rm nmo}^2(SH) = V_{S0}^2(1+2\gamma) , \tag{25}$$

with

$$\sigma = \left(\frac{V_{P0}}{V_{S0}}\right)^2 (\epsilon - \delta). \tag{26}$$

It should be emphasized that these equations are valid for arbitrary strength of the anisotropy. The effective coefficient σ was introduced by Tsvankin and Thomsen (1994a) as the most influential parameter in the SV-wave moveout and velocity equations. Here, I distuinguish between the original Thomsen coefficients and their combinations, which are convenient to use in different applications; the latter will be called "effective" parameters.

In this paper, we are mostly concerned with P-wave signatures. Using equation (23), formula (22) can be rewritten for the P-wave as

$$V_{\rm nmo}^2 = V_{\rm RMS}^2\,(1+2\xi) , \tag{27}$$

where $V_{\rm RMS}$ is the root-mean-square average of the true vertical velocities $V_{P0}^{(i)}$, and ξ is the value of δ averaged over the stack of layers.

$$\xi = \frac{1}{V_{\rm RMS}^2\, t_0}\,\sum_{i=1}^{N}(V_{P0}^{(i)})^2\,\delta^{(i)}\,\Delta t^{(i)}$$

Equation (27) reduces to the conventional Dix (1955) formula only if the average value of δ is zero. Hence, if we try to derive interval vertical velocities $V_{P0}^{(i)}$ from equation (27) by applying the Dix formula, we instead get the NMO velocities, which contain a contribution of the anisotropic parameter δ. Ignoring the influence of δ in conventional processing thus leads to errors in time-to-depth conversion (e.g., Banik, 1984).

Therefore, Thomsen notation makes it possible to obtain concise expressions for normal-moveout velocities that are valid for arbitrary strength of the anisotropy and are symmetric for all wave types. For the P-wave, the NMO velocity depends on the vertical velocity and the parameter δ responsible for near-vertical velocity variations.

Dip-dependence of NMO velocity

Reflection moveout from dipping interfaces is important both in the inversion of reflection data (Alkhalifah and Tsvankin, 1994) and in developing of dip-moveout (DMO) algorithms for anisotropic media (Anderson and Tsvankin, 1994). Conventional constant-velocity DMO is usually based on the cosine-of-dip correction for moveout velocity valid for homogeneous, isotropic media (Levin, 1971):

$$V_{nmo}(\phi) = V_{nmo}(0)/\cos\phi \,, \tag{28}$$

where $V_{nmo}(\phi)$ is the normal-moveout velocity for a reflector dipping at the angle ϕ, and $V_{nmo}(0)$ is the zero-dip NMO velocity, equations (23) through (25). For homogeneous isotropic media, reflection moveout is purely hyperbolic, and equation (28) is exact for any spread length.

Equation (28) cannot be expected to hold in anisotropic media. However, numerical studies of moveout velocities of reflections from dipping boundaries beneath transversely isotropic media (Levin, 1990; Larner, 1993) have shown no apparent correlation between the conventional measures of anisotropy and errors in DMO correction. Here, we present an analytic study of NMO velocities based on the results by Tsvankin (1993a).

Let us consider a common-midpoint (CMP) gather over a homogeneous anisotropic medium with the CMP line perpendicular to the strike of the reflector (Figure 2). The only assumption made about anisotropy is that the incidence (sagittal) plane represents a plane of symmetry. For instance, this treatment is valid for any plane containing the symmetry axis in transversely isotropic media (plus the isotropy plane), as well as for symmetry planes in orthorhombic media. Under this assumption, the normal-moveout velocity for all wave types can be expressed through the phase velocity as (Tsvankin, 1993a)

$$V_{\rm nmo}(\phi) = \frac{V(\phi)}{\cos\phi}\,\frac{\sqrt{1+\frac{1}{V(\phi)}\frac{d^2V}{d\theta^2}}}{1-\frac{\tan\phi}{V(\phi)}\frac{dV}{d\theta}}\,, \tag{29}$$

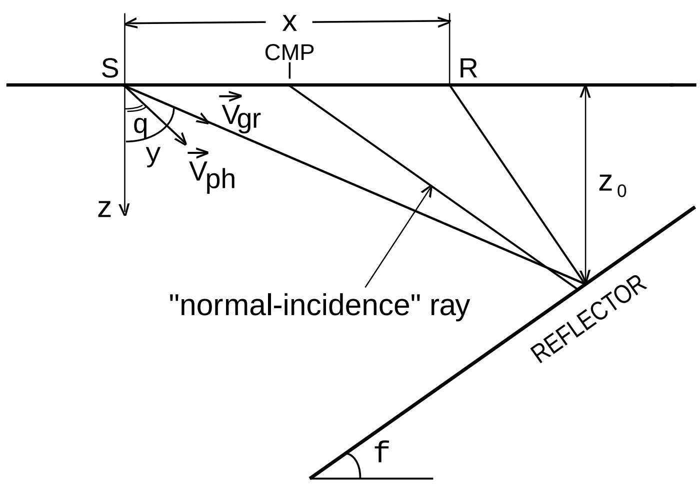

FIG. 2. Common-midpoint gather over a homogeneous anisotropic medium. $\vec{V}_{gr}$ and $\vec{V}_{ph}$ are the group and phase velocity vectors, respectively. The phase-velocity vector of the zero-offset ray (rather than the ray itself) is normal to the reflector. For brevity, in the text $\vec{V}_{ph}$ is referred to just as $\vec{V}$.

where the derivatives should be evaluated at the dip angle ϕ. Alkhalifah and Tsvankin (1994) generalize this equation for layered anisotropic media.

Formula (29) reduces to the known analytic expressions for the normal-moveout velocity from a horizontal reflector beneath a tranversely isotropic medium with a vertical symmetry axis, discussed in the previous section, and from dipping reflectors beneath elliptically anisotropic media (Byun, 1982; Uren et al., 1990). Byun (1984) obtained an analytic expression for normal-moveout velocity in general transversely isotropic media by applying a local elliptical fit to the wavefront. However, my results indicate that Byun's formula deviates from the exact NMO velocity for non-elliptical models.

Levin (1990) showed numerically that the isotropic P-wave DMO formula remains accurate for TI media with a symmetry axis perpendicular to the reflector. Equation (29) gives a clear analytic explanation for this result. If the reflector's normal coincides with the symmetry direction, formula (29) reduces to the isotropic equation (28). However, this conclusion is valid for normal (zero-spread) moveout velocities rather than for moveout velocities measured on finite spreads. In the presence of anisotropy, reflection moveout is nonhyperbolic (Tsvankin and Thomsen, 1994a), and equation (28) may become inaccurate with increasing spread length. Application of formula (29) remains straightforward for an arbitrary orientation of the symmetry axis with respect to the reflector.

In the following, I concentrate on transversely isotropic media with a vertical symmetry axis (VTI). The most convenient way to understand the influence of anisotropy on NMO velocity is to use the weak-anisotropy approximation. Using Thomsen's equation for the P-wave phase velocity (12) and further linearizing in ϵ and δ yields

$$\frac{V_{\rm nmo}(\phi)\cos\phi}{V_{\rm nmo}(0)} = 1 + \delta\sin^2\phi\cos^2\phi + \epsilon\sin^4\phi + 2(\epsilon-\delta)\sin^2\phi\,(1+2\cos^2\phi)\,. \qquad (30)$$

Equation (30) describes the anisotropy-induced distortions in the dip-dependence of P-wave NMO velocity. The P-wave dip-moveout error in transversely isotropic media can be split into two major components, which may be called the "elliptical error" and "non-elliptical error." Since elliptical anisotropy corresponds to $\epsilon = \delta$, the latter component is represented by the last term in expression (30). Analysis of the trigonometric coefficients in equation (30) shows that this non-elliptical term usually makes the most significant contribution to the total error. Thus, the difference $\epsilon - \delta$ determines, to a large degree, the angular behavior of the P-wave NMO velocity. For typical positive values of the difference $\epsilon - \delta$, the cosine-of-dip corrected moveout velocity is usually higher than the moveout velocity for a horizontal reflector.

Equation (30) gives a clear analytic explanation for the numerical results reported by Levin (1990). Note that the distortions of the isotropic cosine-of-dip dependence are not correlated with the magnitude of the velocity variations, which are determined by the individual values of ϵ and, to a lesser degree, δ.

What is the influence of the S-wave vertical velocity V_{S0} on the P-wave normal moveout velocity? If the reflector is horizontal, P-wave $V_{\rm nmo}$ depends just on V_{P0} and δ [equation (23)]. The weak-anisotropy approximation for the dip-dependent NMO velocity (30) does not contain V_{S0} either. However, from the phase-velocity equation (13) it is clear that V_{S0} does have an influence on the terms that are quadratic in ϵ and δ in the NMO equation (29).

Figure 3 shows the dependence of the P-wave NMO velocity on V_{S0} for several combinations of ϵ and δ, including those corresponding to strongly anisotropic models. The NMO velocity in Figure 3 is multiplied with $\cos\phi$, as is conventionally done in the isotropic DMO correction. If the medium were isotropic, the cosine-of-dip corrected normal-moveout velocity would be independent of angle. Due to the influence of the anisotropy, the corrected NMO velocity increases with angle.

Although the two curves on each plot correspond to the extreme values of V_{S0}, the influence of the S-wave vertical velocity is relatively weak. The maximum difference between the two curves reaches about 7 percent at steep dips for the model with a large $\epsilon = 0.5$, and $\delta = 0.1$. This is one of the rare cases when the term quadratic in the anisotropies (that contains V_{S0}) has a noticeable magnitude. For large $\epsilon - \delta$ and steep dips, the contribution of the anisotropy to the NMO velocity may even exceed the isotropic term. Indeed, for the model with $\epsilon = 0.5$ and $\delta = 0.1$ (Figure 3),

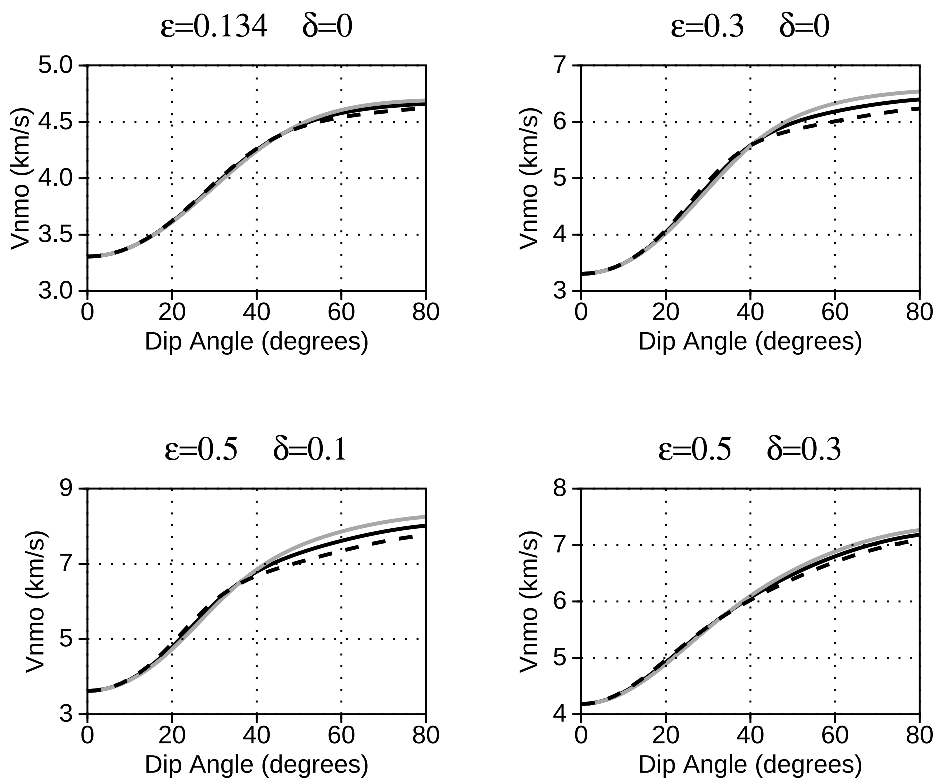

FIG. 3. The influence of V_{S0} on the cosine-of-dip corrected P-wave normal moveout velocity computed from formula (29). The dashed curve on all plots corresponds to $V_{P0}/V_{S0} = 1.5$, the black curve to $V_{P0}/V_{S0} = 1.82$, the gray curve to $V_{P0}/V_{S0} = 2.5$. The plot in the upper left corner is calculated for the parameters V_{P0}, ϵ, and δ of the shale-limestone model from Thomsen (1986) ($V_{P0} = 3.306$ km/s, $\epsilon = 0.134$, $\delta = 0$). Then I vary ϵ and δ keeping V_{P0}=3.306 km/s the same on all plots.

the anisotropic term $\frac{\tan\phi}{V(\phi)}\frac{dV}{d\theta}$ in the denominator of the NMO equation (29) becomes comparable with unity at steep dips. However, NMO velocities at steeps dips become so high that a variation in $V_{\rm nmo}$ of 7 percent would hardly change the traveltimes in a measurable way.

Figure 3 also shows that the influence of V_{S0} becomes more pronounced with increasing difference $\epsilon - \delta$ and is relatively insensitive to the individual values of the anisotropies. On the whole, the contribution of V_{S0} to P-wave NMO velocities is insignificant and, for most practical purposes, can be ignored.

As pointed out above, in a homogeneous medium V_{P0} is just a scaling coefficient for the P-wave phase velocity, if ϵ and δ are kept constant. Therefore, V_{P0} does not change the dependence of the P-wave normal moveout velocity V_{nmo} on the dip angle ϕ. This is illustrated by Figure 4, which shows that the normalized P-wave NMO velocity $V_{nmo}(\phi)/V_{nmo}(0)$ is independent of the vertical velocity V_{P0}.

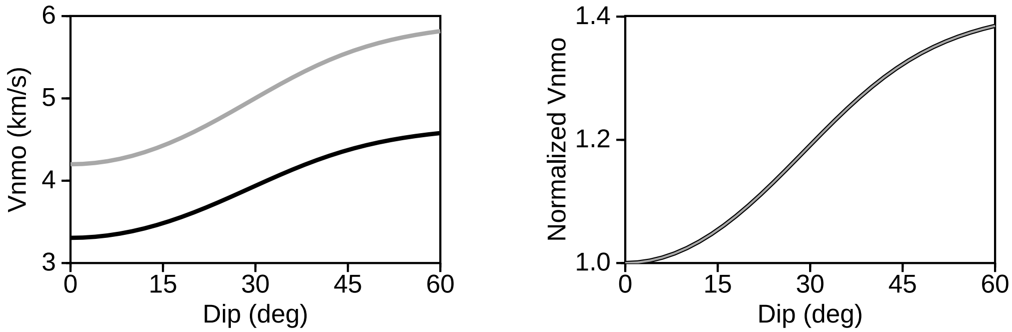

FIG. 4. The influence of V_{P0} on the cosine-of-dip corrected P-wave normal moveout velocity calculated from formula (29). The black curve corresponds to the shale-limestone model from Thomsen (1986) with V_{P0}=3.306 km/s, ϵ=0.134, and δ=0. The gray curve is for the model with V_{P0}=4.200 km/s, and the same ϵ and δ. The curves on the right plot are normalized by the NMO velocity for a horizontal reflector $V_{nmo}(0)$.

The influence of ϵ and δ on the P-wave NMO velocity is illustrated by Figures 5 and 6. Each plot in Figures 5 and 6 contains three curves: the moveout velocity calculated from ray-traced $t^2 - x^2$ functions over a spread length equal to the distance from the CMP to the reflector (solid); the exact analytic normal moveout velocity computed from equation (29) (dotted); and the weak-anisotropy approximation for V_{nmo} (dashed). Comparison between the first two curves makes it possible to estimate the influence of nonhyperbolic moveout on the moveout velocity for a typical short spread. The difference between the second and third curves shows the error of the weak-anisotropy approximation.

For elliptically anisotropic models ($\epsilon - \delta = 0$, Figure 5), the DMO error is moderate: the difference between the corrected moveout velocity and the zero-dip V_{nmo} for $\phi < 60$ degrees, $|\epsilon| < 0.2$, $|\delta| < 0.2$ is less than 15 percent.

If $\epsilon - \delta$ is positive (the most common case, Figure 6), the anisotropy causes a pronounced increase in the cosine-of-dip corrected moveout velocity with dip angle. Even for relatively small $\epsilon - \delta = 0.1$ (not shown here), the dip-moveout error reaches 25 percent at 45-degree dip and 30-35 percent at a dip of 60 degrees. For $\epsilon - \delta = 0.2$ (Figure 6), the corrected moveout velocity at 60-degree dip is consistently about 60 percent higher than the zero-dip moveout velocity!

Figures 5 and 6 show that the P-wave DMO signature is controlled, to a significant degree (although not entirely), by the difference $\epsilon - \delta$. The dominant role of $\epsilon - \delta$ is particularly pronounced for the most typical case $\epsilon - \delta > 0$.

The weak-anisotropy approximation for the NMO velocity is sufficiently accurate

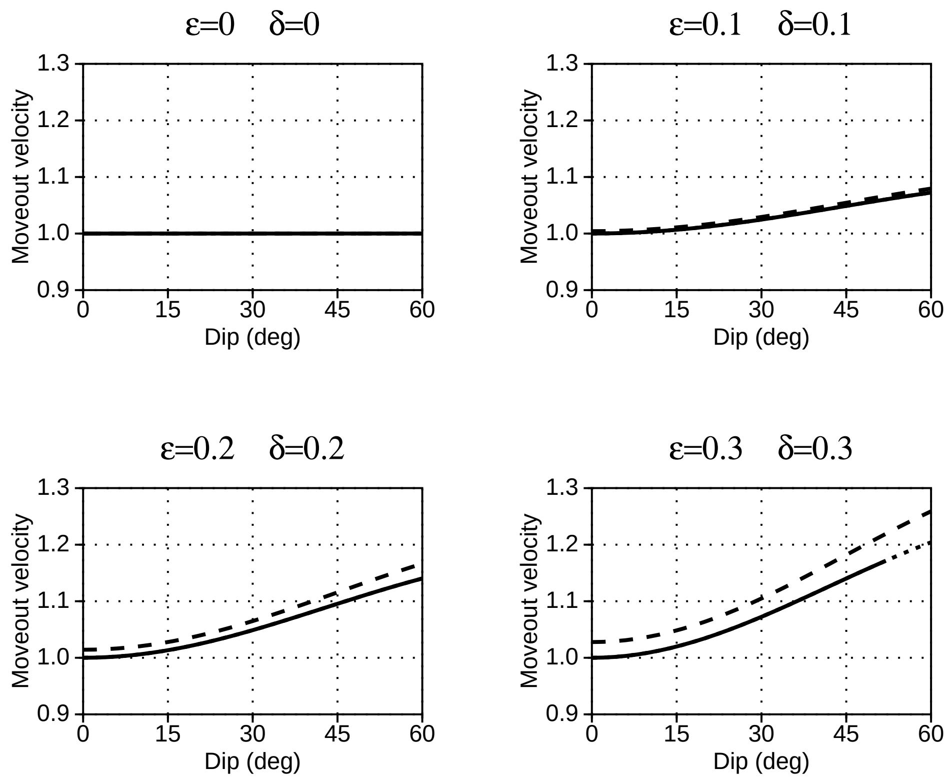

FIG. 5. Cosine-of-dip corrected P-wave moveout velocity for models with $\epsilon - \delta = 0$ (elliptical anisotropy). The solid curve is the moveout velocity calculated from $t^2 - x^2$ curves over a spread length equal to the distance from the CMP to the reflector; the dotted curve is the exact NMO velocity from formula (29); and the dashed curve is the weak-anisotropy approximation.

for small and moderate values of ϵ and δ. The error of the weak-anisotropy result (as compared with the exact NMO velocity) does not exceed 5 percent for $|\epsilon| \leq 0.2$, $|\delta| \leq 0.2$ (except for media with $\delta < -0.15$).

The above suite of plots also allows us to estimate the moveout-velocity distortions at various dips caused by nonhyperbolic moveout. The difference between the moveout velocity on a finite spread and the NMO velocity changes sign with increasing dip, but the influence of nonhyperbolic moveout for steep reflectors is typically smaller than for zero dip. If $|\epsilon - \delta| < 0.15$ to 0.2, nonhyperbolic moveout does not seriously distort the P-wave moveout velocity on short spreads.

NMO velocity as a function of ray parameter

In the above analysis, we examined the NMO velocity as a function of the dip angle ϕ. However, since reflection data do not carry any explicit information about the dip angle, for the purposes of seismic processing and inversion $V_{\rm nmo}$ should be

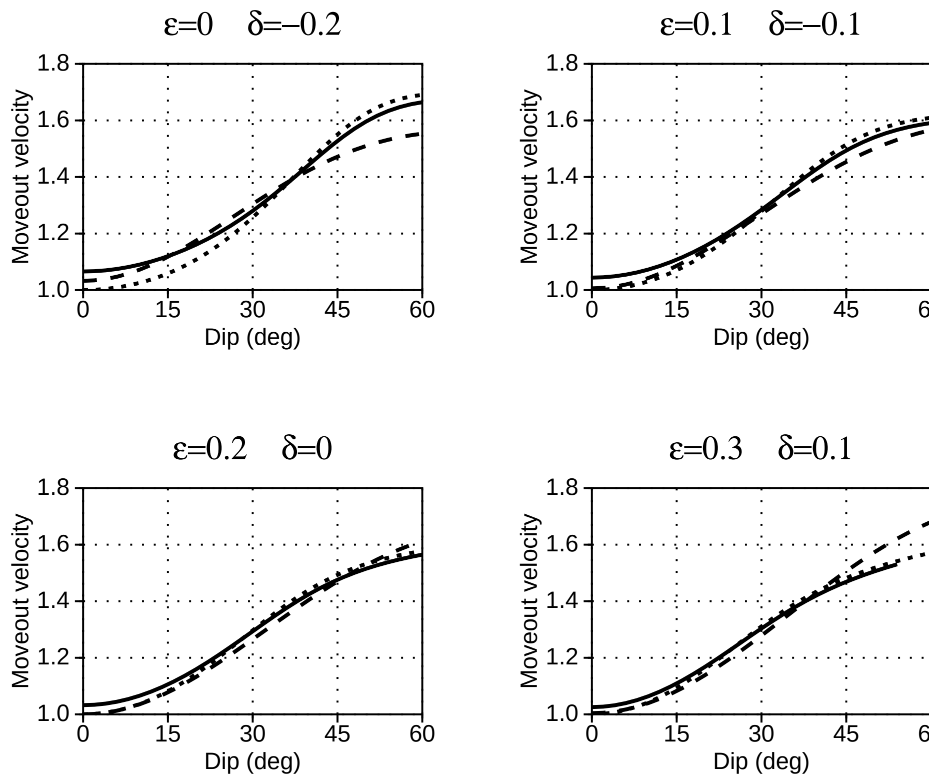

FIG. 6. Cosine-of-dip corrected P-wave moveout velocity for models with $\epsilon - \delta$=0.2.

expressed through the ray parameter $p(\phi)$ corresponding to the zero-offset reflection:

$$p(\phi) = \frac{1}{2}\,\frac{dt_0}{dy} = \frac{\sin\phi}{V(\phi)}\,, \tag{31}$$

where $t_0(y)$ is the two-way traveltime on zero-offset data.

The replacement of the angle ϕ by the ray parameter p (horizontal slowness) can be done in a straightforward fashion using the phase-velocity equations for transverse isotropy (Alkhalifah and Tsvankin, 1994). The NMO velocity (29) as a function of ray parameter for weak transverse isotropy reduces to (Alkhalifah and Tsvankin, 1994)

$$V_{\rm nmo}(p) = \frac{V_{\rm nmo}(0)}{\sqrt{1-y}}\left[1 + (\epsilon - \delta) f(y)\right], \tag{32}$$

where

$$f(y) \equiv \frac{y\,(4y^2 - 9y + 6)}{1-y}, \;\; y \equiv p^2 V_{\rm nmo}^2(0)\,.$$

It is interesting to compare formula (32) with the corresponding weak-anisotropy NMO equation as a function of the dip angle (30). Although I have emphasized the difference $\epsilon - \delta$ as the most influential parameter in the NMO equation (30), $V_{\rm nmo}(\phi)$ does contain separate contributions of ϵ and δ. However, if the dip angle is replaced with the ray parameter, the weak-anisotropy approximation for the NMO velocity contains only the combination $\epsilon - \delta$ and the zero-dip NMO velocity $V_{\rm nmo}(0)$.

Although the analysis based on equation (32) is valid for weak transverse isotropy, the numerical study by Alkhalifah and Tsvankin (1994) leads to similar results for transversely isotropic media with arbitrary strength of the anisotropy. The exact $V_{\rm nmo}(p)$ depends just on $V_{\rm nmo}(0)$ and a new effective parameter

$$\eta = \frac{\epsilon - \delta}{1 + 2\delta}, \tag{33}$$

which reduces to the difference $\epsilon - \delta$ for weak anisotropy. For elliptical anisotropy ($\epsilon = \delta$, $\eta = 0$), NMO velocity is the same function of the ray parameter as in isotropic media.

Alkhalifah and Tsvankin (1994) show that this conclusion has important implications in the inversion of the dip-dependence of the NMO velocity for the anisotropic coefficients. The P-wave NMO velocities for two distinct dips provide enough information to recover the two effective parameters and reconstruct the NMO velocity as a function of ray parameter. In the most common case when the zero-dip NMO velocity has been found by conventional NMO analysis, a single dipping reflector makes it possible to recover the parameter η. However, it is clearly impossible to resolve the trade-off between V_{P0}, ϵ and δ from P-wave NMO velocities, no matter how many reflectors are used.

The importance of the family of models with the same $V_{\rm nmo}(0)$ and η will be discussed in more detail below.

Dip-dependence of NMO velocity in vertically inhomogeneous media

Next, I consider factorized transversely isotropic (FTI) media with linear variation in vertical velocity with depth (Figure 7). In terms of the notation used here, the velocity V_{P0} linearly increases with depth, while the V_{P0}/V_{S0} ratio and the anisotropic coefficients ϵ and δ remain constant. The moveout velocity is calculated from $t^2 - x^2$ curves generated using Larner's (1993) ray-tracing algorithm.

Comparison of Figure 7 with Figure 6 shows that for typical positive values of $\epsilon - \delta$, angular variations of the cosine-of-dip corrected moveout velocity are substantially suppressed by the velocity gradient, and the accuracy of the simplest constant-velocity DMO formula (28) is satisfactory. This means that for FTI models with $\epsilon - \delta > 0$ and a typical vertical velocity gradient, the DMO correction that ignores both anisotropy and inhomogeneity is often more accurate than V(z) DMO that ignores anisotropy.

Another important conclusion from Figure 7 is that in factorized vertically inhomogeneous VTI media the P-wave moveout velocity is still primarily controlled by the difference between ϵ and δ, rather than by the individual values of these parameters.

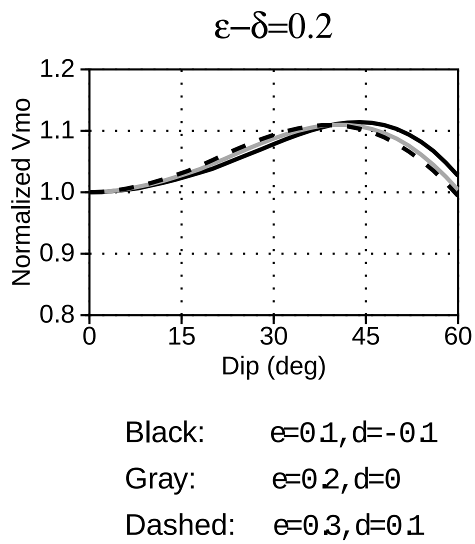

FIG. 7. P-wave moveout velocity corrected for the cosine of the dip angle for VTI models with $\epsilon - \delta = 0.2$ and a velocity gradient of 0.6 s^{-1}. The curves are normalized by the moveout velocity for a horizontal reflector. Each curve corresponds to a different pair of ϵ, δ. The distance from the CMP to the reflector and the spread length are 3000 m; the RMS vertical velocity down to 3000 m is 3500 m/s.

However, in V(z) media, dip-dependence of the moveout velocity is also a function of the velocity gradient, the RMS vertical velocity, and the depth of the reflector. The influence of the shear-wave vertical velocity on the P-wave NMO velocity in vertically inhomogeneous media remains insignificant.

Nonhyperbolic reflection moveout

The standard hyperbolic approximation for reflection moveouts in inhomogeneous media is accurate only for relatively short spreads, even in the absence of anisotropy. Angle-dependent velocity makes moveout nonhyperbolic even in a horizontal homogeneous layer. Here, I consider nonhyperbolic moveout in a horizontally homogeneous transversely isotropic medium using the results by Tsvankin and Thomsen (1994a).

Qualitative description of nonhyperbolic moveout on "intermediate" spreads (offset-to-depth ratio $x/z < 1.7 - 2$) can be given in terms of the quartic Taylor series expansion (19). The quartic moveout term for a single transversely isotropic layer is expressed through Thomsen parameters (for arbitrary strength of the anisotropy) as follows (Tsvankin and Thomsen, 1994a):

$$A_4 = -\frac{2(\epsilon - \delta)}{t_{P0}^2 V_{P0}^4} \frac{1 + \frac{2\delta}{f}}{(1 + 2\delta)^4}, \tag{34}$$

where $f = 1 - V_{S0}^2/V_{P0}^2$ is a function of V_{S0} given by equation (8). Clearly, the influence of the S-wave vertical velocity V_{S0} on the quartic term (34) is relatively weak because, as before, V_{S0} is contained only in the term quadratic in the anisotropies.

Deviations from hyperbolic moveout can be measured by the relative magnitude of the quartic term as a function of the normalized offset $\bar{x} = x/2z$:

$$\frac{A_4 x^4}{A_0 + A_2 x^2} = \bar{x}^4 \, \frac{-2(\epsilon - \delta)}{(1+2\delta)^4} \, \frac{1 + \frac{2\delta}{f}}{1 + \frac{\bar{x}^2}{1+2\delta}} \, . \tag{35}$$

The strength of the P-wave nonhyperbolic moveout is proportional to the magnitude of the difference $\epsilon - \delta$, i.e., to deviations from the elliptical model. If anisotropy is elliptical ($\epsilon = \delta$), moveout is purely hyperbolic. For fixed $\epsilon - \delta$ and conventional (small negative or positive) δ, deviations from nonhyperbolic moveout increase with *decreasing* δ. Hence, there is no simple correlation between the degree of velocity anisotropy and nonhyperbolic moveout.

The quartic moveout equation rapidly loses accuracy with increasing offset and should be replaced by a more accurate moveout formula presented by Tsvankin and Thomsen (1994a):

$$t^2(x) = t_{P0}^2 + A_2 x^2 + \frac{A_4 x^4}{1 + A x^2} \, . \tag{36}$$

$$A = \frac{A_4}{\frac{1}{V_h^2} - A_2} \, ,$$

where V_h is the horizontal velocity. For a single layer,

$$V_h = V_{P0} \sqrt{1 + 2\epsilon} \, . \tag{37}$$

Equation (36) is valid not only for the single-layer model but also for stratified transversely isotropic media, provided the appropriate coefficients A_2, A_4, and A are used. Tsvankin and Thomsen (1994a) show that the moveout formula (36) remains numerically accurate for long spreads (2-3 times, and more, the reflector depth) and pronounced anisotropy.

Since the P-wave horizontal velocity V_h (37) and the quadratic moveout term A_2 are independent of the S-wave vertical velocity, V_{S0} can slightly change P-wave moveout (36) only through the quartic coefficient A_4, but this influence is practically negligible.

Let us now rewrite the moveout equation (36) using the effective parameters $V_{\rm nmo}(0)$ and η suggested by Alkhalifah and Tsvankin (1994). Substituting the parameters $V_{\rm nmo}(0)$ and η into formula (36) and ignoring the contribution of V_{S0} to the quartic term A_4, we obtain

$$t^2(x) = t_{P0}^2 + \frac{x^2}{V_{\rm nmo}^2(0)} - \frac{2\eta x^4}{V_{\rm nmo}^2(0)[t_{P0}^2 V_{\rm nmo}^2(0) + (1+2\eta)x^2]} . \tag{38}$$

Thus, P-wave long-spread moveout can be adequately described by the vertical traveltime and two effective parameters — $V_{\rm nmo}(0)$ and η, with no separate dependence on V_{P0}, ϵ, or δ. If $\eta = 0$, the medium is elliptical and the moveout is purely hyperbolic.

The nonhyperbolic moveout equation for horizontally layered VTI media is represented by the same equation (36), with the quadratic (A_2) and quartic (A_4) moveout coefficients and V_h calculated for the stack of layers above the reflector. Tsvankin and Thomsen (1994a) show that the coefficients A_2, A_4, and V_h for a layered medium are determined by the values of A_2, A_4, and V_h in the individual layers. Since the single-layer coefficients A_2, A_4, and V_h are functions of the two effective parameters $V_{\rm nmo}(0)$ and η, the P-wave moveout curve for a layered medium depends on the values of $V_{\rm nmo}(0)$ and η averaged over the stack of layers.

Although equations (36) and (38) describe moveout for a horizontal reflector, they also can be regarded as the diffraction curves, accurate to a certain dip on the zero-offset (or stacked) section. Since time migration is based on collapsing such diffraction curves to their apex, the values of $V_{\rm nmo}(0)$ and η should be sufficient to generate an accurate time-migration impulse response. All poststack migration errors described by Alkhalifah and Larner (1994) for homogeneous media will then be eliminated. Poststack depth migration may produce depth errors if the value V_{P0} is inaccurate, but this is a different issue. These ideas are discussed in more detail by Aklhalifah and Tsvankin (1994).

POLARIZATION VECTOR

In order to obtain the polarization vector, we have to substitute the expression for a steady-state plane wave into the wave equation and solve the resulting simultaneous equations (the so-called Christoffel equations), which involve the components of the slowness and displacement vectors. For P and SV waves in the $[x_1, x_3]$ plane of a transversely isotropic medium (the x_3-direction coincides with the symmetry axis), the Christoffel equations reduce to (Musgrave, 1970)

$$G_{11}U_1 + G_{13}U_3 = 0\,, \tag{39}$$

$$G_{13}U_1 + G_{33}U_3 = 0\,, \tag{40}$$

where U_1 and U_3 are the components of the displacement vector, and G is the Christoffel matrix. The components of the Christoffel matrix can be expressed in terms of phase velocity as follows:

$$G_{11} = c_{11}\frac{\sin^2\theta}{V^2} + c_{44}\frac{\cos^2\theta}{V^2} - \rho\,,$$

$$G_{33} = c_{44}\frac{\sin^2\theta}{V^2} + c_{33}\frac{\cos^2\theta}{V^2} - \rho\,,$$

$$G_{13} = (c_{13} + c_{44}) \frac{\sin\theta\cos\theta}{V^2} .$$

For the P or SV-wave phase velocity $G_{11}G_{33} = G_{13}^2$, and either of the equations (39) or (40) can be used to find the polarization direction. Substituting G_{11}, G_{33}, and G_{13} into (39) or (40), we obtain the polarization angle γ as

$$\tan\gamma = \frac{U_1}{U_3} = \frac{\sin\theta\cos\theta\,(c_{13} + c_{44})}{\rho V^2 - c_{11}\sin^2\theta - c_{44}\cos^2\theta} . \tag{41}$$

Equation (41) is valid for any strength of the anisotropy.

Using the weak-anisotropy approximation for the phase velocity (12) and carrying out further linearization in ϵ and δ, we get a concise expression for the P-wave polarization angle (Tsvankin, 1993b):

$$\tan\gamma = \tan\theta\,\{1 + B\,[2\delta + 4(\epsilon - \delta)\sin^2\theta]\} , \tag{42}$$

$$B = \frac{1}{2f} = \frac{1}{2(1 - V_{S0}^2/V_{P0}^2)} .$$

If a P-wave is generated by a point source, the polarization at any receiver location can be obtained by applying equation (42) at the phase angle corresponding to the group (source-receiver) direction. In Thomsen's (1986) paper it is stated that the departure of the polarization angle from the phase angle for weak anisotropy is negligible. In fact, the P-wave polarization angle is closer to the group angle than to the phase angle. For the purpose of comparison, I rewrite here the weak-anisotropy approximation for the group angle ψ (18):

$$\tan\psi = \tan\theta\,[1 + 2\delta + 4(\epsilon - \delta)\sin^2\theta] .$$

The expressions for the group (18) and polarization (42) angles are quite similar; the difference between the two is in the quantity B, which is dependent on the V_{S0}/V_{P0} ratio. Since for plausible values of V_{S0}/V_{P0}, B belongs to the interval $0.5 < B < 1$, the polarization vector lies between the phase and group-velocity vectors, usually being closer to the group vector. This analytic result is in good agreement with the numerical analysis of P-wave polarizations given by Tsvankin and Chesnokov (1990a), who show that the P-wave polarization and group-velocity vectors usually remain close to each other even in more complicated orthorhombic models.

Unlike the weak-anisotropy formulas for the P-wave phase and NMO velocity, equation (42) does depend on the shear-wave vertical velocity through the parameter B. However, for a realistic range of V_{S0} and moderate anisotropies $|\epsilon| \leq 0.2$, $|\delta| \leq 0.2$, the influence of V_{S0} on the P-wave polarization angle is weak.

The analysis above is valid for typical transversely isotropic media with $c_{13} + c_{44} > 0$. Helbig and Schoenberg (1987) show that for "abnormal" media that have negative $c_{13} + c_{44}$, P-wave polarization vector may become even perpendicular to the phase-velocity vector. Indeed, it can be shown using equation (42) that if $c_{13} + c_{44} < 0$, the tangent of the polarization angle becomes negative for phase angle within the 0-90 degree range.

DYNAMIC PROPERTIES

Radiation pattern

The shape of body-wave radiation patterns is important in many applications both in earthquake seismology and seismic exploration. Here, I will use the results of Tsvankin (1993b) to discuss the influence of anisotropic radiation patterns on amplitude-versus-offset (AVO) analysis — one of the few exploration methods capable of direct detection of hydrocarbons. It is well known that the angular dependence of reflection coefficients may be significantly distorted in the presence of elastic anisotropy. However, AVO signatures (e.g., AVO gradient) in anisotropic media are also distorted by the redistribution of energy along the wavefront of the wave travelling down to the reflector and back up to the surface (Figure 8). Significant anisotropy above the target horizon may be rather typical of sand-shale sequences commonly encountered in AVO analysis.

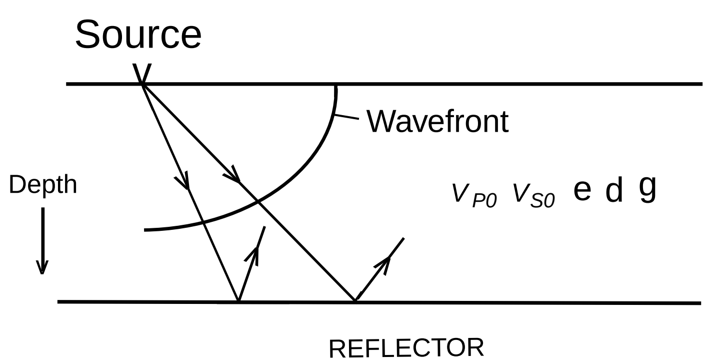

FIG. 8. Reflection from the bottom of a transversely isotropic layer. The redistribution of energy along the wavefront of the incident wave may distort the AVO response.

Far-field point-source radiation in isotropic homogeneous, non-attenuating media is determined just by the source directivity factor and spherical divergence of amplitude (Aki and Richards, 1980). The far-field approximation for source radiation in anisotropic media (Tsvankin and Chesnokov, 1990a; Ben-Menahem et al., 1991; Gajewski, 1993), is a much more complicated function that depends on the shape of the slowness surface. The most significant distortion of radiation patterns in anisotropic media is caused by the phenomena defined by Tsvankin and Chesnokov (1990a) as "focusing" and "defocusing" of energy. Energy increases (focuses) in parts of the wavefront with high concentration of group-velocity vectors of elementary plane waves (which comprise point-source radiation). Conversely, defocusing corresponds to areas with low concentration of group-velocity vectors. Often (but not always), focusing takes place near velocity maxima, while defocusing is often associated with velocity minima.

Tsvankin (1993b) shows that the far-field radiation pattern of P, SV, or SH-waves from a point force for weak transverse isotropy ($\epsilon \ll 1, \delta \ll 1, \gamma \ll 1$) reduces to

$$U(R,\theta) = \frac{F_u}{4\pi\rho V^2(\theta) R} \frac{1}{\sqrt{\frac{\sin\psi}{\sin\theta}\left(1 + \frac{1}{V}\frac{d^2V}{d\theta^2}\right)}}, \tag{43}$$

where U is the magnitude of the displacement, V is the phase velocity, and $R = \sqrt{z^2 + r^2}$ (z is the receiver depth, r is the horizontal source-receiver offset). Equation (43) does not take the influence of the free surface into account. The source term F_u is the projection of the force on the displacement (polarization) vector. Expression (43) should be evaluated at the phase angle θ, corresponding to a given ray (group-velocity) angle $\psi = \tan^{-1}(r/z)$ of the incident wave.

Formula (43) clearly demonstrates how point-source radiation is distorted by velocity anisotropy. The term $F_u/(4\pi\rho V^2 R)$ formally coincides with the well-known expression for the far-field point-force radiation in isotropic media (Aki and Richards, 1980). However, the phase velocity in equation (43) is angle-dependent; also, the source term F_u may be distorted by the anisotropy because the polarization direction deviates from the ray. The term under the radical represents the pure contribution of the anisotropy to the radiation pattern.

The distinction between the source term F_u and the rest of formula (43) is very important. While F_u is itself distorted by the anisotropy, the existence of the remaining anisotropic terms means that the redistribution of energy along the wavefront happens not only in the source layer, but also in any other anisotropic layer along the raypath.

Further linearization of equation (43) in the anisotropies ϵ and δ leads to the following expression for the P-wave:

$$U_P(R,\theta) = \frac{F_u}{4\pi\rho V_{P0}^2 R} \frac{1 - 2(\epsilon - \delta)\sin^2 2\theta + \delta\sin^2\theta}{1 + 2\delta}. \tag{44}$$

From formula (44) it is clear that transverse isotropy distorts the amplitude even in the symmetry direction ($\theta = 0$ for vertical transverse isotropy). By using the stationary-phase expression from Tsvankin and Chesnokov (1990a), it can be shown that the weak-anisotropy approximation (44) is exact for $\theta = 0$. The distortions of the P-wave amplitude in the symmetry direction depend on just one anisotropic coefficient – δ, and are directly related to the character of the velocity variations. If

$\delta < 0$, the velocity function has a maximum at $\theta = 0^\circ$, and the amplitude at vertical incidence increases due to the focusing of energy; conversely, if $\delta > 0$, a velocity minimum leads to lower amplitudes at $\theta = 0^\circ$ due to the defocusing.

While distortions of absolute amplitude are diagnostic of anisotropy, the property of radiation patterns of most importance in AVO analysis is the anisotropic correction to the angular amplitude distribution. Equation (44) shows that the lowest-order anisotropic angular correction to the radiation pattern near vertical is determined by the difference $\epsilon - \delta$. If $\epsilon - \delta > 0$ (the most common case), transverse isotropy causes the P-wave amplitude to decrease away from vertical. For elliptical anisotropy ($\epsilon = \delta$), the term $2(\epsilon - \delta)\sin^2 2\theta$ vanishes, and the anisotropic angular correction reduces to $\delta \sin^2 \theta$. This implies that for elliptical anisotropy P-wave angular amplitude and velocity variations are in good correlation with each other because both depend similarly on the same parameter δ. However, the magnitude of the anisotropy-induced angular correction (given by $\delta \sin^2 \theta$) between 0 and 40 degrees is relatively small, unless δ is unusually large. Finally, if $\epsilon - \delta < 0$, we can expect an increase in the P-wave amplitude with angle due to transverse isotropy.

Angular distortions of the radiation pattern may also be caused by the source term F_u, which depends on the polarization vector [equations (41) and (42)]. However, for moderate anisotropies $|\epsilon| \leq 0.2$, $|\delta| \leq 0.2$, the distortions of the point-force directivity factor F_u in the angular range 0-40 degrees are limited to a few percent. For more complex sources, such as dislocations or explosions, the dependence of the source term on anisotropy is more complicated, and can make a more significant contribution to the distortions of the angular amplitude distribution (Tsvankin and Chesnokov, 1990b).

Clearly, the influence of transverse isotropy on P-wave amplitudes is mostly determined by the anisotropies ϵ and δ. However, P-wave radiation pattern also depends on the vertical velocities V_{P0} and V_{S0}. While the P-wave velocity V_{P0} is just a scaling coefficient that does not change the shape of the radiation pattern (for constant ϵ, δ, and V_{P0}/V_{S0}), the contribution of the S-wave velocity V_{S0} to the P-wave amplitude needs to be evaluated.

I have shown in the previous sections that the influence of the S-wave vertical velocity on P-wave phase and group velocities is practically negligible, even for strong anisotropy. The situation with the amplitudes looks more complicated. Although there is no explicit dependence on V_{S0} in the weak-anisotropy approximation (44), the source term F_u is somewhat dependent on V_{S0} through the polarization direction [equations (41) and (42)]. Therefore, in this case the influence of V_{S0} is not entirely limited to the term quadratic in the anisotropies ϵ and δ.

However, as illustrated by Figure 9, not only the velocity but also the far-field amplitude of the P-wave is practically independent of the S-wave vertical velocity. For V_{P0}/V_{S0} ratio varying from 1.73 to 2.2, the exact far-field amplitude in the 0-40-degree range changes by less than 1.5 percent. In Figures 9, 10, and 11, I show P-wave radiation patterns from a vertical force calculated from the stationary-phase

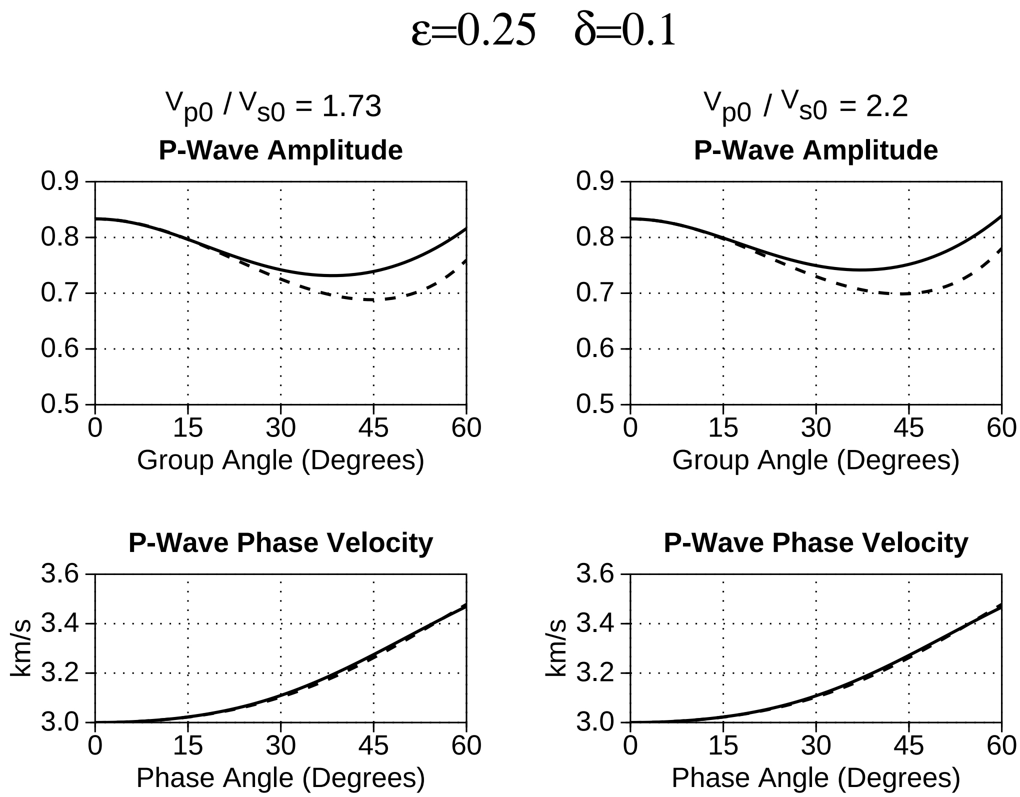

FIG. 9. The influence of the S-wave vertical velocity on P-wave amplitudes for the model with $\epsilon = 0.25$, $\delta = 0.1$, and V_{P0}=3 km/s. The solid curve is the exact far-field amplitude; the dashed curve is the weak-anisotropy approximation (44). The amplitude curves are normalized by the radiation pattern in the corresponding isotropic model ($\epsilon = 0$, $\delta = 0$). The plots at the bottom show the exact phase velocity (solid curve) and its weak-anisotropy approximation (12) (dashed curve).

expression from Tsvankin and Chesnokov (1990a) that gives the exact far-field amplitude (solid curve), and the weak-anisotropy approximation (44) (dashed curve). Both curves are normalized by the radiation pattern in the corresponding isotropic medium (ϵ=0, δ=0). The exact phase velocity (solid curve) and its weak-anisotropy approximation (12) (dashed curve) are shown at the bottom.

In elliptically anisotropic media with positive $\epsilon = \delta$ (Figure 10), the normalized amplitude increases away from vertical but the amplitude variations in the angular range 0-40 degrees remain mild, even for models with significant velocity anisotropy. This conclusion is confirmed by the exact analytic expression for radiation pattern in elliptically anisotropic media (Ben-Menahem, 1990; Tsvankin, 1993b).

For models with $\epsilon - \delta > 0$, believed to be typical for subsurface formations, transverse isotropy may cause the P-wave amplitude to drop by 30 percent and more over the first 40 degrees from vertical (Figure 11). If the difference $\epsilon - \delta$ is positive, the influence of the anisotropy becomes stronger with increasing $\epsilon - \delta$ and, for fixed

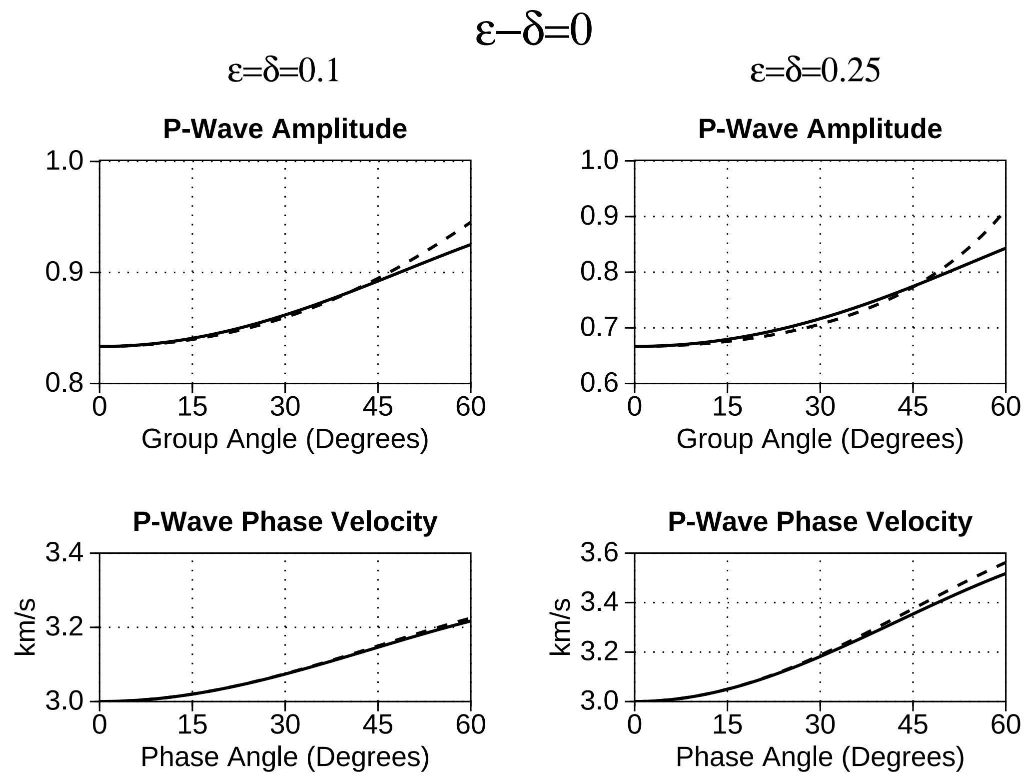

FIG. 10. Normalized P-wave amplitude from a vertical force for models with $\epsilon - \delta = 0$ (elliptical anisotropy).

$\epsilon - \delta$, with decreasing values of ϵ and δ.

Thus, there is no direct correlation between the strength of the velocity anisotropy and the amplitude anomalies. The character of the P-wave angular amplitude variations in the range of angles commonly used in AVO analysis (0-40 degrees) is controlled mostly by the difference between the anisotropies ϵ and δ.

The weak-anisotropy approximation for P-wave radiation (44) is more accurate for models with $\epsilon \leq \delta$ than for media with positive $\epsilon - \delta$. Still, for models with $0 < \epsilon - \delta < 0.2$ (believed to be most typical), the accuracy of the weak-anisotropy approximation is sufficiently high.

Reflection coefficient

The presence of elastic anisotropy on either side of the reflector may significantly distort the angular dependence of reflection coefficients (e.g., Keith and Crampin, 1977; Banik, 1987; Wright, 1987). Banik (1987) and Thomsen (1993) developed analytic approximations for reflection coefficients at a boundary between two transversely isotropic media with a vertical symmetry axis. Thomsen (1993) gives the following approximation for the P-wave reflection coefficient, in the limit of weak anisotropy

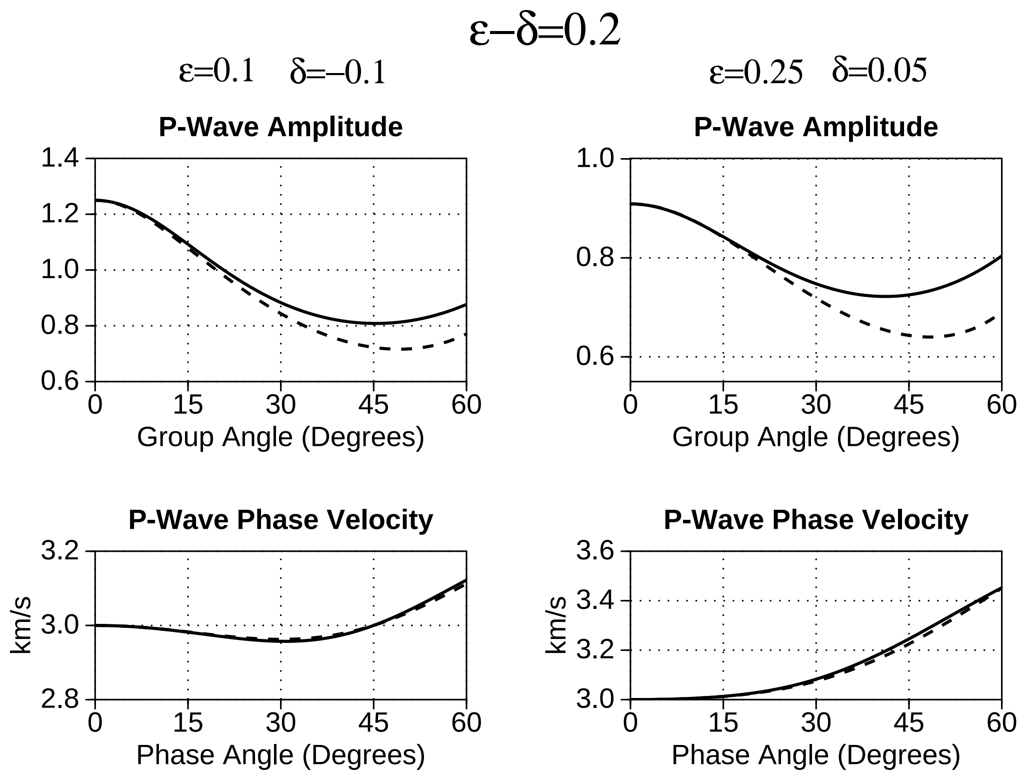

FIG. 11. Normalized P-wave amplitude from a vertical force for models with $\epsilon - \delta = 0.2$.

and of small velocity and density contrasts across the reflector:

$$R(\theta) = R_{isot}(\theta) + R_{anis}(\theta)\,, \tag{45}$$

where $R_{isot}(\theta)$ is the reflection coefficient in the absence of anisotropy ($\epsilon = 0$, $\delta = 0$), and

$$R_{anis}(\theta) = \frac{1}{2}(\delta_2 - \delta_1)\sin^2\theta + \frac{1}{2}(\delta_1 - \delta_2 + \epsilon_2 - \epsilon_1)\sin^2\theta\tan^2\theta\,. \tag{46}$$

Subscripts 1 and 2 refer to the media above and below the reflector respectively. One of the convenient features of equations (45) and (46) is the separation of the "isotropic" and "anisotropic" parts of the reflection coefficient. Unlike Banik's (1987) approximation, formula (46) is not resricted to small incidence angles.

In contrast with radiation pattern, the reflection coefficient at normal incidence is not distorted by transverse isotropy, i.e., $R_{anis} = 0$ if $\epsilon = \delta = 0$. Note that the lowest-order angular correction to the reflection coefficient depends only on the change in δ across the reflector (no dependence on ϵ), while the lowest-order angular term in the radiation pattern (44) contains the difference between ϵ and δ.

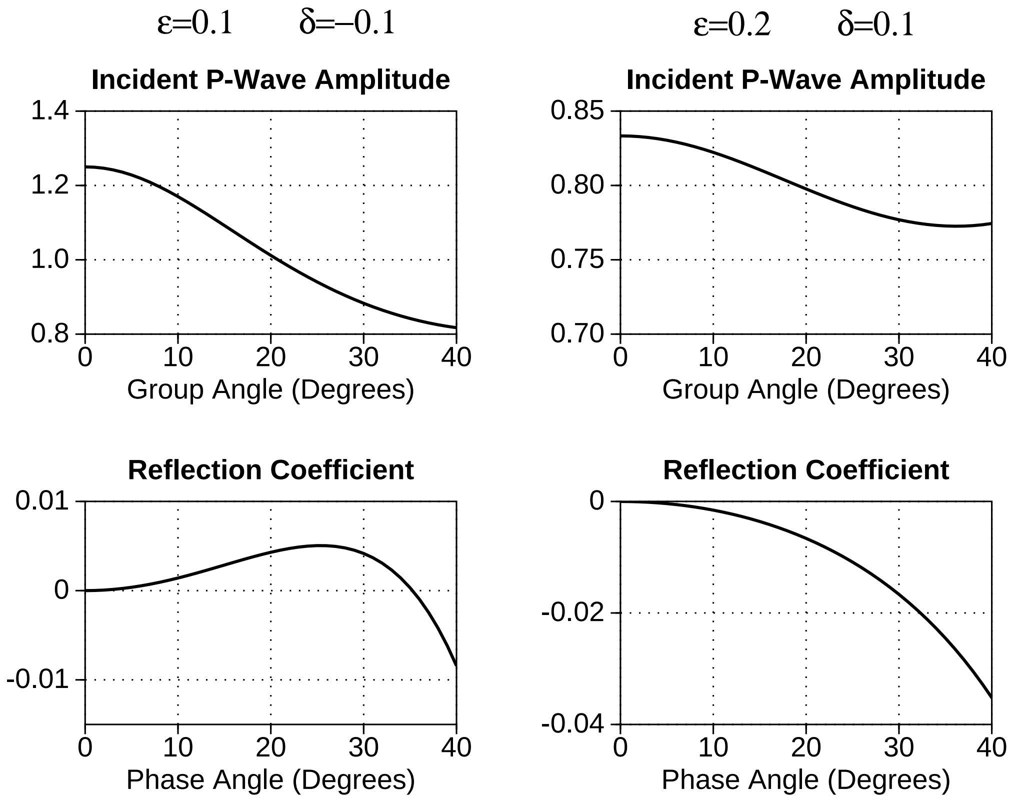

FIG. 12. Comparison of the influence of transverse isotropy on the P-wave radiation pattern from a vertical force and on the reflection coefficient.
(Top) The exact far-field amplitude of the incident wave normalized by the amplitude in the corresponding isotropic medium with $\epsilon = \delta = 0$. (Bottom) The angular variation in the reflection coefficient caused solely by the anisotropy [equation (47)]. It is assumed that the transversely isotropic models with ϵ and δ shown on the plot overlie an isotropic medium.

It should be emphasized that the influence of the S-wave verical velocity V_{S0} on the P-wave reflection coefficient is limited to the isotropic term $R_{isot}(\theta)$, at least for weak transverse isotropy. The anisotropic correction to the reflection coefficient $R_{anis}(\theta)$, derived in the weak-anisotropy approximation, is entirely independent of V_{S0}.

In AVO analysis, it is important to compare the distortions of the P-wave radiation pattern with the influence of transverse isotropy on the reflection coefficient. To make such a comparison, we assume that the medium below the reflector is isotropic (e.g., the shale/sand AVO model discussed by Kim et al., 1993). Then the anisotropic part of the reflection coefficent is determined just by ϵ and δ of the overlying medium:

$$R_{anis}(\theta) = -\frac{1}{2}\delta_1 \sin^2\theta + \frac{1}{2}(\delta_1 - \epsilon_1)\sin^2\theta \tan^2\theta \,. \tag{47}$$

For the two models shown in Figure 12, the importance of the radiation phenomena is quite different. If $\epsilon = 0.1$ and $\delta = -0.1$, the anisotropy causes a 35-percent drop in the amplitude from 0 to 40 degrees, while the anisotropy-induced angular variations in the reflection coefficient do not exceed ± 0.008. Hence, for a typical value of the isotropic part of the reflection coefficient of about 0.1, transverse isotropy would make less than a 10 percent change in the total reflection coefficient. Hence, for the model on the left ($\epsilon = 0.1$, $\delta = -0.1$) the redistribution of energy above the reflector is likely to be a more important factor in AVO than the is influence of the anisotropy on the reflection coefficient, while for the model on the right ($\epsilon = 0.2$, $\delta = 0.1$) the opposite is true.

More examples of P-wave reflection coefficients in transversely isotropic media are provided by Kim et al. (1993), who considered a shale/sand boundary under the assumption that only the shale (the medium above the reflector) is anisotropic. Obviously, it is difficult to make a general comparison between the influence of anisotropy on the propagation phenomena and the reflection coefficient. The angular variations in the reflection coefficient depend on the difference in the anisotropic parameters across the reflector, while the radiation pattern is entirely determined by the properties of the incidence medium. Also, the influence of anisotropy on the reflection coefficient depends on the impedance contrast, i.e., it is more pronounced for weak reflectors. However, it is clear that the two phenomena are often of the same order of magnitude. For "bright spots" with a large amplitude of the normal-incidence reflection and a relatively slow increase in the absolute value of the reflection coefficient with angle (Class 3 sands from Kim et al., 1993), amplitude distortions above the reflector may even reverse the sign of the AVO gradient!

DISCUSSION AND CONCLUSIONS

The description of P-wave signatures given in the paper included phase, group, and normal-moveout velocities, nonhyperbolic moveout, polarizations, radiation patterns, and reflection coefficients. The analysis was based on a series of analytic solutions valid for transversely isotropic media with arbitrary strength of anisotropy (the only exception is the reflection coefficient that was studied in the weak-anisotropy approximation). Although these solutions are convenient to implement numerically, some of them are not simple enough to elucidate the relation between various seismic phenomena and the parameters of the anisotropic velocity field. In order to gain insight into the influence of the anisotropic parameters on seismic signatures, I have systematically applied the weak-anisotropy approximation and checked its accuracy by comparing with the exact results.

A proper choice of parametrization was critically important to this study. Here, I have used the notation of transversely isotropic media developed by Thomsen (1986). Thomsen has demonstrated the following advantages of this notation in his original paper:

1. The anisotropic coefficients ϵ, δ, and γ are dimensionless and vanish for isotropic media. Therefore, they are easy to use in estimating the strength of ve-

locity anisotropy and in developing the weak-anisotropy approximation for velocities and polarizations.

2. The condition satisfied by elliptically anisotropic models is extremely simple: for elliptical anisotropy $\epsilon = \delta$.

3. The parameters ϵ and δ are responsible for P-wave velocity in different ranges of propagation angles. The coefficient δ determines the near-vertical velocity variations, which are of most importance in reflection seismology.

4. The expressions for normal moveout velocities for a horizontal reflector are concise and symmetric for all wave types.

It should be emphasized that all these advantages are valid for arbitrary strength of the anisotropy. However, the main misconception about Thomsen parameters that still persists in the literature is that this notation is useful only for weak anisotropy. Here, I have applied Thomsen parameters in a number of different wave-propagation problems and have shown that they are much more convenient to use than the standard notation (stiffness coefficients c_{ij}), irrespective of the degree of the anisotropy. Some of the results of this paper, which can serve as additional arguments in favor of Thomsen notation, can be summarized as follows:

1. It is possible to cut down on the number of independent parameters needed to describe P-wave propagation because the shear-wave vertical velocity V_{S0} has a weak (usually negligible) influence on P-wave signatures, even in media with strong velocity variations. Although the P-wave reflection coefficient does depend on the jump in V_{S0} across the interface, the contribution of transverse isotropy to the reflection coefficient is practically independent of V_{S0}. Therefore, the influence of transverse isotropy on P-wave propagation is controlled just by the P-wave vertical velocity V_{P0} and two anisotropies, ϵ and δ, with V_{P0} being no more than a scaling coefficient, for homogeneous media. This is extremely important in facilitating our understanding of anisotropic wave propagation and in implementing inversion and processing algorithms in transversely isotropic media. In fact, traveltime inversion of P-wave data using the conventional notation is always ambiguous because the trade-off between the parameters c_{13} and c_{44} can never be resolved, unless some independent information about one of these coefficients is available. The explanation for this ambiguity is simple: c_{13} and c_{44} influence P-wave phase and group velocity only through their combination δ.

2. Not just NMO velocity (as shown in Thomsen's (1986) paper), but also the phase velocity (a fundamentally important function), the quartic moveout coefficient, and nonhyperbolic P-wave moveout for horizontal reflectors can be concisely expressed through the anisotropies ϵ and δ. These analytic developments are valid for arbitrary strength of the anisotropy.

3. The influence of transverse isotropy on all P-wave signatures considered here is much easier to understand if Thomsen parameters are used, because we have to deal just with two dimensionless anisotropies ϵ and δ.

4. Thomsen notation is ideally suitable for developing the weak-anisotropy approximation of different seismic signatures. By systematically applying the weak-anisotropy approximation, I have shown that it provides valuable analytic insight into the dependence of P-wave velocities, polarizations, and amplitudes on the parameters of transverse isotropy.

The results for a wide range of seismic phenomena show that there is no apparent correlation between the strength of P-wave velocity anisotropy and the influence of transverse isotropy on reflection moveouts and amplitudes. The magnitude of the P-wave velocity variations is usually determined by the value of ϵ (unless $\epsilon \ll \delta$), while signatures used in reflection seismology depend either on δ or on a certain combination of ϵ and δ. Therefore, the terms "weak anisotropy" or "strong anisotropy" are meaningless without a reference to a particular problem. For instance, while the model with $\epsilon = 0.1$, $\delta = -0.1$ is weakly anisotropic in terms of velocity variations, it can be characterized as strongly anisotropic regarding the distortions of the P-wave radiation pattern.

P-wave signatures of most interest in reflection seismology can be divided into two main groups. For the first group that includes the normal moveout velocity for horizontal reflectors, small-angle reflection coefficient, and point-source radiation in the vertical (symmetry) direction, the influence of transverse isotropy is entirely determined by the parameter δ.

The second group comprises the dip-dependence of NMO velocity, magnitude of nonhyperbolic moveout, time-migration impulse response, and the shape of the radiation pattern in the angular range 0-40 degrees. The influence of transverse isotropy on these signatures is determined by both anisotropies (ϵ and δ) and is primarily governed by the difference $\epsilon - \delta$, i.e., by deviations from the elliptically anisotropic model. Due to the high sensitivity of P-wave signatures to $\epsilon - \delta$, application of the elliptical-anisotropy approximation in P-wave processing may lead to unacceptable errors, even if the medium is relatively close to elliptical.

As shown by Alkhalifah and Tsvankin (1994), one of the most important effective parameters in seismic processing is $\eta = (\epsilon - \delta)/(1 + 2\delta)$. The parameter η and the normal moveout velocity for a horizontal reflector $V_{\text{nmo}}(0)$ are sufficient not only to obtain P-wave NMO velocity as a function of ray parameter, but also to describe long-spread (nonhyperbolic) reflection moveout for a horizontal reflector, and calculate poststack and prestack time-migration impulse responses. Alkhalifah and Tsvankin (1994) also developed an inversion procedure designed to obtain η and $V_{\text{nmo}}(0)$ from P-wave NMO velocities measured for two different dips. This inversion technique provides enough information to perform all essential time-processing steps.

However, the parameter η cannot fully describe the influence of transverse isotropy on all signatures belonging to the second group. For instance, distortions of P-wave radiation patterns are not governed by η. Also, the P-wave normal-moveout velocity as a function of the dip angle for typical $\epsilon - \delta > 0$ is almost entirely determined by $\epsilon - \delta$ rather than by η.

It should be also mentioned that time-to-depth conversion should be carried out with accurate values of all parameters, which cannot be found from P-wave NMO velocities alone. In some cases, the vertical velocity can be determined directly if check shots or well logs are available. Then, P-wave NMO velocity can be used to obtain ϵ and δ. Another source of additional information is the short-spread moveout velocities of SV or $P-SV$ waves and long-spread SV-wave moveout (Tsvankin and Thomsen, 1994b).

Although the discussion here was centered on vertical transverse isotropy, generalization of most analytic results to transversely isotropic media with a tilted in-plane symmetry axis is straightforward.

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