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3-D description of normal moveout in anisotropic inhomogeneous media

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ABSTRACT

We present a new equation for normal-moveout (NMO) velocity that describes azimuthally dependent reflection traveltimes of pure modes from both horizontal and dipping reflectors in arbitrary anisotropic inhomogeneous media. With the exception of anomalous areas such as those where common-midpoint (CMP) reflection time decreases with offset, the azimuthal variation of NMO velocity represents an ellipse in the horizontal plane, with the orientation of the axes determined by the properties of the medium and the direction of the reflector normal. In general, a minimum of three azimuthal measurements is necessary to reconstruct the best-fit ellipse and obtain NMO velocity in all azimuthal directions. This result provides a simple way to correct for the azimuthal variation in stacking velocity often observed in 3-D surveys. Even more importantly, analytic expressions for the parameters of the NMO ellipse can be used in the inversion of moveout data for the anisotropic coefficients of the medium.

For homogeneous transversely isotropic media with a vertical axis of symmetry (VTI media), our equation for azimuthally dependent NMO velocity from dipping reflectors becomes a relatively simple function of phase velocity and its derivatives. We show that the zero-dip NMO velocity $V_{\text{nmo}}(0)$ and the anisotropic coefficient η are sufficient to describe the P-wave NMO velocity for any orientation of the CMP line with respect to the dip plane of the reflector. Using our formalism, $V_{\text{nmo}}(0)$ and η (the only parameters needed for time processing) can be found from the dip-dependent NMO velocity at any azimuth or, alternatively, from the azimuthally dependent NMO for a single dipping reflector.

We also apply this theory to more complicated azimuthally anisotropic models with the orthorhombic symmetry used to describe fractured reservoirs. For reflections from horizontal interfaces in orthorhombic media, the axes of the normal moveout ellipse are aligned with the vertical symmetry planes. Therefore, azimuthal P-wave moveout measurements can be inverted for the orientation of the symmetry planes (typically determined by the fracture direction) and the NMO velocities within them. If the vertical velocity is known, symmetry-plane NMO velocities make it possible to estimate two anisotropic parameters equivalent to Thomsen's coefficient δ for transversely isotropic media.

INTRODUCTION

Normal moveout (NMO) in anisotropic media is influenced by angular velocity variations; therefore, it contains information about the parameters of the anisotropic velocity field. For instance, P-wave NMO velocity from horizontal reflectors in transversely isotropic models with a vertical symmetry axis (VTI media) depends on the Thomsen (1986) anisotropic parameter δ. The difference between NMO and vertical velocities in VTI media (for $\delta \neq 0$) leads to mis-ties in time-to-depth conversion routinely observed in many exploration areas. If the vertical velocity is known (e.g., from check shots or well logs), P-wave NMO velocity can be used to obtain the coefficient δ as a function of depth (e.g., Alkhalifah et al., 1996).

The dip dependence of NMO velocity provides additional information for seismic inversion and governs the performance of dip-moveout (DMO) algorithms in the presence of anisotropy. An exact equation for NMO velocity from both horizontal and dipping reflectors valid in symmetry planes of anisotropic media was presented by Tsvankin (1995). Applying this NMO expression to vertical transverse isotropy, Alkhalifah and Tsvankin (1995) showed that the dip-dependent P-wave NMO velocity expressed through the ray parameter p is determined by the zero-dip NMO velocity from a horizontal reflector and a single anisotropic coefficient denoted as η. They also proved that these two parameters are responsible for all P-wave time-processing steps, including NMO, DMO, and

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time migration. Both parameters can be obtained from surface P-wave data using NMO velocities measured for two different dips. This algorithm, however, works only for a common-midpoint (CMP) line confined to the dip plane of the reflector.

In azimuthally anisotropic media, typically caused by vertical or dipping fractures, NMO velocity depends on the azimuth of the CMP line, even if the reflector is horizontal. Lynn et al. (1996) and Corrigan et al. (1996) presented case studies documenting substantial azimuthal variations in NMO over fractured reservoirs. Tsvankin (1997a) gave an exact expression for NMO velocity from horizontal reflectors in transversely isotropic media with a horizontal symmetry axis (HTI media). P-wave NMO velocity in an HTI layer depends on only three parameters (vertical velocity, orientation of the symmetry axis, and anisotropic coefficient $\delta^{(V)}$) and varies elliptically as a function of azimuth in the horizontal plane. The elliptical dependence of NMO velocity in a layer with a horizontal symmetry plane was obtained by Sayers (1995a,b), who developed a representation of long-spread moveout based on an expansion of group velocity in spherical harmonics. The coefficients of the moveout expansion derived in Sayers (1995b), however, are difficult to relate to the medium parameters.

Here, we present a new NMO equation based on a more general formalism than the previous results. It provides an efficient way to obtain the exact NMO velocity for models with arbitrary anisotropy and inhomogeneity. Azimuthally dependent NMO velocity is represented by an elliptical curve in the horizontal plane and can be reconstructed using a minimum of three azimuthal moveout measurements. The new equation is used to describe normal moveout of out-of-plane reflections in VTI media and to invert P-wave NMO velocity measured on lines with different azimuthal orientation for the key anisotropic parameter η. We also obtain the parameters of the NMO ellipse for horizontal reflectors in orthorhombic media and show that the exact P-wave NMO velocity represents a simple function of two anisotropic coefficients ($\delta^{(1)}$ and $\delta^{(2)}$) introduced in Tsvankin (1997b).

3-D NMO EQUATION FOR INHOMOGENEOUS ANISOTROPIC MEDIA

Analytic formulation

Here, we give an exact analytic representation of NMO velocity in anisotropic inhomogeneous media and prove that the azimuthal dependence of V_{nmo} typically has a simple elliptical form. We consider CMP lines with different azimuthal directions over a medium with arbitrary anisotropy and inhomogeneity (Figure 1). Azimuthally dependent NMO velocity $V_{nmo}(\alpha)$ is defined in the conventional way through the initial slope of the $t^2(x^2)$ curve (t is the reflection traveltime; x is the offset) on the CMP line with azimuth α [equation (A-5)].

The derivation of the NMO equation, given in Appendix A, is based on expanding the one-way traveltime from the zero-offset reflection point to the surface in a double Taylor series in the horizontal coordinates $[x_1, x_2]$. As a result, we find the following expression for the exact NMO velocity of any pure (nonconverted) mode:

$$\frac{1}{V_{nmo}^2(\alpha)} = W_{11}\cos^2\alpha + 2W_{12}\sin\alpha\cos\alpha + W_{22}\sin^2\alpha. \tag{1}$$

Here, α is the azimuth of the CMP line and $\underset{\sim}{\mathbf{W}}$ is a symmetric matrix defined as $W_{ij} = \tau_0(\partial p_i/\partial x_j)$, where $p_i = \partial\tau/\partial x_i$ $(i = 1, 2)$ are the horizontal components of the slowness vector for rays between the zero-offset reflection point and the surface location $[x_1, x_2]$, $\tau(x_1, x_2)$ is the one-way traveltime from the zero-offset reflection point, and $\tau_0 = \tau(0, 0)$ is the one-way traveltime at the CMP location $x_1 = x_2 = 0$ (Figure 1). The derivatives needed to obtain the matrix $\underset{\sim}{\mathbf{W}}$ are evaluated at the common midpoint.

Therefore, the exact NMO velocity of any pure mode at a given spatial location is fully controlled by just three parameters—the components of the matrix $\underset{\sim}{\mathbf{W}}$. Plotting the value of NMO velocity in each azimuthal direction α yields a certain curve in the horizontal plane. To determine the type of this "NMO curve," we express equation (1) through the eigenvalues λ_1 and λ_2 of the matrix $\underset{\sim}{\mathbf{W}}$ as

$$\frac{1}{V_{nmo}^2(\alpha)} = \lambda_1\cos^2(\alpha - \beta) + \lambda_2\sin^2(\alpha - \beta), \tag{2}$$

where β is the rotation angle determined by one of the eigenvectors of $\underset{\sim}{\mathbf{W}}$ (Appendix A). Typically, the eigenvalues λ_1 and λ_2 are positive. Indeed, if $\lambda_1 < 0$ or $\lambda_2 < 0$, then V_{nmo}^2 in some azimuthal directions becomes negative as well, implying that CMP reflection traveltime decreases with offset. For positive λ_1 and λ_2, equation (2) [fully equivalent to equation (1)] describes an ellipse with the axes rotated by the angle β with respect to the original coordinate frame $[x_1, x_2]$.

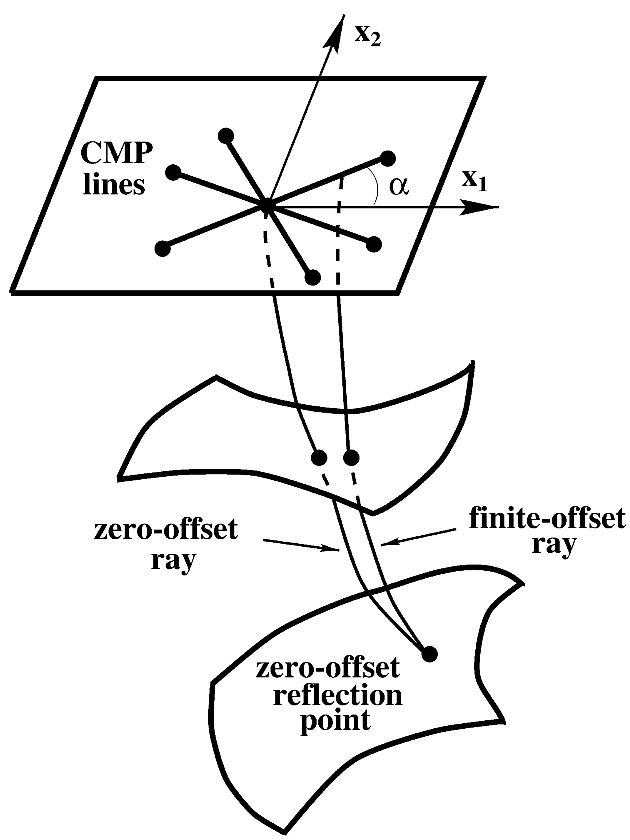

FIG. 1. CMP gathers with different azimuthal orientation around the same CMP location. The medium can be arbitrary anisotropic and inhomogeneous. Reflection point dispersal is ignored because it does not influence NMO velocity (see Appendix A).

Thus, the azimuthally dependent NMO velocity $V_{\text{nmo}}(\alpha)$ from equation (1) usually has an elliptical shape in the horizontal plane. The orientation of the ellipse and the values of its semi-major and minor axes are determined by the derivatives of the ray parameter with respect to the horizontal coordinates. For relatively simple models, these derivatives can be evaluated as functions of phase velocity using the approach suggested by Tsvankin (1995, 1997a); we apply this method to azimuthal moveout analysis in transversely isotropic media. For more complicated media with lower anisotropic symmetries and/or inhomogeneous velocity fields, it is preferable to express the parameters of the NMO ellipse through the components of the slowness vector following the formalism developed by Cohen (1998).

Because we have not made any specific assumptions about the model (see Appendix A), the NMO-velocity equation (1) is valid for any sufficiently smooth reflectors and general anisotropic inhomogeneous velocity fields. We have assumed, though, that the traveltime field exists at any azimuth near the CMP point (an assumption that breaks down in shadow zones) and can be adequately described by a Taylor series expansion for the squared traveltime $t^2(x_\alpha^2)$, where x_α is the offset on the survey line with azimuth α. While this expansion is routinely used in seismic processing, it quickly degenerates in the presence of strong lateral velocity variation. Also, analytic approximations for reflection traveltimes may break down for shear waves in anisotropic media (e.g., Tsvankin and Thomsen, 1994), especially in the vicinity of triplications (cusps) on the wavefront. For typical subsurface models, however, the Taylor series expansion provides a good approximation for the traveltime, and the quadratic term parameterized by the NMO velocity (i.e., the hyperbolic moveout approximation) usually is sufficiently accurate on conventional-length CMP spreads; this will be corroborated by numerical examples below.

For NMO velocity to represent an ellipse, the eigenvalues λ_1 and λ_2 of the matrix $\underset{\sim}{\mathbf{W}}$ should be positive. However, in some anomalous cases the reflection traveltime may decrease with offset (e.g., for turning waves, as discussed in Hale et al., 1992), which corresponds to negative values of λ_1 or λ_2. Then the squared NMO velocity in certain azimuthal directions is also negative and, clearly, V_{nmo} cannot be described by an elliptical curve. Equation (2) remains formally valid, but the hyperbolic moveout approximation may not be accurate even on relatively short spreads. If λ_1 or λ_2 equals zero, the reflection traveltime along one of elliptical axes is constant and the NMO velocity in this direction is infinite. For instance, NMO velocity goes to infinity on the dip line of a vertical reflector in homogeneous isotropic, VTI, and HTI media. For TI media with a tilted symmetry axis, infinite V_{nmo} can be recorded (also on the dip line) for steep dips either below or above 90°, depending on the anisotropic parameters of the medium (Tsvankin, 1997c). For $\lambda_1 = 0$ or $\lambda_2 = 0$, the NMO ellipse degenerates into two straight lines parallel to the direction in which $V_{\text{nmo}} = \infty$ (discussed in more detail later). Such cases, however, cannot be considered as common, and equation (2) should describe an ellipse for most models of practical interest in reflection seismology.

Examples of the NMO ellipse

The simplest example of the elliptical azimuthal dependence of NMO velocity is the well-known NMO equation given by Levin (1971) for a dipping reflector beneath a homogeneous isotropic medium:

$$V_{\text{nmo}}^2(\alpha, \phi) = \frac{V^2}{1 - \cos^2\alpha \sin^2\phi}, \tag{3}$$

where V is the velocity, ϕ is the reflector dip, and α is the azimuth of the CMP line with respect to the dip direction. To demonstrate that equation (3) represents an ellipse, it can be rewritten as

$$\frac{1}{V_{\text{nmo}}^2(\alpha, \phi)} = \frac{\sin^2\alpha}{V^2} + \frac{\cos^2\alpha}{(V/\cos\phi)^2}. \tag{4}$$

Clearly, the elliptical axes are parallel to the dip and strike directions, with the semi-axes given by V (strike line) and $V/\cos\phi$ (dip line). Equation (4) can be easily obtained as a special case of the general NMO expression (1).

Another example, this time for a much more complicated azimuthally anisotropic model, is shown in Figure 2. Here, the medium consists of two orthorhombic layers between interfaces with different dips and azimuths. For this model, the

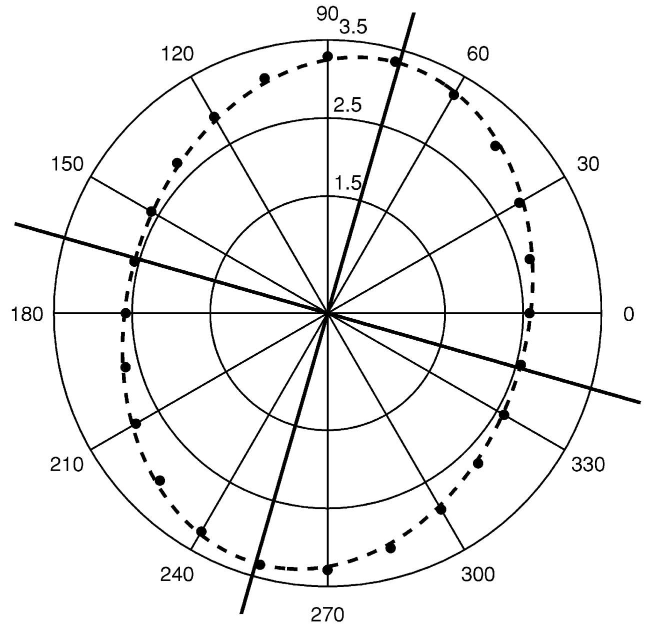

FIG. 2. Azimuthal dependence of moveout velocity (in km/s) for the P-wave reflected from a dipping interface beneath two orthorhombic layers. The dots represent the moveout (stacking) velocity calculated at each azimuth by least-squares fitting of a hyperbola to the exact traveltimes on spread length equal to the distance between the CMP and the reflector. [Traveltimes were computed using a ray-tracing code of Obolentseva and Grechka (1989).] The dashed curve is the best-fit ellipse that approximates the azimuthal dependence of the moveout velocity. The parameters, given in the notation introduced by Tsvankin (1997b), are discussed in the text. Upper layer: $V_{P0} = 2.0$ km/s, $\epsilon^{(1)} = 0.10$, $\epsilon^{(2)} = 0.15$, $\delta^{(1)} = 0.15$, $\delta^{(2)} = -0.05$, $\delta^{(3)} = 0$. Lower layer: $V_{P0} = 3.0$ km/s, $\epsilon^{(1)} = 0.12$, $\epsilon^{(2)} = 0.10$, $\delta^{(1)} = 0.10$, $\delta^{(2)} = -0.07$, $\delta^{(3)} = 0$. Both layers have a horizontal symmetry plane; the azimuth (shown around the circle) is measured from the $[x_1, x_3]$ symmetry plane, which has the same orientation in both layers. The azimuth of the dip plane of the reflector is $\alpha_2 = 90°$, the dip $\phi_2 = 30°$; for the intermediate interface, $\alpha_1 = 60°$ and $\phi_1 = 40°$.

orientation of the NMO ellipse is determined by both the symmetry of the medium and the geometry of the interfaces. The dots in Figure 2 represent the P-wave moveout (stacking) velocity computed from the exact traveltimes for spread length equal to the reflector depth. Due to the influence of nonhyperbolic moveout, the finite-spread moveout velocity may be somewhat different from the analytic zero-spread NMO velocity described by equations (1) and (2). However, even in this complex model, the measured moveout velocity is close to the best-fit ellipse (dashed curve) for the full range of azimuths (the maximum difference does not exceed 1.8%). The elliptical semi-major axis ($\alpha = 74^\circ$) deviates from the dip plane of the reflector because of the influence of both azimuthal anisotropy and the orientation of the intermediate interface.

DIPPING REFLECTORS IN VTI MEDIA

Description of normal moveout

The NMO equation (1) can be used to give an analytic description of the NMO velocity from both horizontal and dipping reflectors in anisotropic media with any symmetry. Here, we obtain dip-dependent NMO velocity as a function of azimuth for the simplest anisotropic model: transverse isotropy with a vertical symmetry axis. Let us consider a dipping reflector beneath a horizontally homogeneous (but maybe vertically inhomogeneous) VTI medium. NMO velocity on the dip line is given by Tsvankin (1995) for homogeneous models and by Alkhalifah and Tsvankin (1995) for vertically inhomogeneous media above the reflector. Here, we present an exact NMO equation valid for arbitrary orientation of the CMP line with respect to the reflector strike.

Because of the axial symmetry of the overburden, the model under consideration has a vertical symmetry plane that coincides with the dip plane of the reflector. (In the special case of a horizontal reflector, the properties of all vertical planes are identical, and NMO velocity is azimuthally independent.) If we align one of the coordinate axes (x_1 or x_2) with the dip plane, the NMO equation (1) yields an ellipse without any rotation because in this case $W_{12} = \partial^2\tau/\partial x_1 \partial x_2$ goes to zero. Therefore, one of the axes of the NMO ellipse for any model with a vertical symmetry plane is always aligned with the symmetry-plane direction.

Clearly, for the VTI medium considered here, the second axis of the ellipse should be parallel to the strike line of the reflector. We will measure the azimuth α from the dip plane and denote the semi-axes of the NMO ellipse as $V_{\rm nmo}(\alpha = 0, \phi)$ (dip line) and $V_{\rm nmo}(\alpha = \pi/2, \phi)$ (strike line). Then the NMO equation (1) becomes

$$V_{\rm nmo}^{-2}(\alpha, \phi) = V_{\rm nmo}^{-2}(0, \phi)\,\cos^2\alpha + V_{\rm nmo}^{-2}\left(\frac{\pi}{2}, \phi\right)\,\sin^2\alpha. \tag{5}$$

Equation (5) is a remarkably simple result that allows us to obtain the exact azimuthally dependent NMO velocity from a dipping reflector overlain by a vertically inhomogeneous VTI medium.

We now restrict ourselves to a single homogeneous VTI layer above the reflector; vertically inhomogeneous models will be discussed in a sequel paper. The dip-line NMO velocity of any pure mode is derived by Tsvankin (1995) as the following function of phase velocity V in the dip plane and reflector dip ϕ:

$$V_{\rm nmo}(0, \phi) = \frac{V(\phi)}{\cos\phi}\,\frac{\sqrt{1 + \dfrac{1}{V(\phi)}\dfrac{d^2V}{d\theta^2}\Big|_{\theta=\phi}}}{1 - \dfrac{\tan\phi}{V(\phi)}\dfrac{dV}{d\theta}\Big|_{\theta=\phi}}, \tag{6}$$

where θ is the angle between the phase-velocity (or slowness) vector and vertical. Equation (6) is valid not only for vertical transverse isotropy but also in vertical symmetry planes of any other homogeneous anisotropic models.

To construct the NMO ellipse [equation (5)], we also need the NMO velocity on the line parallel to the strike of the reflector. Using the approach suggested by Tsvankin (1995, 1997a), we derived the following analytic expression for the strike-line NMO velocity (Appendix B):

$$V_{\rm nmo}\left(\frac{\pi}{2}, \phi\right) = V(\phi)\sqrt{1 + \frac{1}{V(\phi)\tan\phi}\frac{dV}{d\theta}\Big|_{\theta=\phi}}. \tag{7}$$

Since the rays recorded on the strike line deviate from the vertical incidence plane, the derivation of equation (7) was based on the specific form of the phase-velocity function in the axially symmetric VTI model. Therefore, in contrast to the more general dip-plane NMO formula (6), equation (7) is limited to vertical transverse isotropy. Equations (5)–(7) are sufficient to obtain NMO velocity in homogeneous VTI media for all three pure modes (P, SV, SH).

If the medium is isotropic, the derivatives of phase velocity vanish, and the NMO velocities in dip and strike directions [equations (6) and (7)] reduce to the expressions given in Levin (1971) [equation (4)]

$$V_{\rm nmo}(0, \phi) = \frac{V(\phi)}{\cos\phi} = \frac{V}{\cos\phi}$$

and

$$V_{\rm nmo}\left(\frac{\pi}{2}, \phi\right) = V(\phi) = V.$$

Elliptical and weak anisotropy

To gain insight into the dependence of NMO velocity on the anisotropic parameters, we examine the special cases of elliptical and weak anisotropy. The VTI model will be described by the vertical velocities of P- and S-waves (V_{P0} and V_{S0}, respectively) and Thomsen's (1986) anisotropy parameters ϵ, δ, and γ. (For a detailed overview of notation, see Tsvankin, 1996.) In elliptically anisotropic media ($\epsilon = \delta$), the P-wave slowness surface and wavefront have an elliptical shape, while SV-wave velocity is fully independent of propagation angle (i.e., the slowness surface is spherical). For SH-waves in transversely isotropic media, anisotropy is always elliptical, with the magnitude of velocity variations determined by the parameter γ.

The dip-line NMO velocity in elliptical media, first derived by Byun (1982), can be written as (Tsvankin, 1995)

$$V_{\rm nmo}(0, \phi) = \frac{V_{\rm nmo}(0)}{\cos\phi}\frac{V(\phi)}{V_0}, \tag{8}$$

where $V_{\rm nmo}(0) \equiv V_{\rm nmo}(\alpha, 0)$ is the azimuthally independent (for this model) NMO velocity from a horizontal reflector and V_0

is the vertical velocity. Hence, deviations from the isotropic cosine-of-dip dependence of NMO velocity [equation (4)] in elliptical media are tied directly to the angular phase-velocity variations.

Substitution of the phase-velocity function for elliptical anisotropy (e.g., Thomsen, 1986) into the strike-line NMO equation (7) yields

$$V_{\text{nmo}}\left(\frac{\pi}{2}, \phi\right) = V_{\text{nmo}}(0) = \text{const} \tag{9}$$

for any dip ϕ. Therefore, NMO velocity in the strike direction is equal to the zero-dip NMO velocity, as in isotropic media, but does not coincide with the true vertical velocity. In VTI media, equations (8) and (9) are always valid for the SH-wave only; for P- and SV-waves, elliptical anisotropy is regarded as rather atypical (Thomsen, 1986).

For general (nonelliptical) VTI media, a convenient way to study the dependence of NMO velocity on the anisotropy parameters is to use the weak anisotropy approximation ($|\epsilon, \delta, \gamma| \ll 1$). For P-waves, the dip-line NMO velocity [equation (6)], linearized in the anisotropic parameters, is given by Tsvankin (1995):

$$V_{\text{nmo}}(0, \phi) = \frac{V_{\text{nmo}}(0)}{\cos\phi}[1 + \delta \sin^2\phi + 3(\epsilon - \delta)\sin^2\phi(2 - \sin^2\phi)]. \tag{10}$$

Analysis of the trigonometric factors in equation (10) shows that $V_{\text{nmo}}(0, \phi)$ is mostly governed by the difference between parameters ϵ and δ. This conclusion turns out to be even more accurate for the NMO velocity measured in the strike direction. In weakly anisotropic VTI media, $V_{\text{nmo}}(\pi/2, \phi)$ [equation (7)] for P-waves contains parameters ϵ and δ only in the form of the difference $\epsilon - \delta$:

$$V_{\text{nmo}}\left(\frac{\pi}{2}, \phi\right) = V_{\text{nmo}}(0)[1 + (\epsilon - \delta)\sin^2\phi(2 - \sin^2\phi)]. \tag{11}$$

For vertical transverse isotropy with typical positive values of $\epsilon - \delta$, P-wave NMO velocity on the dip line grows with ϕ more rapidly than the isotropic cosine-of-dip dependence [see equation (10) and Tsvankin, 1995]. Likewise, the strike-line NMO velocity increases with dip if $\epsilon - \delta > 0$ [equation (11)], rather than being constant as in isotropic (and any elliptically anisotropic) media. However, since the term that contains $\epsilon - \delta$ is three times smaller in the strike-line expression, the NMO velocity in the strike direction is less sensitive to the anisotropy.

Concise weak anisotropy approximations represent just one of the advantages of Thomsen notation (Tsvankin, 1996). We will use Thomsen parameters to describe VTI media with arbitrary strength of the anisotropy.

P-wave NMO velocity as a function of ray parameter

For reflection data processing, NMO velocity should be represented as a function of the ray parameter p (horizontal slowness) corresponding to the zero-offset reflection. While it is impossible to find the reflector dip ϕ without knowledge of the velocity field, the ray parameter is directly the slope of reflections on zero-offset (or stacked) sections. In general, all kinematic signatures of P-waves in VTI media depend on three parameters: the vertical P-wave velocity V_{P0} and the coefficients ϵ and δ, with the influence of the vertical shear-wave velocity V_{S0} being practically negligible (Tsvankin and Thomsen, 1994; Tsvankin, 1996). Furthermore, Alkhalifah and Tsvankin (1995) prove that the dip-line P-wave NMO velocity given by equation (6), expressed as a function of p, is determined by just two parameters: the zero-dip NMO velocity,

$$V_{\text{nmo}}(0) = V_{P0}\sqrt{1 + 2\delta}, \tag{12}$$

and the anellipticity coefficient η,

$$\eta \equiv \frac{\epsilon - \delta}{1 + 2\delta}. \tag{13}$$

For elliptically anisotropic media, $\eta = 0$ ($\epsilon = \delta$) and the dependence of the dip-line NMO velocity on the ray parameter has exactly the same form as in isotropic media:

$$V_{\text{nmo}}(0, p) = \frac{V_{\text{nmo}}(0)}{\sqrt{1 - p^2 V_{\text{nmo}}^2(0)}}. \tag{14}$$

Our numerical analysis of equation (7) shows that the strike-line NMO velocity is controlled by $V_{\text{nmo}}(0)$ and η as well (Figure 3). This conclusion is also suggested by the form of the

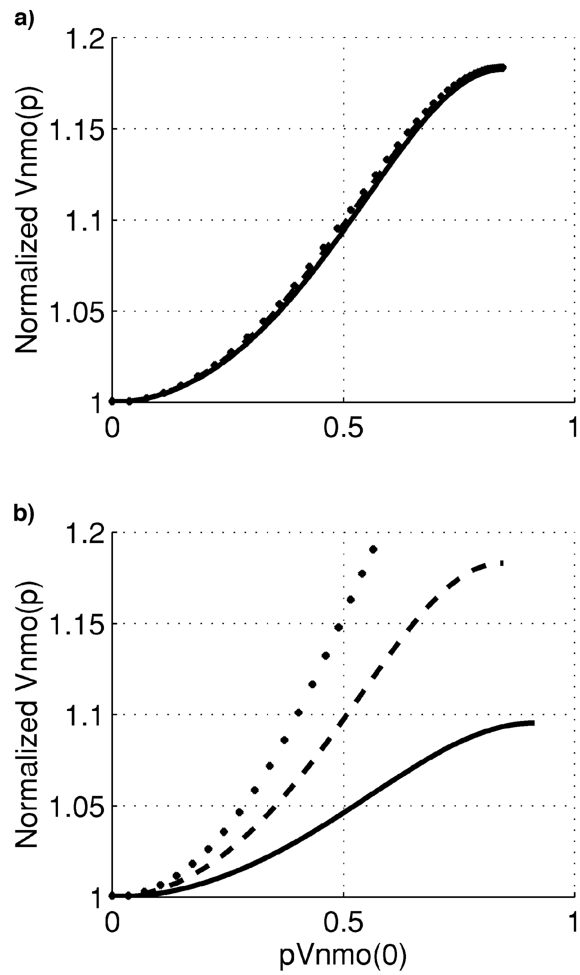

FIG. 3. The strike-line P-wave NMO velocity $V_{\text{nmo}}(\pi/2, p)$ from equation (7) (normalized by the zero-dip NMO velocity) as a function of the ray parameter p for dips ranging from 0° to 90°. (a) Different models with the same $\eta = 0.2$: $\epsilon = 0.1$, $\delta = -0.071$ (solid); $\epsilon = 0.2$, $\delta = 0.0$ (dashed); $\epsilon = 0.3$, $\delta = 0.071$ (dotted). (b) Models with different η: $\eta = 0.1$ (solid); $\eta = 0.2$ (dashed); $\eta = 0.3$ (dotted).

weak anisotropy approximation [equation (11)]. In elliptical media ($\eta = 0$), the NMO velocity on the strike line is independent of dip and equal to $V_{\rm nmo}(0)$ [equation (9)]. Because the NMO ellipse is fully governed by the NMO velocities in the strike and dip directions, parameters $V_{\rm nmo}(0)$ and η are sufficient to determine the P-wave NMO velocity $V_{\rm nmo}(\alpha, p)$ for any mutual orientation of the CMP line and reflector strike.

While the coefficient η is responsible for anisotropy-induced distortions of the NMO ellipse in all azimuthal directions, the influence of η decreases away from the dip plane. For instance, comparison of Figure 3 with the results of Alkhalifah and Tsvankin (1995) shows that the sensitivity of NMO velocity to η is considerably higher on the dip line than on the strike line.

Inversion of azimuthally varying *P*-wave NMO velocity

Although the NMO velocity was derived in the zero-spread limit, it accurately describes P-wave moveout for conventional spread lengths equal to (or even somewhat larger than) the distance between the CMP and the reflector. The example in Figure 4 shows that the "zero-spread" NMO velocity in VTI media is practically indistinguishable from the moveout velocity calculated on a typical finite spread. (Also note the high accuracy of the weak anisotropy approximation for the model from Figure 4 that has moderate values of ϵ, δ, and η.) Hence, $V_{\rm nmo}(\alpha, p)$ can be found via commonly used hyperbolic semblance velocity analysis and inverted for the parameter η. The importance of this inversion cannot be overestimated because,

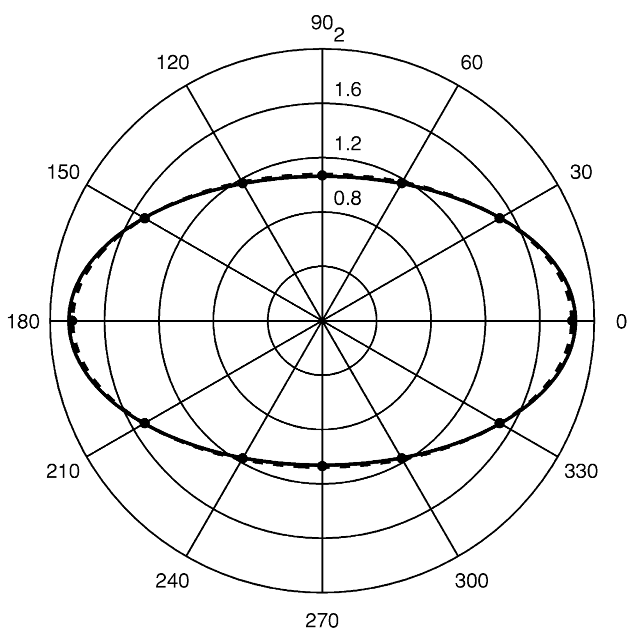

FIG. 4. Azimuthally varying P-wave moveout velocity in a VTI model with $V_{\rm nmo}(0) = 0.926$ km/s and $\eta = 0.2$ ($V_{P0} = 1.0$ km/s, $\epsilon = 0.1$, $\delta = -0.071$) for the reflector dip $\phi = 45°$. The azimuthal angle α is measured with respect to the dip plane of the reflector. The dots represent the moveout (stacking) velocity obtained by fitting a hyperbola to the exact traveltimes computed on spread length equal to the CMP-reflector distance; the solid curve is the ellipse calculated using the exact NMO equations (5)–(7); the dashed curve is the NMO ellipse in the weak anisotropy approximation [equations (10) and (11)].

in combination with $V_{\rm nmo}(0)$, η is sufficient to perform all time-processing steps (NMO, DMO, time migration) in VTI media.

Alkhalifah and Tsvankin (1995) suggest obtaining $V_{\rm nmo}(0)$ and η from equation (6) using NMO velocities measured for two different dips (e.g., a horizontal and a dipping reflector) in the dip plane of the reflector(s). Equation (5) can be used to extend this algorithm to arbitrary azimuthal direction of the CMP line; alternatively, $V_{\rm nmo}(0)$ and η can be found from the azimuthal dependence of NMO velocity for a single dipping reflector.

Estimation of η on a single CMP line.—Let us discuss the inversion for η using the NMO velocity measured on a CMP line with arbitrary azimuthal orientation. The results of the previous sections [i.e., equation (5) and the analysis of the dip- and strike-line NMO velocities] imply that the P-wave NMO velocity from a dipping reflector is a function of four parameters:

$$V_{\rm nmo} = f(\alpha, p, V_{\rm nmo}(0), \eta). \tag{15}$$

Suppose the angle α between the CMP line and the dip plane of a certain reflector is known. Then we can find the ray parameter (horizontal slowness) p of the zero-offset reflection ray by using zero-offset reflection traveltimes along this line. Since the zero-offset reflection slope is equal to the projection of the ray parameter on the CMP line, p can be obtained as

$$p = \frac{1}{2\cos\alpha}\frac{dt_0(y)}{dy}, \tag{16}$$

where t_0 is the two-way zero-offset traveltime on the line with azimuth α and y is the CMP coordinate. If the zero-dip value $V_{\rm nmo}(0)$ has been determined using a horizontal event, NMO velocity from a single dipping reflector is sufficient to obtain the parameter η, which remains the only argument in the NMO function (15).

To carry out the inversion for η, we must be able to calculate the azimuthally dependent NMO velocity as a function of the ray parameter p. The semi-axes of the NMO ellipse [equations (6) and (7)], however, are expressed through the reflector dip ϕ. Therefore, as the first step in our NMO-velocity computation, we obtain the dip ϕ as a function of the ray parameter p using phase-velocity equations for transverse isotropy (Alkhalifah and Tsvankin, 1995) and substitute ϕ into equations (6) and (7). Alternatively, the semi-axes of the NMO ellipse can be computed directly through the slowness components using the results of Cohen (1998) and equation (B-18) (see Appendix B). Then the NMO velocity as a function of p can be found for arbitrary azimuthal direction from equation (5).

The inversion procedure based on the traveltime measurements in a single azimuthal direction is illustrated by the synthetic example in Figure 5. The exact NMO equation (5) was inverted for η using α, p, $V_{\rm nmo}(0)$, and $V_{\rm nmo}(\alpha, p)$ as the input parameters. Because the P-wave phase velocity (or slowness), needed to calculate NMO velocity, is formally a function of four parameters [we used $V_{\rm nmo}(0)$, η, V_{S0}, and δ], it is strictly necessary to specify two more parameters—V_{S0} and δ—for computation purposes. However, in agreement with our earlier conclusion that NMO velocity depends only on $V_{\rm nmo}(0)$ and η, use of intentionally incorrect values of V_{S0} and δ in the inversion did not prevent our algorithm from recovering the exact value of η (for error-free input parameters). The main purpose of the example in Figure 5 was to study the stability of η estimation by introducing errors in the three parameters [p, $V_{nmo}(0)$, and $V_{nmo}(\alpha, p)$] that should be obtained from reflection traveltimes. In Figure 5a, we held $V_{nmo}(\alpha, p)$ at the correct value and examined the influence of errors in p and $V_{nmo}(0)$, while in Figure 5b the ray parameter p was assumed to be exact. Figure 5 that indicates the accuracy of the inversion for η in the presence of realistic errors in p, $V_{nmo}(0)$, and $V_{nmo}(\alpha, p)$ is quite sufficient. For instance, if the error in $V_{nmo}(0)$ does not exceed $\pm 2.5\%$ (a reasonable value for semblance velocity analysis) and the other parameters are exact, the maximum error in η is just ± 0.03. (Given that $\eta \ll 1$, we cannot expect small relative errors in this parameter.)

Our results, obtained on the line at azimuth 30° from the dip direction, are similar to the error estimates made by Alkhalifah and Tsvankin (1995) for the dip plane of the reflector. The stability of the inversion procedure decreases, however, as the CMP line nears the strike direction because the NMO velocity becomes less sensitive to η.

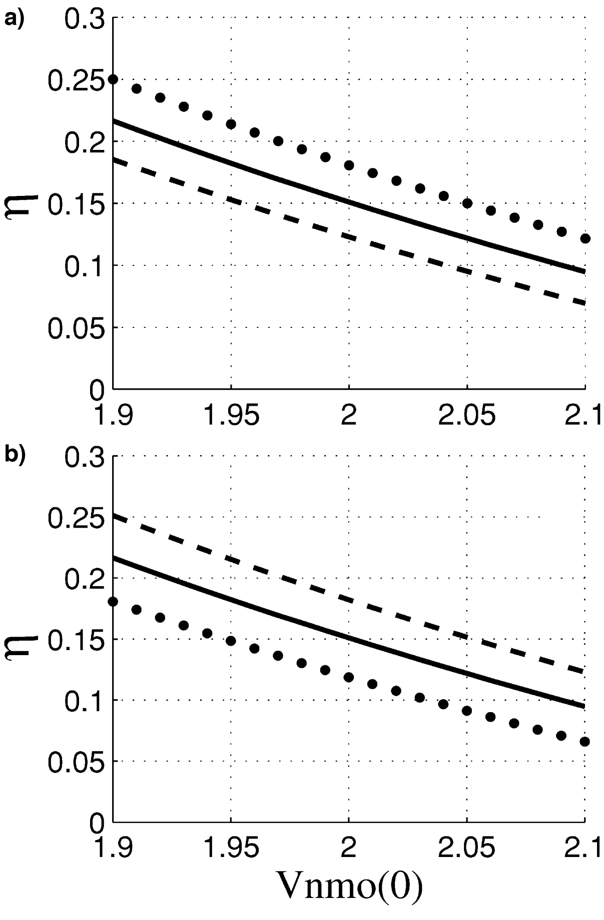

FIG. 5. Stability of η estimation using the P-wave NMO velocity measured on the CMP line that deviates by $\alpha = 30°$ from the dip direction. The actual parameters are $V_{nmo}(0) = 2.0$ km/s, $\eta = 0.15$ ($V_{S0} = 1.2$ km/s, $\delta = 0.0$), $p = 0.35$ s/km, $V_{nmo}(\alpha, p) = 3.24$ km/s. For the inversion, we intentionally used the incorrect values $V_{S0} = 0.8$ km/s and $\delta = 0.2$. (a) $V_{nmo}(\alpha, p) = 3.24$ km/s (correct value) and $p = 0.33$ s/km (dotted line), $p = 0.35$ s/km (solid), $p = 0.37$ s/km (dashed). (b) $p = 0.35$ s/km (correct value) and $V_{nmo}(\alpha, p) = 3.08$ s/km (dotted), $V_{nmo}(\alpha, p) = 3.24$ s/km (solid), $V_{nmo}(\alpha, p) = 3.40$ s/km (dashed).

Inversion using multiple azimuths.—In general, when the orientation of the CMP line with respect to the dip plane is unknown, we need lines with three different azimuthal directions to reconstruct the NMO ellipse from moveout measurements. However, the minimum number of azimuths can be reduced from three to two by using, in addition to NMO, zero-offset reflection traveltimes. Indeed, it is clear from equation (16) that the zero-offset reflection slopes for two different azimuths can be used to find the ray parameter p and the dip direction of the reflector (i.e., the azimuth of one of the elliptical axes). With the orientation of the NMO ellipse obtained from zero-offset reflection traveltimes, the NMO velocities on the same two lines are sufficient to uniquely determine the elliptical semi-axes. Then the dip-line and strike-line NMO velocities for a single dipping event can be inverted for key parameters $V_{nmo}(0)$ and η using equations (6) and (7). If $V_{nmo}(0)$ has already been found using horizontal events, independent estimates of η can be obtained from the NMO velocities in both dip and strike directions, which provides useful redundancy in the inversion.

Clearly, it is preferable to include as many different azimuths as possible to enhance stability in reconstructing the NMO ellipse. The inversion of four azimuthal moveout measurements from a single dipping reflector in the VTI model of Dog Creek Shale is shown in Figure 6. We have found the azimuth of the larger semi-axis of the NMO ellipse ($\alpha_0 = 19°$; the actual value is 20°) and the semi-axes themselves by fitting an ellipse to the

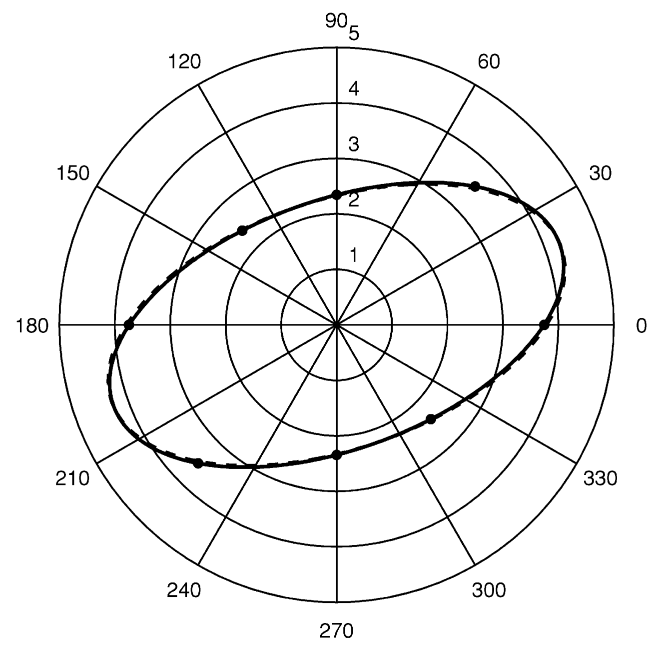

FIG. 6. Reconstruction of the P-wave NMO ellipse from the exact reflection traveltimes (obtained by anisotropic ray tracing) in the VTI model of Dog Creek Shale. The azimuth of the dip plane of the reflector is $\alpha_0 = 20°$, and dip $\phi = 50°$. The dots represent the moveout (stacking) velocity calculated at four azimuths (0°, 45°, 90°, 135°) by least-squares fitting of a hyperbola to the exact traveltimes on spead length equal to the CMP-reflector distance. The dashed curve is the best-fit ellipse that approximates the four values of moveout velocity; the solid curve is the theoretical NMO ellipse for this model. The model parameters are $V_{nmo}(0) = 2.054$ km/s and $\eta = 0.104$ ($V_{P0} = 1.875$ km/s, $\epsilon = 0.225$, $\delta = 0.100$). The semi-axes of the best-fit ellipse are 4.280 km/s (the theoretical value is 4.259 km/s) and 2.243 km/s (the theoretical value is 2.238 km/s).

moveout velocities obtained from the exact traveltimes. Then the inversion of dip-line and strike-line NMO velocities [equations (6) and (7)] using the simplex method yielded the values $V_{\rm nmo}(0) = 2.065$ km/s (instead of the actual value 2.054 km/s) and $\eta = 0.100$ (instead of 0.104). The small errors in both parameters result from the influence of nonhyperbolic moveout on the input values of finite-spread moveout velocity.

Inversion for steep dips.—The inversion method of Alkhalifah and Tsvankin (1995) is limited to the dip plane of the reflector, so it experiences difficulties in handling steeply dipping reflectors (e.g., flanks of salt domes) because of the small magnitude of their reflection moveout in the dip plane. Although the dip-line velocity $V_{\rm nmo}(0, \phi)$ for steep dips remains sensitive to η, the determination of the NMO velocity itself from the reflection traveltimes becomes less stable. Our 3-D algorithm can overcome this problem by using the NMO velocity in the direction parallel to the reflector strike. Indeed, for a vertical reflector in a homogeneous VTI medium, the dip-line NMO velocity [equation (6)] is infinite and the NMO equation (5) reduces to

$$V_{\rm nmo}\left(\alpha, \frac{\pi}{2}\right) = \frac{V_{\rm nmo}\left(\frac{\pi}{2}, \frac{\pi}{2}\right)}{\sin\alpha}. \qquad (17)$$

According to equation (17), the NMO ellipse degenerates into two straight lines parallel to the dip plane of the reflector. The strike-line NMO velocity (7) for a vertical reflector becomes equal to the horizontal velocity:

$$V_{\rm nmo}\left(\frac{\pi}{2}, \frac{\pi}{2}\right) = V\left(\frac{\pi}{2}\right). \qquad (18)$$

For the P-wave,

$$V_{\rm nmo}\left(\frac{\pi}{2}, \frac{\pi}{2}\right) = V_{P0}\sqrt{1+2\epsilon} = V_{\rm nmo}(0)\sqrt{1+2\eta}. \qquad (19)$$

Therefore, the strike-line NMO velocity $V_{\rm nmo}(\pi/2, \pi/2)$ from a vertical reflector makes it possible to obtain η, provided $V_{\rm nmo}(0)$ has been determined from horizontal events. In general, equation (17) allows us to find $V_{\rm nmo}(\pi/2, \pi/2)$ and η from the NMO velocity measured at any known azimuth (except for the dip line), but the accuracy of the moveout-velocity estimation decreases away from the strike direction.

HORIZONTAL REFLECTORS IN ORTHORHOMBIC MEDIA

Description of the orthorhombic model

Vertical transverse isotropy, considered in the previous section, is an azimuthally isotropic model in which the orientation of the NMO ellipse is determined fully by the geometry of the reflector. In contrast, seismic velocities in orthorhombic media are azimuthally dependent, and the symmetry of the medium has a direct influence on the direction of the elliptical axes.

The orthorhombic (orthotropic) symmetry system describes several models typical for fractured reservoirs, including those containing a system of parallel vertical cracks in a VTI background (Figure 7), as well as two orthogonal crack systems. Media with orthorhombic symmetry have three mutually orthogonal planes of mirror symmetry; for the model with a single crack system shown in Figure 7, the vertical symmetry planes are defined by the directions parallel and normal to the cracks. The simplest azimuthally anisotropic model—transverse isotropy with a horizontal axis of symmetry—can be considered a special type of orthorhombic media.

The velocities and polarizations in the symmetry planes of orthorhombic media are given by the same equations as for vertical transverse isotropy. (Body-wave amplitudes in the symmetry planes, however, are influenced by the azimuthal velocity variations and require a special treatment.) Tsvankin (1997b) takes advantage of the limited equivalence between orthorhombic and VTI media to introduce dimensionless anisotropic parameters similar to the well-known Thomsen's (1986) coefficients ϵ, δ, and γ for vertical transverse isotropy. He shows that all kinematic signatures of P-waves in orthorhombic models, both within and outside of the symmetry planes, are determined by the vertical velocity (a scaling coefficient in homogeneous media) and five new anisotropy parameters, as compared to nine stiffnesses in the conventional notation. The parameters responsible for P-wave kinematics (here, we concentrate on P-waves) are represented through the stiffness components c_{ij} and density ρ in the following way:

1) V_{P0}—the vertical velocity of the P-wave:

$$V_{P0} \equiv \sqrt{\frac{c_{33}}{\rho}}. \qquad (20)$$

2) $\epsilon^{(2)}$—the VTI parameter ϵ in the symmetry plane $[x_1, x_3]$ normal to the x_2 axis (close to the fractional difference between the P-wave velocities in the x_1 and x_3 directions):

$$\epsilon^{(2)} \equiv \frac{c_{11} - c_{33}}{2c_{33}}. \qquad (21)$$

3) $\delta^{(2)}$—the VTI parameter δ in the $[x_1, x_3]$ plane (responsible for near-vertical P-wave velocity variations, also influences SV-wave velocity anisotropy):

$$\delta^{(2)} \equiv \frac{(c_{13}+c_{55})^2 - (c_{33}-c_{55})^2}{2c_{33}(c_{33}-c_{55})}. \qquad (22)$$

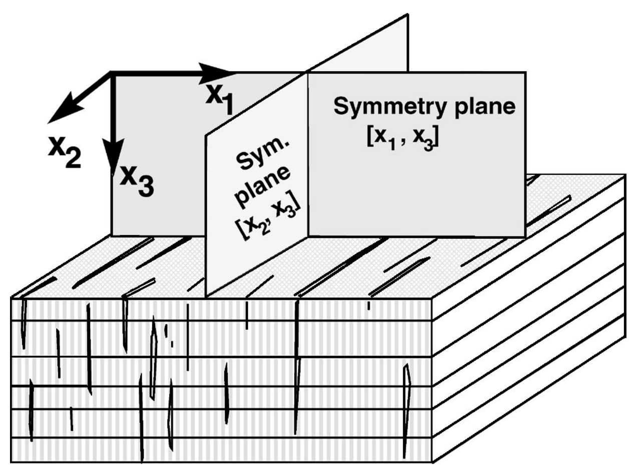

FIG. 7. Orthorhombic media have three mutually orthogonal planes of mirror symmetry. One reason for orthorhombic anisotropy is a combination of parallel vertical cracks and vertical transverse isotropy (e.g., due to thin horizontal layering) in the background medium.

4) $\epsilon^{(1)}$—the VTI parameter ϵ in the $[x_2, x_3]$ plane:

$$\epsilon^{(1)} \equiv \frac{c_{22} - c_{33}}{2c_{33}}. \tag{23}$$

5) $\delta^{(1)}$—the VTI parameter δ in the $[x_2, x_3]$ plane:

$$\delta^{(1)} \equiv \frac{(c_{23} + c_{44})^2 - (c_{33} - c_{44})^2}{2c_{33}(c_{33} - c_{44})}. \tag{24}$$

6) $\delta^{(3)}$—the VTI parameter δ in the $[x_1, x_2]$ plane (x_1 plays the role of the symmetry axis):

$$\delta^{(3)} \equiv \frac{(c_{12} + c_{66})^2 - (c_{11} - c_{66})^2}{2c_{11}(c_{11} - c_{66})}. \tag{25}$$

Tsvankin's notation can be conveniently used to describe seismic velocities and polarizations in the symmetry planes of orthorhombic media by known VTI equations expressed through Thomsen parameters. Here, we consider a homogeneous orthorhombic layer with a horizontal symmetry plane $[x_1, x_2]$. The exact P-wave NMO velocity from a horizontal reflector in the $[x_1, x_3]$ symmetry plane (on the CMP line parallel to the x_1-axis) can be found by analogy with VTI media as (Tsvankin, 1997b)

$$V_{\text{nmo}}^{(2)} = V_{P0}\sqrt{1 + 2\delta^{(2)}}. \tag{26}$$

Likewise, the P-wave NMO velocity on the line parallel to the x_2-axis is given by

$$V_{\text{nmo}}^{(1)} = V_{P0}\sqrt{1 + 2\delta^{(1)}}. \tag{27}$$

NMO velocity outside the symmetry planes, however, is influenced by azimuthal velocity variations and should be studied using the general formalism developed here.

P-wave NMO velocity from horizontal reflectors

As shown above for the VTI model with a dipping reflector, one of the axes of the NMO ellipse for any medium with a vertical symmetry plane is parallel to the symmetry-plane direction. Therefore, in a horizontal orthorhombic layer both elliptical axes lie in the vertical symmetry planes, and the NMO ellipse takes the form [see equation (5)]

$$V_{\text{nmo}}^{-2}(\alpha, 0) \equiv V_{\text{nmo}}^{-2}(\alpha) = \left[V_{\text{nmo}}^{(2)}\right]^{-2}\cos^2\alpha + \left[V_{\text{nmo}}^{(1)}\right]^{-2}\sin^2\alpha, \tag{28}$$

where α is the azimuthal angle of the CMP line with respect to the x_1-axis. Since the velocities $V_{\text{nmo}}^{(1)}$ and $V_{\text{nmo}}^{(2)}$ can be obtained easily by analogy with vertical transverse isotropy [i.e., for P-waves from equations (26) and (27)], equation (28) provides a concise analytic description of azimuthally dependent normal moveout of all three pure (nonconverted) modes in orthorhombic media.

By fitting an ellipse to azimuthal measurements of moveout velocity for any pure wave, we can determine the orientation of the symmetry planes and the NMO velocities within them. After reconstructing the NMO ellipse, we also can obtain the NMO velocity in arbitrary azimuthal direction. For most common orthorhombic models with a single system of fractures embedded in a VTI background or with two orthogonal fracture systems, the vertical symmetry planes are parallel and normal to the fractures. Therefore, azimuthal moveout analysis can be used to supplement or even replace shear-wave splitting in detecting the predominant fracture orientation(s) in the subsurface. Also, for orthorhombic media with a single crack system, the ratio of the P-wave NMO velocities in the symmetry planes is related to the crack density.

Further inversion of the symmetry-plane NMO velocities (semi-axes of the ellipse) for medium parameters depends on the recorded wave types. In the following, we focus on moveout analysis of P-waves; shear-wave NMO velocities and joint inversion of P and S data will be described in forthcoming publications. If the vertical P-wave velocity is known (e.g., from check shots or well logs), the symmetry-plane NMO velocities of P-waves [equations (26) and (27)] can be used to obtain the anisotropic coefficients $\delta^{(1,2)}$. These coefficients, responsible for P-wave velocity variations near vertical, are of primary importance in the analysis of amplitude variation with offset (AVO) in orthorhombic media (Rüger, 1996).

Substitution of equations (26) and (27) into equation (28) allows us to rewrite P-wave NMO velocity as an explicit function of the anisotropic parameters:

$$V_{\text{nmo}}^2(\alpha) = V_{P0}^2 \frac{\left(1 + 2\delta^{(1)}\right)\left(1 + 2\delta^{(2)}\right)}{1 + 2\delta^{(2)}\sin^2\alpha + 2\delta^{(1)}\cos^2\alpha}. \tag{29}$$

Clearly, only two anisotropic coefficients ($\delta^{(1)}$ and $\delta^{(2)}$) influence P-wave NMO velocity from horizontal reflectors. The other three anisotropic parameters ($\epsilon^{(1)}$, $\epsilon^{(2)}$, and $\delta^{(3)}$), however, contribute to the quartic (nonhyperbolic) moveout term (Tsvankin, 1997b) and, therefore, to the moveout velocity determined on finite-length spreads.

NMO velocities for vertical and horizontal transverse isotropy can be obtained as special cases of equation (29). For instance, if the medium is transversely isotropic with a horizontal symmetry axis pointing in the x_1 direction, $[x_2, x_3]$ represents the isotropy plane (in which all velocities are independent of propagation direction) and $\delta^{(1)} = 0$. In this case, equation (29) becomes

$$V_{\text{nmo}}^2(\alpha) = V_{P0}^2 \frac{1 + 2\delta^{(2)}}{1 + 2\delta^{(2)}\sin^2\alpha}, \tag{30}$$

which is equivalent to the NMO expression for HTI media presented by Tsvankin (1997a), who denotes the δ coefficient in the vertical plane that contains the symmetry axis ("symmetry-axis plane") by $\delta^{(V)}$. (Any orthorhombic medium with $\delta^{(1)} = 0$ or $\delta^{(2)} = 0$ is fully equivalent to horizontal transverse isotropy in terms of the azimuthally dependent P-wave NMO velocity from horizontal reflectors.) Since the NMO velocity in the isotropy plane coincides with the true vertical velocity, P-wave normal moveout in this case is sufficient to find both V_{P0} and $\delta^{(2)}$, provided we can distinguish between the isotropy and symmetry-axis planes using moveout data. Usually, the isotropy plane can be identified by the higher value of the NMO velocity because the coefficient $\delta^{(2)}$ ($\delta^{(V)}$) is predominantly negative (Tsvankin, 1997a).

In the weak-anisotropy approximation ($|\delta^{(1)}, \delta^{(2)}| \ll 1$), we can drop the quadratic terms in the anisotropic coefficients in equation (29) to obtain

$$V_{\text{nmo}}(\alpha) = V_{P0}\left(1 + \delta^{(2)}\cos^2\alpha + \delta^{(1)}\sin^2\alpha\right). \tag{31}$$

This expression also can be derived from the symmetry-plane NMO equation (6) by substituting $\phi = 0$ and using the linearized weak-anisotropy approximation for phase velocity given by Tsvankin (1997b); for weak anisotropy, out-of-plane phenomena can be ignored, and any vertical plane for reflections from horizontal interfaces can be regarded as a plane of symmetry.

NMO velocity in horizontally layered orthorhombic media with throughgoing vertical symmetry planes (i.e., the symmetry planes in all layers have the same azimuthal directions) can be found from the elliptical equation (28) introduced for a single layer. In this case, the values of $V_{\rm nmo}^{(1)}$ and $V_{\rm nmo}^{(2)}$ should be determined by Dix's (1955) rms averaging of the interval NMO velocities in the symmetry planes (Tsvankin, 1997b).

Reflection moveout for both transversely isotropic and orthorhombic media is generally nonhyperbolic, and the analytic NMO velocity may differ from the moveout (stacking) velocity measured on finite CMP spreads. For typical VTI media, however, the hyperbolic moveout equation parameterized by the NMO velocity adequately describes P-wave reflection moveout on conventional-length spreads, which are comparable to the reflector depth (see Figure 4 and examples in Tsvankin, 1995). Because of the analogy with vertical transverse isotropy, this conclusion is entirely valid in the symmetry planes of orthorhombic media.

The numerical example in Figure 8 illustrates *azimuthally dependent* distortions in P-wave moveout velocity caused by

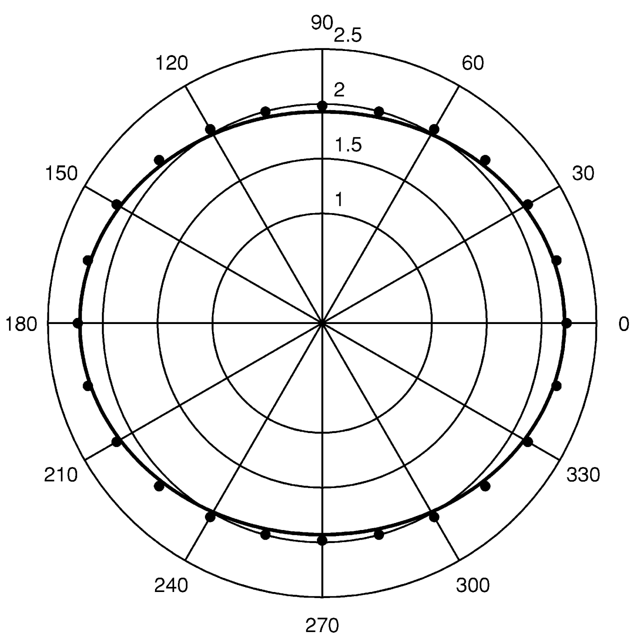

FIG. 8. Accuracy of the hyperbolic moveout equation parameterized by the NMO velocity in the description of P-wave traveltimes for a homogeneous orthorhombic layer. The dots represent the moveout velocity on spread length equal to the reflector depth; the solid curve is the NMO ellipse calculated using equations (26), (27), and (28). The medium parameters are $V_{P0} = 2.0$ km/s, $\epsilon^{(1)} = 0.110$, $\delta^{(1)} = -0.035$, $\epsilon^{(2)} = 0.225$, $\delta^{(2)} = 0.100$, $\delta^{(3)} = 0$. The vertical symmetry plane at zero azimuth has the properties of the VTI model of Dog Creek Shale, while the second vertical symmetry plane is equivalent to the VTI model of Taylor Sandstone.

nonhyperbolic moveout on a typical spread length equal to the reflector depth. We have computed the difference between the analytic NMO velocity and the moveout velocity obtained from the exact traveltimes for an orthorhombic medium whose vertical symmetry planes have the properties of two typical VTI models: Dog Creek Shale and Taylor Sandstone (Thomsen, 1986). As expected from the limited equivalence with vertical transverse isotropy, the error in moveout velocity caused by nonhyperbolic moveout in the symmetry planes is the same as that found in Tsvankin and Thomsen (1994) for the corresponding VTI models (0.8% for Dog Creek Shale and 2.7% for Taylor Sandstone). More importantly, the error changes monotonically with azimuth between the symmetry planes, and the finite-spread moveout velocity remains close to the theoretical NMO ellipse for any orientation of the CMP line. The same observation is made in Tsvankin (1997a) for P-waves in HTI media.

We conclude that the analytic expression for NMO velocity provides sufficient accuracy in describing P-wave reflection moveout on conventional-length spreads in typical orthorhombic media. For models with unusually pronounced deviations from hyperbolic moveout, NMO velocity can be obtained by nonhyperbolic semblance analysis using, for instance, the equation developed by Tsvankin and Thomsen (1994).

DISCUSSION AND CONCLUSIONS

We have presented a general equation for NMO velocity that provides a new framework for traveltime modeling and inversion in anisotropic media. The azimuthal dependence of NMO velocity for pure (nonconverted) modes from horizontal and dipping reflectors in arbitrary anisotropic inhomogeneous media is described by just three parameters and typically represents an ellipse in the horizontal plane. Three or more moveout measurements in different azimuthal directions (one exception is described below) are needed to reconstruct the NMO ellipse and find the NMO velocity in arbitrary direction. The coefficients of the NMO ellipse (e.g., its orientation and semi-axes) are determined by the spatial derivatives of the ray parameter at the CMP location and can be obtained analytically for a range of practically important anisotropic models.

For typical anisotropic media, the hyperbolic moveout equation parameterized by the analytic NMO (zero-spread) velocity given here is sufficiently accurate in the description of P-wave reflection traveltimes on conventional-length CMP spreads. As demonstrated on synthetic examples for transversely isotropic and orthorhombic media, the influence of nonhyperbolic moveout becomes significant only at relatively large offsets exceeding reflector depth. In principle, the nonhyperbolic portion of the moveout curve can be included in the inversion procedure, but a more detailed discussion of long-spread moveout is outside the scope of this work.

The azimuthal variation of normal moveout from horizontal reflectors is usually ignored in 3-D surveys, with semblance velocity analysis performed simultaneously for source-receiver pairs with different azimuthal orientation. In the presence of azimuthal anisotropy, the conventional approach produces an "average" stacking velocity, which may be smaller or larger than the actual value for any given azimuth (Lynn et al., 1996). Our result suggests a simple way to avoid this distortion by building the NMO ellipse and picking the correct stacking velocity for azimuthally binned data.

Application of the new equation to dipping reflectors in transversely isotropic media with a vertical symmetry axis (VTI media) and horizontal reflectors in orthorhombic media yields concise analytic expressions for the azimuthally dependent NMO velocity. In both cases, the orientation of the NMO ellipse is defined by the symmetry of the model, which has at least one vertical plane of mirror symmetry.

For VTI media, the elliptical axes are always parallel to the dip and strike directions of the reflector. We obtained the NMO velocity on the strike line and combined it with the dip-line equation of Tsvankin (1995) to find the NMO ellipse as a simple function of phase velocity and its derivatives. The azimuthally dependent P-wave NMO velocity, expressed through the ray parameter of the zero-offset reflection, is fully described by the zero-dip NMO velocity $V_{\mathrm{nmo}}(0)$ and the anisotropic coefficient η—the same two parameters that control P-wave time processing as a whole. This allows us to extend the 2-D inversion algorithm of Alkhalifah and Tsvankin (1995), designed to obtain η from the NMO velocity in the dip plane, to CMP lines with arbitrary orientation. The problems experienced by the dip-line inversion in handling near-vertical reflectors can be overcome by using CMP azimuths close to the strike direction. We also demonstrated that the NMO ellipse for VTI media can be found using just two differently oriented lines since the direction of the dip plane (and that of the NMO ellipse) can be recovered from the slopes of zero-offset reflections measured for two azimuths.

Azimuthal moveout analysis of horizontal events in orthorhombic media gives the directions of the symmetry planes (usually associated with the predominant fracture orientation) and the NMO velocities within them. If only P-wave data are available, symmetry-plane NMO velocities from horizontal reflectors can be used to determine two anisotropic coefficients, $\delta^{(1)}$ and $\delta^{(2)}$, provided the vertical velocity is known. The parameters $\delta^{(1)}$ and $\delta^{(2)}$ also influence the P-wave reflection coefficients in orthorhombic media and play an important role in the AVO inversion (Rüger, 1996).

For more complex symmetries and/or horizontally inhomogeneous media, the properties of the NMO ellipse have a less transparent physical meaning but can still be obtained as functions of the medium parameters (Tsvankin et al., 1997).

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APPENDIX A

DERIVATION OF THE 3-D NMO EQUATION

In this appendix, we show that the azimuthal dependence of NMO velocity for any inhomogeneous anisotropic medium can be represented as a second-order curve in the horizontal plane with the coefficients determined by the derivatives of the ray parameter with respect to the horizontal coordinates. Since reflection point dispersal (i.e., the movement of the reflection point with offset) has no influence on the NMO velocity (Hubral and Krey, 1980), we ignore the difference between the true (specular) reflection point and the zero-offset reflection point R (Figure A-1). This allows us to obtain reflection moveout of any pure (nonconverted) reflected mode through the one-way traveltime $\tau(x_1, x_2)$ between the zero-offset reflection point and the surface location $[x_1, x_2]$. Extending the approach first suggested by Hale et al. (1992) for the 2-D problem, we express $\tau(x_1, x_2)$ as a double Taylor series in the vicinity of the CMP:

$$\tau_\pm \equiv \tau(\pm x_1, \pm x_2) = \tau_0 \pm \frac{\partial \tau}{\partial x_1} x_1 \pm \frac{\partial \tau}{\partial x_2} x_2 + \frac{\partial^2 \tau}{\partial x_1^2}\frac{x_1^2}{2} + \frac{\partial^2 \tau}{\partial x_1 \partial x_2} x_1 x_2 + \frac{\partial^2 \tau}{\partial x_2^2}\frac{x_2^2}{2} + \cdots. \quad \text{(A-1)}$$

Here, $\tau_+ = \tau(+x_1, +x_2)$, $\tau_- = \tau(-x_1, -x_2)$, τ_0 is the one-way zero-offset traveltime, $(\pm x_1, \pm x_2)$ are the coordinates of source A^+ and receiver A^-, and the derivatives are evaluated at CMP location O (Figure A-1). The one-way traveltime $\tau(x_1, x_2)$ is assumed to be a function differentiable at least twice at the CMP point $(x_1 = x_2 = 0)$.

Keeping only the quadratic and lower-order terms in the Taylor series expansion (A-1), we can represent the squared two-way CMP traveltime as

$$t^2(x_1, x_2) \equiv (\tau_+ + \tau_-)^2 = t_0^2 + 4\big(W_{11} x_1^2 + 2W_{12} x_1 x_2 + W_{22} x_2^2\big), \quad \text{(A-2)}$$

where $t_0 = 2\tau_0$ is the two-way zero-offset traveltime and $\mathbf{W}$ is a symmetric matrix given by $W_{ij} = \tau_0(\partial^2\tau/\partial x_i \partial x_j)$.

The coordinates x_1 and x_2 can be expressed through the azimuth α of the CMP line (Figure A-1) and source-receiver half-offset, h:

$$x_1 = h\cos\alpha$$
and
$$x_2 = h\sin\alpha. \quad \text{(A-3)}$$

The squared two-way CMP traveltime from equation (A-2) now becomes

$$t^2(h, \alpha) = t_0^2 + 4\big(W_{11}\cos^2\alpha + 2W_{12}\sin\alpha\cos\alpha + W_{22}\sin^2\alpha\big)h^2. \quad \text{(A-4)}$$

Using the definition of the NMO velocity $V_{\rm nmo}$,

$$t^2(h, \alpha) = t_0^2 + \frac{4h^2}{V_{\rm nmo}^2(\alpha)} + \cdots, \quad \text{(A-5)}$$

we obtain from equation (A-4)

$$V_{\rm nmo}^2(\alpha) = \big[W_{11}\cos^2\alpha + 2W_{12}\sin\alpha\cos\alpha + W_{22}\sin^2\alpha\big]^{-1} = \frac{2}{t_0}\left[\frac{\partial p_1}{\partial x_1}\cos^2\alpha + \left(\frac{\partial p_1}{\partial x_2} + \frac{\partial p_2}{\partial x_1}\right)\sin\alpha\cos\alpha + \frac{\partial p_2}{\partial x_2}\sin^2\alpha\right]^{-1}, \quad \text{(A-6)}$$

where $p_i = \partial\tau/\partial x_i$, $(i = 1, 2)$ are the horizontal components of the slowness vector $\mathbf{p}$ of rays emanating from the zero-offset reflection point; p_1 and p_2 are measured at the surface.

To simplify equation (A-6), let us align the coordinate axes with the eigenvectors of the matrix $\mathbf{W}$. The required rotation angle β is given by (assuming $W_{12} \neq 0$; if $W_{12} = 0$, no rotation is needed)

$$\beta = \tan^{-1}\left[\frac{W_{22} - W_{11} + \sqrt{(W_{22} - W_{11})^2 + 4W_{12}^2}}{2W_{12}}\right]. \quad \text{(A-7)}$$

Introducing the eigenvalues $\lambda_{1,2}$ of the matrix $\mathbf{W}$,

$$\lambda_{1,2} = \tfrac{1}{2}\Big[W_{11} + W_{22} \pm \sqrt{(W_{11} - W_{22})^2 + 4W_{12}^2}\Big], \quad \text{(A-8)}$$

we represent equation (A-6) after the rotation as

$$V_{\rm nmo}^{-2}(\alpha) = \lambda_1\cos^2(\alpha - \beta) + \lambda_2\sin^2(\alpha - \beta). \quad \text{(A-9)}$$

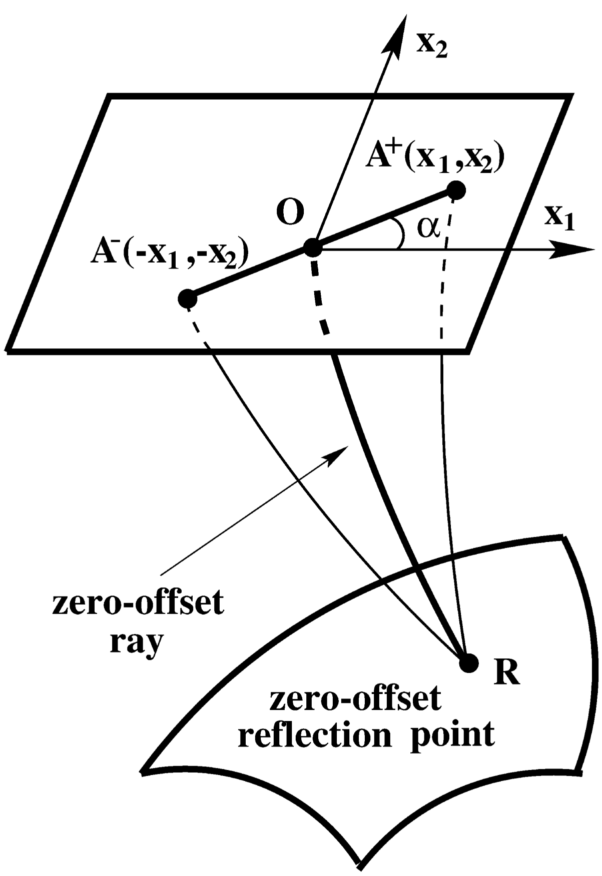

FIG. A-1. In the derivation of NMO velocity, the CMP reflection raypath from A^+ to A^- (not shown on the plot) can be replaced with a nonspecular raypath going through the zero-offset reflection point.

APPENDIX B

STRIKE-LINE NMO VELOCITY IN VTI MEDIA

Let us consider a dipping reflector beneath a homogeneous transversely isotropic medium with a vertical symmetry axis (Figure B-1). The NMO velocity on the CMP line in the dip plane of the reflector is obtained by Tsvankin (1995) as a special case of his general symmetry-plane NMO equation. Here, we derive the NMO velocity in the strike direction needed to construct the NMO ellipse for this model.

The NMO velocity of any pure mode on the strike line is given by [see equation (A-6)]

$$V_{\rm nmo}^2\left(\frac{\pi}{2}, \phi\right) = \frac{2}{t_0}\left.\frac{dh}{dp_h}\right|_{h=0}, \tag{B-1}$$

where $t_0 = 2\tau_0$ is the two-way zero-offset traveltime, h is half the source-receiver offset, and p_h is the projection of the ray parameter (horizontal slowness) on the CMP (strike) line. As discussed in the main text, we can ignore reflection-point dispersal, which has no influence on NMO velocity. Following the approach suggested in Tsvankin (1995, 1997a), we evaluate the derivative in equation (B-1) by representing h and p_h as functions of the phase angle θ corresponding to rays that emanate from the zero-offset reflection point:

$$V_{\rm nmo}^2\left(\frac{\pi}{2}, \phi\right) = \frac{2}{t_0}\frac{dh}{d\theta}\left.\left(\frac{dp_h}{d\theta}\right)^{-1}\right|_{\theta=\phi}, \tag{B-2}$$

where ϕ is the dip of the reflector and θ is measured from the (vertical) symmetry axis (Figure B-1).

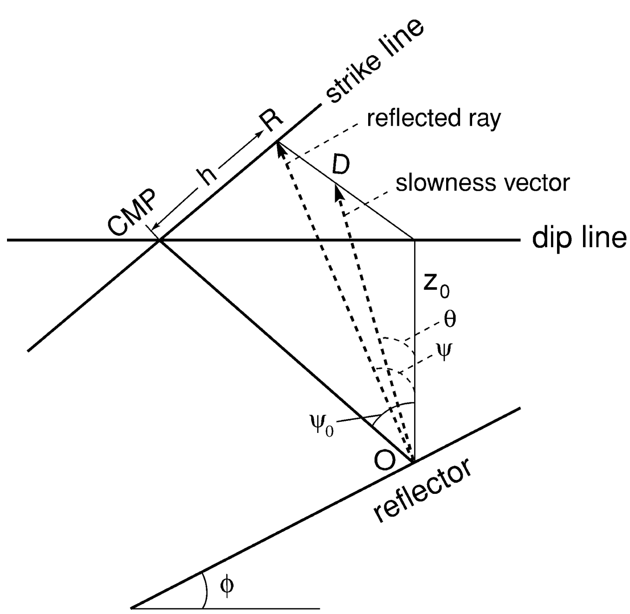

FIG. B-1. Geometry of the group- and phase-velocity vectors of reflected waves on the CMP (strike) line perpendicular to the dip plane of the reflector. The specular reflection point is assumed to coincide with the zero-offset reflection point O. In VTI media, the reflected ray OR (with the group angle ψ) and the corresponding slowness vector OD (with the phase angle θ) lie in the same vertical plane. The group angle of the zero-offset ray is ψ_0, and z_0 is the depth of the zero-offset reflection point.

To relate the half-offset h to the phase angle, it is convenient to introduce the group angle ψ measured from vertical (Figure B-1). Then h can be represented through the depth of the zero-offset reflection point z_0 as

$$h = z_0\sqrt{\tan^2\psi - \tan^2\psi_0}, \tag{B-3}$$

where ψ_0 is the group (ray) angle of the zero-offset ray.

Differentiating equation (B-3) yields

$$\frac{dh}{d\theta} = \frac{z_0\tan\psi}{\sqrt{\tan^2\psi - \tan^2\psi_0}}\frac{d\tan\psi}{d\theta}. \tag{B-4}$$

For vertical transverse isotropy, the slowness vector (parallel to the phase-velocity vector) always lies in the vertical plane that contains the group-velocity vector. Therefore, the projection of the slowness vector on the CMP line is given by (Figure B-1)

$$p_h = \frac{\sin\theta}{V}\frac{\sqrt{\tan^2\psi - \tan^2\psi_0}}{\tan\psi}$$

and

$$\frac{dp_h}{d\theta} = \left(\frac{\cos\theta}{V} - \frac{\sin\theta}{V^2}\frac{dV}{d\theta}\right)\frac{\sqrt{\tan^2\psi - \tan^2\psi_0}}{\tan\psi} + \frac{\sin\theta}{V}\frac{\tan^2\psi_0}{\tan^2\psi\sqrt{\tan^2\psi - \tan^2\psi_0}}\frac{d\tan\psi}{d\theta}. \tag{B-5}$$

Substituting equations (B-4) and (B-5) into equation (B-2) and taking into account that the derivatives should be evaluated at $\psi = \psi_0$, we find

$$V_{\rm nmo}^2\left(\frac{\pi}{2}, \phi\right) = \frac{2z_0}{t_0}\frac{\tan\psi_0}{\sin\phi}V(\phi). \tag{B-6}$$

Expressing t_0 through the group velocity v_g of the zero-offset ray [$t_0 = 2z_0/(v_g(\psi_0)\cos\psi_0)$] reduces equation (B-6) to

$$V_{\rm nmo}^2\left(\frac{\pi}{2}, \phi\right) = \frac{V(\phi)}{\sin\phi}v_g(\psi_0)\sin\psi_0. \tag{B-7}$$

The product $v_g(\psi_0)\sin\psi_0$ represents the horizontal component of the group-velocity vector given by (e.g., Thomsen, 1986)

$$v_g(\psi_0)\sin\psi_0 = V\sin\phi + \left.\frac{dV}{d\theta}\right|_{\theta=\phi}\cos\phi. \tag{B-8}$$

Substitution of equation (B-8) into equation (B-7) leads to the final result:

$$V_{\rm nmo}\left(\frac{\pi}{2}, \phi\right) = V(\phi)\sqrt{1 + \frac{1}{V(\phi)\tan\phi}\left.\frac{dV}{d\theta}\right|_{\theta=\phi}}. \tag{B-9}$$

Since in the inversion procedure we operate with the ray parameter (horizontal slowness), it is convenient to express the strike-line NMO velocity [equation (B-9)] through the components of the slowness vector. The dip-line NMO velocity as a function of the ray parameter is given by Cohen (1998) as

$$V_{\mathrm{nmo}}^2(0,\,p) = \sqrt{\frac{q''}{pq' - q}}, \tag{B-10}$$

where q is the vertical component of the slowness vector,

$$q \equiv q(p) \equiv \frac{\cos\theta}{V(\theta)}, \tag{B-11}$$

and

$$q' \equiv \frac{dq}{dp}; \qquad q'' \equiv \frac{d^2q}{dp^2}.$$

Here, we obtain a similar expression for the strike-line NMO velocity. The derivative $dV/d\theta$ from equation (B-9) can be rewritten through the ray parameter p in the following way:

$$\frac{dV}{d\theta} = \frac{dV}{dp}\frac{dp}{d\theta} \tag{B-12}$$

and

$$p \equiv \frac{\sin\theta}{V(\theta)}. \tag{B-13}$$

Differentiating the identity

$$V^{-2} = p^2 + q^2, \tag{B-14}$$

we obtain

$$\frac{dV}{dp} = -V^3(p + qq'). \tag{B-15}$$

From equation (B-13) we find the derivative $dp/d\theta$ needed in equation (B-12):

$$\frac{dp}{d\theta} = V^{-2}\left(V\cos\theta - \frac{dV}{d\theta}\sin\theta\right). \tag{B-16}$$

Substituting equations (B-15) and (B-16) into equation (B-12) and using equations (B-13) and (B-11) yields

$$\frac{dV}{d\theta} = V\frac{qq' + p}{pq' - q}. \tag{B-17}$$

Finally, we substitute equations (B-14) and (B-17) into equation (B-9) and, taking into account that when $\theta = \phi$, $\tan\phi = p/q$, obtain

$$V_{\mathrm{nmo}}\left(\frac{\pi}{2},\,p\right) = \sqrt{\frac{q'}{p(pq' - q)}}. \tag{B-18}$$

All quantities in equation (B-18) should be evaluated for the zero-offset ray.