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Transverse isotropy versus lateral heterogeneity in the inversion of *P*-wave reflection traveltimes

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ABSTRACT

Nonelliptic transverse isotropy may cause pronounced nonhyperbolic moveout of long-spread *P*-wave reflection data. Lateral heterogeneity may alter the moveout in much the same way, and one can expect that a given *P*-wave reflection moveout may be interpreted equally well in terms of parameters of homogeneous transversely isotropic (TI) or laterally heterogeneous (LH) isotropic models.

Here, the common-midpoint (CMP) moveout of a *P*-wave reflected from a horizontal interface beneath a weakly laterally heterogeneous medium that is also weakly transversely isotropic is represented analytically in the form similar to that in homogeneous TI media. Both the normal-moveout (NMO) velocity and the quartic moveout coefficient contain derivatives of the zero-offset traveltime t_0 and the NMO velocity V_{nmo} with respect to the lateral coordinate.

Despite the presence of heterogeneity, nonhyperbolic velocity analysis can be performed in the same way as in homogeneous TI models. If all parameters of the medium are linear functions of the lateral coordinate, heterogeneity does not influence the results of inversion for the anisotropic parameter η. However, to find η in the case of general lateral heterogeneity, the second derivative of V_{nmo} and the fourth derivative of t_0 are needed. Since these high-order derivatives are calculated from the data measured at discrete points by numerical differentiation, stability of η estimation is further reduced as compared to that in homogeneous TI media. Consequently, the trade-off between anisotropy and heterogeneity significantly complicates the inversion of *P*-wave reflection traveltimes, even in the simplest model of a single plane layer.

INTRODUCTION

The presence of seismic anisotropy in different rocks and on various scales makes it necessary to generalize conventional isotropic velocity analysis to account for anisotropy. This has recently been done and a number of approaches to anisotropic traveltime inversion and velocity analysis have been developed (Byun et al., 1989; Sena, 1991; Tsvankin and Thomsen, 1995; Alkhalifah and Tsvankin, 1995; Alkhalifah, 1996). Alkhalifah and Tsvankin (1995) showed that the normal-moveout velocity V_{nmo} and the anisotropic "anellipticity" parameter η, which can be obtained using surface *P*-wave reflections from interfaces with different dips, allow one to perform all time-processing procedures in transversely isotropic (TI) media, including normal moveout (NMO), dip moveout (DMO), and time migration. Although this DMO-inversion approach provides a stable way of estimating the parameter η, it requires the presence of at least one dipping (and one horizontal) reflector and cannot be used for dips exceeding 70–80°. In principle, the same parameters, V_{nmo} and η, can be extracted from long-spread nonhyperbolic *P*-wave moveout from horizontal interfaces. Alkhalifah (1996) suggested estimating V_{nmo} and η using a 2-D semblance search based on the nonhyperbolic moveout equation derived in Alkhalifah and Tsvankin (1995). Grechka and Tsvankin (1996) performed an error study for the nonhyperbolic velocity analysis and showed that the accuracy of η estimation is not sufficient for lithology discrimination because the inverted values of η are sensitive to small long-period traveltime errors. They also found that the horizontal velocity is better constrained by the long-spread moveout than is η and, therefore, the horizontal velocity rather than η should be extracted.

Although existing velocity-analysis methods can handle vertical velocity variations, they are based on the assumption of lateral homogeneity of TI models on the scale of a common-midpoint gather. Here, under the assumption of weak anisotropy and weak heterogeneity, we examine the simultaneous influence of TI and lateral heterogeneity (LH) on the traveltimes of *P*-waves reflected from a horizontal interface.

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The assumption that anisotropy and heterogeneity are weak allows us to apply linear perturbation theory and derive concise linearized P-wave moveout equations in TI LH media. These equations show that a given long-spread moveout in weakly TI medium can be reproduced exactly in a number of kinematically equivalent isotropic media with weak lateral heterogeneity. The derived equations also indicate that a linear lateral velocity gradient does not influence common midpoint (CMP) reflection traveltimes, and, therefore, nonhyperbolic velocity analysis for the key anisotropic parameter η in such models can be performed in exactly the same way as in homogeneous TI models. For general lateral heterogeneity, however, the second derivative of V_{nmo} and the fourth derivative of the zero-offset traveltime t_0 with respect to the common-midpoint (CMP) coordinate are needed. Since these high-order derivatives are estimated from discrete data, inversion for the anisotropic parameter η will be less stable than that in homogeneous TI models. Consequently, one can hardly expect to separate TI from LH based on P-wave reflection traveltime data.

APPROXIMATIONS OF *P*-WAVE REFLECTION MOVEOUT IN HOMOGENEOUS WEAKLY TI MEDIA

Tsvankin and Thomsen (1994) obtained an approximate equation for the P-wave moveout $t(x)$ in a CMP gather for reflections from a horizontal interface in a homogeneous TI layer with a vertical symmetry axis. This expression was written in terms of exact coefficients A_0, A_2, A_4 of the Taylor series expansion of $t^2(x)$ near $x = 0$, and the horizontal velocity V_{hor}. It was rewritten by Alkhalifah and Tsvankin (1995) as

$$t^2(x) = t_0^2 + \frac{x^2}{V_{\text{nmo}}^2} - \frac{2\eta x^4}{V_{\text{nmo}}^2\left[t_0^2 V_{\text{nmo}}^2 + (1+2\eta)x^2\right]}. \quad (1)$$

In this equation, x is the offset, t_0 is the two-way zero-offset traveltime, η is the anisotropic parameter, which is related to Thomsen's (1986) parameters ϵ and δ as

$$\eta = \frac{\epsilon - \delta}{1 + 2\delta}, \quad (2)$$

and V_{nmo} is the normal moveout velocity given by (Thomsen, 1986)

$$V_{\text{nmo}}^2 = V_0^2(1 + 2\delta), \quad (3)$$

where V_0 is the vertical velocity.

Under the assumption of weak anisotropy, i.e., when

$$\{|\epsilon|, |\delta|, |\eta|\} \ll 1, \quad (4)$$

equation (1) can be linearized in ϵ, δ, η in the manner done by Tsvankin and Thomsen (1994), yielding

$$t^2(x) = t_0^2 + \frac{x^2}{V_{\text{nmo}}^2} - \frac{2\eta x^4}{V_{\text{nmo}}^2\left(t_0^2 V_{\text{nmo}}^2 + x^2\right)}, \quad (5)$$

where η from equation (2) reduces to the difference

$$\eta = \epsilon - \delta. \quad (6)$$

Although the linearized moveout equation (5) is less accurate than Alkhalifah and Tsvankin's (1995) equation (1), it can still be used to gain insight into the influence of anisotropy on P-wave reflection traveltimes. Figure 1 compares, for two different TI media, the traveltime $t(x)$ calculated using equation (5) and traveltime $t_{\text{num}}(x)$ computed numerically without the weak-anisotropy assumption. It shows that equation (5) gives a good approximation of the correct moveout up to relative offset $x/z = 1.5$ whereupon the approximation error increases rapidly.

It is instructive to derive equation (5) one more time applying linear-perturbation theory with respect to the velocity variations. The same approach will be used in the following sections to obtain an analytic expression for reflection moveout in laterally heterogeneous media.

Let us begin with the equation for CMP reflection moveout $t_{\text{iso}}(x)$ in a homogeneous isotropic medium (Figure 2)

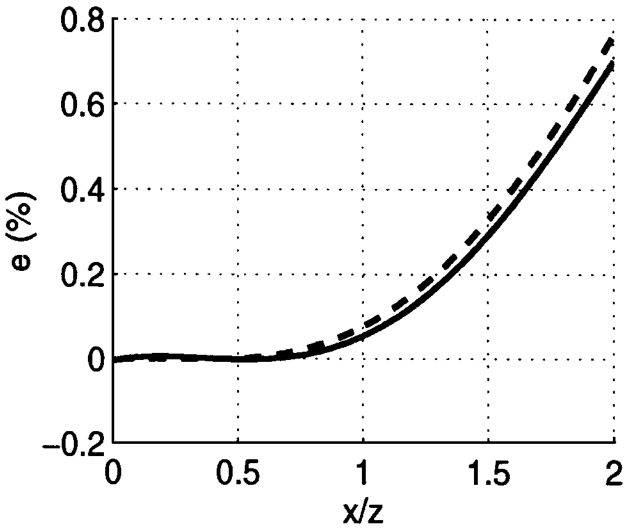

FIG. 1. Relative error $e(x/z) = t_{\text{num}}(x)/t(x) - 1$ (in %) of the traveltime approximation $t(x)$ [equation (5)] as a function of the normalized offset x/z (z is the reflector depth). Taylor sandstone (V_0 = 3.368 km/s, ϵ = 0.110, δ = −0.035, η = 0.145) (solid line); Dog Creek shale (V_0 = 1.875 km/s, ϵ = 0.225, δ = 0.100, η = 0.125) (dashed line). The exact traveltime $t_{\text{num}}(x)$ was computed using the two-point, ray-tracing code described in Obolentseva and Grechka (1989).

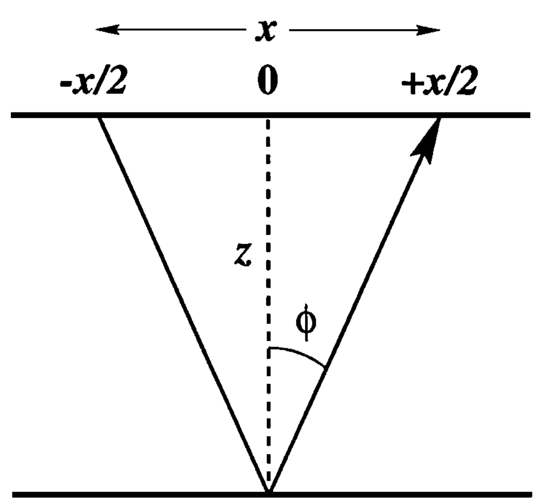

FIG. 2. Notation for the CMP geometry.

$$t_{\text{iso}}(x) = \frac{\sqrt{x^2 + a^2}}{V_0}, \tag{7}$$

where V_0 is the P-wave velocity, $a = 2z$ is the doubled reflector depth, and x is the offset. Consider a small velocity perturbation ΔV along a ray such as

$$V = V_0\left(1 + \frac{\Delta V}{V_0}\right), \tag{8}$$

where

$$|\Delta V/V_0| \ll 1. \tag{9}$$

Linear-perturbation theory suggests that the traveltime for the medium with the perturbed velocity can be approximated by the first two terms of the Taylor series in $\Delta V/V_0$

$$t(x) = \frac{\sqrt{x^2 + a^2}}{V} \approx \frac{\sqrt{x^2 + a^2}}{V_0}\left[1 - \frac{\Delta V}{V_0}\right]. \tag{10}$$

It follows from Fermat's principle (e.g., Backus and Gilbert, 1969) that, if the quadratic and higher-order terms with respect to $\Delta V/V_0$ in equation (10) are neglected, the perturbations of the ray trajectory can be neglected as well.

Note that equation (10) does not assume any specific velocity perturbation or source of the perturbation; only inequality (9) has to be satisfied. We will apply equation (10) to velocity perturbations caused by anisotropy and lateral heterogeneity. In this section, we consider only anisotropic velocity perturbations.

It is important to mention that equation (10) is linear with respect to $\Delta V/V_0$, although ΔV itself may be an arbitrary function of the anisotropic velocity perturbation expressed via parameters ϵ and δ. Here, I choose ΔV as a linear function of ϵ and δ and basically repeat the derivation of equation (5) given in Appendix A of Tsvankin and Thomsen (1994). Consider a homogeneous TI layer with a vertical symmetry axis. The phase velocity $v(\theta)$ as a function of the phase angle θ is expressed, in the weak anisotropy limit ($|\epsilon| \ll 1$, $|\delta| \ll 1$), as (Thomsen, 1986)

$$v(\theta) = V_0(1 + \delta \sin^2\theta \cos^2\theta + \epsilon \sin^4\theta), \tag{11}$$

where V_0 is the vertical velocity. Although the phase angle θ differs from the group angle ϕ in a weakly anisotropic TI medium, the group velocity $V(\phi)$ as the function of the group angle ϕ has the same form as in equation (11):

$$V(\phi) = V_0(1 + \delta \sin^2\phi \cos^2\phi + \epsilon \sin^4\phi). \tag{12}$$

The group angle ϕ in equation (12) can be expressed through the offset x and the doubled reflector depth a (Figure 2):

$$\sin^2\phi = x^2/(x^2 + a^2). \tag{13}$$

Equation (12) then becomes

$$V = V_0\left[1 + \frac{x^2(x^2\epsilon + a^2\delta)}{(x^2 + a^2)^2}\right], \tag{14}$$

where the second term in the brackets can be treated as the small velocity perturbation $\Delta V/V_0$ in equation (8). This means that we can directly apply general equation (10) to the velocity function (14) and obtain the P-wave reflection traveltime in the form

$$t(x) = \frac{\sqrt{x^2 + a^2}}{V_0}\left[1 - \frac{x^2(x^2\epsilon + a^2\delta)}{(x^2 + a^2)^2}\right]. \tag{15}$$

The next step is to show that equation (15) is equivalent to equation (5). Raising equation (15) to the fourth power to eliminate the denominator in the second term in the brackets in equation (15) and keeping only terms linear in ϵ and δ, we obtain

$$t^4(x) = \frac{1}{V_0^4}[a^4 + 2a^2x^2(1 - 2\delta) + x^4(1 - 4\epsilon)]. \tag{16}$$

Denoting $t_0^2 = a^2/V_0^2$ and $V_{\text{nmo}}^2 = V_0^2(1 + 2\delta)$, we find from equation (16)

$$t^4(x) = \left(t_0^2 + x^2/V_{\text{nmo}}^2\right)^2 - 4x^4(\epsilon - \delta)/V_{\text{nmo}}^4. \tag{17}$$

Finally, a linear approximation in ϵ and δ for the square-root of equation (17) gives

$$t^2(x) = t_0^2 + \frac{x^2}{V_{\text{nmo}}^2} - \frac{2(\epsilon - \delta)x^4}{V_{\text{nmo}}^2(t_0^2 V_{\text{nmo}}^2 + x^2)},$$

which is exactly equation (5).

The derivation above justifies the approach of obtaining traveltimes in homogeneous TI media based on the velocity perturbations. Below, the same method is applied to account for lateral heterogeneity.

INTERPRETATION OF TI MOVEOUT IN TERMS OF LATERAL HETEROGENEITY

Now let us examine moveout (5) from a somewhat different standpoint. Assume that we do not know anything about the origin of this equation and try to interpret it in terms of lateral velocity heterogeneity for an isotropic medium. Although equation (5) corresponds to the angle-dependent group velocity (12), equation(14) shows that this angle dependence can be treated equivalently as the dependence of velocity on the *offset*. Therefore, equation (14) suggests that laterally varying velocity can mimic the influence of transverse isotropy on the P-wave moveout. Moreover, equation (14) indicates that lateral heterogeneity, which affects traveltimes similarly to weak anisotropy, must be weak as well. Hence, we will consider the following form of the velocity function in an isotropic LH medium:

$$V(y) = V_0[1 + c(y)], \tag{18}$$

where y is the CMP (lateral) coordinate and $|c(y)| \ll 1$ is a dimensionless function.

Since the velocity (18) has the generic perturbation form (8), the approximation of traveltimes can be expressed in a form similar to that of equation (10). To obtain this approximation, we write the CMP reflection traveltime (see Figure 2) in LH media as

$$t(y, x) = \int_{y-x/2}^{y+x/2} \frac{d\xi}{V(\xi)\sin\phi} = \frac{\sqrt{x^2 + a^2}}{x} \int_{y-x/2}^{y+x/2} \frac{d\xi}{V(\xi)}. \tag{19}$$

Equation (19) assumes that, in the case of weak heterogeneity, integration can be performed along straight rays. For simplicity, choose the CMP coordinate $y = 0$. After substitution of equation (18) into equation (19) and linearization with respect to the velocity perturbation c, we obtain

$$t(0,x) \equiv t(x) = \frac{\sqrt{x^2+a^2}}{V_0}\left[1 - \frac{1}{x}\int_{-x/2}^{x/2} c(\xi)\,d\xi\right], \quad (20)$$

where division by zero for $x \to 0$ does not occur because

$$\lim_{x\to 0}\frac{1}{x}\int_{-x/2}^{x/2} c(\xi)\,d\xi = c(0). \quad (21)$$

Equation (20) gives an approximation for the reflection moveout within a CMP gather in isotropic weakly LH media. Its form is similar to that in equation (10), with the second term in the brackets expressing average relative velocity perturbation along a ray because of the influence of the lateral heterogeneity.

Let us return to the initial problem of this section and find an isotropic lateral velocity distribution that reproduces moveout (5) in a homogeneous TI medium. Comparison of small terms in equations (15) and (20) gives the following equivalence

$$\int_{-x/2}^{x/2} c(\xi)\,d\xi = \frac{x^3(x^2\epsilon + a^2\delta)}{(x^2+a^2)^2}. \quad (22)$$

It is important to note that the integration of the odd component $c_{\mathrm{odd}}(y)$ (with respect to CMP position) of the function $c(y)$ is zero. That is, in the linear theory odd components of the lateral velocity distribution do not influence the CMP moveout curve. Therefore, differentiating equation (22) with respect to x, and noting that

$$\frac{d}{dx}\int_{-x/2}^{x/2} c(\xi)\,d\xi = 2\,\frac{d}{dx}\int_0^{x/2} c_{\mathrm{even}}(\xi)\,d\xi = c_{\mathrm{even}}\left(\frac{x}{2}\right),$$

we obtain

$$c_{\mathrm{even}}(y) = c(y) = \frac{d}{dy}\left[\frac{y^3(y^2\epsilon + z^2\delta)}{(y^2+z^2)^2}\right], \quad (23)$$

where $z = a/2$ is the reflector depth, and x has been replaced by y because, for the common-midpoint located at $y = 0$, the velocity dependence $c(x)$ on the offset x is equivalent to the dependence on the coordinate y along the CMP line.

In the linear approximation, equation (23) becomes

$$c(y) = \frac{\delta\tau_0^2 V_{\mathrm{nmo}}^2 y^2\left(3\tau_0^2 V_{\mathrm{nmo}}^2 - y^2\right) + \epsilon y^4\left(5\tau_0^2 V_{\mathrm{nmo}}^2 + y^2\right)}{\left(\tau_0^2 V_{\mathrm{nmo}}^2 + y^2\right)^3}, \quad (24)$$

where $\tau_0 = t_0/2$ is the one-way zero-offset traveltime, and we have replaced z by $V_{\mathrm{nmo}}\tau_0$. Note that, here, δ and ϵ are parameters of an *isotropic*, laterally heterogeneous velocity field. Figure 3 shows the functions $c(y)$ that give, in the linear approximation, the same reflection traveltimes as those for homogeneous TI models of Taylor sandstone and Dog Creek shale, for a CMP gather at $y = 0$.

Figure 4 compares traveltimes in TI media and traveltimes in corresponding LH media that were computed using the Chebyshev ray-tracing method (Grechka and McMechan, 1996; 1997), which gives high accuracy for smooth velocity distributions. The curves show that, for ratios of offset to depth as large as 2, these traveltimes are remarkably close for the two media considered. This justifies use of the above approximations. On the other hand, this result indicates that the traveltimes in homogeneous TI and isotropic LH media are practically indistinguishable and the moveout in a single CMP gather can be equivalently interpreted in terms of either a TI or LH model.

We can make the above statement even more evident by writing moveout (20) in LH media in the same form as moveout

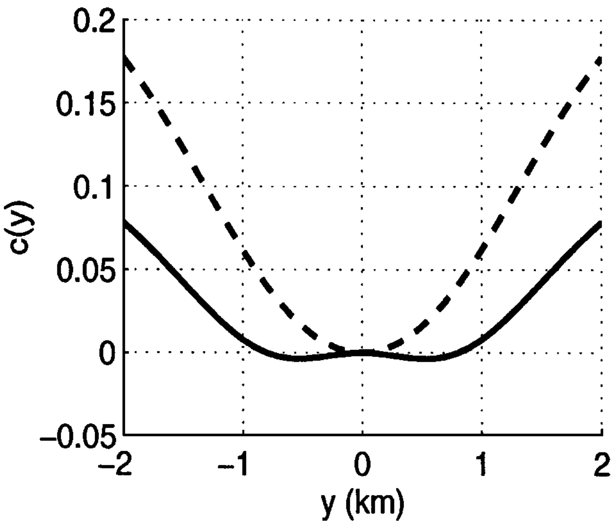

FIG. 3. Functions $c(y)$ describing the lateral velocity variation in two isotropic models kinematically equivalent to homogeneous TI models of Taylor sandstone (solid line) and Dog Creek shale (dashed line). The reflector depth $z = 2$ km, the maximum offset $x = 2z$.

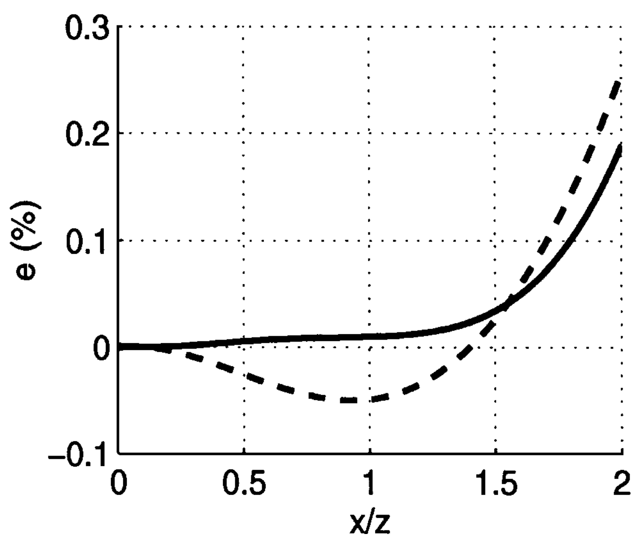

FIG. 4. Relative difference $e(x/z) = t_{\mathrm{LH}}/t_{\mathrm{TI}} - 1$ (in %) between the traveltimes t_{TI} computed in TI media using approximation (5) and exact traveltimes t_{LH} computed in LH media with the velocity distribution given in equations (18) and (24): Taylor sandstone (solid line); Dog Creek shale (dashed line). The traveltimes in the LH models were computed using the algorithm described in Grechka and McMechan (1996; 1997).

(5) in TI media. To do so, use a Taylor series for $c(y)$ in the vicinity of the midpoint $y = 0$

$$c(y) = \frac{1}{V_0}\sum_{k=1}^{\infty} V_0^{(k)}\frac{y^k}{k!}, \tag{25}$$

where the kth derivatives $V_0^{(k)}$ of velocity (18) with respect to y are taken at the midpoint and the inequalities $|V_0^{(k)}y^k/V_0| \ll 1$ hold for all k.

Substitution of equation (25) into moveout equation (20) and subsequent linearization with respect to derivatives $V_0^{(k)}$ in the same way as was done for TI media yields (Appendix A)

$$t^2(x) = t_0^2 + \frac{x^2}{V_{\rm nmo}^2} - \frac{2\eta_{\rm eff}x^4}{V_{\rm nmo}^2\left(t_0^2V_{\rm nmo}^2 + x^2\right)}, \tag{26}$$

with the NMO velocity given by

$$V_{\rm nmo}^2 = V_0^2\left(1 + \frac{1}{12}t_0^2V_0V_0''\right), \tag{27}$$

and the effective "anellipticity" coefficient

$$\eta_{\rm eff} = \frac{1}{24}t_0^2V_0\left(V_0'' + \frac{1}{80}t_0^2V_0^2V_0^{(IV)}\right). \tag{28}$$

Note that only the second- and fourth-order velocity derivatives influence the reflection moveout in LH media up to the quartic term. Bear in mind, however, that this result, obtained under the assumption of weak and smooth lateral heterogeneity, is not expected to be valid for arbitrary heterogeneity, which may not be described accurately enough by the first four terms of the Taylor series expansion (25).

The fact that equations (5) and (26) have identical form is not surprising. This form could have been deduced even before actual derivation. As follows from equation (20), the CMP traveltime is a symmetric function of the offset x. Therefore, we should have x^4 with some coefficient in the numerator of the term following $x^2/V_{\rm nmo}^2$. The denominator of this term, because of the procedure of deriving equations (5) and (26), should contain a hyperbolic portion of the moveout with a coefficient that makes the whole term have dimensionality of squared traveltime. Since equations (5) and (26) have been derived under assumptions of weak anisotropy and weak lateral heterogeneity, there is a freedom of putting V_0 instead of $V_{\rm nmo}$ in the denominator of the last term. We have chosen $V_{\rm nmo}$ because it gives better accuracy for the approximation of exact traveltimes.

Since moveout equations (5) and (26) have exactly the same form, the traveltimes in TI and isotropic LH media coincide (in the linear approximation) if

$$V_0'' = 24\frac{\delta}{t_0^2V_0}, \tag{29}$$

and

$$V_0^{(IV)} = 1920\frac{\epsilon - 2\delta}{t_0^4V_0^3}. \tag{30}$$

Equations (29) and (30) describe the lateral velocity variation at the midpoint that produces the same moveout as that in a corresponding homogeneous TI medium. The velocity derivatives (29) and (30) correspond (in the linear approximation) to the second- and the fourth-derivatives of $c(y)$ given by equation (24).

The moveout equation (26) in LH media [as well as equation (5) for TI media] is determined by only three parameters t_0, $V_{\rm nmo}$, and $\eta_{\rm eff}$ (which corresponds to η in the TI case) and does not depend on V_0, V_0'', and $V_0^{(IV)}$ separately. This means that we can vary the LH medium parameters V_0, V_0'', and $V_0^{(IV)}$ without changing the moveout, as long as t_0, $V_{\rm nmo}$, and $\eta_{\rm eff}$ are held constant. Therefore, inversion of the moveout equation (26) is inherently ambiguous. The nature of this ambiguity is similar to that in TI media: moveout (5) does not allow one to resolve V_0 and δ separately; only their combination $V_{\rm nmo} = V_0\sqrt{1+2\delta}$ can be found. If the vertical velocity V_0 is unknown, one possible choice is to assume that $V_0 = V_{\rm nmo}$ and $\delta = 0$. For such values of V_0 and δ, the corresponding LH medium will also be different from that found for the correct values of V_0 and (nonzero) δ. Equations (29) and (30) indicate that for $\delta = 0$ we have $V_0'' = 0$ and $V_0^{(IV)} = 1920\epsilon/(t_0^4V_0^3)$.

Figure 5 shows several functions $c(y)$ for different values of δ. Although all these $c(y)$ yield kinematically equivalent LH models, the degree of heterogeneity differs considerably among the various models. We see that the same reflection traveltimes can be obtained in the medium with 17% (for $\delta = 0.10$) and with 2% (for $\delta = -0.10$) velocity heterogeneity. This illustrates the ambiguity of traveltime inversion for LH media and shows that it is not necessary to have a pronounced lateral velocity variation to reproduce TI reflection moveout at a given CMP location.

P-WAVE REFLECTION MOVEOUT IN THE PRESENCE OF TRANSVERSE ISOTROPY AND LATERAL HETEROGENEITY

The main conclusion of the previous section was that we are not able to distinguish between transverse isotropy and lateral heterogeneity by looking at a P-wave reflection moveout at a single CMP location. Moreover, a range of kinematically

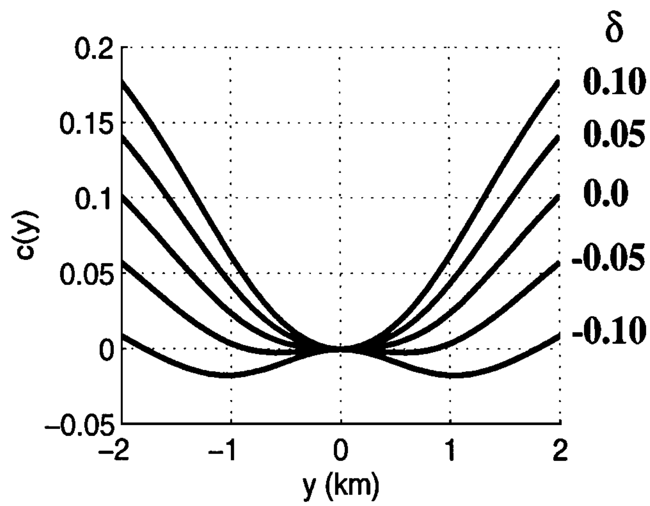

FIG. 5. The set of functions $c(y)$ corresponding to kinematically equivalent LH models giving the same reflection moveout as in the homogeneous TI model of Dog Creek shale. For all these models, t_0, $V_{\rm nmo}$, and $\eta_{\rm eff}$ are the same. Different values of the parameter δ (indicated on the right), corresponding to different ratios $V_{\rm nmo}/V_0$, result in different values of V_0 and the derivatives V_0'', $V_0^{(IV)}$.

equivalent LH models can be chosen to produce the same moveout curve. In practice, however, we have a number of overlapping moveout measurements along a CMP line. This information is useful in helping to separate the influence of anisotropy and heterogeneity. The schematic curves in Figure 6 show possible results for velocity interpretation of several CMP moveouts in a laterally heterogeneous anisotropic medium. Interpretation is made assuming isotropic LH models in Figure 6a and assuming homogeneous TI models in Figure 6b. In both cases we expect to obtain inconsistent velocity profiles because the actual model is both TI and LH. A more consistent profile (Figure 6c) can be found if both anisotropy and lateral heterogeneity are properly taken into account.

Let us again assume that both heterogeneity and anisotropy are present, but weak. Then, the linearized group velocity in TI LH media can be written by combining equations (14) and (18) as

$$V(y,x) = V_0\left[1 + c(y) + \frac{x^2(x^2\epsilon(y) + a^2\delta(y))}{(x^2+a^2)^2}\right]. \quad (31)$$

To obtain an expression for common-midpoint P-wave reflection moveout in weakly LH TI media, we use the linear perturbation theory again with respect to the small quantities c, ϵ, and δ. Then, the moveout takes a form similar to that of equation (20):

$$t(y,x) = \frac{\sqrt{x^2+a^2}}{V_0}\left[1 - \frac{1}{x}\int_{y-x/2}^{y+x/2} c(\xi)\,d\xi - \frac{x}{(x^2+a^2)^2}\int_{y-x/2}^{y+x/2}\{x^2\epsilon(\xi) + a^2\delta(\xi)\}\,d\xi\right], \quad (32)$$

containing one additional term caused by anisotropy. Using the expansions of the functions $c(y)$, $\epsilon(y)$, and $\delta(y)$ in a Taylor series in the vicinity of the midpoint, equation (32) can be written as (Appendix B)

$$t^2(y,x) = t_0^2(y) + \frac{x^2}{V_{\rm nmo}^2(y)} - \frac{2\eta_{\rm eff}(y)x^4}{V_{\rm nmo}^2(y)\big(t_0^2(y)V_{\rm nmo}^2(y) + x^2\big)}, \quad (33)$$

where $t_0(y) \equiv t(y,0)$ is the two-way zero-offset traveltime, and

$$V_{\rm nmo}^2(y) = V^2(y,0) \times \left(1 + 2\delta(y) - \frac{1}{12}t_0(y)V^2(y,0)t_0''(y)\right), \quad (34)$$

$$\eta_{\rm eff}(y) = \eta(y) + \frac{1}{24}t_0^2(y)V_{\rm nmo}(y) \times \left[V_{\rm nmo}''(y) + \frac{7}{240}t_0(y)V_{\rm nmo}^3(y)t_0^{(IV)}(y)\right]. \quad (35)$$

The parameter $\eta(y)$ is given by

$$\eta(y) = \epsilon(y) - \delta(y), \quad (36)$$

and the superscripts $(\cdot)''$ and $(\cdot)^{(IV)}$ denote the second- and the fourth-derivatives of corresponding functions with respect to the CMP coordinate.

Clearly, equations (5) and (33) have the same form. For lateral heterogeneity, however, all moveout parameters (t_0, $V_{\rm nmo}$, and $\eta_{\rm eff}$) vary laterally. If, instead, the medium were homogeneous, the derivatives in equations (34) and (35) would vanish and equation (33) would reduce to equation (5).

Since the form of equations (5) and (33) is identical, nonhyperbolic velocity analysis developed for homogeneous TI media (Alkhalifah, 1996) can be applied for TI LH media as well. This technique allows one to determine t_0, $V_{\rm nmo}$, and η using a semblance scan over these parameters. In the case of TI LH media, instead of η we estimate $\eta_{\rm eff}$, which contains contributions of both anisotropy and lateral heterogeneity [equation (35)]. The influence of LH is expressed in terms of the derivatives of the NMO velocity and the zero-offset traveltime with respect to the lateral coordinate. The derivatives can be estimated if $V_{\rm nmo}(y)$ and $t_0(y)$ are found along the CMP line. These derivatives, however, have to be calculated numerically from the discrete values obtained at the midpoint locations. Any errors in the numerical differentiation of $V_{\rm nmo}$ and t_0 (which can be expected to be significant) will directly propagate into the

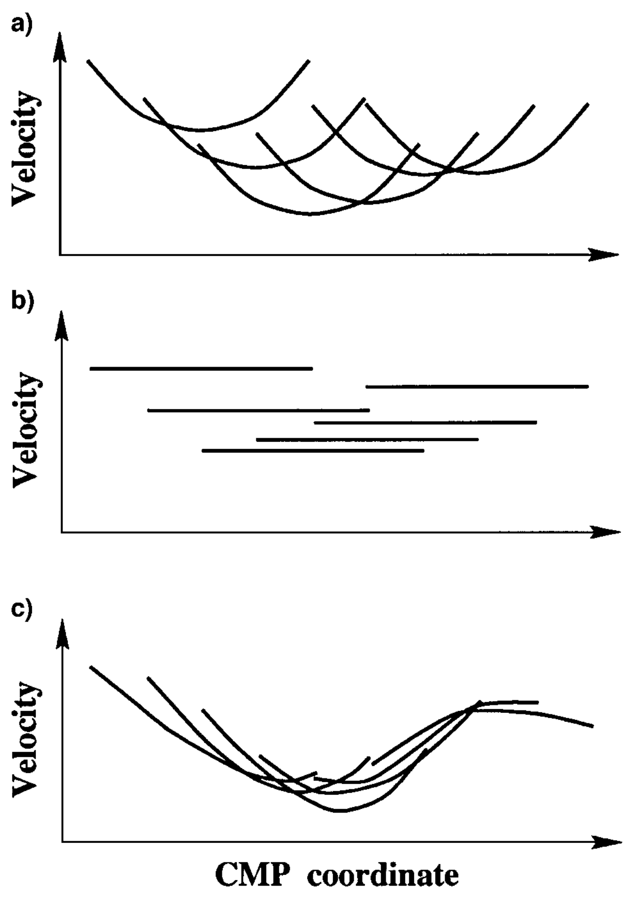

FIG. 6. Schematic curves of possible results of velocity interpretation for different interpretation models: (a) isotropic LH model assumed, (b) homogeneous TI model assumed, and (c) both TI and LH taken into account.

estimate of η, as indicated by equation (35). As was shown in Grechka and Tsvankin (1996), even in homogeneous TI media, the parameter η found from nonhyperbolic velocity analysis is extremely sensitive to small long-period traveltime errors because of the trade off between V_{nmo} and η. Clearly, in the presence of lateral heterogeneity, the stability of η estimation decreases even further.

There is, however, one special case that deserves special consideration. Note that equations (34) and (35) contain only even-order derivatives of t_0 and V_{nmo}. This means that a constant lateral gradient in the vertical velocity and in the anisotropic parameters does not influence the CMP moveout in TI LH media, so V_{nmo} and η are given by the same equations as in homogeneous TI media. Therefore, nonhyperbolic velocity analysis, initially developed for homogeneous TI media, is entirely valid for TI media with constant lateral gradient in velocity. In the latter case, the information about lateral velocity heterogeneity can be found directly from the variation of t_0 because $t_0(y) = a/V(y, 0)$.

DISCUSSION AND CONCLUSIONS

In this paper, we examined the simultaneous influence of transverse isotropy and lateral heterogeneity on P-wave reflection moveout from a horizontal interface. It was shown that the dependence of group velocity on the ray direction in homogeneous TI media can be equivalently treated as the offset dependence of *isotropic* velocity for a specific laterally heterogeneous model. Therefore, it is not surprising that transverse isotropy and lateral heterogeneity can mimic each other's influence and produce similar traveltime behavior. As a result, long-spread nonhyperbolic P-wave reflection moveout at a single CMP location can be interpreted in terms of either transverse isotropy or lateral heterogeneity.

It is important to point out that the lateral heterogeneity needed to reproduce nonhyperbolic reflection moveout in weakly TI media turns out to be weak as well. In contrast, vertical velocity heterogeneity must be rather strong to cause noticeable nonhyperbolicity of the moveout (e.g., Alkhalifah, 1996) because the nonhyperbolicity in vertically heterogeneous media is attributable to ray bending only.

If a TI medium is laterally heterogeneous, reflection moveout contains additional terms responsible for the lateral variations in NMO velocity and zero-offset traveltime in the vicinity of the midpoint. Even in an isotropic medium, these terms may cause pronounced nonhyperbolicity of the moveout.

Nonhyperbolic velocity analysis in LH TI media can be performed using equation (33) in exactly the same fashion as in homogeneous or vertically inhomogeneous TI media (Alkhalifah, 1996; Grechka and Tsvankin, 1996). This analysis provides estimates of the normal-moveout velocity V_{nmo} and the coefficient η_{eff} responsible for nonhyperbolic moveout. However, in the presence of lateral heterogeneity, the coefficient η_{eff} contains not only the actual anisotropic parameter η, but also the terms dependent on heterogeneity [the derivatives of the zero-offset traveltime $t_0^{(IV)}(y)$ and the NMO velocity $V''_{\mathrm{nmo}}(y)$ with respect to the CMP coordinate y]. Calculation of these derivatives may seem straightforward; however, numerical differentiation of t_0 and V_{nmo} determined from the data may lead to substantial errors that will distort values of η. Furthermore, as was shown in Grechka and Tsvankin (1996), the estimation of the quartic term η_{eff} itself (η_{eff} equals η for homogeneous TI media) suffers from the trade off between η_{eff} and V_{nmo} which may lead to relative errors in η_{eff} of about 50–80% even for the simplest single-layer model. The combination of these two errors, which are of different nature, may give rise to entirely unrealistic estimates of anisotropic parameter η.

The only model that does not suffer from the trade off between anisotropy and heterogeneity is TI media with a constant lateral gradient in all parameters. In this case, TI and LH are completely decoupled and the coefficient η_{eff} of the quartic moveout term is influenced by anisotropy only.

The values of η obtained using NMO velocities from dipping reflectors in homogeneous TI media (Alkhalifah and Tsvankin, 1995) are more accurate than those found from nonhyperbolic velocity analysis of horizontal events. A similar approach using information from dipping reflectors is expected to give better results in the presence of lateral heterogeneity as well.

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APPENDIX A

DERIVATION OF EQUATION (26)

Substituting equation (25) into equation (20) for the CMP traveltime and integrating, we obtain

$$t(x) = \frac{\sqrt{x^2+a^2}}{V_0}\left(1 - \frac{V_0'' x^2}{24V_0} - \frac{V_0^{(IV)} x^4}{1920V_0} - \cdots\right), \quad \text{(A-1)}$$

where powers of x, which influence quadratic and quartic moveout terms, are explicitly written.

Raising equation (A-1) in the fourth power and keeping the zero-order and linear terms with respect to the ratios V_0''/V_0 and $V_0^{(IV)}/V_0$, we get

$$t^4(x) = t_0^4 + \left(\frac{2t_0^2}{V_0^2} - \frac{t_0^4 V_0''}{6V_0}\right)x^2 + \left(\frac{1}{V_0^4} - \frac{t_0^2 V_0''}{3V_0^3} - \frac{t_0^4 V_0^{(IV)}}{480V_0}\right)x^4. \quad \text{(A-2)}$$

This equation can be rewritten in the form

$$t^4(x) = \left(t_0^2 + \frac{x^2}{V_{\text{nmo}}^2}\right)^2 - \frac{t_0^2 x^4}{6V_0^3}\left(V_0'' + \frac{1}{80}t_0^2 V_0^2 V_0^{(IV)}\right), \quad \text{(A-3)}$$

where the NMO velocity is given by equation

$$V_{\text{nmo}}^2 = V_0^2\left(1 + \frac{1}{12}t_0^2 V_0 V_0''\right), \quad \text{(A-4)}$$

and we assume again that V_0''/V_0 is small.

Taking the square root of equation (A-3) in the linear approximation with respect to V_0''/V_0 and $V_0^{(IV)}/V_0$, we obtain the equation for the squared CMP traveltime in isotropic laterally heterogeneous media in exactly the same form as that in TI media [equation (5)]

$$t^2(x) = t_0^2 + \frac{x^2}{V_{\text{nmo}}^2} - \frac{2\eta_{\text{eff}} x^4}{V_{\text{nmo}}^2\left(t_0^2 V_{\text{nmo}}^2 + x^2\right)}, \quad \text{(A-5)}$$

where the coefficient for the quartic term

$$\eta_{\text{eff}} = \frac{1}{24}t_0^2 V_0\left(V_0'' + \frac{1}{80}t_0^2 V_0^2 V_0^{(IV)}\right). \quad \text{(A-6)}$$

APPENDIX B

THE MOVEOUT EQUATION IN LH TI MEDIA

We assume, for simplicity, that the midpoint coordinate y in equation (32) is equal to zero. Bearing in mind that the final result should look similar to the polynomial-like equations (5) or (26), we expand the functions $c(y)$, $\epsilon(y)$, and $\delta(y)$ in a Taylor series in the vicinity of the midpoint $y = 0$:

$$c(y) = \frac{1}{V_0}\sum_{k=1}^{\infty} V_0^{(k)}\frac{y^k}{k!}, \quad \text{(B-1)}$$

$$\epsilon(y) = \sum_{k=0}^{\infty} \epsilon^{(k)}\frac{y^k}{k!}, \quad \text{(B-2)}$$

and

$$\delta(y) = \sum_{k=0}^{\infty} \delta^{(k)}\frac{y^k}{k!}. \quad \text{(B-3)}$$

The quantities $V_0^{(k)}$, $\epsilon^{(k)}$, and $\delta^{(k)}$ are the derivatives of the order k of vertical velocity $V(y,0)$ and of anisotropic parameters $\epsilon(y)$, $\delta(y)$ with respect to y:

$$V_0^{(k)} \equiv \left.\frac{d^k V(y,0)}{dy^k}\right|_{y=0}, \quad \text{(B-4)}$$

$$\epsilon^{(k)} \equiv \left.\frac{d^k \epsilon(y)}{dy^k}\right|_{y=0}, \quad \text{(B-5)}$$

and

$$\delta^{(k)} \equiv \left.\frac{d^k \delta(y)}{dy^k}\right|_{y=0}. \quad \text{(B-6)}$$

Notice that the summation begins with 1 in equation (B-1) and with 0 in equations (B-2) and (B-3). This is because the zero-order term, which is equal to 1 in equation (B-1), has already been explicitly taken into account in equation (32).

After substituting series (B-1)–(B-3) into the moveout equation (32), evaluating the integrals, and performing the same steps as those that have been done for equation (15), we obtain the nonhyperbolic moveout equation in weakly LH TI media in the form

$$t^2(x) = t_0^2 + \frac{x^2}{V_{\text{nmo}}^2} - \frac{2\eta_{\text{eff}} x^4}{V_{\text{nmo}}^2\left(t_0^2 V_{\text{nmo}}^2 + x^2\right)}, \quad \text{(B-7)}$$

where t_0 is the two-way zero-offset traveltime. The normal-moveout velocity V_{nmo} and the coefficient η_{eff} are given by

$$V_{\text{nmo}}^2 = V_0^2\left(1 + 2\delta + \frac{t_0^2 V_0}{12}V_0''\right), \quad \text{(B-8)}$$

$$\eta_{\text{eff}} = \eta + \frac{1}{24}t_0^2 V_{\text{nmo}}^2\left(\delta'' + \frac{V_0''}{V_{\text{nmo}}}\right) + \frac{1}{1920}t_0^4 V_{\text{nmo}}^3 V_0^{(IV)}, \quad \text{(B-9)}$$

and

$$\eta = \epsilon - \delta. \quad \text{(B-10)}$$

The superscripts $(\cdot)''$ and $(\cdot)^{(IV)}$ denote the second- and the fourth-derivatives of the corresponding functions with respect to the CMP-coordinate at the midpoint $y = 0$.

Since, in the linear approximation for a horizontal reflector,

$$\frac{t_0^{(k)}}{t_0} = -\frac{V_0^{(k)}}{V_0} \quad \text{(B-11)}$$

for any $k > 0$, we can rewrite equation (B-8) as

$$V_{\mathrm{nmo}}^2 = V_0^2\left(1 + 2\delta - \frac{t_0 V_0^2}{12} t_0''\right). \tag{B-12}$$

The second-derivative δ'' in equation (B-9) can be found by linearizing equation (B-12) and taking its second derivative. After replacing V_0 with V_{nmo} (in the linear approximation), we obtain

$$\delta'' = -\frac{V_0''}{V_0} + \frac{V_{\mathrm{nmo}}''}{V_{\mathrm{nmo}}} + \frac{1}{24} t_0 V_{\mathrm{nmo}}^2 t_0^{(IV)}. \tag{B-13}$$

Application of equation (B-11) yields

$$\delta'' = \frac{t_0''}{t_0} + \frac{V_{\mathrm{nmo}}''}{V_{\mathrm{nmo}}} + \frac{1}{24} t_0 V_{\mathrm{nmo}}^2 t_0^{(IV)}. \tag{B-14}$$

Substitution of equation (B-14) into equation (B-9) gives the final expression for η_{eff}:

$$\eta_{\mathrm{eff}} = \eta + \frac{1}{24} t_0^2 V_{\mathrm{nmo}} \left[V_{\mathrm{nmo}}'' + \frac{7}{240} t_0 V_{\mathrm{nmo}}^3 t_0^{(IV)}\right]. \tag{B-15}$$